Non-Perturbative Effects in Complex Gravitationally Bound Systems

Final Report

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1 Overview

General relativity, the geometric formulation of gravitational interactions, today is well accepted to properly describe the dynamics of our universe on large scales [1, 2, 3]. Despite its successes, some observational facts seem to indicate a disagreement with the predictions of general relativity or force scientists to ad-hoc assumptions that remain unsatisfactory. The three most important effects of this type are:

Dark matter: From the measured velocity profiles (rotation curves) of stars in galaxies or of galaxies in galaxy clusters, the mass distribution within the galaxy or galaxy cluster can be estimated if the underlying theory of gravitational interaction is known. All measurements seem to indicate that the visible matter (the stars) can only contribute about a quarter to the total mass of our universe, cf. Fig. 1. The additional mass needed to explain the measured rotation curves is called dark matter. Mainly gravitational lensing, the fact that a localized mass distribution according to general relativity bends light and can act as a lens, provides an independent way to determine the mass of galaxies. The findings from these measurements seem to agree with the results obtained via velocity profiles. Moreover, dark matter plays an important role in the standard picture of structure formation in the early universe.

Despite these various aspects, dark matter remains an ad-hoc concept. Till today, no direct evidence for dark matter has been found, but various candidates of such particles have been postulated within the context of so-called grand unified theories. For a recent review on the standard view of dark matter see e.g. [4].

Dark energy: Since its discovery in 1964 the cosmic microwave background, a background radiation with a temperature of 2.7K interpreted as "echo" from the big bang, has been measured with a very high resolution, mainly thanks the NASA's WMAP satellites. This information together with the measured velocities of distant galaxies suggest that our universe is almost flat and in a state of accelerated expansion. To reconcile these findings with the standard model of cosmology, visible and dark matter only contribute a fraction of the total energy density of the universe. The missing part is called dark energy. Its nature is essentially unknown, though some candidates, in the simplest form a so-called cosmological constant, have been proposed ([5] and references therein). A summary of the estimated contributions of visible matter, dark matter and dark energy to the total energy density is given in Fig. 2.

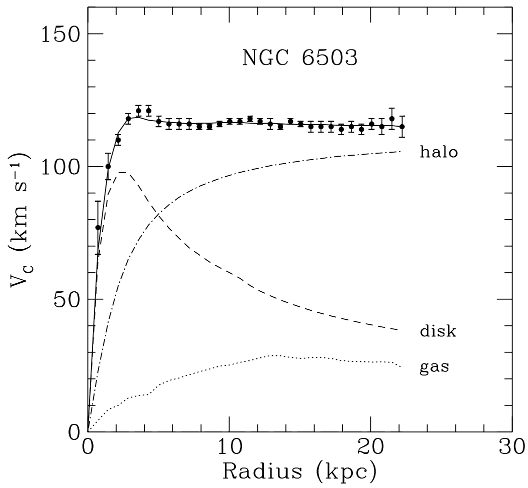

Figure 1: Rotation curve of NGC5603. Dotted, dashed and dash-dotted curves indicate the contributions from stars, gas and dark matter, resp., according to the standard paradigm of dark matter. Picture taken from [6].

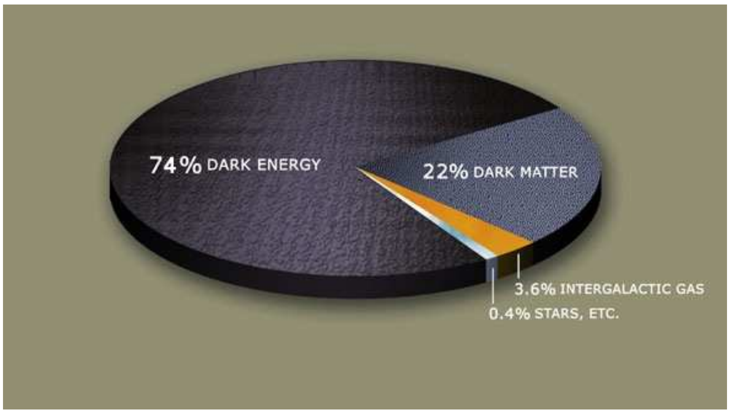

Figure 2: Estimated contribution of dark energy, dark matter and visible matter to the total energy density of the universe.

Pioneer anomaly: While dark matter and dark energy mainly affect the dynamics of the universe on large scales, a third class of apparent discrepancies between theoretical predictions and measurements concern the dynamics of the Solar System [7]. Best known is the so-called Pioneer anomaly [8, 9], which is interpreted as an anomalous acceleration of the Pioneer spacecraft towards the center of the galaxy.

There exist alternatives to the ad-hoc postulates of additional matter or energy to reconcile the measurements with theoretical predictions. It has been suggested that rather the laws of gravitational interaction should be changed than the mass and energy densities. As most famous example, the idea of MOND (modified Newtonian dynamics) [10] and its covariant extension [11] changes Newton's second law of dynamics, $F = ma$, in such a way as to explain the rotation curves of galaxies without the need of dark matter, while keeping Newton's law unchanged where it undoubtedly applies. The main disadvantage of these ideas is the fact that they risk to cast out the daemon by the ruler of the daemons. Indeed, the modification of Newton's law also is an ad-hoc assumption and except the mentioned observational fact there exists no independent motivation to implement this modification.

Only recently new proposals to explain the dynamics of our universe have been proposed, which go without the need of ad-hoc postulates since they completely rely on the known theories, here general relativity. To understand the apparent contradiction to the aforementioned facts, some details about the derivation of these results have to be explained.

General relativity constitutes a set of 10 coupled, *non-linear* partial differential equations, which departs quite drastically from the Newtonian description defined via a linear Newtonian Poisson equation for the gravitational potential [12]. One consequence thereof is the practical impossibility to analyze or simulate many body systems within fully fledged general relativity. Almost all results supporting the existence of dark energy or dark matter thus rely on *approximation techniques*, where general relativity effects are seen as a correction to the Newtonian result rather than descending from a theory on its own. Indeed, approximations of this type seem to be justified when modeling galaxies, galaxy clusters or even the universe: Formally, Newton's theory of gravity is obtained as a limit of general relativity where the speed of light approaches infinity and the mass densities tend to zero. For all situations discussed here, these restrictions apply very well since the typical velocities stay small compared to the speed of light and the matter densities only approach critical values in the center of the galaxy and some additional isolated points (neutron stars or black holes.) Thus, the dynamics of galaxies usually are determined by a systematic expansion: the dominant contribution is assumed to be the result from Newtonian dynamics, general relativity effects are included as corrections, which in principle can be calculated order by order in the small parameters. This technique essentially is equivalent to the Taylor series of a function which states that

$$f(x+\epsilon) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x)}{n!} \epsilon^n \; , \tag{1}$$

where $f^{(n)}$ is the n-th derivative of f with respect to x. However, the equality of both sides in Eq. (1) is only guaranteed for a restricted class of functions, in general a representation of this type is not permitted even for small ϵ. This essentially is the loophole used to obtain "anomalous" contributions to the dynamics within general relativity. It is claimed that complex, extended systems like a galaxy do not permit a systematic expansion since there exist relevant *non-perturbative* effects, which cannot be taken into account properly if an expansion starting with Newtonian dynamics is imposed.

Therefore several authors, most prominently Cooperstock and Tieu (CT), have argued that taking general relativistic effects properly into account might provide an explanation for the deviation of the rotation curves in spite of the (ad-hoc) introduction of dark matter. The difficulty of this approach lies in a correct choice of a model that serves as a sample galaxy: the model should be simple enough to be solvable exactly within general relativity, but still realistic enough to make significant statements about the dynamics of real galaxies. Although the original model by Cooperstock and Tieu [13] could adapt the whole dark-matter content within a general relativistic setting, i.e. modeling the galaxy as stationary axis-symmetric dust, it turned out that in addition to the physical matter it contained unphysical (negative-mass) matter in the galactic plane. Although this casts serious doubts on the viability of the model, nevertheless the basic idea of "taking general relativity more serious" rather than introducing ad-hoc matter is quite intriguing.

2 The model

The choice of the specific model to describe a galaxy provides an important first step. Given the complexity of Einstein's field equations, exact solutions describing a simplified galaxy model are restricted to special configurations of continuous mass distribution (liquid or dust, cf. [14].) Our specific model starts from an axis-symmetric stationary dust solution (cf. A.1), but we take a different approach than [13] thereby circumventing the singularity in the equatorial plane of the galaxy [15] (cf. Appendix B for details.) Thanks to the high symmetry of the solution, the Einstein field equations decompose into a simpler set of (in general non-linear) partial differential equations. Most importantly, the velocity profile, more precisely its potential, obeys a linear equation, which may be solved and then can be used as input for the remaining equations, thereby determining the mass-distribution. Upon comparison with the analogous Newtonian situation we obtain a significant reduction (about 30 percent) of dark-matter. Thus our calculation does not support the view of [13] that general relativity may account completely for dark matter.

It may be worth to explain one point about this result more in detail. Indeed, if a surprisingly large deviation between the Newtonian result and general relativity occurs in galaxies, why does the Newtonian limit work so well in the Solar System, which is also part of a galaxy? Certainly, on scales like the Solar System the approximation of a galaxy as dust is no longer permitted and thus the result may not apply. Still, our model provides a satisfactory answer to this question. Given a certain velocity profile, the mass density from general relativity can always be matched to the Newtonian one *locally* by an appropriate choice of parameters. However, this specific choice of parameters is different in different regions of the galaxy and thus a matching cannot be found *globally*. In other words, the dynamics of general relativity locally can always be approximated by the Newtonian dynamics, but globally such an approximation fails to provide a suitable result.

3 Problems with the model

Although our model has several attractive features there are deficiencies as well. In particular from the mathematical side it can be shown that the velocity profile develops a singularity along the *whole* axis (cf. A.2,) which may be interpreted to be of cosmic-string type.

The second more physical problem is connected with the absence of an outside observer. This simply means that our model refers to the galaxy from within (cf. A.3). Therefore, the model should only be applied to observational evidence of dark matter within our galaxy. Though the observed dynamics within the Milky Way show a systematic disagreement with the expected result from Newtonian physics, which again is interpreted as dark matter [12], the main source of observational data is obtained from distant galaxies.

This last problem might appear as a minor issue, but the observer dependence of observational data, here the measured velocity profiles, is a highly complex problem in general relativity. A first step towards a resolution of this problem would be the choice of a different model, which includes an extended distribution of mass, interpreted as galaxy, but also an outer space.

Recent investigations, in particular the work of Wiltshire [5] on a related problem, namely dark energy, show that a too naive treatment of observation may result in a gross misinterpretation of observational data. In particular the apparent acceleration of distant supernovae can be accounted for as an observer selection effect, since our universe in the present epoch seems to be dominated by large empty regions, so-called voids, whereas matter and thereby observers are mainly found on filament-like structures surrounding these voids. Taking into account the different time-flow between those regions gives rise to apparent acceleration of highly distant objects. Therefore the position of the observer relative to the observed object plays a crucial role in interpreting observational data correctly.

4 New developments

In order to deal with the problems of our original model an exact solution of Einstein's field equations, the so-called Meinel-Neugebauer disc of dust [16], was investigated. This solution is asymptotically flat, therefore containing an outside observer, and contains a very thin (thickness zero), axi-symmetric and stationary disc of pressure-free matter. In contrast to the original model discussed in Sect. 2, this solution is free of singularities.

The most important aspect of the model for our investigation is that there is a difference between the (four-dimensional) velocity of the asymptotic observer and that of the co-rotating dust particles in the disc. It manifests itself in relative time-dilatation and can therefore give rise to velocity measurements. This time dilatation depends on the location (more exactly, the radial distance from the center of the disc) of the dust particle. Once again these are precisely effects of the sort that allow to identify dark-energy as an observer selection effect (cf. A.4).

5 Applications to the original model

From the above problems and the new developments mentioned in the previous paragraphs we have taken first steps to adapt our model in an appropriate way.

In order to incorporate the position of the observer we have cast our original model in a form that explicitly displays this fact. This step amounts to a so-called "dressing" of parameters, that is a mapping from the original parameters referring to the co-rotating dust particles to those relative to the asymptotic, i.e. outside observer (cf. A.5). We believe that this is an important first step to relating the model to observational data.

Moreover, inspired by the investigations on dark-energy, where quasi-local *gravitational energy* plays a major role, we also investigated a simple model containing a negative cosmological constant, which effectively mimics gravitational potential energy. In this model the negative cosmological constant affects fall-off behavior of the velocity profile in the expected manner. However, further work that goes beyond this rather ad-hoc identification between potential gravitational energy and a negative cosmological constant is required (cf. A.6).

6 Conclusion

Starting from the idea of Ref. [13] that non-linear effects from general relativity might account for dark matter, the possibility of such large non-linear effects in regions of weak gravitation has been studied within this project. It was already criticized before that the model of Ref. [13] contains serious mathematical problems, nonetheless large non-linear effects can be found in different systems as well. Most prominently, this model can be modified in such a way as to circumvent the mentioned technical issues. In this way a non-negligible amount of dark matter could be explained as an effect of general relativity [15].

However, there remain complications of both, mathematical as well as physical nature, in our model. Considering the first point, the model still contains singularities, which were interpreted as a jet-like structure of the galaxy. These singularities might appear since the model just considers a pressureless fluid (dust), an approximation which must break down close to the center of the galaxy. A more complicated setup taking pressure into account might resolve or at least reduce this problem.

More important appeared the fact that our model does not contain an observer outside of the galaxy. Thus, the conclusions still apply for our own galaxy, but the model is not able to straightforwardly explain the anomalous rotation curves of distant galaxies. This observation forced us to consider a different approach, which resulted in a dressing of the original parameters. As a simple model of a galaxy embedded in outer space the exact solution by Meinel and Neugebauer, a thin rotating disc of dust, may be used. Remarkably enough, this solution does not contain any singularities. From this model one can derive a structural difference between the internal dust-velocity and the velocity measured by an external observer due to gravitational time-dilatation, which however remains small in regions with weak gravitation.

Still, the Meinl-Neugebauer disk might not yet be a suitable model for an observer on Earth measuring the rotation curves of distant galaxies, as we are not asymptotic observers but ourselves trapped in a gravitational field. This led us to make a link to a recent work by Wiltshire [5] on a similar problem, where dark energy is explained as an observer selection effect. A simple toy model was analyzed to illustrate this effect.

At this point it may be worth to re-consider a point already mentioned in the introduction, namely the fact that there is independent evidence for dark matter from gravitational lensing and primordial fluctuations of the microwave background. On the one hand this supports our result that general relativity cannot explain dark matter completely. On the other hand it should be remarked that the models of structure formation may suffer from the same incompleteness as discussed here for galaxies and that gravitational lensing can as well occur due to spacetime-curvature effects in empty, i.e. vacuum, regions.

Certainly, the questions how and where large non-linear effects can appear in regions of weak gravitation will remain a interesting topic in the future. The setup of this study was too limited to provide a conclusive answer, but some important points have been highlighted during the study:

- Since non-linear effects by definition are not visible within standard approximation techniques, the choice of the model (here the sample galaxy) remains one of the most challenging steps. The model must be realistic enough to describe in good approximation the real situation (here a galaxy,) but it still needs to be solvable within full general relativity to make the non-linear effects visible.

- It would be interesting to study carefully the role of the dynamics of the observer with respect to the galaxy and to discriminate between observers within a galaxy and asymptotic observers. A seemingly small change of the dynamics of the observer can have a large effect due to the complicated non-linear behavior of general relativity.

- The exact technique of the measurement is important as well. In astrophysical observations velocities are measured by means of red and blue shifts of light. Conclusions drawn for this type of measurements need not apply for different techniques, e.g. direct measurements of a displacement.

From all this it has to be concluded that the eventual existence of large non-linear effects has to be re-considered carefully for each new situation at hand, defined as the whole setup of observed object, observer and observation method. Our conclusions may provide the basis of many fascinating results still to be found within general relativity.

A Appendix

This appendix contains mathematical details of the calculations discussed in the main part of this report. Our notation and conventions mainly follow the book by Wald [3].

A.1 Symmetries of the model geometry

For an idealized model of a galaxy, we assume that the solution is axisymmetric and stationary. A symmetry of a space-time with metric g_{ab} is defined by so-called Killing condition:

$$L_\psi g_{ab} = 0 \qquad \Longleftrightarrow \qquad \nabla_a \psi_b + \nabla_b \psi_a = 0$$

Geometrically the Killing vector field ψ^a defines a symmetry direction of the spacetime, i.e. motions that leave the metric g_{ab} and thereby all geometrical quantities invariant. Hence in the present case, we have

- Time translation symmetry: $\psi_1^a := \xi^a = \partial_t^a$.
- Axial symmetry: $\psi_2^a := \eta^a = \partial_\phi^a$.

With this information one can deduce the most general form of the symmetry reduced metric [3]. We work with a metric ansatz in the Lewis-Weyl-Papapetrou (LWP) form:

$$ds^2 = V^2(dt - Nd\phi)^2 + \frac{\rho^2}{V^2} d\phi^2 + \Omega^2(d\rho^2 + \Lambda dz^2)\,, \qquad (2)$$

where V, N, Ω and Λ are functions of the two variables ρ and z. In addition we will impose reflection symmetry with respect to the $z = 0$ plane on our model, which is interpreted as galactic plane.

Idealized matter model Furthermore, we make some simplifying assumptions for the galactic matter. First of all, the matter distribution in our model galaxy is smoothed out, i.e. we work with an idealized fluid. Since the interaction of particles in a galaxy outside of the central region is negligible, we are save to assume that the pressure in the idealized fluid vanishes, in other words our matter is represented by an energy momentum tensor for dust

$$T^{ab} = \rho_m u^a u^b,$$

where u^a is the four-velocity of the dust particles.

The local conservation law for energy and momentum $\nabla_a T^{ab} = 0$ immediately yields two constraints

$$\rho_m u^a \nabla_a u^b = 0 \,, \qquad \nabla_a(\rho_m u^a) = 0 \,.$$

The first condition is the requirement for a geodesic motion of all dust particles while the second condition tells us that no sources or sinks of particles are present.

A.2 Distributional contributions of $\mathbf{N} = \rho^2/\sqrt{\rho^2+\mathbf{z}^2}^3$

In order to check whether our velocity N really solves the homogeneous equations [15] we have to evaluate the corresponding equation distributionally. In the present example we thus have to calculate the distributional "Laplacian" of $\rho^2/\sqrt{\rho^2+z^2}^3$

$$(L^2 \frac{\rho^2}{\sqrt{\rho^2+z^2}^3}, \tilde{\varphi}) := (\frac{\rho^2}{\sqrt{\rho^2+z^2}^3}, (L^\dagger)^2 \tilde{\varphi}) \,,$$

where we used

$$\tilde{\varphi} = (\varphi\rho) d\rho d\phi dz \,, \qquad (L^\dagger)^2 \tilde{\varphi} = ((\varphi\rho)_{\rho\rho} + \varphi_\rho + \rho\varphi_{zz}) d\rho d\phi dz \,.$$

The right hand side of the definition may be taken to be a regular distribution and therefore calculated via integration. Since the bulk terms vanish as we deal with a classical solution of the system, we only have to take into account the surface contributions:

$$\begin{aligned} -\int_\epsilon^\infty \frac{\rho^3}{\sqrt{\rho^2+z^2}^3} \varphi_z|_{-\epsilon}^{\epsilon} d\rho d\phi = \\ = \int_1^{1/\epsilon} d\rho d\phi \frac{\epsilon}{\sqrt{\rho^2+1}^3} (\varphi_z(\epsilon\rho e_\rho, \epsilon) - \varphi(\epsilon\rho e_\rho, -\epsilon)) = 0 \end{aligned} \quad (3)$$

$$\begin{aligned} \int_\epsilon^\infty \frac{3z\rho^3}{\sqrt{\rho^2+z^2}^5} \varphi|_{-\epsilon}^{\epsilon} d\rho d\phi = \\ = \int_1^\infty d\rho \frac{6\rho}{\sqrt{\rho^2+1^2}^5} \varphi(0,0) 2\pi = 2\pi \frac{1}{\sqrt{2}} \varphi(0,0) \end{aligned} \quad (4)$$

$$(\int_{-\infty}^{-\epsilon}+\int_{\epsilon}^{\infty})dzd\phi\frac{\epsilon^2}{\sqrt{\epsilon^2+z^2}^3}\varphi(\epsilon e_r,z)=4\pi\int_1^\infty dz\frac{1}{\sqrt{1+z^2}^3}\varphi(0,0)$$
$$=4\pi(1-\frac{1}{\sqrt{2}})\varphi(0,0) \tag{5}$$

$$(\int_{-\infty}^{-\epsilon}+\int_{\epsilon}^{\infty})dzd\phi\frac{\epsilon^2}{\sqrt{\epsilon^2+z^2}^3}(\varphi(\epsilon e_\rho,z)+\epsilon\varphi_\rho(\epsilon e_\rho,z))=$$
$$=4\pi\int_1^\infty dz\frac{1}{\sqrt{1+z^2}^3}\varphi(0,0)=4\pi(1-\frac{1}{\sqrt{2}})\varphi(0,0) \tag{6}$$

$$(\int_{-\infty}^{-\epsilon}+\int_{\epsilon}^{\infty})dzd\phi\frac{\epsilon^2(\epsilon^2-2z^2)}{\sqrt{\epsilon^2+z^2}^5}\varphi(\epsilon e_r,z) \quad = \quad 4\pi\int_1^\infty dz\frac{1-2z^2}{\sqrt{1+z^2}^5}\varphi(0,0) \tag{7}$$

Even without explicitly calculating the last integral (7) it can be shown that

$$L^2\frac{\rho^2}{\sqrt{\rho^2+z^2}^3}=\alpha\delta^{(2)}(x)\delta(z),$$

where $\alpha=$ *const.* Applying this to the general solution with arbitrary weight gives

$$L^2N=\frac{1}{2}\int dyC(y)L^2(\frac{\rho^2}{\sqrt{\rho^2+(z\pm y)}^3})=\frac{\alpha}{2}\delta^{(2)}(x)C(\mp z),$$

which yields a contribution along the whole axis and not only at the discontinuities. In our case this indicates the presence of additional (constant) matter along the whole axis, even far away from $z=0$ and thus far away from the galactic plane.

A.3 Geodesic deviation of co-rotating dust-particles

Since in our model the four-velocity u^a of the dust-particles coincides with the Killing-vector ξ^a the geodesic deviation equation becomes

$$(\xi\nabla)^2\eta^a+R^a{}_{bcd}\xi^b\xi^d\eta^c=0,$$

where η^a denotes the deviation vector-field which is Lie-transported along the dust-particle trajectory. From being Killing ξ^a satisfies

$$\nabla_a\nabla_b\xi_c=\xi_dR^d{}_{cba},$$

which upon rewriting

$$R^a{}_{bcd}\xi^d = \xi^d R_{dcb}{}^a = \nabla^a \nabla_b \xi_c$$

turns the deviation equation into

$$(\xi\nabla)^2 \eta^a + \nabla^a \xi^b (\eta\nabla)\xi_b = 0,$$

or equivalently

$$(\xi\nabla)^2 \eta^a + \nabla^a \xi_b (\xi\nabla)\eta^a = 0,$$

where we took the Lie-transport condition on η^a into account. The last form exhibits the analogy with the Lorentz-force law in electrodynamics. In particular $\nabla^a \xi_b$ plays the role of electromagnetic field-strength and $(\xi\nabla)\eta^a$ represents the relative-velocity of the dust-particles.

Since the deviation equation preserves the inner product between the deviation field η^a and the four-velocity ξ^a along the geodesic

$$(\xi\nabla)(\xi^a \eta_a) = \xi^a (\xi\nabla)\eta_a = \frac{1}{2}(\eta\nabla)\xi^2 = 0,$$

we may take η^a to be in the orthogonal space of ξ^a, where both particle start at equal time. Since $\epsilon_{abc} := \xi^d \epsilon_{dabc}$ is preserved along the geodesic the motion is purely "magnetic" with $\nabla_a \xi_b = \epsilon_{acb} B^c$, i.e.

$$(\xi\nabla)^2 \eta^a = \epsilon^a{}_{bc} (\xi\nabla)\eta^b B^c \, .$$

Note that $B^a = -1/2\epsilon^{abc}\nabla_b \xi_c = -1/2\xi_d \epsilon^{dabc}\nabla_b \xi_c$ vanishes as soon as ξ^a is hypersurface-orthogonal. This shows that the magnetic field captures the relative acceleration of dust particles due to rotation.

A.4 Velocity definition and the Meinel-Neugebauer disc

A.4.1 A very special choice for the dust four velocity – Co-rotation

In [15] as well as in [13] we can find a very special choice for the four velocity of the dust particles:

$$u^a \propto \xi^a$$

From the normalization we find the form

$$u^a = \frac{1}{V}\xi^a. \tag{8}$$

From the geodeticity of u^a we deduce that ξ^a is a geodesic vector field too. Since the norm of a geodetic Killing field is covariantly conserved

$$\nabla_a(\xi^b \xi_b) = 2\xi^b \nabla_a \xi_b = -2\xi^b \nabla_b \xi_a = 0,$$

we can also absorb $V(r, z)$ into the definition of the metric functions. This completes the steps of symmetry reducing and simplifying the ansatz for the metric and energy momentum tensor. The metric is finally in the form

$$ds^2 = -(dt - N d\phi)^2 + \rho^2 d\phi^2 + \Omega^2(d\rho^2 + dz^2). \tag{9}$$

We are now in a position where we can feed our model with 'observational' input, for example the rotation curve of a typical Sc-Galaxy and ask for the matter density distribution, which is a method often referred to as back-scattering. As pointed out in [15] the three-velocity definition of [13] can be geometrically explained as ADM split with respect to the spacelike hypersurfaces.

A.4.2 ADM-split and ADM velocity

Stationary, axisymmetric space-times have a special feature first observed by [20]. There exists a *unique* future pointing timelike, hypersurface orthogonal, normalized vector field n^a, which is a linear combination of the two Killing fields

$$\begin{aligned} n^a &= a(\rho, z) E^a_t + b(\rho, z) E^a_\phi \\ a > 0\,, \quad b = \pm\sqrt{a^2 - 1} \quad &\longrightarrow \quad \frac{a_{\rho/z}}{a} = \frac{b_{\rho/z}}{b}\,, \\ dn = \nu \wedge n \qquad &\text{(Frobenius theorem.)} \end{aligned}$$

That means that there exists a *unique* family of spacelike hypersurfaces, which are the best analog of the Newtonian absolute space at hand in the sense that they provide a *unique* way of deciding whether an object is rotating or not. We now shed light on this feature from two perspectives. The first one is, to our knowledge, nowhere presented in the literature, while the second one is the approach chosen by Bardeen [20].

From the conditions above we obtain the following equations

$$\begin{aligned} \frac{\partial a}{\partial z} &= \mp \frac{a\sqrt{a^2 - 1}V^2}{\rho} \frac{\partial N}{\partial z} + a(a^2 - 1)\frac{2}{V}\frac{\partial V}{\partial z}\,, \\ \frac{\partial a}{\partial \rho} &= \mp \frac{a\sqrt{a^2 - 1}V^2}{\rho} \frac{\partial N}{\partial \rho} + a(a^2 - 1)\Big(\frac{2}{V}\frac{\partial V}{\partial \rho} - \frac{1}{\rho}\Big)\,. \end{aligned}$$

Then we perform a substitution of variables

$$\mathcal{V} = \frac{V^2}{\rho} , \qquad \frac{\mathcal{V}_\rho}{\mathcal{V}} = \left(\frac{2}{\rho V}V_\rho - \frac{1}{\rho}\right) , \qquad \frac{\mathcal{V}_z}{\mathcal{V}} = \frac{2}{\rho V}V_z ,$$

and write the coefficient a as a function of $(N, \mathcal{V})$ which yields the equations

$$\frac{\partial a}{\partial N} = \mp a\sqrt{a^2 - 1}\mathcal{V} , \qquad \frac{\partial a}{\partial \mathcal{V}} = \frac{a(a^2 - 1)}{\mathcal{V}} .$$

These equations can be solved with the ansatz

$$a = \sqrt{\frac{f(N, \mathcal{V})}{g(N, \mathcal{V})}}$$

which yields

$$a = \sqrt{\frac{\rho^2}{\rho^2 - (NV^2)^2}} , \qquad b = -\sqrt{\frac{(NV^2)^2}{\rho^2 - (NV^2)^2}} ,$$

and hence

$$n_a = -\gamma dt = -\sqrt{\frac{(\rho V)^2}{\rho^2 - (NV^2)^2}}dt.$$

Thus the level surfaces of the time function t determine the hypersurfaces and the Killing field ∂_t can be seen as the foliation vector field.

A.4.3 Bardeens velocity definition and "ZAMOS"

Bardeens ansatz [20] is more physically motivated. An axial Killing vector field ∂_ϕ^a allows the definition of angular momentum per unit rest mass for an object with four velocity n^a:

$$L := g_{ab}\eta^a n^b$$

Asking for the four velocity with vanishing angular momentum (referred to as the Zero Angular Momentum Observer) with the ansatz

$$n^a = \frac{1}{\gamma}(\xi^a + \omega\eta^a)$$

and $n^a n_a = -1$ gives exactly the hypersurface orthogonal vector field found above

$$n^a = \frac{\sqrt{\eta^2}}{\rho}(\xi^a - \frac{\xi \cdot \eta}{\eta^2}\eta^a),$$

from which we can read off the lapse γ and the shift N^a

$$\gamma = \frac{\rho}{\sqrt{\eta^2}}, \quad N^a = -\omega\eta^a,$$

which describe the time dilation and the shift of coordinates within the hypersurface from one time step to the other respectively. The coefficient ω denotes the angular velocity as *seen* by an asymptotic observer following the time Killing field.

The ZAMO distinguishes an orthonormal tetrad (referred to as the Locally Non Rotating Frame)

$$E_0^a = n^a \ , \qquad E_1^a = \frac{1}{\Omega}\partial_\rho^a \ , \qquad E_2^a = \frac{1}{\sqrt{\eta^2}}\eta^a \ , \qquad E_3^a = \frac{1}{\Omega\sqrt{\Lambda}}\partial_z^a \ ,$$

which is non rotating in the sense that the spatial part is not twisting in contrast to an adapted tetrad read-off from (2) and *dragged* in direction of $E_0^a = n^a$.

The ZAMO measures the velocity of the dust particles by decomposing it with respect to the LNRF. We start with the ansatz

$$u^a = \frac{1}{\alpha}(\xi^a + \tilde{\omega}\eta^a) \ ,$$

where $\tilde{\omega}$ again has the interpretation as angular velocity as seen by an asymptotic observer, and find

$$u^a = \tilde{\gamma}(n^a + \frac{\eta^2}{\rho}(\tilde{\omega} - \omega)E_2^a).$$

Hence in general the ADM velocity is

$$v_{ADM} = \frac{\eta^2}{\rho}(\tilde{\omega} - \omega). \tag{10}$$

In [15] and [13] $\tilde{\omega}$ is set to zero and $\xi^2 = -1$, therefore (10) reduces to the shift coefficient

$$v = -\rho\omega = \frac{N}{\rho}. \tag{11}$$

Thus within the model of [15] and [13] a given velocity distribution completely determines the metric coefficient N. So far the two approaches coincide concerning the matter type, the dust particle velocity and the metric ansatz, although they arrived at this preliminary results on quite different routes. The main differences of the two papers lie in the usage of linearized Einstein equations in [13] while [15] keep the full theory of general relativity and in the z-dependence of their input function N.

They both obtain a space time filled with dust orbiting the symmetry axis on circles with constant z. Ref. [13] is able to explain the dynamics of their model galaxy without any need of exotic dark matter, while [15] could reduce the necessary amount of exotic dark matter by one third in comparison with an analog Newtonian model. Now both models face major problems. In the case of [13] one can show that due to the specific choice of the z-dependence of N the energy momentum tensor on the hypersurface $z = 0$ violates all physically reasonable energy conditions, i.e. matter with negative mass is sitting there, which is repulsive. The problem encountered in the model presented in [15] is a space time singularity residing on the *whole* symmetry axis, essentially a cosmic string, which they found to be a generic feature independent of the model building.

Note that these two obstacles, a repulsive matter distribution at $z = 0$ as well as a large amount of matter at $\rho = 0$ are a possible physical a posteriori explanation for some (at first sight unphysical) assumptions taken in the model building. We assumed that pressure is negligible, but wanted our dust particles to be confined to $z =$ *const.* surfaces. To provide such a motion without pressure we need some "attraction" from above or some "repulsion" from below, which is exactly what is found in [15]. Another argument in favor of such an interpretation of the respective deficiencies of the two models is provided by the well known Meinel-Neugebauer disk, where no such problems occur. In that particular solution to the Einstein equations the energy momentum tensor is the one for dust, its motion is circular, its four velocity is Killing but the whole matter is completely confined to the $z = 0$ inside a finite radius, while out side there is vacuum.

For future research it would be important to confirm in a rigorous way that the existence of the singularity or the negative mass plane respectively is due to our assumptions, in particular the absence of pressure. Furthermore one could introduce pressure in the z-direction mimicking the observed oscillatory motion in z-direction of stars, which allows the visible galactic disk to be thickened.

A.4.4 Light signals and Doppler shift

In this section we will present a different operational notion of rotation velocity which is closer to the practice of measuring velocities, namely the Doppler shift. The problem of galactic rotation curves became most prominent 1979/1980 after the publication of [17], where Doppler shifts of HI regions in 21 Sc Galaxies where analyzed.[1] Similar methods are used when the dynamics of the Milky Way is analyzed via the methods developed by Oort [12]. Thus it is natural and necessary to implement this notion of rotation in a galaxy model.

The frequency of a light signal measured by a specific observer is given by the contraction of the tangent vector to the photon trajectory k^a, which for our purposes will be taken to be geodetic, and the four velocity of the observer u^a

$$\omega_O = k^a u_a,$$

where the subscript O denotes 'observer', while E will denote emitter. The relative red shift factor z is defined as

$$z = \frac{\omega_E}{\omega_O} - 1\ .$$

An inside view Since in the models [15] and [13] there is no "outside" region where an observer performing measurements as in [17] could reside we can only ask for the outcome of observations performed by inside observers, for example astronomers on Earth taking a look at the HI regions in our Milky way. Again the simplifying assumptions taken, in particular vanishing pressure together with the co-rotation requirement have severe consequences, namely that simply no Doppler shift is present. The simple reason for that is that the contraction of a geodetic vector field with a Killing vector field is constant along the geodesic, in fomulae

$$k^a \nabla_a (\xi^b k_b) = k^b k^a \nabla_a \xi_b + \xi^b k^a \nabla_a k^b = 0,$$

where the first term vanishes due to the Killing equation and second vanishes due to the geodeticity of k^a. Thus the above presented models can not reproduce the observed Doppler shifts, although a differential ADM velocity was implemented in the models.

[1] Sc galaxies are a special class of spiral galaxies. HI regions are interstellar clouds composed of neutral atomic hydrogen. Mapping HI regions via their characteristic emission line is a technique used for determining the structure of spiral galaxies (cf. e.g. [18].)

If we allow nonvanishing pressure as was suggested in Section A.4.2, then the fluid velocity becomes

$$u^a = \frac{1}{V}\xi^a$$

and the observed Doppler shift is

$$z = \frac{V_O}{V_E} - 1.$$

Note that V depends on ρ and z only, thus the measured Doppler shifts of signals from HI regions on the same circular orbit observed at a particular position are all the same, which again is not the case in our Milky Way. This in turn is an indication for a four velocity of the form

$$u^a = \frac{1}{\gamma}(\xi^a + \tilde{\omega}(\rho, z)\eta^a), \tag{12}$$

which accounts for the different outcome of Doppler shift measurements of different HI-spots on the same orbit, since the contraction of the photon tangent with the four velocities does not drop out.

An outside view Suppose we had a stationary axisymmetric asymptotically flat solution of the Einstein equations. Consider an observer at infinity following the ξ^a trajectory, measuring Doppler shifts of light signals emitted by circular moving objects with a trajectory of the form (12).

For any fluid element in circular motion emitting a photon with energy $E := -k^a\xi_a$ and angular momentum $j := k^a\eta_a$ the red shift measured by the asymptotic observer is

$$z = \frac{1}{\gamma} - \tilde{\omega}\frac{j}{E} - 1 \ .$$

Now let us consider fluid elements in the $z = 0$ plane emitting photons confined to that plane, whose tangents initially are linear combinations of the two Killings, i.e. the photon is emitted tangential to the fluid orbit in forward or backward direction:

$$k^a = \dot{t}\xi^a + \dot{\phi}\eta^a$$

From the normalization of k^a we obtain

$$\dot{t}_{1,2} = \frac{\dot{\phi}}{\xi^2}(-\xi \cdot \eta \pm \rho) \ .$$

Since k^a should be future pointing $\dot{t}$ should be positive, therefore we have for a photon emitted in forward direction ($\dot{\phi} > 0$) as seen by the emitter

$$\dot{t}_+ = \frac{\dot{\phi}_+}{\xi^2}(-\xi \cdot \eta - \rho) ,$$

and for the backward direction

$$\dot{t}_- = \frac{\dot{\phi}_-}{\xi^2}(-\xi \cdot \eta + \rho) .$$

For the angular momentum and the energy we find

$$j_\pm = \frac{\rho\phi_\pm}{\xi^2}(-\rho \mp \xi \cdot \eta) , \qquad E_\pm = \mp\rho\phi_\pm ,$$

and thus for the red shift

$$z_\pm = \frac{1}{\gamma} - 1 + \tilde{\omega}\frac{\xi \cdot \eta \pm \rho}{\gamma\xi^2} . \tag{13}$$

A.4.5 Meinel-Neugebauer rigidly rotating disc of dust

To examine the different notions of rotation velocity given above - Doppler shift and ADM velocity - we want to discuss a particular exact solution of the Einstein equations known as the Meinel-Neugebauer disk. The Meinel-Neugebauer disk represents the metric field of a infinitely thin annular disk of rigidly rotating dust, residing in the $z = 0$-plane inside the radius ρ_0. An approximate solution to this problem was first given in [21] using numerical methods. The exact solution was given by Meinel and Neugebauer more than 20 years later. A more recent review by the same authors can be found in [16]. The solution was constructed as the disk limit of a rigidly rotating spheroid of an ideal fluid with pressure using the method of back-scattering. In the weak field - small velocity limit it approaches the classical MacLaurin disk (see e.g. [22]), while the ultra-relativistic limit is extreme Kerr. We will show that also in this example the ADM velocity and the velocity deduced from Doppler shift experiments are different, at least in the relativistic regime. Note that from the perspective of our project the Meinel-Neugebauer disk, as a metric field of an extended rotating source, merely represents a counter example to our proposal to identify some aspects of non-Keplerian dynamics of galaxies as consequences of the non-linearity of General Relativity, since the deviation from the MacLaurin disk, for typical mass and angular velocity values for a galaxy, is completely negligible. Nevertheless, it can be objected that the Meinel-Neugebauer disk is highly idealized. Furthermore the rim orbit is unstable; it is marginally bound.

For a rigidly rotating disk of dust as discussed in [16] and [21] the fluid velocity is given by

$$u^a = e^{-V_0}(\xi^a + \tilde{\omega}\eta^a) \; . \tag{14}$$

The solution depends on two parameters, $\tilde{\omega}$ and the so-called centrifugal parameter $\mu = 2\rho_0^2\tilde{\omega}^2 e^{-2V_0}$, where ρ_0 is the coordinate radius of the disk. The function V_0 depends on μ only [23]

$$V_0(\mu) = \sinh^{-1}(-\mu - \frac{1+\mu^2}{\mathcal{P}[I(\mu), \frac{4}{3}\mu^2 - 4, -\frac{8}{3}\mu(1+\frac{\mu^2}{9})] - \frac{2}{3}\mu}), \tag{15}$$

where $\mathcal{P}$ is the Weierstrass P-function (a class of elliptic functions [24]) and $I(\mu)$ is given by

$$I(\mu) = \frac{1}{\pi}\int_0^\mu \frac{\ln(x+\sqrt{1+x^2})}{\sqrt{(1+x^2)(\mu-x)}} \; . \tag{16}$$

The disk metric in the $t-\phi$-block is given by

$$\xi^2 = -\exp\{2V_0(\mu[1-\frac{\rho^2}{\rho_0^2}])\} + \frac{\mu\rho^2}{2\rho_0^2} \; , \tag{17}$$

$$\tilde{\omega}\xi\cdot\eta = -\exp\{V_0(\mu)\}exp\{V_0(\mu[1-\frac{\rho^2}{\rho_0^2}])\} - \xi^2 \; , \tag{18}$$

$$\tilde{\omega}^2\eta^2 = -\frac{1}{\xi^2}(\frac{\mu\rho^2}{2\rho_0^2}e^{-2V_0} + (\tilde{\omega}\xi\cdot\eta)^2) \; . \tag{19}$$

Describing the $\rho - z$-block is much more complicated since derivatives with respect to μ are involved. However, it is not necessary for the discussion presented here.

The behavior of ξ^2 is presented in Figure 3. In this figure one can see the development of the ergosphere on the disk, i.e. where ξ^a becomes spacelike. This corresponds to the zeros in the denominator of equation (15).

The metric functions $\tilde{\omega}\xi\cdot\eta$ and $\tilde{\omega}^2\eta^2$ are shown in Figure 4.

Figure 5 displays the frame dragging effect. In the extreme limit $\mu = 4.629...$ the angular velocity of the LNRF equals the angular velocity of the fluid as seen by the asymptotic observer, i.e. perfect dragging occurs. This is an illusion due to the infinite time dilation between the asymptotic observer and the ZAMO, which explains why the relative angular velocity of the fluid measured by the ZAMO remains finite. ZAMOs measure a physical distance to the symmetry axis via the circumferential radius $R_p = \sqrt{\eta^2}$, while the relative angular velocity measured

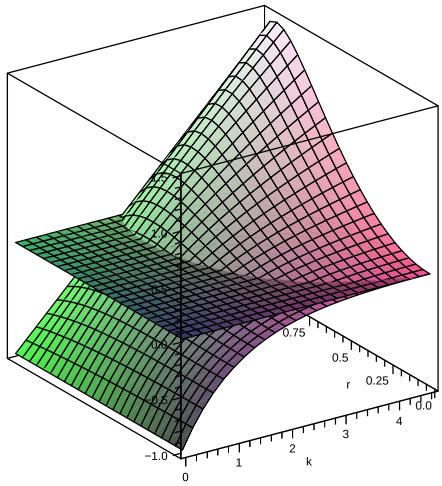

Figure 3: ξ^2 as a function of k and r, where k refers to the centrifugal parameter and r to the (re-scaled) radial coordinate. For a better understanding of the figure the null plane is indicated as well.

by the asymptotic observer has to be multiplied by the inverse shift in the LNRF. We can then recast (10) in the form

$$v_{ADM} = R_p \tilde{\omega}_p. \tag{20}$$

The ADM velocities measured by the ZAMO for different centrifugal parameters are displayed in Figure 6. In this figure the ADM velocity versus the coordinate radius is plotted, which for the ZAMO is without meaning. To illustrate this fact, the two curves of the ADM velocity for $\mu = 1.34$ versus coordinate radius and circumferential radius, resp., are presented in Figure 7.

Finally the redshift of photons emitted in the plane of the disk (13) measured by the asymptotic observer are presented in Figure 8, where the upper and lower branches represent the photons emitted in backward and forward direction respectively. The peak in the redshift for $\mu = 1.34$ corresponds to the peaks in the metric functions. For $\mu = 1.68..$ the first zero in ξ^2 occurs, i.e. there is a ring like ergosphere, and the red shift for photons emitted in backward direction becomes infinite. However, when interpreting this plot, one has to bear in mind that the null congruence of photons has non vanishing expansion, i.e. the emitters might not appear on a radial straight line for the asymptotic observer. Thus more detailed analysis of particle trajectories would be necessary, in order to clarify the appearance of a relativistically rotating disk of dust.

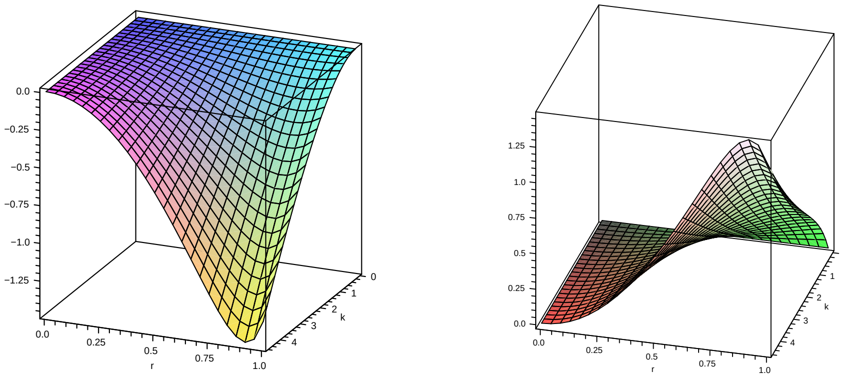

Figure 4: The metric functions $\tilde{\omega}\xi \cdot \eta$ and $\tilde{\omega}^2\eta^2$ as functions of k and r (cf. Fig. 3).

Even though in the Newtonian limit the relativistic effects are negligible, there are two points which should be emphasized. From the point of view of the asymptotic observer the observational effects are significant in the outer part of this extended rotating source of gravitation. The Doppler shift curve is flattened there. The ergosphere starts to develop near the rim of the disk. Also for the velocity measured by the ZAMO the effects are more significant towards the rim. On the other hand there *appears* to be more frame dragging towards the center of the disk, which is in accordance with the Machian objection to the Newtonian bucket experiment, if we consider the inner part disk as the water and the outer part as the thickened bucket walls, which should have an effect on the inertial forces, which act on the water [25].
Another conclusion we can draw from the excursion to the Meinel-Neugebauer disk is, that the actual frame dragging effect and gravitational red shift in galaxies must be very small. The rotation curves obtained from the observations by [17] are highly point symmetric, while the curves shown above are not, which is a generic feature, exactly due to this relativistic effects. This in turn means that in principle, if we had a sufficiently big sample of Sc galaxies, one could look for systematic deviations from point symmetry between the red and blue branches of the rotation curves to determine the actual gravitational field of Sc galaxies.

A.5 Dressing of parameters

Incorporating the lessons from the Meinel-Neugebauer disc of dust into the original model based on co-rotation, we have to switch to a description relative to an asymptotic (i.e. outside) observer. Thus we have to decompose the dust 4-velocity

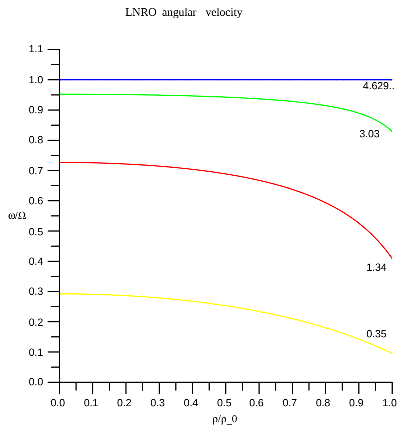

Figure 5: Illustration of the frame dragging effect.

u^a, which corresponds to the motion of the co-rotating observer, with respect to the asymptotic Killing vectors:

$$u^a = \hat{\xi}^a = \xi^a + \Omega\eta^a$$

Here and in the following, hatted variables refer to the co-rotating observer, while ξ^a and η^a are the asymptotic timelike Killing vector and the azimuthal Killing vector, respectively. This change turns the line-element (9) for the co-rotating observer

$$ds^2 = -(d\hat{t} - N d\hat{\phi})^2 + \rho^2 d\hat{\phi}^2 + e^{2\hat{k}}(d\rho^2 + dz^2)$$

into

$$ds^2 = -e^{2U}(dt - a d\phi)^2 + e^{-2U}(\rho^2 d\phi^2 + e^{2k}(d\rho^2 + dz^2)) \ .$$

Since the line elements describe the same geometry we obtain the following "dressed" parameters

$$\begin{aligned} e^{2U} &= (1 + N\Omega)^2 - \Omega^2\rho^2 \ , \\ a &= \frac{(1 + N\Omega)N - \Omega\rho^2}{(1 + N\Omega)^2 - \Omega^2\rho^2} \ , \\ e^{2k} &= e^{2\hat{k}}((1 + N\Omega)^2 - \Omega^2\rho^2) \ , \end{aligned}$$

which in turn (upon ADM-decomposition) gives rise to the "dressed" velocity

$$v_{co} = N/\rho \qquad v_{rig} = v_{co} - \Omega\rho(1 - v_{co}^2) \ .$$

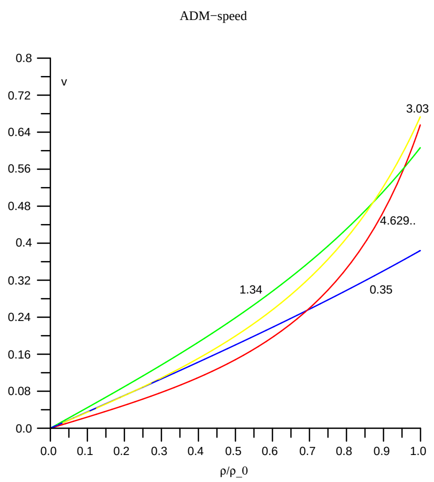

Figure 6: Illustration of the ADM velocity as a function of the coordinate radius.

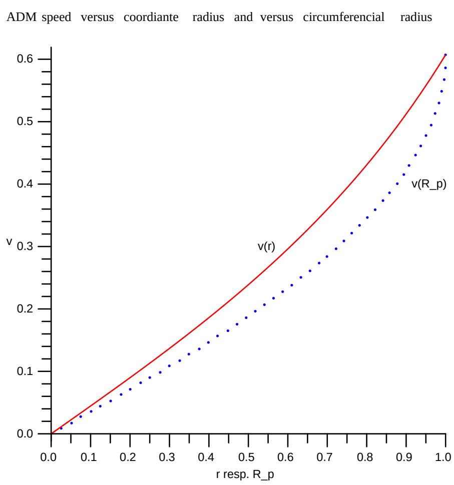

Figure 7: The ADM velocity as a function of the coordinate radius (red curve) and the circumferential radius (blue, dotted curve) for $\mu = 1.34$.

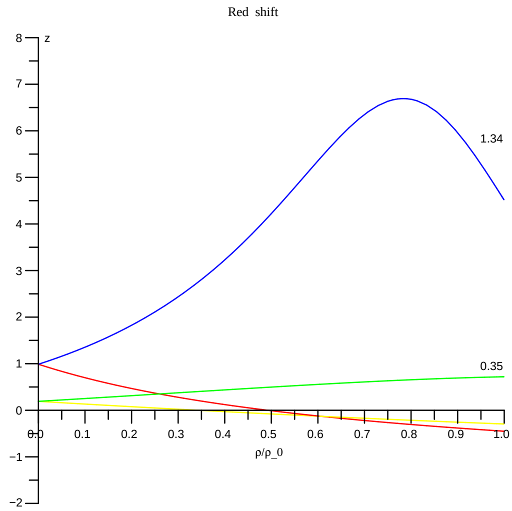

Figure 8: The redshift measured by the asymptotic observer. The upper branch represents photons emitted in the backward direction, the lower branch those emitted in the forward direction.

A.6 New developments

Another route would be one, which is inspired by the pioneering work of G.F.R. Ellis, who introduced the notion of finite infinity in [17] and the more recent work by D. Wiltshire in [5], namely to consider the embeddedness of galaxies in a cosmic fluid. This setting focuses more on the dynamical aspects of space-time. Now one can keep the spacetime filled with a cosmic fluid as in [15] and [13] and one introduces boundaries, which distinguish the bound or even collapsing[2] system from the possibly expanding outside. Of course this is in stark contrast to our stationarity assumption. The main point in Wiltshires approach to explain the observed cosmic acceleration as an apparent effect, is the difference in quasi local gravitational energy of non-expanding regions (essentially regions around super clusters) located at filamentary walls of huge voids and the voids themselves. The energy differences are due to vanishing spatial curvature in the non-expanding regions and negative curvature in the voids. For our purposes here, we would have to consider quasi local energy differences between galactic and cosmic frames. The first step in such a direction would consist in identifying the "decoupling" surface, i.e. to answer the question "*where is finite infinity?*" [5].

A simple toy model, which reveals the differences between a system which is asymptotically flat and one which is not, is the Schwarzschild-(Anti) de Sitter

[2] This would account for the fact that there is still matter falling into galaxies and clusters, causing them to grow [5].

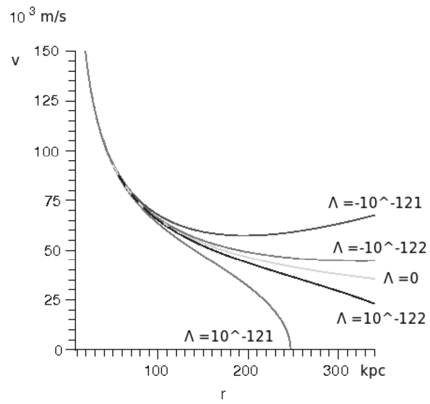

Figure 9: Influence of a cosmological constant on the rotation curves in Schwarzschild-(A)dS spacetime.

spacetime. If we consider particles moving on circular orbits about a center of gravity of 10^{11} masses of sun, sufficiently far away from the black hole horizon, we observe a Keplerian behavior. However, in Schwarzschild-(A)dS spacetime the rotation curves deviate significantly from this behavior with increasing radius. In particular, for a negative cosmological constant the velocity starts to increase again. If the cosmological constant is positive the region where circular orbits can exist is bounded. The effect of a cosmological constant on the rotation curves is illustrated in Figure 9. It should be noted that the absolute value of the cosmological constant used in this plot are of the same magnitude as in ΛCDM models, where dark energy is explained as a cosmological constant.

Of course the deviation from the $\Lambda = 0$ curve is significant far outside the regions where astronomical observations are made[3] but inside the so called virial radius[4], which is used in calculations of the total mass of a galaxy.

[3] For the Milky Way we have data up to a radius of 60 kpc [26]

[4] $r_{\rm vir} = 290$ to 340 kpc for the Milky Way

B Reprint of Ref. [15]

This Appendix contains a reprint of the paper [15], which was published as an invited contribution to the International Journal of Modern Physics D during the Ariadna study and which contains many details of the model discussed in the main part of the report.

Due to copyright issues this Appendix does not appear in the online version of the report. Please refer to the original article, an eprint version of the manuscript is available via `http://arxiv.org/pdf/astro-ph/0602519v3`.

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Fact sheet for Final Report
of
Non-perturbative Effects of Rotation in Gravitationally Bound Systems

Institutions involved:

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Personnel:

- Administrative Coordinator: Herbert Balasin (ITP)
- Scientific Coordinator: Daniel Grumiller (ITP)
- Junior Collaborator: Florian Preis (ITP)
- ESA contact person, technical officer: Luzi Bergamin (ESA)

Time frame:

- Submission of proposal: 30th October 2007
- Kick-off meeting: 30th November 2007 (phone), 13th December (at ESA)
- Official start of project: 1st March 2008
- Mid-term meeting at ITP: 28th - 29th April 2008
- Final presentation at ESA: 22nd July 2008
- Final report: 25th March 2009

Scientific conferences:

In April 2008 the Scientific Coordinator was invited to a Turkish theoretical physics conference, which featured a lot of theory talks on gravity (including the invited talk by the Scientific Coordinator). This provided an opportunity to disseminate some of our results and to foster new contacts with the community. In September 2008 the Junior Collaborator visited the Spanish Relativity meeting (webpage: http://www.usal.es/~ere2008/website/modules/tinyd0/), which was an excellent opportunity to learn about recent developments in gravitational physics, some of which were pertinent to our project.

Papers in peer reviewed journals:

In April 2008 an invited contribution of the Coordinators was published in the peer-reviewed International Journal of Modern Physics. The publication is reprinted in this report as Appendix B. The bibliographical data are:

Non-Newtonian behavior in weak field general relativity for extended rotating sources. H. Balasin and D. Grumiller, Int.J.Mod.Phys.D17:475-488,2008.

Talks and discussions:

The Administrative Coordinator had an extended discussion with one of the key players in the international community, David Wiltshire, who was invited to Vienna in May 2008. The Scientific Coordinator gave a series of seminar talks at MIT (see also 'unexpected spin-offs' below).

Unexpected spin-offs:

The astrophysical community at MIT took notice of our approach. The seminar talks by the Scientific Coordinator engendered a fruitful set of discussions with Bruno Coppi [MIT] and Paola Rebusco [MIT]. They are interested in the dynamics of accretion disks, and it turned out that our approach to set up an inverse problem, i.e., to treat the velocity profile as an input and to predict the mass density as an output, is useful also in other areas of gravitational physics, beyond the ones that we had envisaged in our project (for details see our project proposal and the scientific part of the final report). These contacts may well turn into a long-term collaboration, and the Scientific Coordinator is invited again to MIT for November/December 2008.

Outlook:

The goals that we have formulated require a long-term approach, and it is fair to say that we are not completely there yet. However, despite of the relatively short time-span of our project we have achieved substantial progress on the pertinent questions.