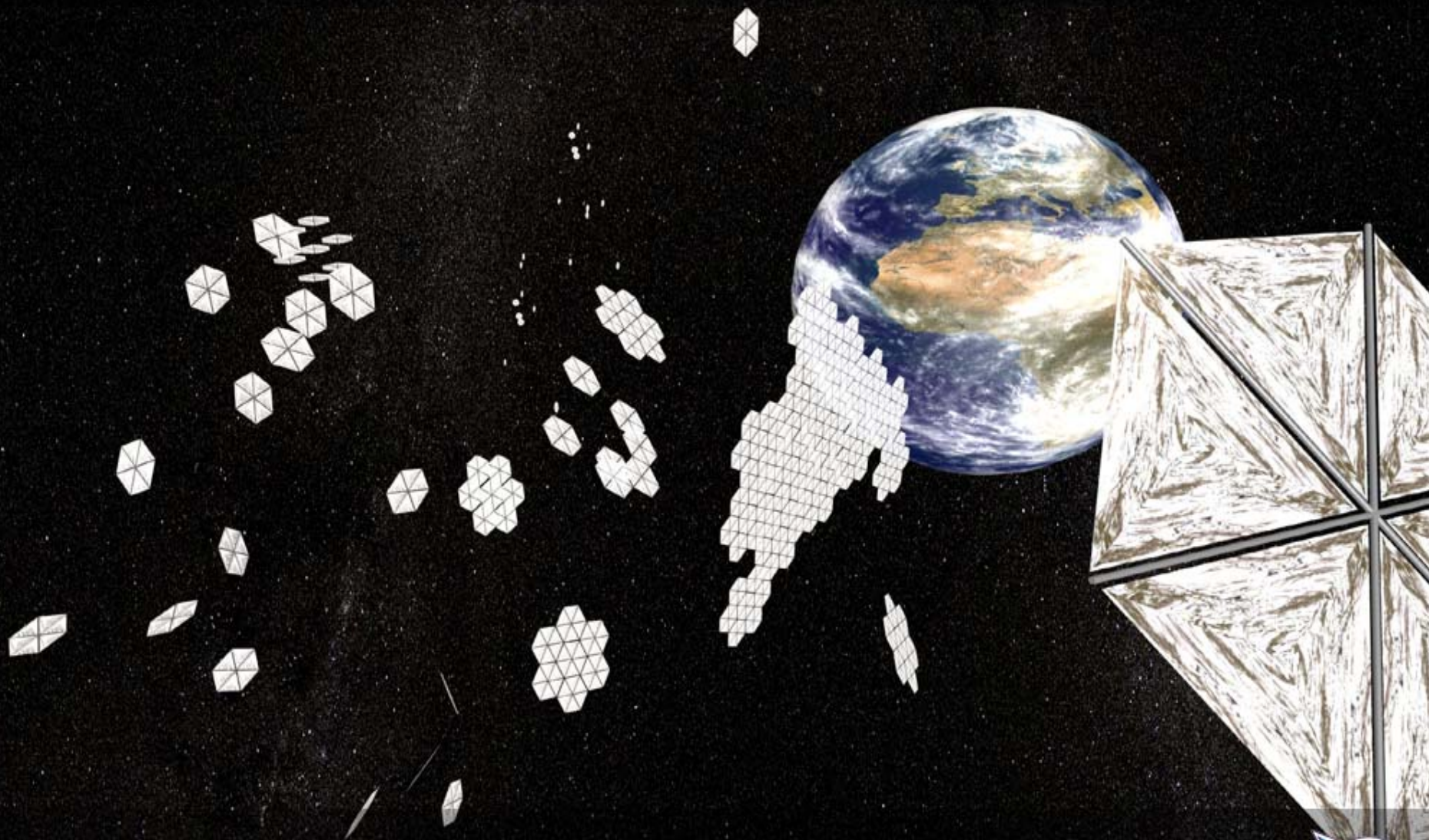


Zarm University of Bremen Advanced Concepts Team European Space Agency
Equilibrium Shaping: navigating swarm using behavioural approaches
Dario Izzo Lorenzo Pettazzi



WorkShop on INNOVATIVE System Concepts - 2006



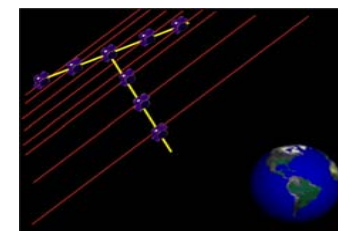
Objective of the work and Applications:

Can useful tasks be accomplished by a homogeneous team of spacecraft without direct communications and using decentralized control?

- Formation Flying



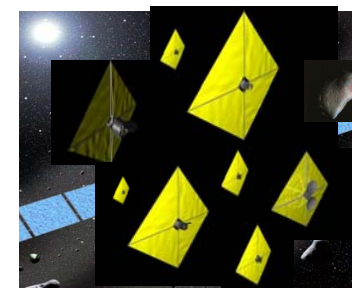
Autonomous acquisition and reconfiguration



- Exploration



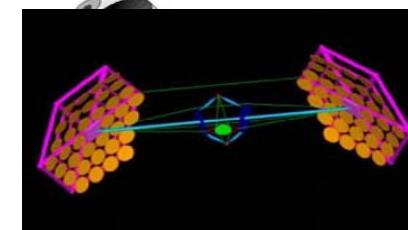
NASA APOLLO mission



- Self-assembly array in space



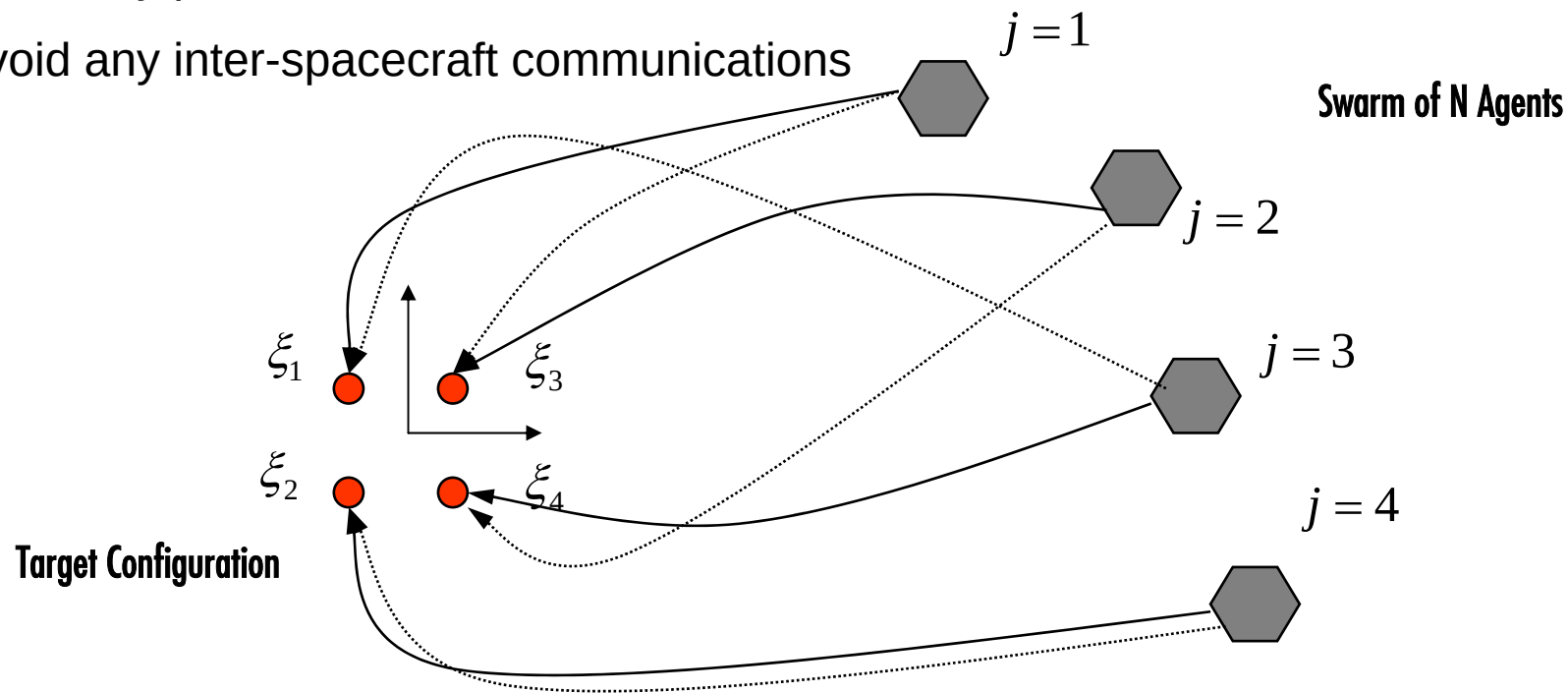
Solar Power Satellite



Problem Definition:

Each satellite belonging to the swarm has to:

- Autonomously decide which position it will have in the final desired formation
- Avoid any possible collision
- Avoid any inter-spacecraft communications



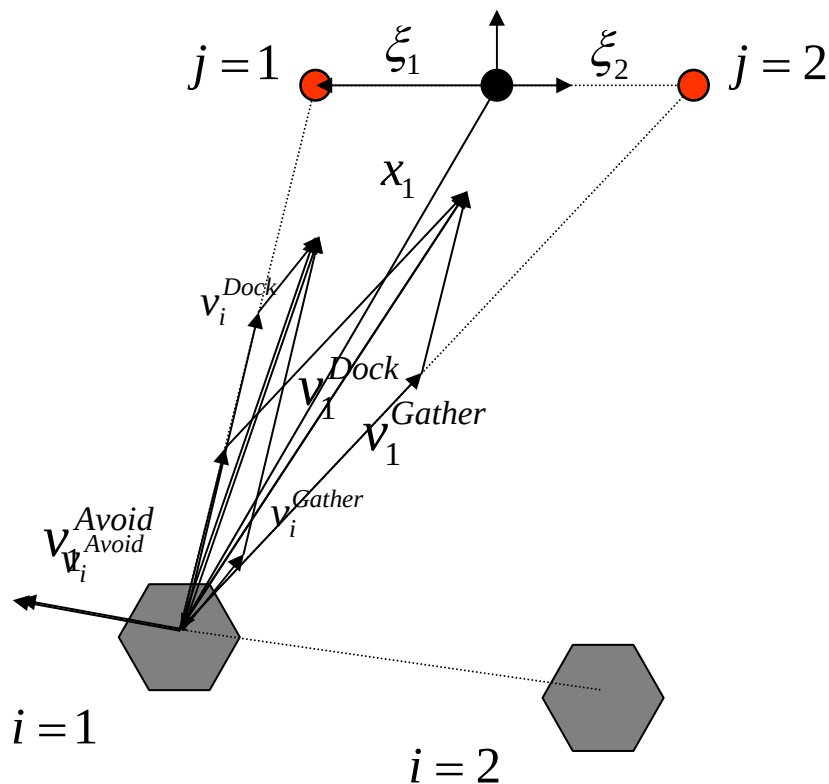
Solution Approach:

Two step approach based on **behaviour based path planning** with behaviours acting on **different length scales**:

- First a method has been developed that defines for each target disposition and each agent neighbourhood configuration a desired velocity vector as a sum of different weighted contributions.
- Different control techniques have been considered that allow each S/C to track the desired kinematical field.

Behaviour Based Approach:

Drawing the inspiration from collective robotics techniques we assign to each satellite a desired velocity as weighted sum of short/long range contributions.



$$v_i^{Gather} = \sum_j c_j \psi_G(\|\xi_j - x_i\|)(\xi_j - x_i)$$

$$v_i^{Dock} = \sum_j d_j \psi_D(\|\xi_j - x_i\|, k_D)(\xi_j - x_i)$$

$$v_i^{Avoid} = \sum_j b \psi_A(\|x_j - x_i\|, k_A)(x_j - x_i)$$

$$v_{d_i} = \dot{x}_i = v_i^{Gather} + v_i^{Dock} + v_i^{Avoid}$$

Equilibrium Shaping (1/2):

We enforce that when the agents have achieved the final desired relative configuration (or any possible permutations of its points) the desired velocity of each agent vanishes.

This procedure “shapes” the equilibrium of the dynamical system.

$$v_{d_i} = \dot{x}_i = v_i^{Gather} + v_i^{Dock} + v_i^{Avoid} = 0$$

$$i = 1 \dots N$$

$$\sum_{j=1}^N [b \psi_A(\|\xi_i - \xi_j\|, k_A) - c_j \psi_G(\|\xi_i - \xi_j\|) - d_j \psi_D(\|\xi_i - \xi_j\|, k_D)] (\xi_i - \xi_j) = 0$$



$$A[c_1, \dots, c_N, d_1, \dots, d_N] = g$$

That is a linear system of 3N equations in 2N unknowns

Equilibrium Shaping (2/2):



Video clip

$$v_{d_i} = 0, i = 1, \dots, N$$

3N equations for 2N unknowns

$$v_{d_i} = 0, v_{d_j} = 0$$

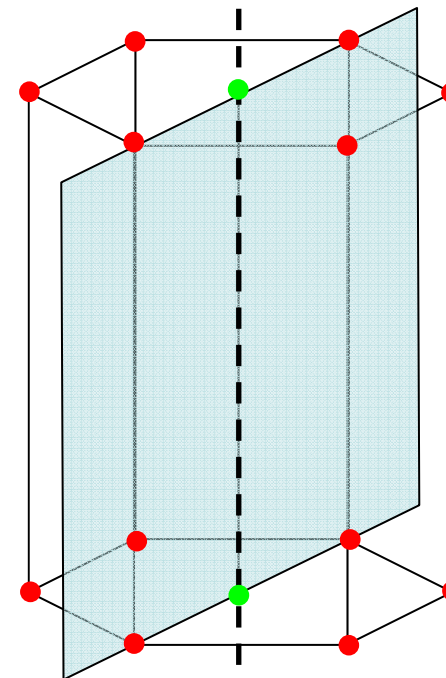
are linearly dependent

$$v_{d_i} = 0$$

Reduces to two independent scalar equations

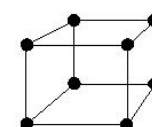
$$v_{d_i} = 0$$

Reduces to one independent scalar equation

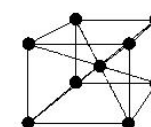


Formations Achievable:

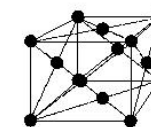
Formation shape	Number of equations	Number of unknowns
Regular solids	1	2
Regular polygons	1	2
Pyramids with a regular basis	3	4
Cubic-P	1	2
Cubic-I	1	4
Cubic-F	2	4
Tetragonal-P	2	2
Tetragonal-I	2	4
Orthorhombic-P	3	2
Orthorhombic-I	3	4
Orthorhombic-C	4	4
Orthorhombic-F	6	4
Hexagonal-P	3	4



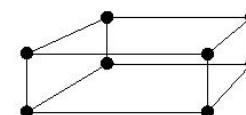
Cubic-P



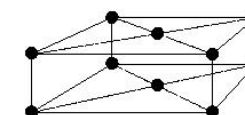
Cubic-I



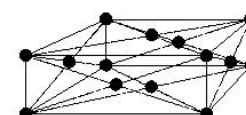
Cubic-F



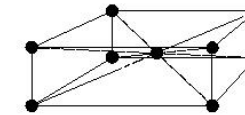
Orthorhombic-P



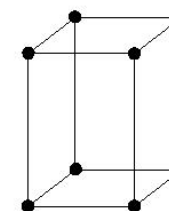
Orthorhombic-C



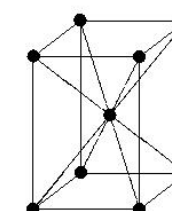
Orthorhombic-F



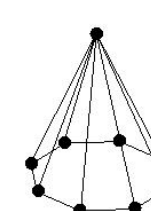
Orthorhombic-I



Tetragonal-P



Tetragonal-I



Pyramid with octagonal basis

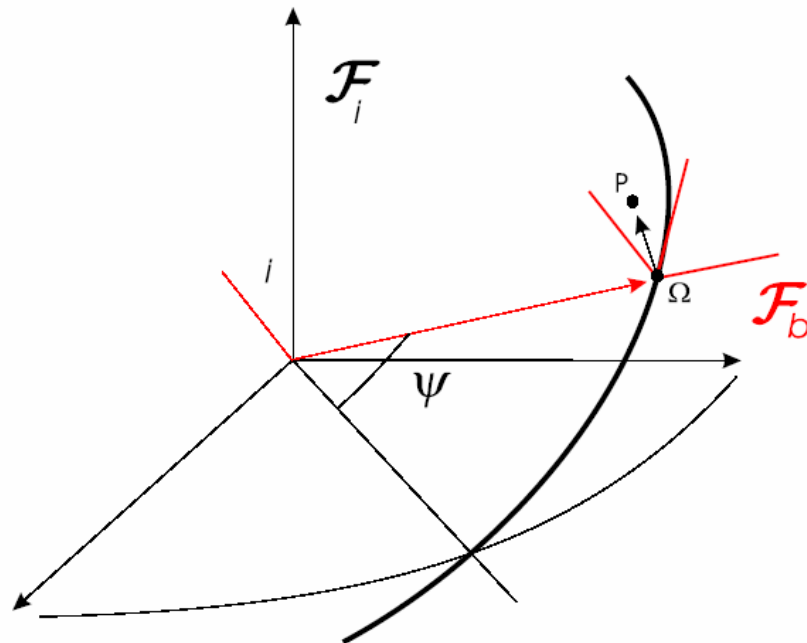
A realistic environment: the gravitational field: (1/2)

The Hill equations

It is the linearized model describing the relative dynamics between satellites.

$$\begin{cases} \ddot{x} - 2n\dot{y} - 3n^2x = 0 \\ \ddot{y} + 2n\dot{x} = 0 \\ \ddot{z} + n^2z = 0 \end{cases}$$

- x,y and z: position of P with respect of Ω
- n: orbital mean motion



A realistic environment: the gravitational field: (2/2)

The analytical solution for Hill equations

$$\begin{bmatrix} \rho \\ \dot{\rho} \end{bmatrix} = \begin{bmatrix} A(\tau) & B(\tau) \\ C(\tau) & D(\tau) \end{bmatrix} \begin{bmatrix} \rho_0 \\ \dot{\rho}_0 \end{bmatrix}$$

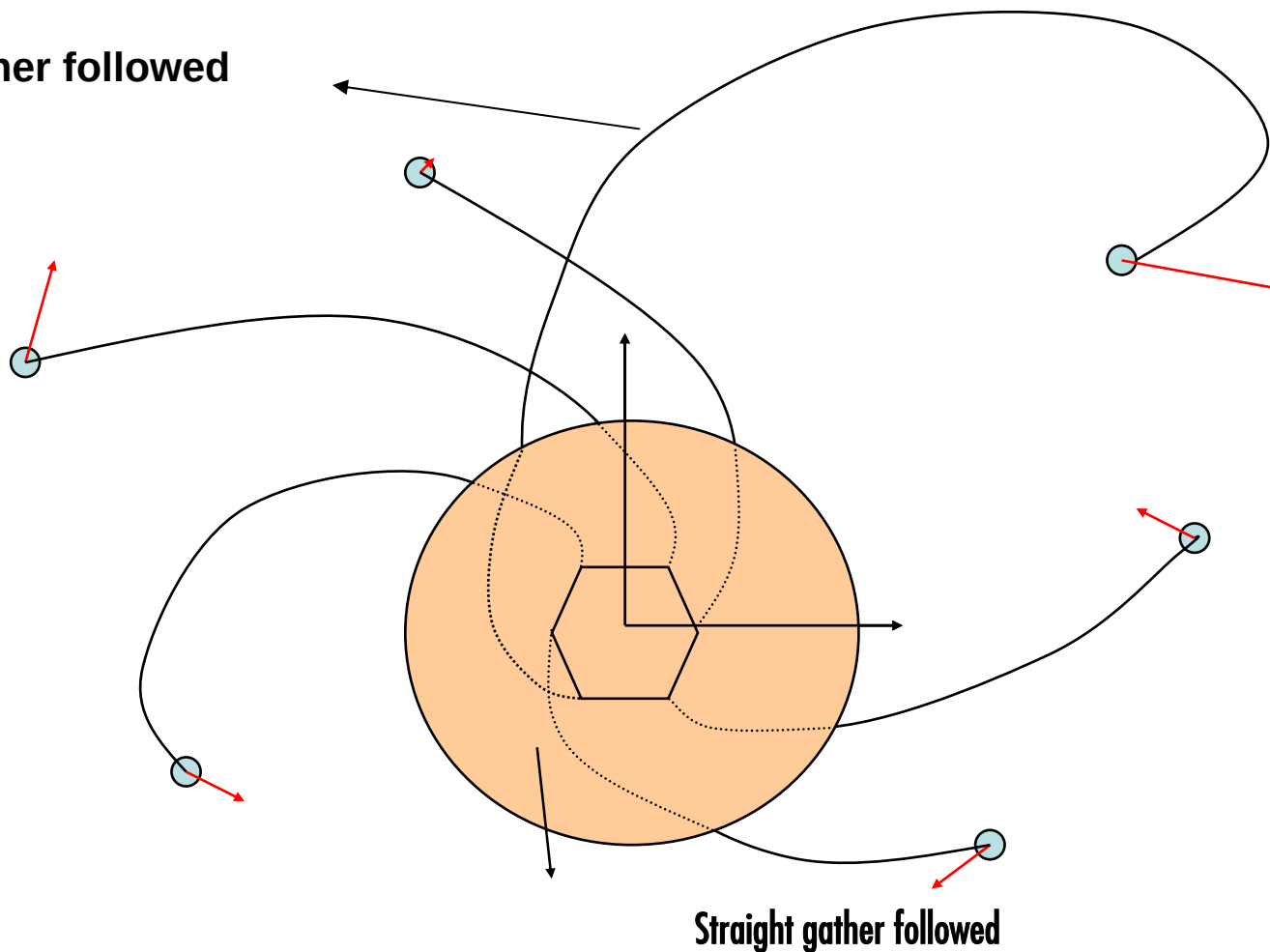
Each spacecraft has to follow a geodetic trajectory that will bring it towards a desired position ρ_d of the target formation in a desired time τ_d .

$$\rho_d = \rho(\tau_d - t) = A(\tau_d - t)\rho(t) + B(\tau_d - t)\dot{\rho}(t)$$

$$v_i^{Gather} = \dot{\rho}(\tau) = \frac{1}{N} B^{-1} \left(\sum_j \xi_j + A(\tau_d - t)x_i \right)$$

Dividing the Kinematic field:

Geodetic gather followed



Control Strategy Implementation (1/2):

Control design techniques applied:

VTBG: Velocity To Be Gained.

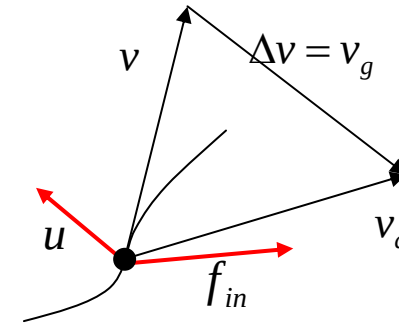
CPSL: Cross Product Steering Law.

SMC: Sliding Mode Control.

APF: Artificial Potential Function.

Control Strategy Implementation:

$$\left\{ \begin{array}{l} v \\ v_d \\ v_g = v_d - v \\ \dot{v} = f_{in} + u \end{array} \right. \quad \begin{array}{l} \text{Actual velocity} \\ \text{Desired velocity} \\ \text{Velocity-to-be-gained} \\ \text{Agent equation of motion} \end{array}$$



$$\kappa > 0 \in \text{Re}$$

VTBG+CPSL

$$\longrightarrow u = \kappa v_g - \dot{v}_d - f_{in}$$

SMC

$$\longrightarrow u = u_0 \text{sign}(v_g) - \dot{v}_d - f_{in}$$

APF

$$\longrightarrow u = \kappa v_g + (1 - \kappa) v_d - f_{in}$$

Introducing a saturation level for the thrust u_{sat} the real control law implemented

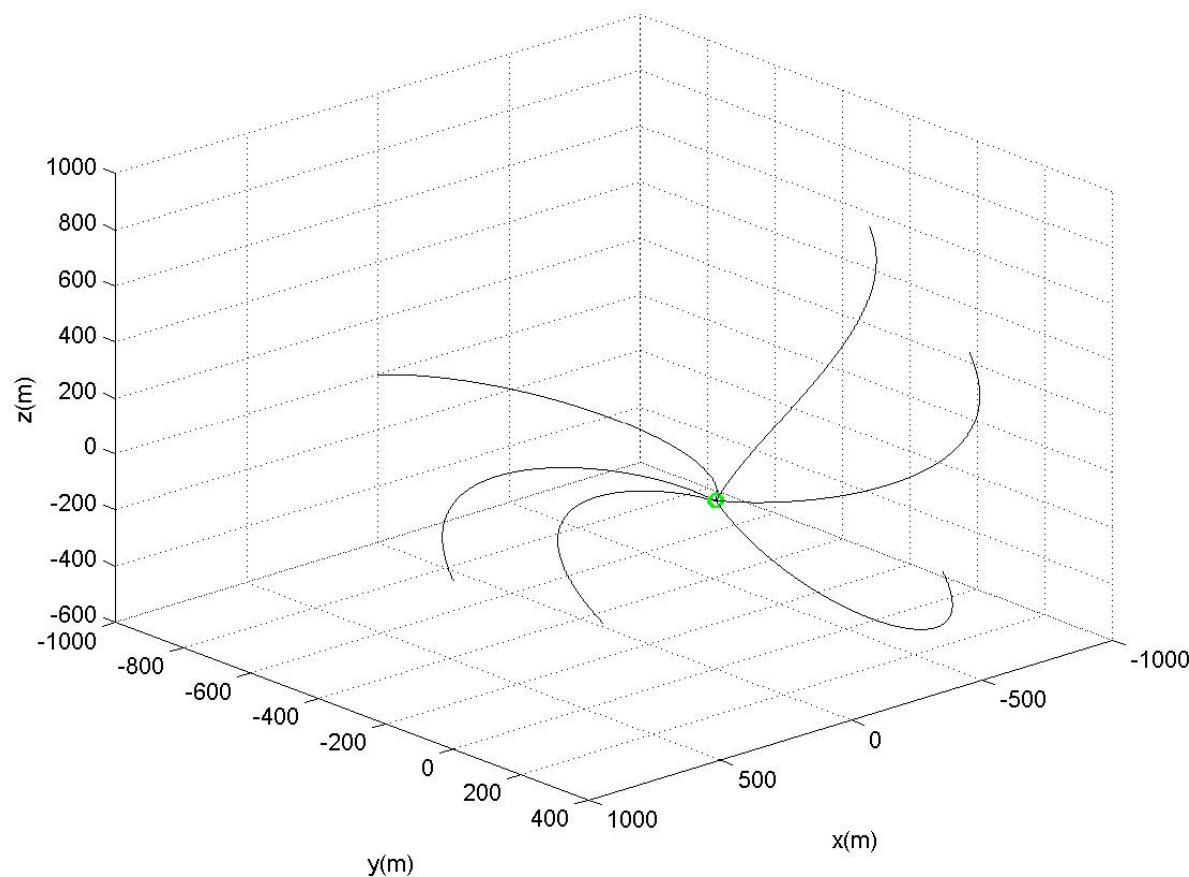


$$u = \max(\|u\|, \|u_{sat}\|) \frac{u}{\|u\|}$$

Simulation Results:



Video clip



Simulation Data:

-Initial average distance=1000m

-Initial velocities=0m/s

-Thrust to mass ratio=.005

-Total acquisition time=20000sec

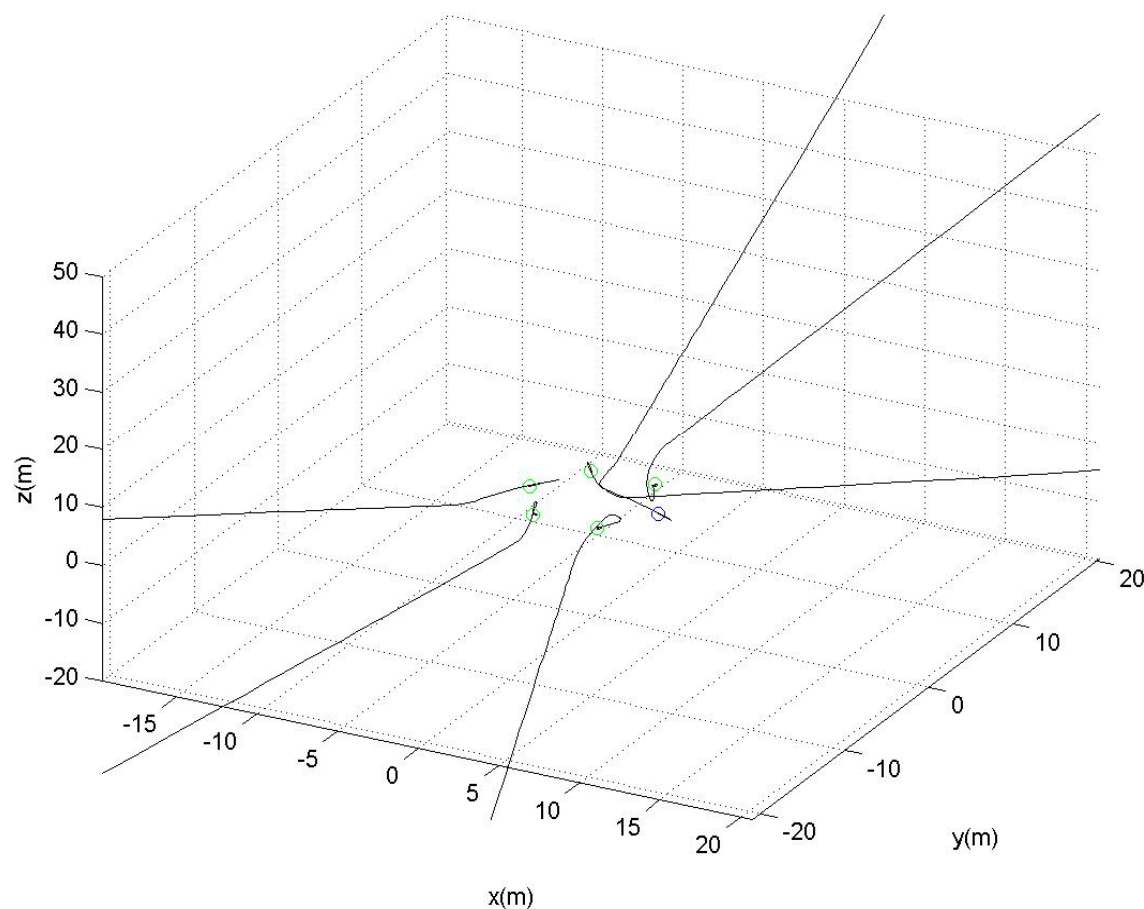
-Radius of the sphere of flat space=30m

-Radius of the final formation=6m

Simulation Results:



Video clip



Simulation Data:

-Initial average distance=1000m

-Initial velocities=0m/s

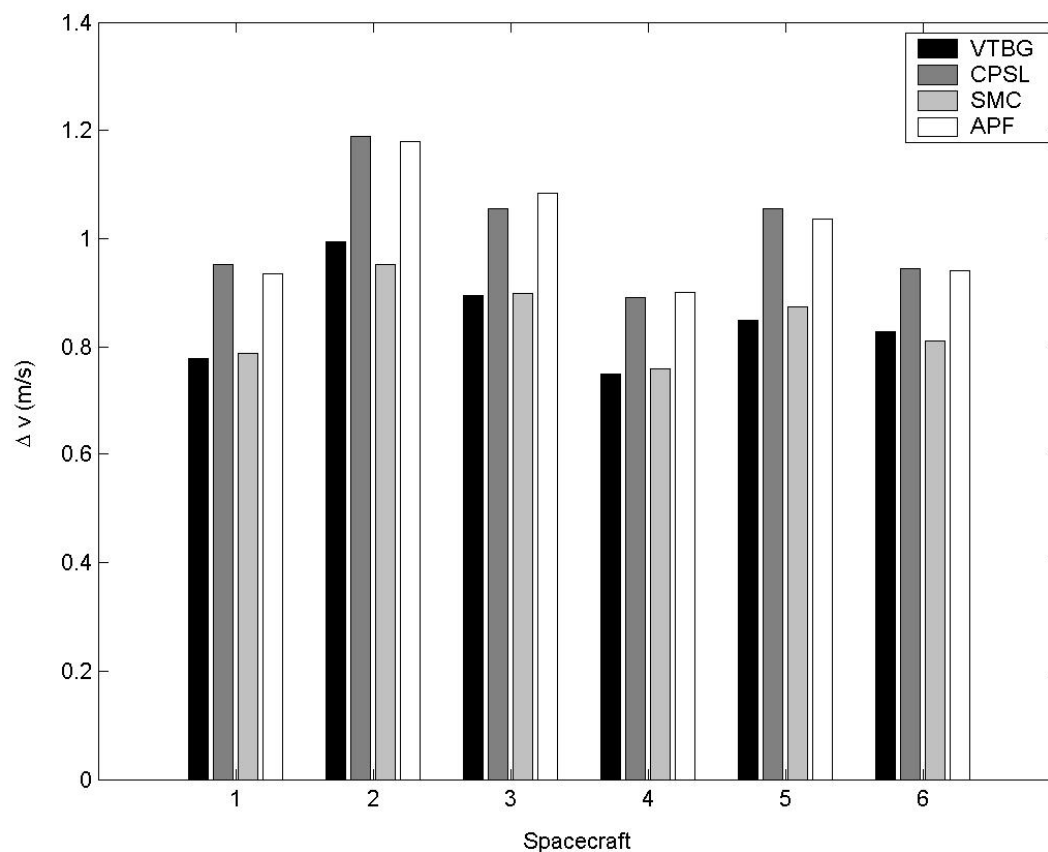
-Thrust to mass ratio=.005

-Total acquisition time=20000sec

-Radius of the sphere of flat space=30m

-Radius of the final formation=6m

A Comparison between different thrusting strategies:



Simulation Data:

-Initial average distance=1000m

-Initial velocities=0m/s

-Thrust to mass ratio=.005

-Total acquisition time=20000sec

-Radius of the sphere of flat space=30m

-Radius of the final formation=6m



Video clip



Video clip

Application to Lattice Structures:

- The agents are divided into two different groups: Seeds (j) and Agents (i).
- A new behavior is defined to avoid collisions between elements belonging to different groups:

$$v_i^{Avoid_{in}} = \sum_j b_{in} \psi_A(\|x_j - x_i\|, k_{A_{in}})(x_j - x_i) \quad \text{with } k_{A_{in}} \text{ properly defined}$$

The equilibrium shaping technique is applied to the seeds with a larger radius of influence.



v_{s_i}

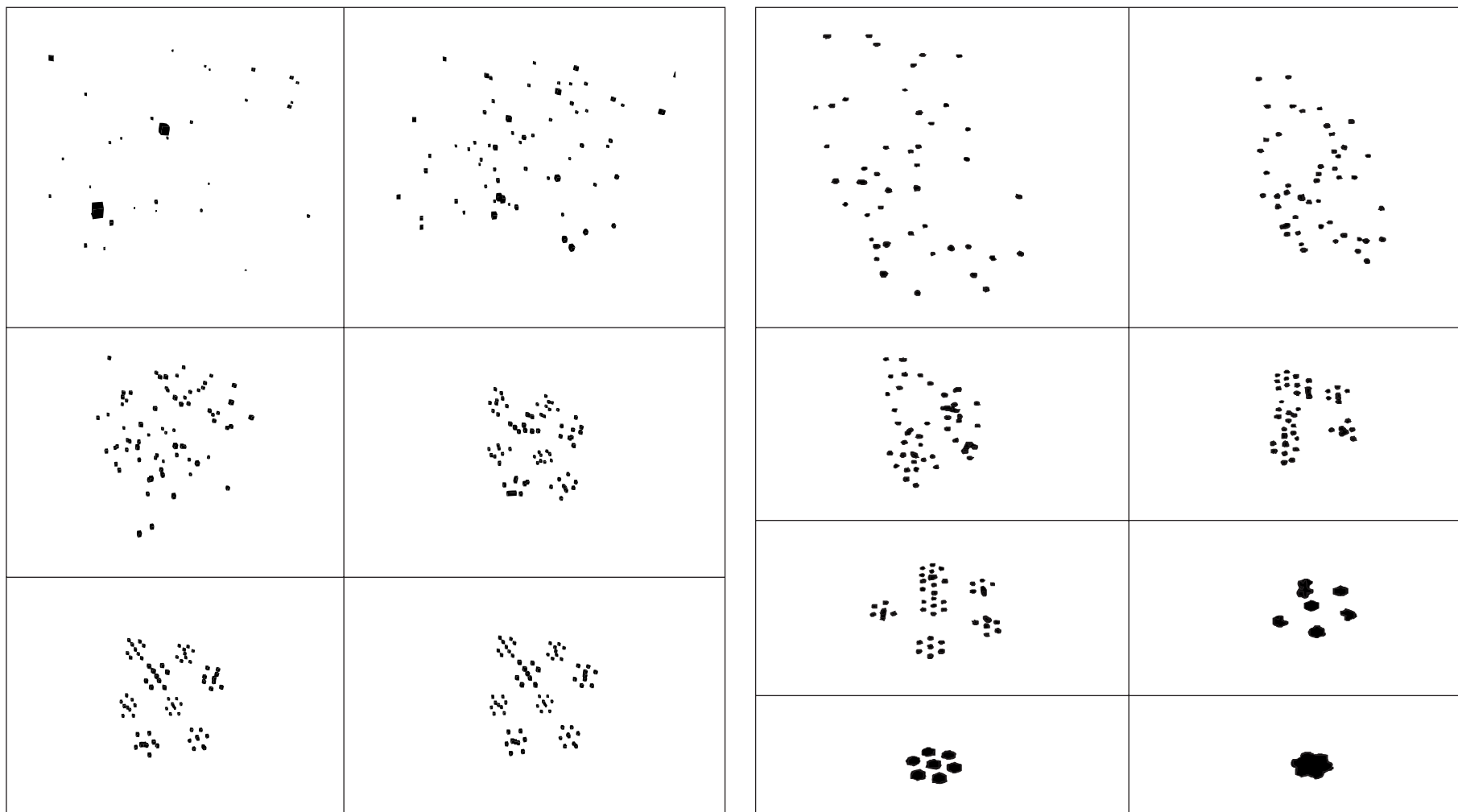
The equilibrium shaping technique is applied to each group of agents.



v_{a_j}

$$V_{tot_j} = v_{s_i} + v_{a_j}$$

For a swarm with N seeds and N groups of n agents, the final configuration is chosen among $(N!)(n!)^N$ possibilities (2.81e41 for the second simulation!!!!).



Conclusions:

- The **Equilibrium Shaping** technique has been developed to design dynamical system that allows for intelligent and coordinated path planning of a swarm of N satellites.
- The set of all the possible target formations achievable with this method has been derived using the theory of symmetry groups.
- The gravitational field has been exploited in order to increase the efficiency of the whole acquisition maneuver.
- Different feedback controls have been introduced adopting **VTBG**, **CPSL**, **SMC** and **APF** techniques. The results in term of fuel consumptions have been compared.
- The introduction of a small amount of communication between the satellites leads to the possibility of creating large and complex **Lattice Formations**.

References:

- Ayre, M., Izzo, D. and Pettazzi, L., "Self Assembly In Orbit Using Intelligent Components", TAROS 2005, proceedings
- Izzo, D. and Pettazzi, L., "Equilibrium Shaping: autonomous and distributed control for satellite swarm", 8th International Symposium on Artificial Intelligence, Robotics and Automation in Space, 5-9 Sept, Munich, Germany 2005.

available on-line at www.esa.int/act

Stochastic local minimum avoidance:

It is possible that an agent becomes stuck in a local minimum position - an algorithm has been designed in order to allow the SWARM to release itself from these undesired configurations.

