Problem Description for the
2^{nd} Global Trajectory Optimisation Competition

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1 Background

The Global Trajectory Optimisation Competition was inaugurated last year by Dario Izzo of the Advanced Concepts Team, European Space Agency. The second competition, GTOC2, organised this year by the Outer Planets Mission Analysis Group of the Jet Propulsion Laboratory, was announced on 05 October 2006. This document reveals the problem that is to be solved for GTOC2.

2 Introduction

The criteria for selecting a problem this year are similar to those used in the first competition:

- Global optimisation over a large design space (e.g. large launch window), with many local optima.
- Unusual objective function or constraints — no canned methods or existing software can likely fully solve the problem.
- Problem is easy enough to tackle in a 3-4 week timeframe for experienced mission designers or mathematicians, including exploration of new algorithms.
- Problem solutions can be easily verified.

The problem chosen this year, like that of last year, involves low-thrust trajectory design. The complexities of gravity assists, which were permitted last year, have been replaced by another sort of combinatorial complexity, and the mission objectives and constraints are of course changed.

3 Problem Description

3.1 Summary

This year's problem is a multiple asteroid rendezvous. A trajectory must be designed for a low-thrust spacecraft which launches from Earth and subsequently performs a rendezvous with one asteroid from each of four defined groups of asteroids. Maximisation of the ratio of final spacecraft mass to flight time is sought.

3.2 The four asteroid groups

The asteroid groups are chosen such that they are sufficiently well populated and sufficiently diverse between themselves (in terms of orbit elements) so as to ensure that there is no obvious best solution and that the trajectories necessarily exhibit large excursions in most of the orbit elements. The composition of the four groups is primarily as follows. The asteroid ephemeris data needed for solving the problem are provided in a separate file.

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Group 1: Jupiter Former Comets, that is, asteroids which are believed to have once been comets whose orbits were altered by close approach to Jupiter. These asteroids have values between 2 and 3 for the Tisserand invariant with respect to Jupiter, and they have low albedo. Number of asteroids: 96.

Group 2: C- or M-class asteroids, that is, asteroids whose Tholen spectral type is C (Carbonaceous) or M (Metallic). Number of asteroids: 176.

Group 3: S-class main belt asteroids, that is, asteroids whose Tholen spectral type is S (Siliceous), and which lie in the Main Belt. Number of asteroids: 300.

Group 4: Aten asteroids, that is, asteroids having semimajor axis less than 1 AU, but aphelion radius greater than 1 AU. Number of asteroids: 338.

4 The mission and engineering parameters

The spacecraft is to launch from the Earth, with a hyperbolic excess velocity (v_∞) of up to 3.5 km/s and of unconstrained direction. The year of launch must lie in the range 2015 to 2035, inclusive. After launch, the spacecraft must rendezvous with one asteroid from each group. Choosing an asteroid from each group is part of the optimisation process. The order in which the asteroids are visited is immaterial. A stay time of at least 90 days is required at each of the first three asteroids. The flight time, t_f, measured from launch up to the point of rendezvous with the fourth asteroid, must not exceed 20 years. Gravity-assists are not permitted. Objective of the optimisation is to maximise the quantity

$$J = m_f / t_f$$

where m_f is the final spacecraft mass.

The spacecraft has a fixed initial mass of 1500 kg (it does not change with launch v_∞). The initial mass includes 1000 kg of available propellant. The propulsion is by means of a thruster which can be turned on or off at will, has a constant specific impulse of 4000 s, and has a maximum thrust level of 0.1 N. There is no constraint on the thrust direction.

5 Dynamical models

The Earth and asteroids are assumed to follow Keplerian (conic) orbits around the Sun, as specified below. The only forces acting on the spacecraft are the Sun's gravity and, when on, the thrust from the propulsion system.

The file `ast_ephem.txt` is an ASCII file giving the asteroid ephemeris in terms of Keplerian orbit elements. The columns in the file are as follows: 1) A unique ID number of the asteroid; 2) semimajor axis in AU; 3) eccentricity; 4) inclination in degrees; 5) longitude of the ascending node in degrees; 6) argument of periapsis in degrees; 7) mean anomaly in degrees; 8) epoch, in modified Julian date, at which the mean anomaly is given; 9) the asteroid's group number. The orbit elements are expressed in the J2000 heliocentric ecliptic frame. The asteroids are listed first by group and then by ID number. [The elements are taken from the public, small-body database maintained by JPL and accessible at `http://ssd.jpl.nasa.gov`. However, the official asteroid groupings and asteroid ephemeris for this problem are those provided in the file.]

The Earth's orbit elements are to be taken from Table 1. Other required constants are shown in Table 2.

Table 1: **Keplerian orbit elements of the Earth, J2000 heliocentric ecliptic reference frame**

semimajor axis, a (AU)	0.999988049532578
eccentricity, e	1.671681163160e-02
inclination, i (deg.)	0.8854353079654e-03
LAN, Ω (deg.)	175.40647696473
Arg. peri., ω (deg.)	287.61577546182
Mean anomaly at epoch, M_0 (deg.)	257.60683707535
Epoch, t_0 (MJD)	54000

Table 2: **Other constants and conversions**

Gravitational parameter of the Sun, μ (km^3/s^2)	1.32712440018e11
Astronomical Unit (AU), (km)	1.49597870691e+08
Standard acceleration due to gravity, g (m/s^2)	9.80665
Day, (s)	86400
Year, (days)	365.25
00:00 01 January 2015, (MJD)	57023.5
24:00 31 December 2035, (MJD)	64693.5

6 Solution Format

Each team should return its best solution by email to Anastassios.E.Petropoulos@jpl.nasa.gov on or before 04 December 2006. Two files must be returned.

The first file should contain:

1. a brief description of the methods used,
2. a summary of the best trajectory found (at least: IDs and group numbers of the asteroids visited, launch date, launch v_∞, arrival and departure dates at the asteroids, spacecraft mass at the asteroids, thrust duration, total flight time, and value of the objective function **in units of kg/year**),
3. a visual representation of the trajectory, such as a projection of the trajectory onto the ecliptic plane.

The file should preferably be in Portable Document Format (PDF) or PostScript (PS) format; Microsoft Word format should also be acceptable.

The second file, which will be used to verify the solution returned, must follow the format and units provided in the ASCII template file `solution_format.txt`. As shown in the file, trajectory data are to be provided at one-day increments for each inter-body phase of the trajectory. The first timepoint for each phase should correspond with body departure; the second timepoint should be one day thence, and so on. If arrival at an asteroid does not fall on a one-day increment, then the last timepoint for the phase should be reported using a partial-day increment from the previous timepoint. The coordinate frame should be the same as that used for the Earth and asteroid ephemeris: the J2000 heliocentric ecliptic frame.

7 Appendix

As a potential aid for the non-astrodynamicist or non-engineer, this appendix provides a set of equations describing the dynamics of this problem along with other background information.

7.1 Nomenclature

Orbit elements and related quantities	
a	semimajor axis, km
e	eccentricity
i	inclination, rad
Ω	longitude of the ascending node (LAN), rad
ω	argument of periapsis, rad
M	mean anomaly, rad
θ	true anomaly, rad
E	eccentric anomaly, rad
r	radius from the Sun, km
γ	the flight path angle, rad
μ	Gravitational parameter of the Sun, $\mathrm{km}^3/\mathrm{s}^2$
Cartesian position and velocity	
x, y, z	the cartesian position coordinates of an orbiting body with respect to the Sun
$\vec{x}$	vector of position coordinates, x, y, z
v_x, v_y, v_z	the cartesian velocity components of an orbiting body with respect to the Sun expressed in an inertial reference frame.
$\vec{v}$	vector of velocity components, v_x, v_y, v_z
Other quantities	
t	time, s
t_f	flight time, measured from launch to beginning of rendezvous with fourth asteroid, years
m	spacecraft mass, kg
I_{sp}	specific impulse, s
T	thrust of propulsion system, N
g	standard acceleration due to gravity at Earth's surface, $\mathrm{m/s}^2$
v_∞	hyperbolic excess velocity, km/s
Subscripts and superscripts	
$()_f$	value of quantity measured at beginning of rendezvous with fourth asteroid
$()_0$	value of quantity at some given instant
$\dot{()}$	time derivative of quantity

7.2 Dynamics and conversions between elements

The motion of the Earth and asteroids around Sun is governed by these equations:

$$\ddot{x} + \mu\frac{x}{r^3} = 0, \quad \ddot{y} + \mu\frac{y}{r^3} = 0, \quad \ddot{z} + \mu\frac{z}{r^3} = 0$$

where

$$r = \sqrt{x^2 + y^2 + z^2} = \frac{a(1 - e^2)}{1 + e\cos\theta}$$

The motion of the spacecraft around the Sun is governed by the same formulas but with the addition of the x, y, z components of the thrust acceleration and an equation for the mass:

$$\ddot{x} + \mu\frac{x}{r^3} = \frac{T_x}{m}, \quad \ddot{y} + \mu\frac{y}{r^3} = \frac{T_y}{m}, \quad \ddot{z} + \mu\frac{z}{r^3} = \frac{T_z}{m}, \quad \dot{m} = -\frac{T}{I_{sp}g}$$

where

$$T = \sqrt{T_x^2 + T_y^2 + T_z^2} \le 0.1\mathrm{N}$$

Conversion from orbit elements to cartesian quantities is as follows:

$$\begin{aligned}
x &= r[\cos(\theta+\omega)\cos\Omega - \sin(\theta+\omega)\cos i \sin\Omega] \\
y &= r[\cos(\theta+\omega)\sin\Omega + \sin(\theta+\omega)\cos i \cos\Omega] \\
z &= r[\sin(\theta+\omega)\sin i] \\
v_x &= v[-\sin(\theta+\omega-\gamma)\cos\Omega - \cos(\theta+\omega-\gamma)\cos i \sin\Omega] \\
v_y &= v[-\sin(\theta+\omega-\gamma)\sin\Omega + \cos(\theta+\omega-\gamma)\cos i \cos\Omega] \\
v_z &= v[\cos(\theta+\omega-\gamma)\sin i]
\end{aligned}$$

where the velocity v is

$$v = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}},$$

the flight path angle is obtained from

$$\tan\gamma = \frac{e\sin\theta}{1 + e\cos\theta},$$

the true anomaly is related to the eccentric anomaly by

$$\tan\frac{E}{2} = \sqrt{\frac{1-e}{1+e}}\tan\frac{\theta}{2},$$

the eccentric anomaly is related to the mean anomaly by Kepler's equation,

$$M = E - e\sin E,$$

and the mean anomaly is related to time and the initial mean anomaly by

$$M - M_0 = \sqrt{\frac{\mu}{a^3}}(t - t_0).$$

Thus, based on the provided ephemeris data, the cartesian positions and velocities of the Earth and asteroids may be computed as a function of time with only the minor nuisance of having to solve Kepler's equation for E by some iterative procedure. (That is, for the Earth, the asteroids, and a non-thrusting spacecraft, the equations of motion do not need to be numerically integrated to find position and velocity at some given time.)

7.2.1 Mathematical definition of launch and rendezvous

Given a launch date, t_L, and a launch hyperbolic excess velocity, $\vec{v}_{\infty L}$, the initial position and velocity of the spacecraft, $\vec{x}_s$ and $\vec{v}_s$, are then given by

$$\vec{x}_s(t_L) = \vec{x}_E(t_L), \quad \vec{v}_s(t_L) = \vec{v}_E(t_L) + \vec{v}_{\infty L}$$

where $\vec{x}_E(t_L)$ and $\vec{v}_E(t_L)$ are the position and velocity of the Earth at time t_L, and

$$|\vec{v}_{\infty L}| \leq 3.5 \text{ km/s}.$$

As mentioned earlier, the effect of Earth's gravity on the spacecraft is to be ignored. (This approximation of the real physics of the problem is very good for heliocentric trajectories.) The mass of the spacecraft at launch is $m(t_L) = 1500$ kg.

Asteroid rendezvous occurs at a time t_R when the spacecraft matches the position and velocity of the asteroid:

$$\vec{x}_s(t_R) = \vec{x}_A(t_R), \quad \vec{v}_s(t_R) = \vec{v}_A(t_R).$$

7.3 Glossary

gravity assist A hyperbolic flyby of a [massive] body for purposes of achieving a desirable course change.

Modified Julian Date (MJD) Has units of days and is defined as MJD=(Julian_Date−2400000.5), where the Julian Date is simply the number of days past some defined point in the past.

rendezvous Meeting a body such as an asteroid by matching its position and velocity. The body is treated as a massless, moving point in space.

stay-time A period of time during which the spacecraft remains in a state of rendezvous with a body.