Reasoning Under an Uncertain Thermal State

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Abstract

We propose a novel approach based on dual filtering techniques for the detection of possible variations of the thermal properties of the spacecraft that result from variations of its physical properties and for determining a complete thermal mapping of the system. System and sensor uncertainties are taken into account in the *lumped parameter* modeling of the thermal system and a dual filter is run on the stochastic model in an alternating optimization fashion to estimate the thermal state and coefficients of the resulting thermal network from the readings of few, strategically placed, thermal sensors.

1 Introduction

In this paper we introduce a novel approach to infer the health status of the spacecraft and its instrumentations from the readings of few, strategically placed thermal sensors. System and sensor uncertainties are taken into account in the *lumped parameter network* (LPN) modeling of the thermal system and a nonlinear dual filter is run on the resulting stochastic model. In the dual filtering configuration [1, 2] the states and the parameters of a dynamic system are estimated simultaneously in an alternating optimization fashion. To cope with the strong nonlinearities of the resulting thermal network we propose to use the "unscented" extension of the well known Kalman filter [3, 4]. The main advantage of the dual filtering based method applied to the thermal network is the possibility of detecting variations in the thermal properties of the spacecraft as a result of variations of its physical properties together with a complete thermal mapping of the system. Events such as faults can be detected by the dual filter as well as new values of system parameters (e.g. radiative couplings) that result from a variation of the spacecraft geometry (e.g. from the deployment of antennas, solar panels, etc.). Finally, the system could be employed as a virtual sensor able to identify anomalous behaviors of a possible faulty physical sensor.

The method proposed would be particular attractive in those network whose state and parameters can be estimated by the filter using a minimal amount of readings. The relation between the network topology and this minimal number is therefore an issue strictly related to the observability of the system which is here approached using graph theory.

The idea of describing in a probabilistic framework the uncertainties in a LPN has been proposed by [5] in order to detect the presence of faulty electrical components. However that approach based on a Markov network and on the Iterative Proportional Fitting [6, 7] is unable in its original formulation to deal with dynamical systems. The dual filtering based method proposed in this paper can be seen as an extension of that approach for dynamical system.

This paper is organized as follows. In Section 2 we introduce the state estimation problem and give a brief description of the unscented Kalman filter and of the dual state estimation configuration. In Section 3 a stochastic modeling of the thermal system based on the LPN and its state-space representation are presented. The problem of parameters observability is discussed in Section 4. In Section 5 we illustrate our method through a simple example and in Section 6 we draw conclusions and discuss future work.

2 Filtering techniques and configurations

In state estimation problems, the state-space representation of the dynamical system is used. This describes the evolution of the system state $\mathbf{x}_k$ over time, and the measurements $\mathbf{y}_k$ as a function of the state[1]:

$$\mathbf{x}_k = \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{w}, \mathbf{v}_{k-1}) \tag{1}$$

$$\mathbf{y}_k = \mathbf{g}(\mathbf{x}_k, \mathbf{w}, \mathbf{n}_k) \tag{2}$$

where $\mathbf{w}$ are the model parameters, $\mathbf{v}_k$ is the state noise, $\mathbf{n}_k$ the measurement noise, and k the sample step counter. For given parameters $\mathbf{w}$ these equations define a probability density function (pdf) for the state transition $p(\mathbf{x}_k|\mathbf{x}_{k-1})$ and for the measurement $p(\mathbf{y}_k|\mathbf{x}_k)$.

Since the system and the measurements are stochastic, the exact state cannot be inferred from the measurements, only the pdf of the state $p(\mathbf{x}_k|\mathbf{y}_{1:k})$ from sample step 1 to k can be de termined. So, the goal of the state estimation problem is to determine $p(\mathbf{x}_k|\mathbf{y}_{1:k})$. Although it is possible to use Bayes' rule to express this conditional density in terms of the state transition pdf $p(\mathbf{x}_k|\mathbf{x}_{k-1})$, and the measurement pdf $p(\mathbf{y}_k|\mathbf{x}_k)$, the evaluation of it requires the evaluation of several integrals, which is not possible (analytically) in general [8]. In principle it is possible to evaluate these integrals numerically (which is done, e.g., in approximate grid-based methods where the state space is discretized [8]), but these methods are in most cases very inefficient.

Under certain assumptions the conditional pdf $p(\mathbf{x}_k|\mathbf{y}_{1:k})$ can be solved (or approximated) by the Kalman filter or its extensions, such as the extended/unscented Kalman filter. Below we give a short overview of the Kalman filter and the unscented Kalman filter and their corresponding assumptions. Note that there are other filtering methods that are not discussed here.

[1] For simplicity, we do not consider inputs that may act on the system. The extension to include inputs is straightforward.

2.1 Filter types

2.1.1 Kalman filter (KF)

Consider a linear system

$$\mathbf{x}_k = \mathbf{F}\mathbf{x}_{k-1} + \mathbf{v}_{k-1}$$
$$\mathbf{y}_k = \mathbf{G}\mathbf{x}_k + \mathbf{n}_k$$

with known and constant system matrices $\mathbf{F}$ and $\mathbf{G}$. The state noise $\mathbf{v}_{k-1}$ and measurement noise $\mathbf{n}_k$ are both assumed to be additive, and assumed to have a zero mean Gaussian distribution. Furthermore independence between noises at different time instants and between the state and measurement noise is assumed: $\text{cov}\{\mathbf{v}_{k_1}, \mathbf{v}_{k_2}\} = 0$ and $\text{cov}\{\mathbf{n}_{k_1}, \mathbf{n}_{k_2}\} = 0$ for $k_1 \neq k_2$, and $\text{cov}\{\mathbf{v}_{k_1}, \mathbf{n}_{k_2}\} = 0$ for any k_1 and k_2.

Under these assumptions the conditional pdf $p(\mathbf{x}_k|\mathbf{y}_{1:k})$ is also Gaussian, and the Kalman filter expresses analytically the mean and covariance of $p(\mathbf{x}_k|\mathbf{y}_{1:k})$ [1]. The Kalman filter is guaranteed to converge if the state noise excites all states and the system $(\mathbf{F}, \mathbf{G})$ is observable [9].

If the process equation and/or the measurement equation is/are nonlinear, modified versions of the Kalman filter must be used. The most common modification is based on the linearization of the nonlinear functions and is referred to as extended Kalman filter (EKF) [10, 11]. Another extension of the Kalman filter for nonlinear systems is the Unscented Kalman Filter (UKF).

2.1.2 Unscented Kalman filter

Contrary to the EKF, the UKF does not use a linearization of the system and the noises are not assumed to be Gaussian [3, 4]. To represent the mean and the covariance of the (conditional) state pdf's, so-called sigma points are defined with appropriate weights attached to each point. The sigma points and the weights are chosen such that their weighted mean and covariance approximate the true mean and covariance of the pdf.

The UKF approximates the mean and the covariance of the posterior pdf with second order (Taylor) accuracy. As the EKF operates with first order accuracy, the UKF can be expected to have better performance and convergence properties. Nevertheless, convergence cannot be guaranteed for the UKF.

The equations of the UKF are given in Table 1. The main assumption here is that the state pdf can be sufficiently described by its mean and covariance.

2.2 Filter configurations

These filters can be used for state estimation, parameter estimation, or for the simultaneous estimation of the state and the parameters. These require different filter configurations, which are summarized below.

2.2.1 State tracking

The model parameters are assumed to be known. The goal of state tracking is to determine the pdf $p(\mathbf{x}_k|\mathbf{y}_{1:k})$ for every k.

2.2.2 Parameter tracking

The model states and measurements are assumed to be known. The state-space model is formed for the evolution of the model parameters $\mathbf{x}_{\mathrm{par},k} = \mathbf{w}_k$, which is often assumed to be a random walk with noise $\mathbf{v}_{\mathrm{par},k}$. The measurement is written as a function of the system state $\mathbf{x}_k$ and the model parameters $\mathbf{x}_{\mathrm{par},k}$, and a "state tracking" filter is run for $\mathbf{x}_{\mathrm{par},k}$:

$$\mathbf{x}_{\mathrm{par},k} = \mathbf{x}_{\mathrm{par},k-1} + \mathbf{v}_{\mathrm{par},k-1} \quad (3)$$

$$\mathbf{y}_k = \mathbf{g}'(\mathbf{x}_k, \mathbf{x}_{\mathrm{par},k}, \mathbf{n}_k) \quad (4)$$

2.2.3 Joint estimation

In joint estimation both the system state and the model parameters are estimated simultaneously. To this end, an augmented state vector is defined consisting of both the system state and the model parameters, $\mathbf{x}_{\mathrm{aug},k} = [\mathbf{x}_k^{\mathrm{T}}, \mathbf{x}_{\mathrm{par},k}^{\mathrm{T}}]^{\mathrm{T}}$. Based on (1)–(4) a new state-space system is formed on which the filter is run.

2.2.4 Dual estimation

Similarly to joint estimation, in dual estimation the system state and the model parameters are estimated simultaneously. However, here the state system (1)–(2) and the parameter system (3)–(4) are kept separately, and two filters are run, one for the state estimation, and one for the parameter estimation. For each sample step k the result of the state estimation of the previous sample step $\mathbf{x}_{k-1}$ is used as an input for the parameter estimator, and vice versa, the result of the parameter estimator of the previous sample step $\mathbf{x}_{\mathrm{par},k-1}$ is used in the state estimatoras shown in Fig. 1.

In [1] it is suggested that the dual filter has better convergence properties than the joint filter.

3 The thermal stochastic state-space model

As already mentioned the thermal system can be represented by a LPN, whose nodes are isothermal volumes where heat can be stored and links represent the heat flow between two nodes. Nodes are characterized by their thermal capacitance and possibly by a heat source whereas links are characterized by a thermal conductance and/or by a radiative exchange factor.

Applying the heat conduction and radiation equations to this network yields to the following system of differential equations.

$$C_i \frac{dT_i}{dt} = \sum_{j \neq i} K_{ij} \left(T_j - T_i\right) + \sum_{j \neq i} R_{ij} \left(T_j^4 - T_i^4\right) + Q_i \quad (5)$$

with $i, j = 1, \ldots, n$, where n is the number of nodes

Table 1: The Unscented Kalman Filter

I. Initialize with:

$$\hat{\mathbf{x}}_0 = E[\mathbf{x}_0], \mathbf{P}_0 = E[(\mathbf{x}_0 - \hat{\mathbf{x}}_0)(\mathbf{x}_0 - \hat{\mathbf{x}}_0)^{\mathrm{T}}], \hat{\mathbf{x}}_0^a = E[\mathbf{x}_0^a],$$
$$\mathbf{P}_0^a = E[(\mathbf{x}_0^a - \hat{\mathbf{x}}_0^a)(\mathbf{x}_0^a - \hat{\mathbf{x}}_0^a)^{\mathrm{T}}] = \mathrm{diag}\{\mathbf{P}_0, \mathbf{P}_{\mathbf{v}}, \mathbf{P}_{\mathbf{n}}\}$$

where $\mathbf{x}_k^a = [\mathbf{x}_k^{\mathrm{T}}\ \mathbf{v}_k^{\mathrm{T}}\ \mathbf{n}_k^{\mathrm{T}}]^{\mathrm{T}}$ is the augmented state vector.

Evaluate steps II, III, and IV below for $k = 1, 2, \ldots$

II. Calculate sigma points:

$$\mathcal{X}_{0,k-1}^a = \hat{\mathbf{x}}_{k-1}^a$$
$$\mathcal{X}_{i,k-1}^a = \hat{\mathbf{x}}_{k-1}^a + \left(\sqrt{(n_x + \lambda)\mathbf{P}_{k-1}^a}\right)_i, \quad \text{for } i = 1, \ldots, n_x$$
$$\mathcal{X}_{i,k-1}^a = \hat{\mathbf{x}}_{k-1}^a - \left(\sqrt{(n_x + \lambda)\mathbf{P}_{k-1}^a}\right)_{i-n_x}, \text{for } i = n_x, \ldots, 2n_x$$

where $\mathcal{X}_k^a = [(\mathcal{X}_k^x)^{\mathrm{T}}\ (\mathcal{X}_k^v)^{\mathrm{T}}\ (\mathcal{X}_k^n)^{\mathrm{T}}]^{\mathrm{T}}$ and $\sqrt{\mathbf{P}_{k-1}^a}$ is a Cholesky factor, and the design parameters selected as $\lambda = \alpha^2(n_x + k) - n_x$, $1 \geq \alpha \geq 10^{-4}, k$ is typically taken to equal $3 - n_x$, and n_x is the dimension of the augmented state, and $(\mathbf{M})_i$ denotes the i-th column of matrix $\mathbf{M}$.

III. Time update :

$$\mathcal{X}_{i,k|k-1}^x = \mathbf{f}(\mathcal{X}_{i,k-1}^x, \mathcal{X}_{i,k-1}^v),$$
$$\hat{\mathbf{x}}_{k|k-1} = \sum_{i=0}^{2n_x} W_i^{(m)} \mathcal{X}_{i,k|k-1}^x,$$
$$\mathbf{P}_{k|k-1} = \sum_{i=0}^{2n_x} W_i^{(c)} [\mathcal{X}_{i,k|k-1}^x - \hat{\mathbf{x}}_{k|k-1}][\mathcal{X}_{i,k|k-1}^x - \hat{\mathbf{x}}_{k|k-1}]^{\mathrm{T}},$$
$$\mathcal{Y}_{i,k|k-1} = \mathbf{g}(\mathcal{X}_{i,k|k-1}^x, \mathcal{X}_{i,k-1}^n),$$
$$\hat{\mathbf{y}}_{k|k-1} = \sum_{i=0}^{2n_x} W_i^{(m)} \mathcal{Y}_{i,k|k-1}.$$

IV. Measurement update:

$$\mathbf{P}_{\mathbf{y}_k\mathbf{y}_k} = \sum_{i=0}^{2n_x} W_i^{(c)} [\mathcal{Y}_{i,k|k-1} - \hat{\mathbf{y}}_{k|k-1}][\mathcal{Y}_{i,k|k-1} - \hat{\mathbf{y}}_{k|k-1}]^{\mathrm{T}},$$
$$\mathbf{P}_{\mathbf{x}_k\mathbf{y}_k} = \sum_{i=0}^{2n_x} W_i^{(c)} [\mathcal{X}_{i,k|k-1}^x - \hat{\mathbf{x}}_{k|k-1}][\mathcal{Y}_{i,k|k-1} - \hat{\mathbf{y}}_{k|k-1}]^{\mathrm{T}},$$
$$\mathcal{K}_k = \mathbf{P}_{\mathbf{x}_k\mathbf{y}_k} \mathbf{P}_{\mathbf{y}_k\mathbf{y}_k}^{-1},$$
$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathcal{K}_k(\mathbf{y}_k - \hat{\mathbf{y}}_{k|k-1}),$$
$$\mathbf{P}_{k|k} = \mathbf{P}_{k|k-1} - \mathcal{K}_k \mathbf{P}_{\mathbf{y}_k\mathbf{y}_k} \mathcal{K}_k^{\mathrm{T}},$$

where the weights are: $W_0^{(m)} = \lambda/(n_x + \lambda)$, $W_0^{(c)} = \lambda/(n_x + \lambda) + (1 - \alpha^2 + \beta)$, $W_i^{(m)} = W_i^{(c)} = 1/2(n_x + \lambda)$, for $i = 1, \ldots, 2n_x$.

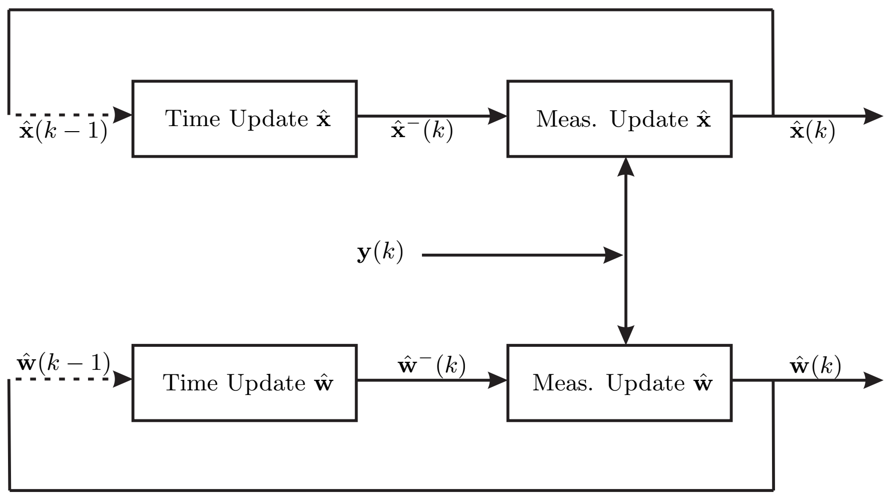

Figure 1: The dual estimation scheme. The two filters use each other's estimation from the previous sample step.

in the network, T_i [°C], C_i [J/°C] and Q_i [W] are the temperature, capacitance and heat source of node i respectively, K_{ij} [W/°C] and R_{ij} [W/°C^4] are the conductance and the radiative exchange factor between nodes i and j.

A discrete time system can be derived by (5) by substituting the time derivatives in the left-hand side by a first order approximation. The discrete time system equations considering an additive noise term v_i that takes into account the modeling uncertainties can be written as follows:

$$\mathrm{T}_{i,k+1} = \frac{t}{C_{i,k}} \left[\sum_{j \neq i} K_{ij,k} \left(T_{j,k} - T_{i,k}\right) + \dots \right.$$

$$\left. + \sum_{j \neq i} R_{ij,k} \left(T_{j,k}^4 - T_{i,k}^4\right) + Q_{i,k} \right] + v_{i,k} \qquad (6)$$

Note that in (6) we introduce a time dependency of the system parameters C_i, K_{ij} and R_{ij} denoted by the subscript k.

Denoting the measured temperature at node i and the measurement noise by $\tilde{T}_i$ and n_i respectively, we can write:

$$\tilde{T}_{i,k} = T_{i,k} + n_{i,k} \qquad (7)$$

with $i = 1, \dots, m$, where m is the number of temperature sensors in the network.

Furthermore the state variable vector, the input vector and the measurement vector are defined as $\mathbf{T} = [T_1, \dots, T_n]^T$, $\mathbf{Q} = [Q_1, \dots, Q_n]^T$, $\tilde{\mathbf{T}} = [\tilde{T}_1, \dots, \tilde{T}_m]^T$, respectively. Then the stochastic thermal model introduced in Equations (6) and (7) can be rewritten in the following compact state-space representation:

$$\mathbf{T}_{k+1} = \mathbf{A}_k \mathbf{T}_k + \mathbf{B}_k \mathbf{T}_k^4 + \mathbf{Q}_k + \mathbf{v}_k \qquad (8)$$

$$\tilde{\mathbf{T}}_k = \mathbf{C}\mathbf{T}_k + \mathbf{n}_k \qquad (9)$$

where

$$\mathbf{A} \in \mathbb{R}^{n\times n},\ a(i,i) = \frac{-t}{C_i}\sum_{j\neq i} K_{ij},\ a(i,j) = \frac{-t}{C_i}K_{ij},$$

$$\mathbf{B} \in \mathbb{R}^{n\times n},\ b(i,i) = \frac{-t}{C_i}\sum_{j\neq i} R_{ij},\ b(i,j) = \frac{-t}{C_i}R_{ij},$$

$$\mathbf{C} \in \mathbb{R}^{m\times n},$$

$$\mathbf{v} = [v_1, \ldots, v_n]^T, \qquad \mathbf{n} = [n_1, \ldots, n_m]^T.$$

The dual filter configuration introduced in Section 2.2.4 can now be applied to the system represented by Equations (8) and (9) to estimate in an alternating optimization fashion the complete thermal states and parameters of the network (e.g. the radiative exchange factors). Note that, by estimating the network parameters, possible variations of the physical properties of the system can be detected as the result of variations of the thermal properties. This feature allows the identification in real time of possible faults in the system as well as of new values of parameters resulting from changes in the spacecraft geometry (e.g. from the deployment of antennas).

The method proposed is particularly attractive in those network whose state and parameters can be estimated by the filter using a minimal amount of readings. The relation between the network topology and this minimal number is therefore an issue strictly related to the observability of the system which is here approached using graph theory and discussed in the next session. Note that the observability of the parameters can be computed analytically because in the dual filtering configuration the parameter tracking filter is run on a liner system.

4 Network observability

The necessary number and the location of the sensors depends on the structure of the network, it can be cast into the question: "Which nodes of a given network must be chosen in order that for each link at least one of the nodes it connects is among the chosen nodes?". In the following we shortly describe a strategy to find an answer to this question. Consider a network with nodes $\mathbf{N} = \{N_\alpha\},\ \alpha = 1, \ldots, n$ and links $\mathbf{L} = \{L_\beta\},\ \beta = 1, \ldots, l$. We denote the adjacency matrix[2] (links) by $\mathbf{L}$ as well and moreover use the incidence matrix $\mathbf{I}$. Our network does not contain any loops and two nodes are connected by not more than one link. Therefore the possible entries of the incidence and of the adjacency matrix are simply 0 and 1, the former matrix in addition is traceless. Finally, a n-vector $\mathbf{M}_0$ is needed, whose entries are the number of links attached to each of the nodes. This vector is e.g. obtained as $\mathbf{M}_0 = \mathbf{L} \times \mathbb{1}_n = \mathbf{I} \times \mathbb{1}_l$.

From the vector $\mathbf{M}_0$ we can derive relatively easy an upper and lower bound for the number S of necessary sensors to ensure observability of the whole thermal network, namely

$$\begin{aligned} &\min_m \left\{ \sum_{\alpha\in\{m\}} N_\alpha \geq m \right\} \leq S \\ &\leq \min_m \left\{ \left(\sum_{\alpha\in\{m\}} N_\alpha - \frac{1}{2}\gamma(\gamma-1) \right) \geq m \right\}\ , \end{aligned} \tag{10}$$

where $\{m\}$ is the set of chosen nodes and m its cardinality (the number of measurements). This formula represents the fact that in the best case we choose m nodes without any link between each other while in the other extreme every chosen node has a link to each of the other chosen nodes.

To obtain a precise result the multiple choices of a certain link must be identified. Whether or not two nodes N_i and N_j are connected by a link can be determined either from the adjacency or from the incidence matrix by

$$\Delta N_{\alpha\beta} = \sum_\gamma I_{\alpha\gamma} I_{\gamma\beta} = L_{\alpha\beta}\ . \tag{11}$$

[2]We use the standard definitions of graph theory. The adjacency matrix $\mathbf{L}$ is a $n \times n$ matrix, whose entries $L_{\alpha\beta}$ are the number of links connecting the node N_α with the node N_β. The incidence matrix is an $n \times l$ matrix. Its entries have values 0, 1 or 2 and define the number of times the node N_α is incident to the link L_β.

Therefore the minimal set of nodes to be chosen may be written as

$$S = \min_{m} \left\{ \left(\sum_{\alpha\in\{m\}} N_\alpha - \sum_{\substack{\alpha,\beta\in\{m\}\\ \alpha>\beta}} L_{\alpha\beta} \right) \geq m \right\} . \qquad (12)$$

We illustrate this result by two examples. Consider a complete network, which means that each node is incident to $n-1$ links. Choosing the node N_1 therefore chooses $n-1$ links. The second node N_2 chooses $n-2$ as the link connected to N_1 does not count any more. The next one chooses $n-3$ and so on. With a set of m nodes therefore

$$(n-1)+(n-2)+\ldots+(n-m-1) = \\ = m(m-1) - \frac{1}{2}m(m-1)$$

links have been chosen. This yields the upper bound in equation (10) and given the total number of links $l = n(n-1)/2$ one obtains that $m = (n-1)$ nodes must be chosen. On the other hand in a minimal network, where each node N_α is just connected to $N_{\alpha-1}$ and $N_{\alpha+1}$ the total number of links is $l = n-1$. Choosing all links $N_{2\alpha}$ we obtain the lower bound in (10).

Obviously equation (12) yet is lacking a clear strategy how to choose the nodes. Here we propose a simple iterative program, based on a series of vectors $\mathbf{M}_a$, $a = 0, 1, 2, \ldots$, where $\mathbf{M}_0$ is the list of numbers of links per node as defined above. Each vector $\mathbf{M}_a$ defines the next node to be chosen and a rule how to determine $\mathbf{M}_{a+1}$ as follows:

1. Determine all nodes with only one link attached to, i.e. the set of elements $\{M_{a,\alpha}\}$ of $\mathbf{M}_a$ with $M_{a,\alpha} = 1\ \forall\alpha$. All nodes connected by a link to one of the elements in $\{M_{a,\alpha}\}$ are chosen in a first step. This is motivated by the fact that for each node with a single link either the node itself must be chosen or the one it is connected to. The latter case is advantageous if one intends to reduce the number of necessary nodes.

2. Define a new vector M_{a+1} as

$$M_{a+1} = M_a - \sum_\alpha \mathbf{L} \times L_\alpha \ , \qquad (13)$$

where L_α is the vector built from the α'th column of $\mathbf{L}$ and α runs over all the nodes that have been chosen in step one. Obviously the elements $M_{a+1,\alpha}$ now are zero. M_{a+1} counts the number of links attached to each node that are not yet chosen. If M_{a+1} again has entries with value $+1$ go back to step 1. If M_{a+1} has entries larger than zero and not unity go to step 3. Else the process terminates and the nodes to be chosen are determined.

3. Choose the largest component among the vector M_a, which we denote by $M_{a,\beta}$. This is the next node to be chosen in the process. It may be possible that more than one node has the maximal number of links. Denoting the set of these nodes by $N_{\max}$ one should choose the node with the least elements pointing to another member of the set, i.e.

$$\min_{\beta\in N_{\max}} \left\{ \sum_{\gamma\in N_{\max}} L_{\beta\gamma} \right\} . \qquad (14)$$

If the answer is not yet unique one could repeat this step by taking the node with the least links to the set of nodes with the second most links and so on.

4. Analogous to point 2 a vector M_{a+1} is defined as

$$M_{a+1} = M_a - \mathbf{L} \times L_\beta \ . \qquad (15)$$

If M_{a+1} has entries larger than zero go back to step one. Else the process terminates and the nodes to be chosen are determined.

5 A simple example

To illustrate our approach we consider the simple three nodes thermal network shown in Fig. 2. According to the arguments presented in Section 4, to preserve the observability of the parameters of the network, two of the three nodes must be chosen as locations for the temperature sensors (represented by the gray color in the figure).

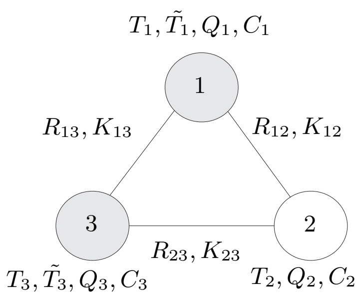

Figure 2: A simple three nodes thermal networks. The gray nodes are those chosen as location for the sensors.

The state variable vector, the measurement vector and the input vectors become $\mathbf{T} = [T_1, T_2, T_3]^T$, $\tilde{\mathbf{T}} = [\tilde{T}_1, \tilde{T}_2]^T$ and $\mathbf{Q} = [Q_1, Q_2, Q_3]^T$, respectively. As an example, let us consider constant the radiative and capacitance parameters and assume that we want to estimate the state vector together with the conductive parameters $\mathbf{K} = [K_{12}, K_{13}, K_{23}]^T$. The state space system for this network can be written as follows:

$$
\begin{aligned}
\begin{pmatrix} T_{1,k+1} \\ T_{2,k+1} \\ T_{3,k+1} \end{pmatrix} &= \mathbf{A}_k \begin{pmatrix} T_{1,k} \\ T_{2,k} \\ T_{3,k} \end{pmatrix} + \mathbf{B} \begin{pmatrix} T_{1,k}^4 \\ T_{2,k}^4 \\ T_{3,k}^4 \end{pmatrix} + \\
& \begin{pmatrix} Q_{1,k} \\ Q_{2,k} \\ Q_{3,k} \end{pmatrix} + \begin{pmatrix} v_{1,k} \\ v_{2,k} \\ v_{3,k} \end{pmatrix}, \qquad (16) \\
\begin{pmatrix} \tilde{T}_{1,k} \\ \tilde{T}_{2,k} \end{pmatrix} &= \mathbf{C} \begin{pmatrix} T_{1,k} \\ T_{2,k} \\ T_{3,k} \end{pmatrix} + \begin{pmatrix} n_{1,k} \\ n_{2,k} \end{pmatrix} \qquad (17)
\end{aligned}
$$

where:

$$
\mathbf{A}_k = \begin{pmatrix} -\frac{K_{12,k}+K_{13,k}}{C_1} & \frac{K_{12,k}}{C_1} & \frac{K_{13,k}}{C_1} \\ \frac{K_{21,k}}{C_2} & -\frac{K_{21,k}+K_{23,k}}{C_2} & \frac{K_{23,k}}{C_2} \\ \frac{K_{31,k}}{C_3} & \frac{K_{32,k}}{C_3} & -\frac{K_{31,k}+K_{32,k}}{C_3} \end{pmatrix},
$$

$$
\mathbf{B} = \begin{pmatrix} -\frac{R_{12}+R_{13}}{C_1} & \frac{R_{12}}{C_1} & \frac{R_{13}}{C_1} \\ \frac{R_{21}}{C_2} & -\frac{R_{21}+R_{23}}{C_2} & \frac{R_{23}}{C_2} \\ \frac{R_{31}}{C_3} & \frac{R_{32}}{C_3} & -\frac{R_{31}+R_{32}}{C_3} \end{pmatrix},
$$

$$
\mathbf{C} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}.
$$

For the sake of this example, we show in Fig. 3 and Fig. 4 the performance of our approach on the simple triangular network. Fig. 3 show the capability of the dual filter to track abrupt variations of the network parameters that might results from variations of the system physical properties. Fig. 4 shows the dynamics of the “true” network temperatures and their estimates.

6 Conclusions and future work

We have proposed a novel approach based on dual filtering for the detection of variations of the thermal properties of the spacecraft and for determining a complete thermal mapping of the system. System and sensor uncertainties are taken into account in the *lumped parameter* modeling of the thermal system and a dual filter is run on the stochastic model to estimate the thermal states and the network parameters from the minimum set of sensor readings that insures system observability. To cope with the strong nonlinearities of the resulting thermal network we have proposed to use an unscented Kalman filter. We have illustrated the approach on a simple three nodes network and shown how the dual filter is able to estimate the system states and at the same time detect changes of the thermal parameters of the network. We are currently working on the application of our method to a satellite thermal network. In the future we

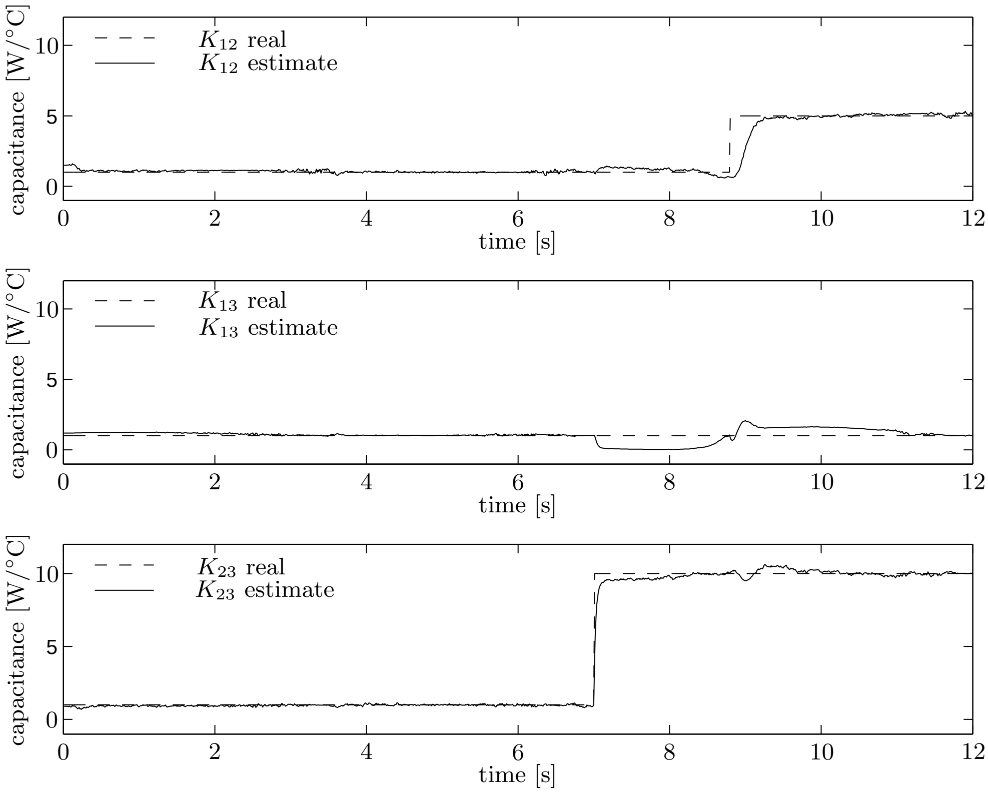

Figure 3: Performances of the dual filter for the tracking of $\mathbf{K} = [K_{12}, K_{13}, K_{23}]^T$. The dashed lines represent the real values of the parameters whereas the continuous lines represents the filter estimates.

will extend our model in order to account for the temperature dependency of the network parameters and to cope with unknown system inputs.

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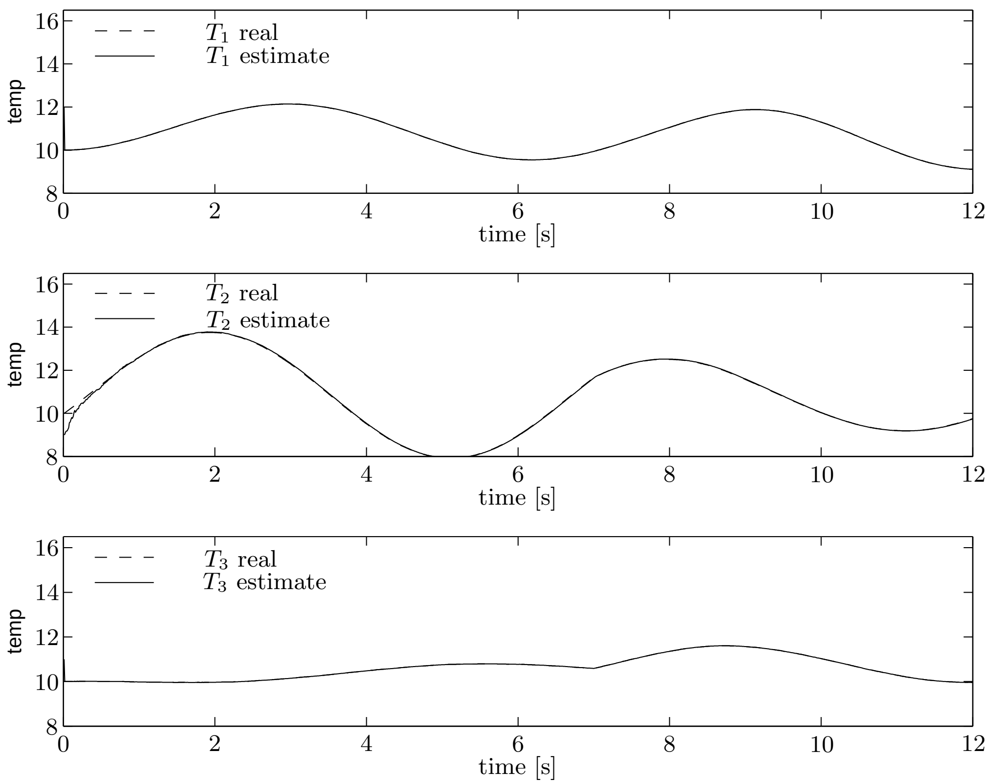

Figure 4: Performances of the dual filter for the tracking of the state $\mathbf{T} = [T_1, T_2, T_3]^T$. The dashed lines represent the real values of states whereas the continuous lines represent the filter estimates.

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