Quantum Metrology for Space-Based Tests of Gravitational Physics

QMAQRO

Final Report

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Abstract Testing the validity of general relativity at cosmological and microscopic scales is a challenging open question. The impediment at cosmological scales is our lack of understanding of dark matter and dark energy. At microscopic scales, it is the unknown effect of gravity on quantum mechanics. Recently, space based matter wave interferometers have been proposed as candidates for addressing certain aspects of both these problems. The MAcroscopic Quantum ResOnators (MAQRO) mission, for instance, can be used to directly detect dark matter and to experimentally bound the effects of gravitational decoherence on quantum superpositions. These phenomena manifest themselves as phase and dephasing in the quantum state of the interferometer. Our study will investigate the enhancements that quantum metrology can provide in the detection of dark matter and the gravitational decoherence in matter wave interferometers. Our methodology will use principles of quantum estimation theory. Using our recent results in this area, we will design the optimal and attainable strategies for detecting phase and dephasing simultaneously in matter wave interferometers. We will evaluate the performance of input states such as thermal squeezed states and propose optimal detector designs. The objective will be to attain in practice a performance as close as possible to the in-principle limits.

CONTENTS

INTRODUCTION

General Relativity The General Theory of Relativity has proven to be highly successful, describing accurately the behaviour of massive bodies on macroscopic scales whose principal interaction is gravitational. As predicted by Einstein in 1916, it has accounted for the anomalous perihelion precession of Mercury's orbit, the deflection of light by the Sun, as well as the redshift of light in a gravitational potential. Astronomical observations by Eddington and others tested all of these to some degree of satisfaction in the period from 1919–25. It was not until 1959, however, that a precise test of general relativity (GR) was performed in the Pound-Rebka experiment on gravitational redshift. The crucial enabling tool for this experiment was Mössbauer spectroscopy, based on the Mössbauer effect —

the resonant and recoil-free emission and absorption of light by atomic nuclei in a lattice. The era of precision testing GR thus began with the aid of a uniquely quantum phenomena

Since the 1960s, several other precision tests of GR have been performed in space, testing further the weak field limit of the theory. For example: the Gravity A & B probes (1970s and 2000s respectively), the unintended reaffirmation of gravitational redshift by global satellite navigation systems [1], as well as the gravitational deflection of light by the Sun calculated using the initial census data from the Gaia spacecraft [2]. Further tests of weak gravity will appear in the coming years beginning with the Atomic Clock Ensemble in Space (ACES) [3] in 2017, that will test gravitational redshift down to an uncertainty of approximately one part in 10^6.

Tests of gravity in the strong field regime have also closely matched the predictions of GR. The decaying orbits of the Hulse–Taylor binary pulsar PSR B1913+16 [4] could be accounted for by a loss of energy, emitted as gravitational radiation, from the system. More recently the first direct detection of gravitational waves from the merger of two black holes by the advanced LIGO (aLIGO) collaboration [5], as well as a neutron star merger [6], were well described by numerical relativity simulations based on classical GR. Further data from this promising field of multi-messenger (gravitational and electromagnetic) astronomy will lead to ever more stringent tests of strong field gravity.

To make these detections, the LIGO experiments had strain sensitivities better than $10^{-23}\,\sqrt{\mathrm{Hz}}^{-1}$ [7, 8]. The Advanced LIGO interferometers which achieved this were engineered to reduce noise to the point that quantum noise sources were dominant, particularly affecting sensitivity to higher frequency gravitational waves [7, 8]. Following the success of the LISA Pathfinder mission [9], the Laser Interferometer Space Antenna (LISA) mission has been approved by the European Space Agency. LISA is a space-based gravitational-wave detector has been approved, that will target lower frequency gravitational waves which are not accessible to ground based detectors and is slated for operations in the 2030s [10]. LISA will be able to take advantage of 2.5×10^6 km long interferometer arms, compared to the 3 km and 4 km of the VIRGO and Advanced LIGO interferometers—which promises much better sensitivities in the 10^{-5} Hz to 10^{-1} Hz frequency range. Gravitational-wave sources in this frequency range including black hole mergers similar to those LIGO has already observed, but also opens up the possibility of direct detection of a gravitational wave background [11], a gravitational analog of the cosmic microwave background.

However, despite these successes, GR is almost certainly only an effective theory of gravity. There are several reasons for this. On the one hand, GR predicts, under generic conditions, the formation of spacetime singularities where the theory itself breaks down. A resolution to this problem will require a complete quantum description of gravity, valid even on microscopic scales. However, precision tests of GR at sub-mm scales that might shed light on the topic are lacking due to the extreme, relative, weakness of the gravitational force. On the other hand there is evidence from astrophysical surveys that up to 85% of the mass, and about 26% of the mass and energy of the observable universe is in the form of so called dark matter. We know little about dark matter save that it dominates the gravitational dynamics of the universe at astronomical distances - as does dark energy at cosmological distances. This means that, without a proper accounting of the dark universe, precision tests of GR at such large scales is also problematic.

Dark matter Dark matter was first proposed to account for the anomalous orbital velocities observed within galaxies and galaxy clusters [12]. Since then additional evidence for its existence has accumulated, for example including its gravitational lensing effects [13], and imprints on the Cosmic Microwave Background [14]. Based on observation of the luminous matter in the galaxy and the virial theorem we can infer that dark matter forms a halo with a local energy density of approximately $\rho \approx 0.4\,\mathrm{GeV/cm^3}$, and follows a non-relativistic Maxwellian velocity profile centred around a velocity $v_{\mathrm{dm}} \approx 220\,\mathrm{km\,s^{-1}}$ in the Galactic rest frame [15, 16]. Taken together, a variety of experiments, astronomical observations, and theoretical considerations have defined a complex web of restrictions on the allowed parameter space of the dark matter particles [17]; in this sense we know quite a lot about what dark matter *is not*.

However, these tests have always been indirect and essentially gravitational in nature, and we know very little about what dark matter actually *is*, or the nature of its (non-gravitational) interactions. This is a primary challenge that besets any attempt to directly detect dark matter; its mass can vary from astronomically large scales to $10^{-22}\,\mathrm{eV}$, a limit set by quantum pressure effects. It is thus clear that no single detector can be deployed that is effective over the entire range, adding to the complication of detecting dark matter directly.

Quantum gravity General relativity and quantum mechanics are the two pillars of twentieth century physics. Together they provide a framework within which the vast majority of physical phenomena can be understood. However, they are valid in mutually exclusive parameter regimes; GR in the regime of large mass and length scales, where gravity dominates, and quantum mechanics in the opposite, microscopic, regime where electroweak and strong forces prevail. The effective decoupling of these regimes has allowed us to make dramatic progress in fundamental physics, but the lack of a single underlying framework has been troubling for both conceptual and now, increasingly, practical reasons.

This is because there a number of phenomena for which both gravitational and quantum mechanical effects are relevant, and these phenomena are starting to become accessible to experiments. For example, both the very early, pre-inflationary, universe, and astrophysical black holes, are being studied in observational astronomy, despite lying well within the quantum gravity regime. These phenomena correspond to macroscopic objects (the universe and stars respectively) that are compressed down to scales at which quantum mechanics is relevant, and whose properties can be observed at a distance. Of particular interest to us, however, is the opposite scenario, in which quantum systems are brought up to the macroscopic scales at which classical gravitational effects start to be relevant, and can be studied locally, in a lab setting.

Collapse models The emergence of classical physics in the macroscopic world has often been seen as conflicting with the quantum nature of the microscopic world. For example, the measurement problem is raised as an objection to the Copenhagen interpretation of quantum mechanics [18, 19], because, while microscopic objects interact with one another under unitary and reversible dynamics, the influence of a macroscopic object or environment gives rise to non-unitary and irreversible evolution i.e. collapse [20, 21]. This is matched by the absence of (and, indeed, by how difficult it is to generate) macroscopic superpositions, wherein large objects are solely observed in classical states, while microscopic systems can be readily manipulated into quantum mechanical superpositions

Karolyhazy [22] suggested a natural collapse of coherence could resolve issues of the measurement problem, Ghirardi *et al.* [20] later proposed what that each particle could be subject to an intrinsic probability of collapse occuring—commonly known as GRW theory. This hypothesised collapse would have negligible effect on individual quantum particles—having too low a rate of $\sim 10^{-16}\,\mathrm{s}^{-1}$ to be observable in small systems—yet cause rapid collapse of macroscopic objects. A macroscopic spatial superposition of an object could be collapsed by any single particle within it collapsing, giving rise to a rate of $\sim 10^{7}\,\mathrm{s}^{-1}$ collapse events occurring within a mole of substance [20]. Diósi [23] proposed a similar damping of coherence in macroscopic systems which shares similarities with the mass-proportional continuous spontaneous localization (CSL) model [24] which itself derived from GRW theory.

The continuous spontaneous localization (CSL) model was a revision of GRW theory, introduced to instead apply a continuous decay of quantum correlations through a continuous dephasing in the position basis, or localisation rate [25, 26] acting like environmental interactions [27]. CSL predicts coherences in the position basis to undergo some decoherence rate of the form

$$\frac{\partial}{\partial t}\rho = -\frac{i}{\hbar}[\hat{H}, \rho] + \zeta \int \mathrm{d}^3\vec{x}[G(\hat{\boldsymbol{Q}} - \hat{\boldsymbol{x}}), [G(\hat{\boldsymbol{Q}} - \hat{\boldsymbol{x}}), \rho]], \tag{1}$$

where $\hat{\boldsymbol{Q}}$ is the centre-of-mass coordinate, and G a function giving a density distribution of number [26] or mass [28].

For small spatial movements the localisation term can be expanded to make the relevant master equation [29–31]

$$\frac{\partial}{\partial t}\rho = -\frac{i}{\hbar}[\hat{H}, \rho] - \Lambda[\hat{x}, [\hat{x}, \rho]], \tag{2}$$

which is a case of the quantum Brownian motion master equation [32–34].

One can get an intuition for the decoherence caused by Λ in this expression by passing to the position basis and evaluating the off-diagonal terms. Then

$$\partial_t \rho(x, x') = (\partial^2_{xx} - \partial^2_{x'x'})\rho(x, x') - \Lambda(x - x')^2\rho(x, x'), \tag{3}$$

which in the limit of large separation $x - x'$, has a solution of the form $\rho \approx \rho_0(x, x') \exp[-\Lambda(x - x')^2 t]$ showing an exponential suppression of correlations with separation [35].

This form of the master equation is widely applicable, providing a good approximation to the effects of a thermal bath of linearly coupled oscillators [36, 37]. Importantly, it describes the dynamics taken by a test mass subjected to collisional decoherence [35] from an environment of lighter particles, such as low mass dark matter.

These theories focus on dark matter detection through the decoherence they introduce, including localisations of the same form as Eq. (2). Interactions with massive particles have similarly been predicted to produce decoherence effects in the same manner as the CSL dynamics which may enable detection of otherwise dark objects [34, 38, 39]. These models are notable as they allow for the detection of low mass dark matter for which we have few alternative direct detection schemes.

It is also believed that one of the consequences of introducing a dynamical spacetime, i.e. gravity, that can interact with quantum systems is the collapse of quantum superpositions. That is to say that gravity, like other sources of noise, is a source of decoherence, and the increased relevance of gravity in the classical regime may mean that gravitational decoherence underlies the quantum to classical transition. There exist a variety of such gravitational collapse models [21, 40] whose predictions can be used to experimentally quantify how gravity affects macroscopic quantum superpositions. What is effectively measured is the rate of dephasing of macroscopic superpositions. Superposing objects of increasingly larger mass provides bounds on the strength of gravitational collapse models. Exploring this effect experimentally, however, requires measuring the extremely small effects of gravity at the atomic or molecular scale, where systems with manifestly quantum properties most often live. This has so far proven challenging due to more dominant sources of noise that outstrip the expected magnitudes of gravitational modifications to quantum mechanics.

Various ground-based experiments have been able to bound collapse rates with nanocantilevers [30], cold atoms [41], and even the LIGO interferometers [31], Vinante *et al.* [42] achieve greater precision and observed an excess noise which could be consistent with the CSL theory of Adler [43]. Data from the space-based LISA Pathfinder experiment, designed as a pre-cursor to LISA and focused towards demonstrating necessary technologies for gravitational wave detection, can also be used to test the predictions of such collapse theories [31, 44].

In terms of mass scales, these experiments have confirmed the predictions of quantum mechanics all the way up to massive molecules of 10^4 u [45] (1 u is approximately equal to the nucleon count). However, this is still orders of magnitude away from the regime where gravitational collapse models predict deviations will arise from standard quantum mechanics [46]. Using more massive test particles on the Earth may soon face limitations, though, because of the limited free fall times in a non-microgravity environment, and background noise; this limit may be reached for interferometric experiments with particles in the mass range between 10^6 u to 10^8 u [47].

MAQRO Demonstrations of quantum phenomena such as the violation of non-locality [48–50] and macrorealism [51] have been performed with increasing clarity and subject to fewer and fewer assumptions. LIGO is one of a number of experiments pushing the limits set by quantum mechanics, with highly sensitive measurements being heavily influenced by quantum-mechanical phenomena such as noise in the vacuum [52]. Quantum mechanics has been similarly probed in macroscopic regimes by generating superpositions over larger distances [41] or of ever larger objects [45].

Such advances in testing the foundations of quantum mechanics and its predictions in an ever-wider set of regimes have been coupled by an advance in quantum technologies. LIGO is now not just limited by quantum noise [7, 8] but also able to improve its sensitivity from the use of non-classical—squeezed—light [53]. Devices for quantum sensing are increasingly moving from the lab towards commercial and real-world applications [54], with commercial quantum key distribution [55] and quantum random number generation [56], along with portable atom-interferometry devices [57, 58]. Some of these technologies have been proposed for space-based experiments [59] and been demonstrated with satellite links [60] or on board sounding rockets [61, 62].

These recent developments in quantum technologies, in particular quantum optomechanics, and the optical trapping of dielectric particles, may allow us to go further in probing macroscopic quantum systems with matter wave interferometry [47]. So too will the move to a space environment with lower pressure and thermal noise. Such considerations has prompted the development of the MAcroscopic Quantum ResOnators (MAQRO) project [63, 64]. MAQRO has been submitted in the past, unsuccessfully, for the European Space Agency's (ESA's) M3 and M4 medium class mission proposals. More recently, MAQRO has been shortlisted for ESA's call for New Science Ideas in 2017 under which it will be provided with support to mature the concept towards a potential future submission.

The goal of MAQRO is to observe high mass matter wave interferometry with particles of varying size and (large) mass, up to 10^{10} atoms, comparing the interference visibility observed with the predictions of gravitational collapse models and with the predictions of standard quantum mechanics [64].

Quantum metrology The field of quantum metrology looks to devise measurement strategies that allow for the highest possible precision when estimating parameters of a system [65–67]. Preparing non-classical states which utilise squeezing [68–71] or entanglement [69, 70, 72] enable more efficient use of particle numbers, to achieve a Heisenberg scaling $\Delta\phi \sim N^{-1}$ over the shot-noise scaling $\Delta\phi \sim N^{-\frac{1}{2}}$ which forms a significant part of the work on metrology [66, 67]. Other significant questions involve the potential improvement available from estimating many parameters simultaneously [73–75], and problems of quantum imaging [76].

Estimating noise processes such as loss and dephasing has been looked at [77–82] primarily with the aim of characterising environmental noises. Interest in applying these techniques to systems optimised for estimation of collapse model parameters has recently grown [83–85].

In the context of the MAQRO proposal [63, 64] the prospect of using phenomena such as recently demonstrated squeezing of a mechanical oscillator [86], could allow the application of squeezed states which have found much usage in quantum optical sensing [68, 69, 87, 88].

Study objectives The goal of this study is to determine the potential for quantum enhancement in the MAQRO space based matter wave interferometry mission. This experiment, made possible by recent developments in quantum technologies, has the potential to explore certain dark matter and gravitational collapse models that live at the boundaries of our knowledge of gravitational physics.

In particular we will evaluate the difference between the quantum Fisher information (QFI) giving a measurement-independent lower bound for the attainable precision, and the classical Fisher information (CFI) corresponding to particular measurement strategies. The latter will be related to the MAQRO detection system, and we will also explore detection systems that get us as close to the fundamental limit set by the QFI as possible.

QUANTUM METROLOGY

The Cramér-Rao bound (CRB) provides a lower bound to the variance of an unbiased estimator $\hat{\Lambda}$ of a parameter Λ [65, 89]

$$\mathrm{Var}\,\hat{\Lambda} \geq \frac{1}{\mathcal{N} F(\Lambda)}, \tag{4}$$

where $\mathcal{N}$ is the number of repetitions performed and $F(\Lambda)$ is the Fisher information, hereinafter CFI, which is derived from the sampled probability distribution $P(\vec{x}|\Lambda)$ as

$$F(\Lambda) = \int \mathrm{d}\vec{x} \frac{1}{P(\vec{x}|\Lambda)} \left(\frac{\partial P(\vec{x}|\Lambda)}{\partial \Lambda} \right)^2 . \tag{5}$$

In the case of estimating a parameter of a quantum state, the parameter is encoded in some state $\rho = \rho(\Lambda)$. By performing a measurement (such as a positive-operator valued measurement (POVM) $\{\Pi_{\vec{x}}\}$) a probability distribution $P(\vec{x}|\Lambda) = \mathrm{Tr}\left(\Pi_{\vec{x}}\Lambda\right)$ is derived from this state. The quantum Cramér-Rao bound (QCRB) gives a measurement-independent lower-bound on the CRB [65, 66]

$$\mathrm{Var}\,\hat{\Lambda} \geq \frac{1}{\mathcal{N}F_Q(\Lambda)}, \tag{6}$$

being a measure of how quickly the state changes with a change in the parameter value. With the QFI, which is given by

$$F_Q(\Lambda) = \lim_{\mathrm{d}\Lambda\to 0}\left\{-4\frac{\partial^2\mathcal{F}(\rho_\Lambda, \rho_{\phi+\mathrm{d}\Lambda})}{\partial(\mathrm{d}\Lambda)^2}\right\}, \tag{7}$$

where $\mathcal{F}$ is the fidelity

$$\mathcal{F}(\rho,\sigma) = \mathrm{Tr}\left(\sqrt{\sqrt{\rho}\sigma\sqrt{\rho}}\right). \tag{8}$$

The QFI can be equivalently defined with respect to the symmetric logarithmic derivatives (SLDs) which are given by

$$\frac{\partial\rho_\Lambda}{\partial\Lambda} = \frac{L_{\rho_\Lambda}\rho_\Lambda + \rho_\Lambda L_{\rho_\Lambda}}{2}, \tag{9}$$

and will produce the QFI as

$$F_Q(\Lambda) = \mathrm{Tr}\left(\rho_\Lambda L^2_{\rho_\Lambda}\right). \tag{10}$$

The hierarchy of CRBs formed is then

$$\mathrm{Var}\,\hat{\Lambda} \geq \frac{1}{\mathcal{N}F(\Lambda)} \geq \frac{1}{\mathcal{N}F_Q(\Lambda)}, \tag{11}$$

where saturation of the first inequality relies on the use of an appropriate unbiased estimator, the maximum likelihood estimator generally suffices in the asymptotic limit of many trials [89, 90], while the latter inequality relies upon the choice of an appropriate POVM to saturate the inequality [65, 91]. For single-parameter estimation the QCRB is saturable through use of the set of projectors formed by the SLD [65, 91].

GAUSSIAN QUANTUM METROLOGY

Gaussian quantum states are prevalent across mechanical and optical states [92, 93] with wide-ranging applications [94] including metrology [68, 87, 95, 96], and cryptography [97, 98]. Gaussian states describe the state of continuous-variable systems whose Wigner functions are determined fully by their first and second order correlations, which facilitates the use of a feasible analytic framework to perform analysis on such states and channels [92, 93].

A general continuous variable system of k modes is made up of the states $\{|n_1, \cdots, n_k\rangle\}$ where $n_i \in \mathbb{Z}_{\geq 0}$, which describes states of a fixed number of excitations such as photons or phonons. Each mode has its own creation and annihilation operators $\hat{a}_i$ and $\hat{a}_i^\dagger$ with commutation relations

$$[\hat{a}_j, \hat{a}_k] = [\hat{a}_j^\dagger, \hat{a}_k^\dagger] = 0 \qquad [\hat{a}_j, \hat{a}_k^\dagger] = -[\hat{a}_j^\dagger, \hat{a}_k] = i\hbar\delta_{j,k} \tag{12}$$

from which the set of quadrature operators can be defined

$$\hat{q}_j = \sqrt{\frac{\hbar}{2}}(\hat{a}_j + \hat{a}_j^\dagger), \qquad \hat{p}_j = -i\sqrt{\frac{\hbar}{2}}(\hat{a}_j - \hat{a}_j^\dagger). \tag{13}$$

For an infinite-dimensional system the continuous spectra of the $\hat{p}$ and $\hat{x}$ operators allows the state to be defined in terms of continuous functions rather than as infinite-dimensional matrices.

The Wigner function can be defined from the $\hat{p}$ and $\hat{x}$ operators as [93]

$$W_\rho(\vec{q}, \vec{p}) = \frac{1}{\pi^N} \int_{\mathbb{R}^N} \langle \vec{q} + \vec{x} | \rho | \vec{q} - \vec{x} \rangle \, e^{2i\vec{x}\cdot\vec{p}} \mathrm{d}^N \vec{x}, \tag{14}$$

where the Wigner function of a state being Gaussian is a necessary and sufficient condition for the state being Gaussian.

For Gaussian states the Wigner function is fully determined by the displacement vector

$$\vec{d}_j = \langle \hat{R}_j \rangle, \tag{15}$$

and the covariance matrix

$$\gamma_{j,k} = \langle \hat{R}_j \hat{R}_k + \hat{R}_k \hat{R}_j \rangle - 2\langle \hat{R}_j \rangle \langle \hat{R}_k \rangle, \tag{16}$$

where $\hat{R}_{2j-1} = \hat{x}_j$ and $\hat{R}_{2j} = \hat{p}_j$. Gaussian channels—which take Gaussian states to Gaussian states—can thus be described in terms of their action on the displacement vector and covariance matrix alone, providing a simpler and more compact description of their evolution.

Gaussian states afford the use of more direct expressions for the QFI, with fixed expressions for the fidelity between any two Gaussian states being known [99]. For single-mode Gaussian states, simpler forms for the Uhlmann fidelity exist [100–102]

$$\mathcal{F}(\rho_1, \rho_2) = \frac{\exp\left[-\frac{1}{4}\delta^T(\gamma_1 + \gamma_2)^{-1}\delta\right]}{\sqrt{\sqrt{D + L} - \sqrt{L}}}, \tag{17}$$

where

$$D = \det(\gamma_1 + \gamma_2), \tag{18}$$

$$L = 4\det\left(\gamma_A + \frac{i}{2}\Omega\right)\det\left(\gamma_B + \frac{i}{2}\Omega\right), \tag{19}$$

with $\delta = \vec{d}_1 - \vec{d}_2$ is the difference of the displacement vectors of ρ_1 and ρ_2 and Ω is given by

$$\Omega_{j,k} = -\frac{i}{\hbar}[\hat{R}_j, \hat{R}_k]. \tag{20}$$

From Eq. (17) the QFI for a general parameter encoded in a single-mode Gaussian state is thus [95]

$$F_Q(\phi) = \frac{\mathrm{Tr}\left((\gamma^{-1}\partial_\phi \gamma)^2\right)}{2(1+\mu^2)} + \frac{2(\partial_\phi \mu)^2}{1-\mu^4} + \frac{1}{2}(\partial_\phi \vec{d})^T \gamma^{-1} \partial_\phi \vec{d} \tag{21}$$

where $\mu = (2\sqrt{\det(\gamma)})^{-1}$ is the purity of a single-mode state.

THE MASTER EQUATION

The models for CSL and dark matter interactions have all been predicted to cause a localisation or decoherence in the evolution of the form

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar}[\hat{H}_0, \rho] - \Lambda[\hat{x}, [\hat{x}, \rho]]. \tag{22}$$

In order to construct an estimate of Λ we must solve for the evolution of Eq. (22), which is also considered in Riedel [34]. To consider a free particle evolution we focus on $\hat{H}_0 = \hat{p}^2/(2m)$, which could be performed with long free-fall times accessible in space [64]. The density operator in the interaction picture is,

$$\tilde{\rho} = e^{\frac{i}{\hbar}\hat{H}_0 t}\rho e^{-\frac{i}{\hbar}\hat{H}_0 t}. \tag{23}$$

Equation (23) can be trivially inverted,

$$\rho = e^{-\frac{i}{\hbar}\hat{H}_0 t}\tilde{\rho} e^{\frac{i}{\hbar}\hat{H}_0 t}. \tag{24}$$

From Eq. (24) we find,

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar}[\hat{H}_0, \rho] + e^{-\frac{i}{\hbar}\hat{H}_0 t}\frac{\partial \tilde{\rho}}{\partial t} e^{\frac{i}{\hbar}\hat{H}_0 t}. \tag{25}$$

Equation (22) with the help of Eq. (25) takes the form,

$$\frac{\partial \tilde{\rho}}{\partial t} = -\Lambda\left(\hat{\tilde{x}}^2\tilde{\rho} - 2\hat{\tilde{x}}\tilde{\rho}\hat{\tilde{x}} + \tilde{\rho}\hat{\tilde{x}}^2\right), \tag{26}$$

which is the master equation in the interaction picture. By $\hat{\tilde{x}}$ we denote the position operator in interaction picture,

$$\hat{\tilde{x}} = e^{\frac{i}{\hbar}\hat{H}_0 t}\hat{x} e^{-\frac{i}{\hbar}\hat{H}_0 t}. \tag{27}$$

FOKKER-PLANCK EQUATION

Fokker-Planck equation describes the time evolution of the Wigner function and can be derived from the correspondent master equation. The way to do that will be described later, now some algebra is required to bring the master equation (26) in a desired form. Note that, since henceforth everything concerns the interaction picture, we will drop the tilde notation. Using the relation,

$$\hat{x} = \sqrt{\frac{\hbar}{2}}(\hat{a} + \hat{a}^\dagger) \tag{28}$$

Eq. (26) can be written,

$$\frac{\partial \rho}{\partial t} = -\frac{\Lambda\hbar}{2}\left(\hat{a}^2\rho + \hat{a}^{\dagger 2}\rho + \hat{a}\hat{a}^\dagger\rho + \hat{a}^\dagger\hat{a}\rho - 2(\hat{a}\rho\hat{a} + \hat{a}\rho\hat{a}^\dagger + \hat{a}^\dagger\rho\hat{a} + \hat{a}^\dagger\rho\hat{a}^\dagger) + \rho\hat{a}^2 + \rho\hat{a}^{\dagger 2} + \rho\hat{a}\hat{a}^\dagger + \rho\hat{a}^\dagger\hat{a}\right). \tag{29}$$

By defining the following superoperators,

$$\mathcal{L}[\hat{a}]\rho = 2\hat{a}\rho\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\rho - \rho\hat{a}^\dagger\hat{a} \tag{30}$$
$$\mathcal{L}[\hat{a}^\dagger]\rho = 2\hat{a}^\dagger\rho\hat{a} - \hat{a}\hat{a}^\dagger\rho - \rho\hat{a}\hat{a}^\dagger \tag{31}$$
$$\mathcal{D}[\hat{a}]\rho = 2\hat{a}\rho\hat{a} - \hat{a}^2\rho - \rho\hat{a}^2 \tag{32}$$
$$\mathcal{D}[\hat{a}^\dagger]\rho = 2\hat{a}^\dagger\rho\hat{a}^\dagger - (\hat{a}^\dagger)^2\rho - \rho(\hat{a}^\dagger)^2. \tag{33}$$

Equation (29) can be written in a more compact form,

$$\frac{\partial \rho}{\partial t} = -\frac{\Lambda\hbar}{2}\left(\mathcal{L}[\hat{a}] + \mathcal{L}[\hat{a}^\dagger] + \mathcal{D}[\hat{a}] + \mathcal{D}[\hat{a}^\dagger]\right)\rho. \tag{34}$$

It is clear that the master equation (34) is not of the same form as the master equation found in [92] (Eq. (4.1)), however by applying the operator-function correspondence in both sides of Eq. (34)

and with the help of the relations found in Ferraro *et al.* [92] (Eqs. 4.2a-4.3c), we can transform the master equation at hand in a Focker-Plank equation. Here, for the reader's ease, we write the operator-function correspondence,

$$\rho \longrightarrow W(\alpha) \tag{35}$$

$$\hat{a}\rho \longrightarrow (\alpha + \frac{1}{2}\partial_{\alpha^*})W(\alpha) \tag{36}$$

$$\rho\hat{a} \longrightarrow (\alpha - \frac{1}{2}\partial_{\alpha^*})W(\alpha) \tag{37}$$

$$\hat{a}^\dagger\rho \longrightarrow (\alpha^* - \frac{1}{2}\partial_{\alpha})W(\alpha) \tag{38}$$

$$\rho\hat{a}^\dagger \longrightarrow (\alpha^* + \frac{1}{2}\partial_{\alpha})W(\alpha). \tag{39}$$

Where $W(\alpha)$ is the time dependent Wigner function. One can write,

$$\mathcal{L}[\hat{a}]\rho \longrightarrow \left(\partial_\alpha\alpha + \partial_{\alpha^*}\alpha^* + \partial^2_{\alpha\alpha^*}\right)W(\alpha) \tag{40}$$

$$\mathcal{L}[\hat{a}^\dagger]\rho \longrightarrow -\left(\partial_\alpha\alpha + \partial_{\alpha^*}\alpha^* - \partial^2_{\alpha\alpha^*}\right)W(\alpha) \tag{41}$$

$$\mathcal{D}[\hat{a}]\rho \longrightarrow -\partial^2_{\alpha^*\alpha^*}W(\alpha) \tag{42}$$

$$\mathcal{D}[\hat{a}^\dagger]\rho \longrightarrow -\partial^2_{\alpha\alpha}W(\alpha). \tag{43}$$

Applying Eqs. (40)-(43) to Eq. (34) we get,

$$\frac{\partial W(\alpha)}{\partial t} = \frac{\Lambda\hbar}{2}\left(2\partial^2_{\alpha\alpha^*} - \partial^2_{\alpha\alpha} - \partial^2_{\alpha^*\alpha^*}\right)W(\alpha). \tag{44}$$

We pass to Cartesian coordinates using the relations,

$$\alpha = \frac{1}{\sqrt{2\hbar}}(x + ip) \tag{45}$$

$$\alpha^* = \frac{1}{\sqrt{2\hbar}}(x - ip) \tag{46}$$

$$\partial_\alpha = \sqrt{\frac{\hbar}{2}}(\partial_x - i\partial_p) \tag{47}$$

$$\partial_{\alpha^*} = \sqrt{\frac{\hbar}{2}}(\partial_x + i\partial_p) \tag{48}$$

and Eq. (44) can be written as,

$$\frac{\partial W(x,p;t)}{\partial t} = \hbar^2\Lambda\partial^2_{pp}W(x,p;t). \tag{49}$$

Equation (49) is the heat equation and we need to solve it with the boundary conditions that $W(x,p;t)$ tends to zero at $x,p \to \pm\infty$. The solution to Eq. (49) is given by integrating over the the variables of the partial differential (p in this instance) the initial Wigner function $W_0(x,p)$ with the heat kernel,

$$W(x,p;t) = \frac{1}{\sqrt{4\pi\Lambda t}}\int dq W_0(x,q)\exp\left[-\frac{(p-q)^2}{4\Lambda t}\right]. \tag{50}$$

Since the heat kernel is a Gaussian function, then for any initial Gaussian state, i.e., a state with Gaussian Wigner function, Eq. (49) can yield only Gaussian Wigner function. Therefore the channel described by Eq. (26) is a Gaussian channel. From now on we fix $\hbar = 1$. A Gaussian Wigner function has the form,

$$W(x,p) = \frac{1}{2\pi\sqrt{\det\gamma}}\exp\left[-\frac{1}{2}(r-d)^T\gamma^{-1}(r-d)\right] \tag{51}$$

where the r is phase space vector $r = (x, p)$, d is the displacement vector, and γ is the covariance matrix which is positive-definite and symmetric.

A single-mode Gaussian state is a squeezed and displaced thermal state $\hat{\rho}_{th}$, i.e,

$$\hat{\rho}_G = D(\alpha)S(\xi)\hat{\rho}_{th}S(\xi)^\dagger D(\alpha)^\dagger,$$

where α is the displacement and $\xi = |\xi|e^{i\phi}$ is the squeezing parameter.

Alternatively, any Gaussian state can be written as

$$\hat{\rho}_G = D(\alpha)R(\phi/2)S(|\xi|)\hat{\rho}_{\rm th}S(|\xi|)^\dagger R(\phi/2)^\dagger D(\alpha)^\dagger \tag{52}$$

that is, squeezing with positive squeezing $|\xi|$, then applying a $R(\phi/2)$ rotation, and then displacing. In this work, we consider as $\hat{\rho}_{\rm th}$ a harmonic oscillator with mass m and frequency ω in the canonical ensemble, i.e., in a thermal bath with inverse temperature $\beta = (k_B T)^{-1}$, where k_B is Boltzmann's constant. The state $\hat{\rho}_{\rm th}$ can be expressed in the occupation basis (Fock basis) as,

$$\hat{\rho}_{\rm th} = \sum_n p_n \left|n\right\rangle \left\langle n\right| . \tag{53}$$

Where p_n is the probability that the system occupies the n-th level of the harmonic oscillator,

$$p_n = (1-s)s^n \tag{54}$$

$$s = \frac{N}{N+1} \tag{55}$$

where N is the mean thermal value of the occupation number,

$$N = \frac{1}{\exp\left(\beta\hbar\omega\right) - 1}. \tag{56}$$

The state $\hat{\rho}_{\rm th}$ in the Wigner representation is

$$W_{\rm th}(x,p) = \sum_n p_n W_n(x,p) \tag{57}$$

where $W_n(x, p)$ is the Wigner representation of the state $|n\rangle\langle n|$,

$$W_n(x,p) = \frac{1}{2\pi\hbar}\exp\left[-\frac{2}{\hbar\omega}\left(\frac{p^2}{2m} + \frac{1}{2}m\omega^2x^2\right)\right] L_n\left[\frac{4}{\hbar\omega}\left(\frac{p^2}{2m} + \frac{1}{2}m\omega^2x^2\right)\right], \tag{58}$$

where L_n is n-th order Laguerre polynomial. It can be shown,

$$W_{\rm th}(x,p) = \frac{1}{2\pi\sqrt{\det\gamma_{\rm th}}}\exp\left[-\frac{1}{2}r^T\gamma_{\rm th}^{-1}r\right]. \tag{59}$$

The initial Wigner function will be the single-mode, Gaussian Wigner function $W_0(x, p)$ of the form (51), derived by squeezing, rotating, and displacing the Wigner function $W_{\rm th}(x, p)$ as Eq. (52) describes but in the phase space, by transforming the first and second moments with the corresponding symplectic transformations

The covariance matrix of $W_0(x, p)$ is,

$$\gamma_0 = \begin{pmatrix}\sigma_{11} & \sigma_{12}\\ \sigma_{12} & \sigma_{22}\end{pmatrix} \tag{60}$$

where

$$\sigma_{11} = \frac{2N+1}{2}\left(m\omega e^{-2|\xi|}\sin^2\frac{\phi}{2} + \frac{e^{2|\xi|}}{m\omega}\cos^2\frac{\phi}{2}\right) \tag{61}$$

$$\sigma_{12} = \frac{2N+1}{2}\frac{1}{2}\left(m\omega e^{-2|\xi|} - \frac{e^{2|\xi|}}{m\omega}\right)\sin\phi \tag{62}$$

$$\sigma_{22} = \frac{2N+1}{2}\left(m\omega e^{-2|\xi|}\cos^2\frac{\phi}{2} + \frac{e^{2|\xi|}}{m\omega}\sin^2\frac{\phi}{2}\right). \tag{63}$$

Note that for $|\xi| = 0$, $\phi = 0$, and $N = 0$, the initial state is in general a squeezed state because of the presence of mass and frequency, i.e., the ground state of the harmonic oscillator is a squeezed state. If we allow $N > 0$, then it is a thermal squeezed state and for $|\xi| > 0$ we account for some external squeezing that can be seen as contributing to an effective mass or frequency.

The evolved covariance matrix, i.e., the covariance matrix of $W(x, p; t)$ with the help of (50) is,

$$\gamma_\Lambda = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} + 2\Lambda t \end{pmatrix} \tag{64}$$

while the displacement vector remains the same. The evolved state is a Gaussian state whose position variance remains unchanged while the momentum variance grows linearly with time.

QUANTUM FISHER INFORMATION FOR THE RATE OF DECOHERENCE

An important remark is that the Uhlmann fidelity is invariant under unitary transformation, i.e., let U a unitary transformations [103], i.e., $\mathcal{F}(\rho_\Lambda, \rho_{\Lambda+d\Lambda}) = \mathcal{F}(U\rho_\Lambda U^\dagger, U\rho_{\Lambda+d\Lambda}U^\dagger)$. Therefore the quantum Fisher information will remain the same under unitaries, which do not depend on the estimated parameter, on the final state (see also Zhong *et al.* [104]). Since the interaction and the Schrödinger pictures are connected with a unitary transformation, i.e., the unitary with generator the free Hamiltonian of the system, one does not have to go back to the Schrödinger picture to calculate the quantum Fisher information. Remaining in the interaction picture suffices for that task.

From Eq. (21) we find the QFI to be,

$$F_Q = \frac{32\sigma_{11}^2 t^2}{16\left(2\Lambda t\sigma_{11} + \sigma_{11}\sigma_{22} - \sigma_{12}^2\right)^2 - 1}. \tag{65}$$

Equation (65) can be written in the more compact form,

$$F_Q = \frac{32\sigma_{11}^2 t^2}{16\det(\gamma_\Lambda)^2 - 1} \tag{66}$$

or in terms of the purity μ, as

$$F_Q = \frac{32\sigma_{11}^2 t^2 \mu(t)^4}{1 - \mu(t)^4} \tag{67}$$

where $\mu(t) = \left(2\sqrt{\det(\gamma_\Lambda(t))}\right)^{-1}$ is the time-dependent purity of the final state and $\sigma_{11}, \sigma_{12},$ and σ_{22} are given in Eqs. (61), (62), and (63).

By maximising the QFI over ϕ we find that if $m\omega < e^{2|\xi|}$ then the optimal angle $\phi = 0$, while if $m\omega > e^{2|\xi|}$ the optimal angle is $\phi = \pi$. Therefore, given if the quantity $m\omega e^{-2|\xi|}$ is greater or less than 1, one can apply the appropriate rotation. This means that the optimal initial covariance matrix always corresponds to a potentially thermal squeezed in the momentum quadrature state, i.e., in the direction that the decoherence effect manifests itself. The QFI for this case is

$$F_Q = \frac{8t^2\chi^2}{4(2t\Lambda\chi+\nu)^2 - \frac{1}{4\nu^2}} \tag{68}$$

where $\nu = (2N+1)/2$ and $\chi = e^{2|\xi|}/(m\omega)$ (for $\phi = 0$) or $\chi = (m\omega)/e^{2|\xi|}$ (for $\phi = \pi$) depending which one gives $\chi > 1$. The quantity χ is dimensionless. This is because χ is the ratio of the position variance σ_{11}^{opt} of the optimal input state and the position variance $\sigma_{11}^{\text{tpi}} = (2N+1)/2$ of a phase-invariant thermal state, with the same N as the optimal input state,

$$\chi = \frac{\frac{e^{2|\xi|}(2N+1)}{2m\omega}}{\frac{2N+1}{2}} = \frac{e^{2|\xi|}}{m\omega} \tag{69}$$

The denominator $2N+1$ in Eq. (69) is the numerator for $\xi = 0$ and mass m numerically equal to $1/\omega$. Therefore both numerator and denominator in the definition of χ, that is after the first equality sign in Eq. (69), being variances of the same quadrature, they have units of length squared.

For $\chi > 1$ there is squeezing in the variance where the χ appears. For $\chi = 1$ the dependence of the QFI on ϕ disappears, as this case corresponds to a phase-invariant thermal state, and the corresponding QFI is sub-optimal.

The QFI increases with χ, achieving a maximum of $F_Q = (2\Lambda)^{-2}$ for $\chi \to \infty$. Note that the high mass limit corresponds to the case of $\phi = \pi$, so that χ has the mass in the numerator, otherwise the limit is meaningless as $\chi > 1$ always. Note that for $t \to \infty$ the dominant term in the covariance matrix will be $2\Lambda t$. Therefore, in this limit, the state will depend only on Λ and it is expected that the QFI, which measures the distance between two states displaced in the parametric space spanned by Λ, will reach a value independent of the parameters of the initial state, and it may depend only on Λ. Indeed we find, $\lim_{t\to\infty} F_Q = (2\Lambda)^{-2}$ which is maximum value of the QFI.

CLASSICAL FISHER INFORMATION AND MEASUREMENT

A question that arises in any estimation problem is which measurement saturates the bound obtained from the theoretic, quantum mechanical, approach. In other words, the question would be which measurement corresponds to CFI which equals the QFI. Moreover, even if one obtains the POVMs which saturate the QFI, then this POVM may be highly impractical in the sense that it would be extremely difficult to actually implement it in the laboratory. In order to keep things in a practical level, we find the CFI which corresponds to homodyne detection on the position quadrature, homodyne detection on the momentum quadrature, heterodyne detection. We will compare the CFI of each aforementioned measurement and we will draw a conclusion on their performance.

Since the measurement will take place in the standing frame, i.e. not in the interaction picture, we have to find the Wigner function of the evolved state in the Schrödinger picture. To this end, we have to find how the first and second moments are transformed between the two pictures. In this point, we will restore the different notation for operators in the interaction picture (IP) and Schrödinger picture (SP), i.e., operators in the IP will bear a tilde. For the problem at hand and for $\hbar = 1$, the position operator in the IP is,

$$\tilde{\hat{x}} = e^{\frac{it}{2m}\hat{p}^2}\hat{x}e^{-\frac{it}{2m}\hat{p}^2}. \tag{70}$$

We applying the commutation relation $[\hat{x},\hat{p}] = i$ k-times we obtain,

$$[\hat{p}^{2k},\hat{x}] = -2ik\hat{p}^{2k-1}. \tag{71}$$

With the aid of Eq. (71) and series expansion we obtain,

$$[e^{\frac{it}{2m}\hat{p}^2}, \hat{x}] = \frac{t}{m} e^{\frac{it}{2m}\hat{p}^2} \hat{p}. \tag{72}$$

Form Eqs. (70) and (72) we derive,

$$\hat{\tilde{x}} = \hat{x} + \frac{t}{m}\hat{p} \tag{73}$$

Since the free Hamiltonian is $\hat{p}^2/(2m)$, it will not affect the momentum operator in the IP,

$$\hat{\tilde{p}} = \hat{p}. \tag{74}$$

Using Eqs. (73) and (74), we can calculate the first and second order moments in the IP relate to those of the SP. The relation between the IP and the SP, for the covariance matrix and the first moments vector respectively is found to be,

$$d = S\mu \tag{75}$$

$$\gamma_\Lambda = S\Gamma_\Lambda S^T \tag{76}$$

where (d, γ_Λ) are the first moments and covariance matrix respectively in the IP while (μ, Γ_Λ) are the first moments and covariance matrix respectively in the SP. The transformation S which connects the two pictures is,

$$S = \begin{pmatrix} 1 & \frac{t}{m} \\ 0 & 1 \end{pmatrix} \tag{77}$$

and is symplectic as it satisfies $S\Omega S^T = \Omega$ where Ω was defined in Eq. (20).

The Wigner function of the evolved state in the SP, is

$$W(r;t) = \frac{1}{2\pi\sqrt{\det \Gamma_\Lambda}} \exp\left[-\frac{1}{2}(r-\mu)^T \Gamma_\Lambda^{-1} (r-\mu)\right] \tag{78}$$

where the dependence on time is in the first and second order moments, while the dependence on the parameter Λ is only in the second order moments.

Homodyne detection

To find the CFI that corresponds to a measurement in a quadrature i, one must find the probability $P(i|\Lambda)$ by integrating the Wigner function (78) over the perpendicular to i quadrature [93] and then apply Eq. (5). In this way we find,

$$F_p = \frac{2t^2}{(2t\Lambda + \sigma_{22})^2}, \tag{79}$$

$$F_x = \frac{2t^6}{(2t^3\Lambda + t^2\sigma_{22} - 2mt\sigma_{12} + m^2\sigma_{11})^2} \tag{80}$$

where the index $i = x, p$ in the CFI F_i, refers to homodyne measurement in the i quadrature.

We note that the CFI, F_p, is the same no matter if we compute it in IP or SP. On the other hand, F_x is not the same in the two pictures. This is due to the fact that the momentum operator remains the same in the two pictures while the position operators changes.

For $\phi = 0$ and $\chi > 1$, i.e., for the class of optimal input states, it can be shown that,

$$\frac{F_p}{F_x} = \left(1 + \frac{m^2\chi^2\nu}{(2t\Lambda\chi + \nu)t^2}\right)^2 > 1 \tag{81}$$

meaning that homodyne measurement on momentum is better than homodyne measurement on position.

For the same class of input states we also have,

$$\frac{F_p}{F_Q} = 1 - \frac{1}{\left(4\nu(2\Lambda t\chi + \nu)\right)^2} < 1 \tag{82}$$

meaning that homodyne detection on momentum does not saturate the quantum limit, but it is close to it for large χ, large time scales, large decoherence rate, or in the limit of high background temperature.

Heterodyne detection

The probability distribution in Eq. (5) that is obtained in an ideal heterodyne measurement is the $Q(x,p)$ representation of the state. The $Q(x,p)$ representation of a state $\hat{\rho}$ is defined as $Q(x,p) = \pi^{-1}\langle\alpha|\hat{\rho}|\alpha\rangle$, where $|\alpha\rangle$ is a coherent state with amplitude $\alpha = (x+ip)/\sqrt{2}$. From its definition, it is understood why the $Q(x,p)$ representation gives the outcome probability of heterodyne measurements: it is the projection of the state on the coherent state basis, i.e., on position and momentum simultaneously.

Knowing the Wigner function of a Gaussian state, one can find the $Q(x,p)$ representation. The $Q(x,p)$ will have the same form as the Gaussian Wigner function but it is required to add $I/2$ to the covariance matrix, where I is the identity matrix of the same dimension as the covariance matrix. This identity matrix accounts for noise introduced by measuring position and momentum simultaneously. The $Q(x,p)$ representation of the state (78) is,

$$Q(r;t) = \frac{1}{\mathcal{K}}\exp\left(-\frac{1}{2}(r-\mu)^T A_\Lambda (r-\mu)\right) \tag{83}$$

where,

$$A_\Lambda = \left(\Gamma_\Lambda + \frac{1}{2}I\right)^{-1} \tag{84}$$

and ν is the normalisation,

$$\mathcal{K} = \frac{2\pi}{\sqrt{\det A}}. \tag{85}$$

The CFI for heterodyne measurement is

$$\begin{aligned} F_{xp} &= \frac{1}{\mathcal{K}^2}\left(\frac{\partial\mathcal{K}}{\partial\Lambda}\right)^2 + \frac{1}{4}\left\langle\left(r^T\frac{\partial A}{\partial\Lambda}r\right)^2\right\rangle_Q + \frac{1}{\mathcal{K}}\frac{\partial\mathcal{K}}{\partial\Lambda}\left\langle r^T\frac{\partial A}{\partial\Lambda}r\right\rangle_Q \\ &= \frac{8\left[(2\sigma_{11}+1)m^2t^3+t\right]^2}{D_{xp}^2} \end{aligned} \tag{86}$$

where

$$\langle f(x,p)\rangle_Q = \int \mathrm{d}x\mathrm{d}p\, Q(x,p)f(x,p) \tag{87}$$

and

$$D_{xp} = 4\Lambda t^3 + 2\sigma_{22}t^2 + 4\left[(2\sigma_{11}+1)\Lambda m^2 - \sigma_{12}m\right]t + \left(4\sigma_{11}\sigma_{22} + 2\sigma_{11} + 2\sigma_{22} - 4\sigma_{12}^2 + 1\right)m^2. \tag{88}$$

For the class of optimal input states, i.e., $\phi = 0$ and $\chi > 1$, the following inequality is valid,

$$F_p > F_{xp} > F_x. \tag{89}$$

The comparison between all CFIs and QFI for $\phi = \pi$ and $\chi = (m\omega)/e^{2|\xi|}$ remains the same.

CONCLUSIONS

We have given analytical expressions for the QFI of the decoherence parameter Λ and for the CFIs corresponding to homodyne detection on position and momentum, and heterodyne detection. From the QFI we see that the sensitivity scales quadratically with time, clearly demonstrating how long free-fall times will be of benefit. The covariance of the position operator is significant, with an improved sensitivity for a reduced variance in the initial position; for pure states this is simply the squeezing of the position degree of freedom. The QFI similarly favours high purity states, with the $\mu \to 0$ limit yielding no sensitivity; as the state is already maximally decohered, any further decoherence will have no effect on the system.

The CFIs for homodyne along the position and momentum quadratures were given. The measurement of either quadrature is found to be sub-optimal, but measurement of momentum being relatively superior. The parameter can be recognised as appearing first in the momentum quadrature in Eq. (64), and only later in the position quadrature through the action of the local Hamiltonian multiplied with a higher power of t as can be recognised from Eq. (73) and Eq. (64). A known artefact (and suggested flaw in CSL theories) of the quantum Brownian motion is a heating effect and superior or optimal measurements may look more like thermometry. In the same way that the optimal measurement in light-interferometric gravitational wave detectors is neither projection onto the quadrature with maximal signal or squeezing [105, 106] an alternative quadrature may yet be optimal, although measuring a superposition of position and momentum operators is more demanding in mechanical systems than optical.

We have derived bounds to the precision when estimating a dephasing or localization rate, which covers a general class of evolutions including (gravitational) collapse models of quantum mechanics and interactions with massive, otherwise dark, particles. Dark matter wind could produce an imaginary component of the decoherence rate (a unitary phase) [38] which could also be of interest for measurement and could be included in an extension of this work to a multi-parameter analysis. An advanced analysis which took into account expected space-based environmental noise may also prove beneficial to understand the realistic nature of detecting such decoherence rates and ensure any observed decoherence can be accurately attributed to new physics.

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