A Reliable Method for Verifying Structural Optimality of Circle Packing Configurations in the Unit Square

Mihály Csaba Markót[1] and Tibor Csendes[2]
[1] European Space Agency, ESTEC, Advanced Concepts Team
Keplerlaan 1, 2201 AZ Noordwijk
The Netherlands
[2] Institute of Informatics, University of Szeged
H-6720 Szeged, Árpád tér 2.
Hungary
Mihaly.Csaba.Markot@esa.int
csendes@inf.u-szeged.hu

Abstract

The talk deals with the problem of packing non-overlapping equal circles with maximal diameter into a unit square. In a recent paper [2] we presented a reliable, interval-based optimization procedure and solved the earlier open problems of finding all the optimal packings of $n = 28, 29,$ and 30 circles. More precisely, we stated that – apart from symmetric cases – all the optimal solutions of the particular problem instances are located in a small $X_n^* \in \mathbf{I}^{2n}$ result box. However, from the geometric point of view this kind of a numerical solution is not complete. To fill this gap, the present talk introduces an interval method verifying also *structural optimality properties* of packing configurations. The structure of a packing configuration determines which circles touch the side of the square and which pairs of circles touch each other (in the form of a system of equations). Our method consists of two main parts: first we prove that the system of equations describing the structure of the conjectured best packing configuration [1,3] has exactly one solution in X_n^*, $n = 28, 29, 30,$ and then we show that this solution corresponds to the only optimal packing in the result box.

References:

[**1**] R. L. Graham and B. D. Lubachevsky; *Repeated patterns of dense packings of equal disks in a square*, Electronic Journal of Combinatorics 3:R16 (1996).

[**2**] M. Cs. Markót and T. Csendes; *A New Verified Optimization Technique for*

the "Packing Circles in a Unit Square" Problems, conditionally accepted for publication in SIAM J. Optimization.

[**3**] K. J. Nurmela and P. R. J. Östergård; *Packing up to 50 circles in a square*, Discrete and Computational Geometry 18 (1997), pp. 111–120.

Keywords: circle packing, structural optimality, computer-aided proof