

Final Rankings and Brief Descriptions of the Returned Solutions and Methods Used for the 2nd Global Trajectory Optimisation Competition

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12 January 2007

Overview

The problem posed for the 2nd Global Trajectory Optimisation Competition was announced on 06 November 2006. Of the 26 registered teams, 15 teams responded by the deadline of 04 December 2006. Eleven of the returned solutions were found to be complete solutions in the sense that they satisfied all of the constraints of the problem, or had only minor or moderate constraint violations which were deemed small enough that no significant penalty on the reported merit function was warranted. These eleven solutions were thus ranked according to the reported merit function, J . Three solutions were either partial or violated the constraints so significantly that it was not clear how to penalise the reported merit function. Hence these solutions were not ranked. Lastly, one response consisted of a proposed method without a reported solution. The rankings are summarised in Table 1. Tables 2 and 3 provide additional information about the solutions returned. All teams visited Group 4 first and Group 1 last, based on increasing orbital energy. Most teams used a countdown group sequence: 4,3,2,1. The remaining sections of this document describe briefly the teams' methods, based on the brief descriptions returned by the teams.

Table 1: Ranking of Returned Solutions

Rank	Team	J (kg/yr)
1	4: Politecnico di Torino	98.64
2	13: Moscow Aviation Institute, and Khrunichev State Research and Production Space Center	87.93
3	10: Advanced Concepts Team, ESA	87.05
4	15: Centre National d'Etudes Spatiales (CNES)	85.43
5	1: GMV Aerospace and Defence	85.28
6	2: German Aerospace Center (DLR)	84.48
7	9: Politecnico di Milano	82.48
8	19: Alcatel Alenia Space	76.37
9	14: Moscow State University	75.08
10	7: Tsinghua University	56.87
11	18: Carnegie Mellon University, J.J. Arrieta-Camacho	27.94
–	17: University of Glasgow, <i>et al.</i>	73.87 ^a
–	21: Technical University of Delft and Dutch Space	15.95 ^b
–	23: Facultes Universitaires Notre-Dame de la Paix (FUNDP)	– ^c
–	26: University of Maribor, Bostjan Eferl	– ^d

^a Significant position and velocity violations at the asteroids and Earth

^b Significant position and velocity violations at the asteroids and Earth, and flight time limit violation

^c Only one leg computed (Earth to Group 4)

^d Only a proposed method described, no solution computed

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Table 2: Asteroids visited and trajectory characteristics

Rank	Team	$v_{\infty L}$ (km/s)	TOF (yrs)	m_f (kg)	Asteroid sequence (SPKID) and group numbers					
1	4	3.50	9.106	898.2	3258076 (4)	2000060 (3)	2000058 (2)	2002959 (1)		
2	13	3.50	10.394	913.9	3250293 (4)	2000149 (3)	2000569 (2)	2002483 (1)		
3	10	2.58	9.523	829.0	3170221 (4)	2000574 (3)	2000209 (2)	2011542 (1)		
4	15	2.45	9.777	835.2	3170221 (4)	2001990 (3)	2000240 (2)	2001754 (1)		
5	1	2.18	10.096	861.0	3017309 (4)	2000443 (3)	2000490 (2)	2001345 (1)		
6	2	3.23	10.170	859.1	3250293 (4)	2000027 (3)	2000110 (2)	2001038 (1)		
7	9	3.50	10.796	890.5	3288933 (4)	2001707 (3)	2000047 (2)	2014569 (1)		
8	19	3.50	10.816	826.1	3329255 (4)	2000232 (2)	2000807 (3)	2001754 (1)		
9	14	2.46	11.509	864.1	3170221 (4)	2000043 (3)	2000074 (2)	2002483 (1)		
10	7	3.50	12.941	735.9	3250293 (4)	2000149 (3)	2000224 (2)	2009661 (1)		
11	18	3.50	19.195	536.3	3343104 (4)	2000169 (3)	2000075 (2)	2000659 (1)		
–	17	–	12.991	959.6	3250293 (4)	2000443 (3)	2000058 (2)	2002959 (1)		
–	21	–	32.25	514.3	3170221 (4)	2001314 (3)	2000395 (2)	2002483 (1)		
–	23	–	–	–	3177202 (4)					

Table 3: Dates at the various bodies

Rank	Team	Earth launch, and asteroid arrival and departure dates (MJD)								
1	4	59870	60283	60373	61979	62069	62647	62737	63196	
2	13	62866	63028	63118	64907	64997	65712	65802	66662	
3	10	57372	57747	57849	59485	59587	60034	60139	60851	
4	15	59574	60104	60194	61749	61839	62306	62396	63145	
5	1	61073	61258	61348	63178	63268	64011	64101	64761	
6	2	58021	58379	58469	60236	60326	60872	60963	61735	
7	9	62201	62454	62544	64444	64534	65394	65484	66144	
8	19	59418	59610	59700	61603	61693	62288	62378	63369	
9	14	57561	57987	58106	59627	59717	60935	61025	61764	
10	7	58448	58752	58846	60826	61048	61991	62232	63175	
11	18	58246	59125	59215	61731	61821	62552	62642	65257	
–	17	58460	58794	58884	60623	60714	62303	62393	63204	
–	21	57755	58659	58749	61861	62190	64925	65200	69534	
–	23	57052	59226							

Rank 1: Team 4, Torino (98.64 kg/yr)

The global search of Team 4 was aided by their observation that, of the Group-1 asteroids, those with low energy and low inclination pass through their perihelia within two-year windows, repeated about every 8 years. Noting that it is efficient to meet the last asteroid just after its perihelion passage, they significantly narrowed down the arrival times at the last asteroid. A database of optimal trajectories to selected Group 4 asteroids (using various performance indices) was built. Group 2 and 3 asteroids were selected based on a modified Edelbaum approximation and phasing. Candidate trajectories from the global search were optimised using the indirect formulation solved by classical shooting.

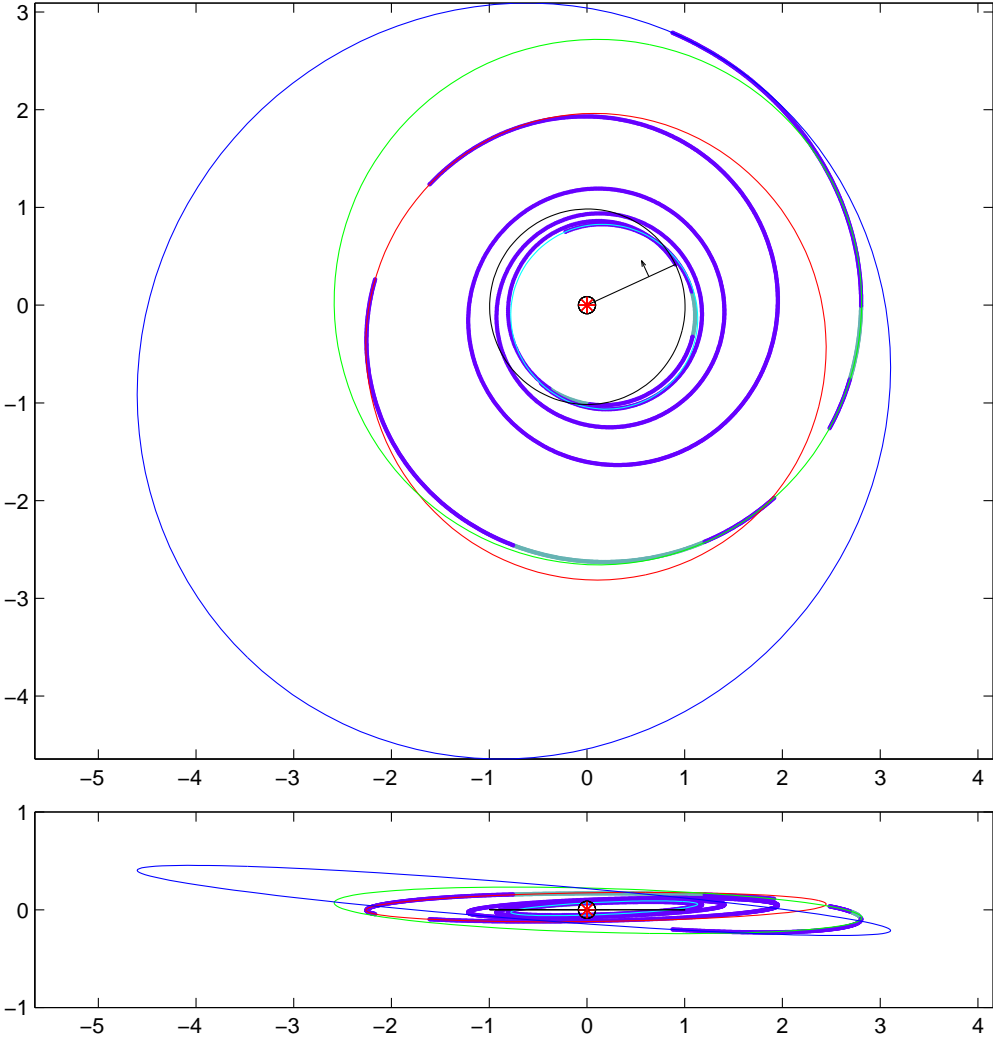


Figure 1: Politecnico di Torino trajectory, xy and xz projections

Rank 2: Team 13, MAI and Khrunichev (87.93 kg/yr)

Lambert solutions between asteroids were used to screen the global search space. Then candidates were optimised using the Maximum Principle, continuation with respect to the boundary conditions, continuation with respect to the gravity parameter, and continuation from a power-limited engine model to the problem's constant-exhaust-velocity engine model.

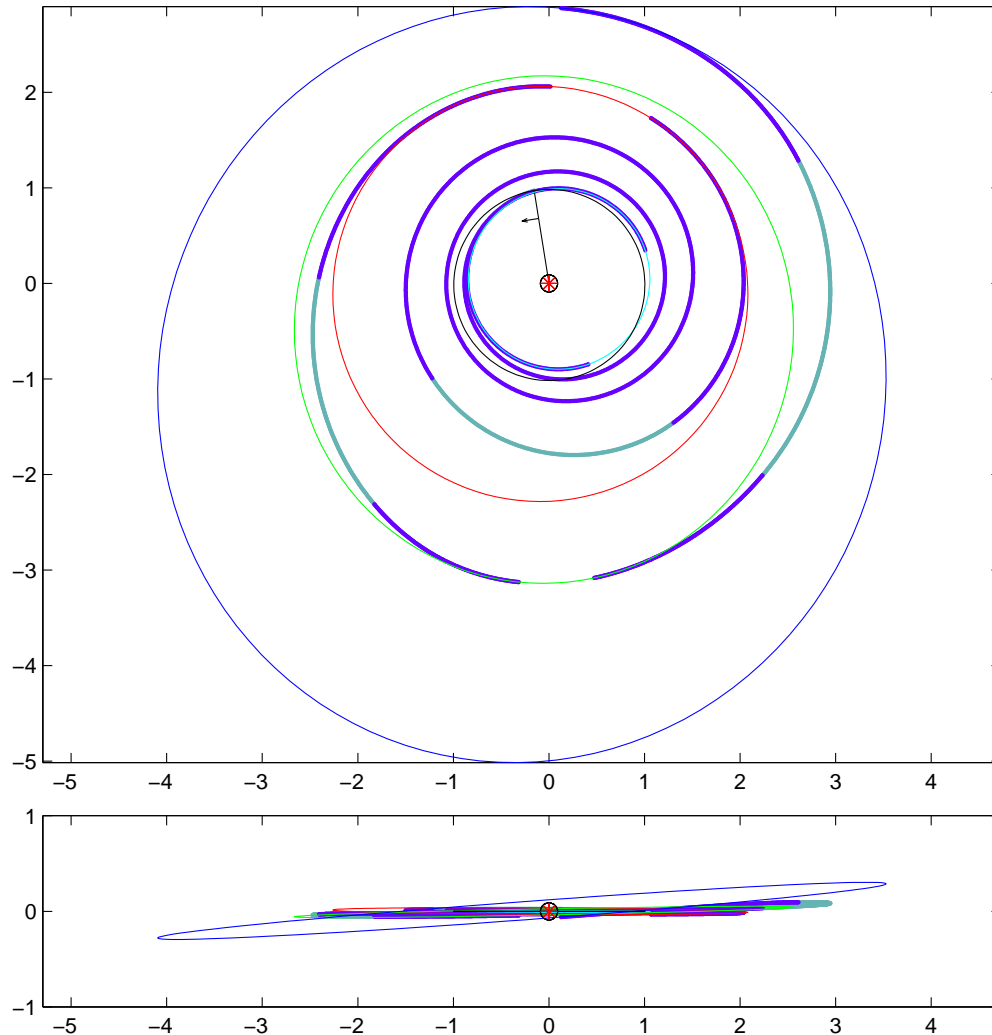


Figure 2: MAI-Khrunichev trajectory, xy and xz projections

Rank 3: Team 10, ESA Advanced Concepts Team (87.05 kg/yr)

A branch-and-prune combinatorial analysis was first performed on all the possible asteroid sequences, where the pruning was based on the ΔV of a Hohmann-like transfer between the asteroids. The result was a list of 13132 asteroid sequences (all 4-3-2-1 or 4-2-3-1). To reduce this list further, the asteroid phasing was approximately taken into account by assuming Lambert arcs between the asteroids with additional time allowed for “spiralling”. A second approach was also used where exponential sinusoid arcs were used instead of Lambert arcs between the first and second asteroids. The free parameters (essentially the various times), were optimised by a differential evolution global optimisation technique. The final locally optimal solution was found using a non-linear programming method with one of the exponential sinusoid trajectories as an initial guess.

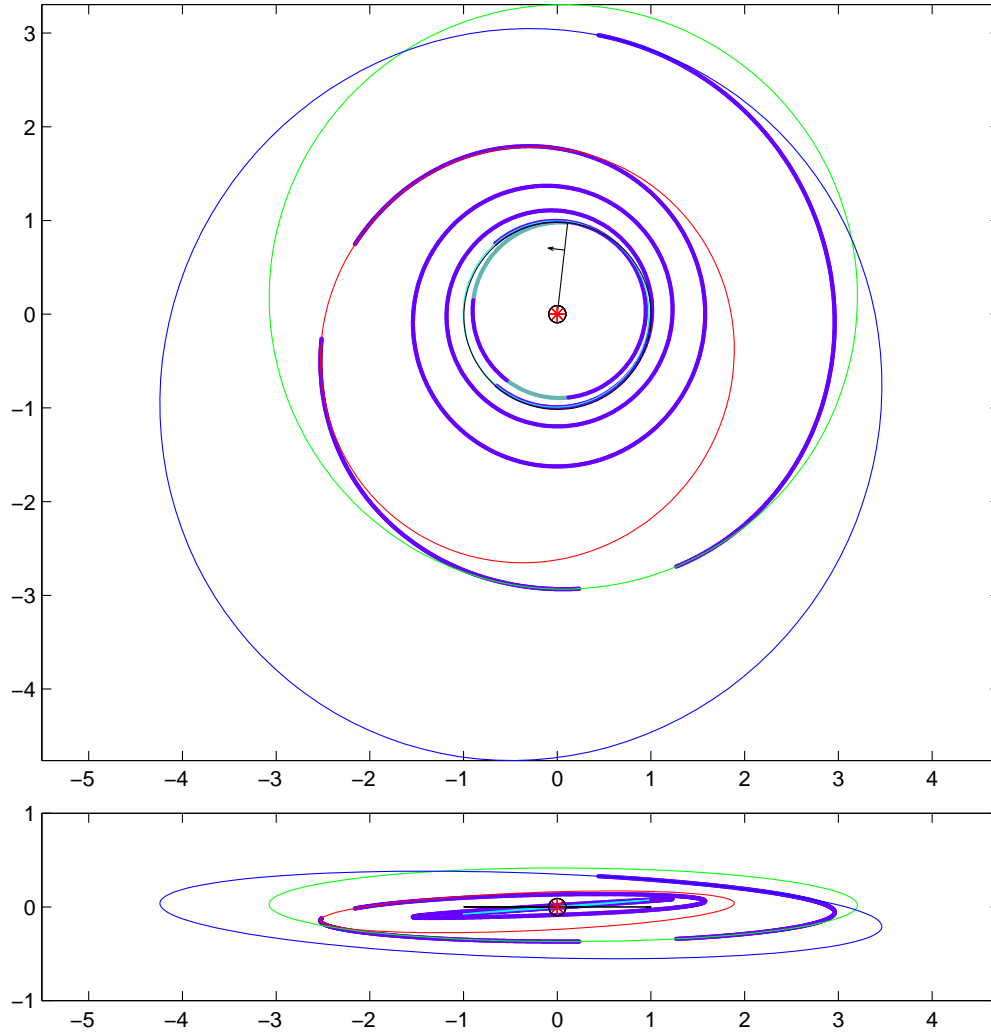


Figure 3: ESA Advanced Concepts Team trajectory, xy and xz projections

Rank 4: Team 15, CNES (85.43 kg/yr)

The potential asteroid sequences were reduced to 1080 by selecting 22 asteroids based on continuously increasing the semi-major axis, minimising the inclination corrections, minimising the transfer time from the first to the second asteroid, and selecting reasonable phasing between asteroids of Groups 3, 2, and 1. Candidate sequences are then assessed using Lambert arcs and impulsive ΔV s, with constraints on the arrival, departure and deep-space ΔV . A simplex Nelder-Mead method (direct, gradient-free) is used. The low-thrust problem is then solved using Pontryagin's Maximum Principle and a decomposition-coordination technique.

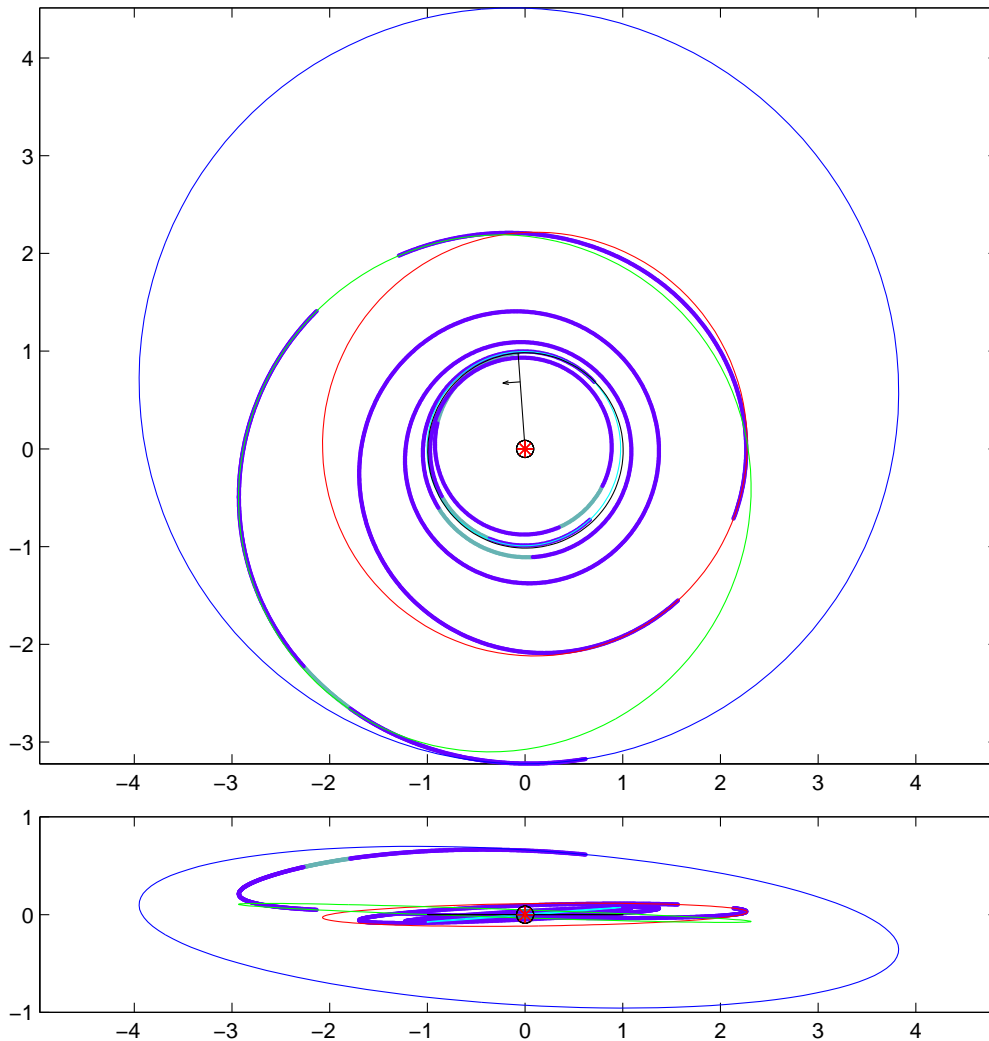


Figure 4: CNES trajectory, xy and xz projections

Rank 5: Team 1, GMV (85.28 kg/yr)

Asteroids were first filtered based on an upper limit in the variation of orbital elements per leg, and on the cost of propellant-optimal, two-impulse, phase-free transfers between asteroid pairs. The next filtering step included asteroid phasing as well as a low-thrust arc between the first and second asteroids (instead of the two-impulse solution). Finally, Lawden's implicit guidance scheme was used, and equality and inequality constraints were incorporated as penalty functions in an augmented objective function. Optimisation was then by means of a genetic algorithm or a simplex Nelder-Mead, derivative-free local optimiser.

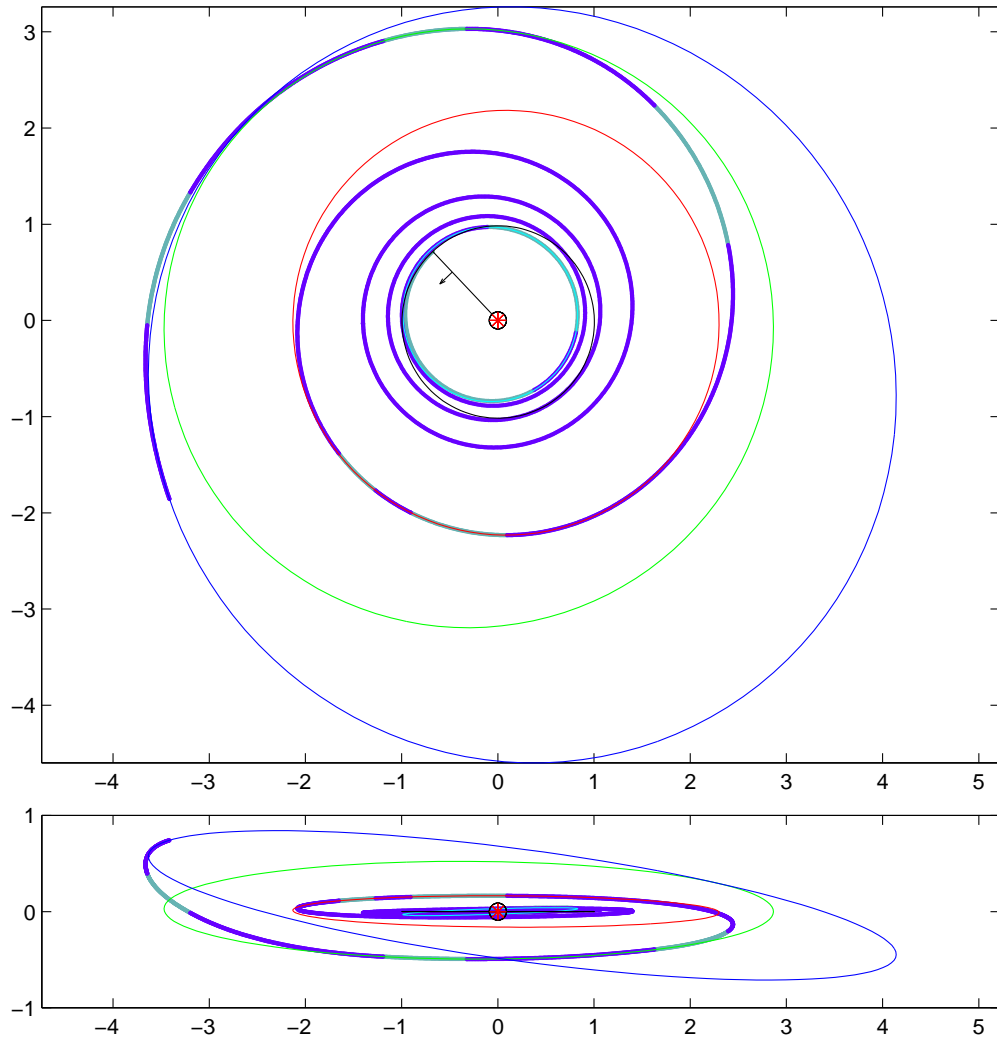


Figure 5: GMV trajectory, xy and xz projections

Rank 6: Team 2, DLR (84.48 kg/yr)

A global search on a per-leg basis (asteroid to asteroid using low thrust) was performed using neural networks with an evolutionary algorithm driver. The four legs of the trajectory were built up in an iterative loop with a local optimiser based on a non-linear-programming formulation solved using the optimisation package SNOPT.

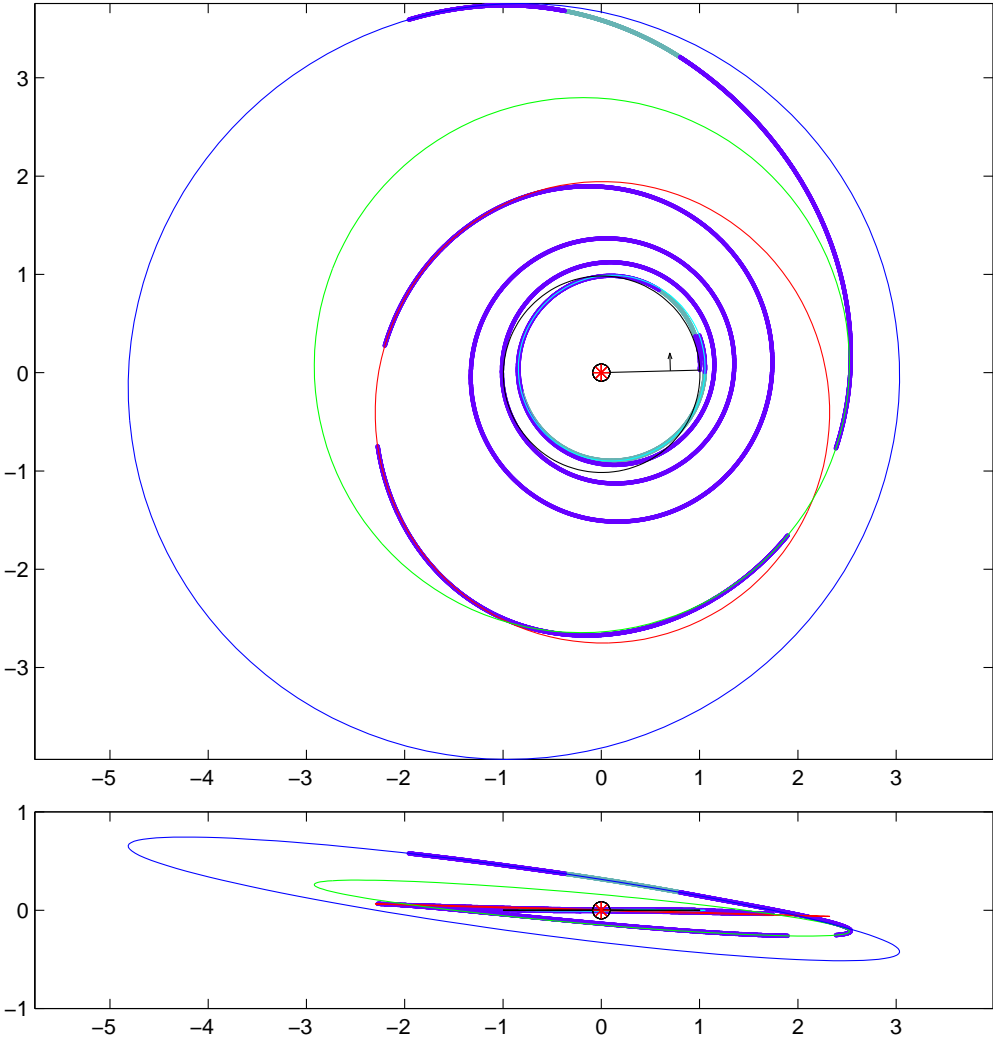


Figure 6: DLR trajectory, xy and xz projections

Rank 7: Team 9, Milano (82.48 kg/yr)

The global search was performed much like Team 10: Lambert arcs were assumed for all legs except for the leg from the first asteroid (Group 4) to the second (Group 2 or 3), where an exponential sinusoid arc was assumed. Optimisation within this simplified model was performed using three different methods: genetic algorithm, particle swarm, and multi-level coordinate search algorithm. Optimisation in the full model was done using a direct method with either multiple shooting or collocation with Lagrange polynomial interpolation.

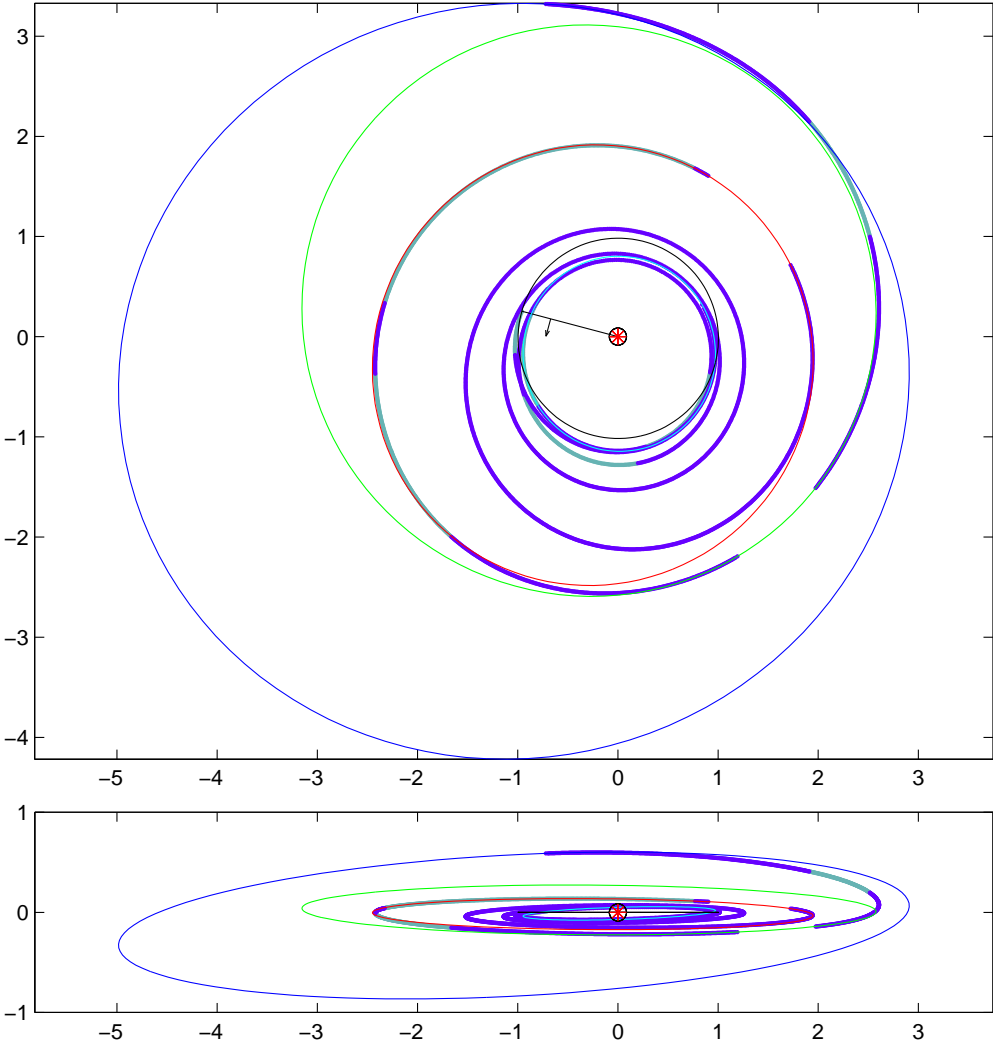


Figure 7: Politecnico di Milano trajectory, xy and xz projections

Rank 8: Team 19, Alcatel Alenia Space (76.37 kg/yr)

A four-tiered process was used to obtain the final solution. The first step screened the asteroid sequences using dynamic programming and a cost function which penalised large differences in angular momentum between successive asteroids in a sequence and also penalised large orbital periods of asteroids in the sequence. This screening resulted in an asteroid group sequence of 4-2-3-1. The second step involved a scan over departure date, with impulsive trajectories between asteroids ranked by ΔV and duration. The third step used dynamic programming to solve the best candidates as a minimum time problem. The fourth step resolved the results from step three as a maximum final mass problem.

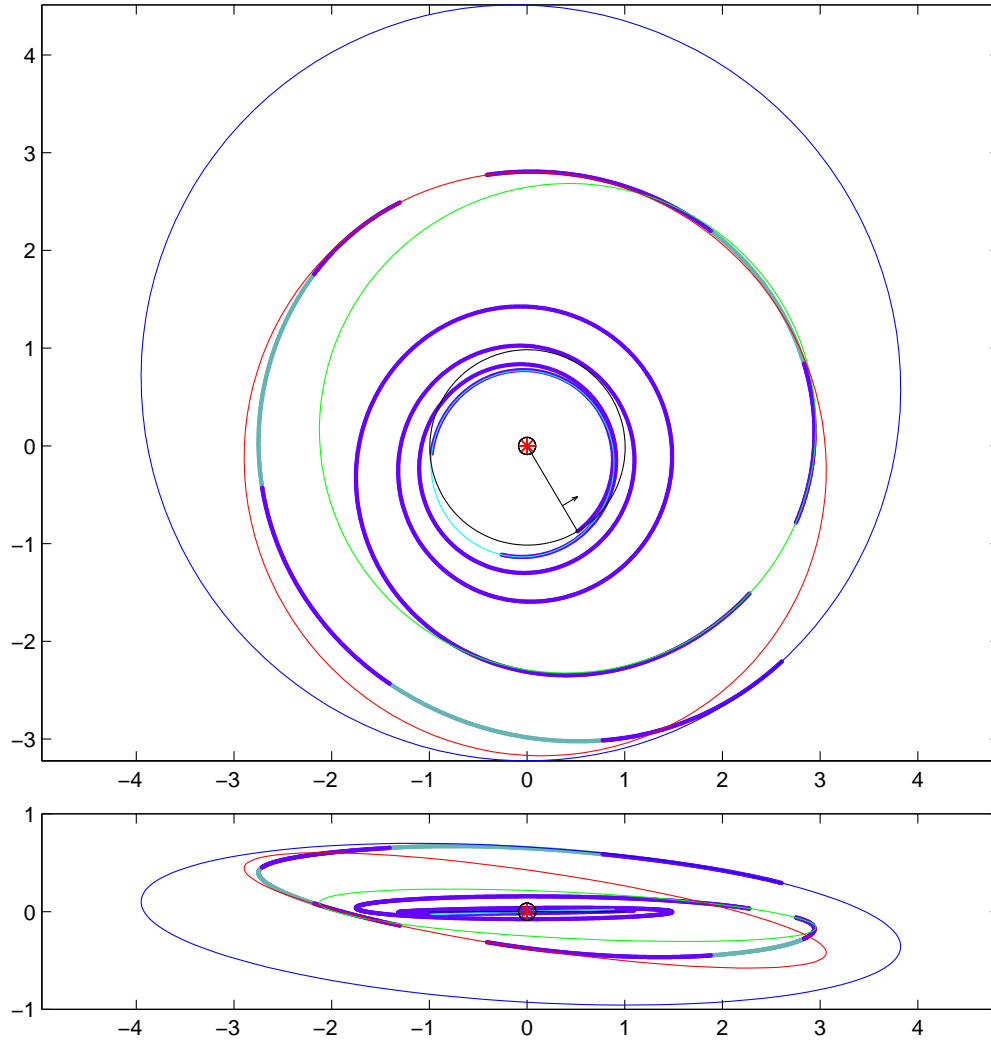


Figure 8: Alcatel Alenia trajectory, xy and xz projections

Rank 9: Team 14, Moscow State University (75.08 kg/yr)

Preliminary selection of asteroids was made based on Lambert solutions for optimal two-impulse transfers between successive bodies. The best asteroid sequences were then optimised based on Pontryagin's Maximum Principle, solved using a shooting method and continuation on a parameter.

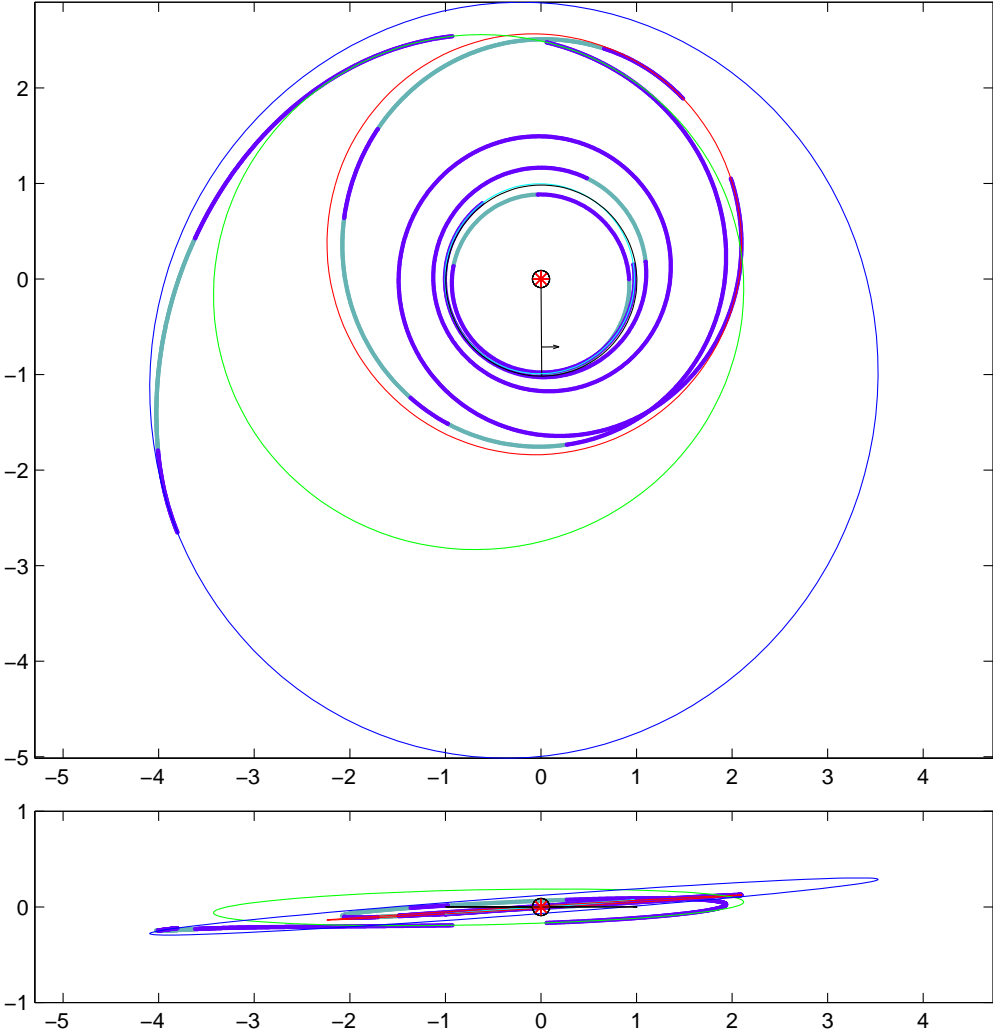


Figure 9: Moscow State University trajectory, xy and xz projections

Rank 10: Team 7, Tsinghua University (56.87 kg/yr)

After selecting an asteroid sequence, the trajectory legs are divided into segments during which the thrust magnitude is held fixed, and the direction varies linearly between initial and final cone and clock angles for the segment. These thrust variables, together with dates and other problem parameters, are optimised using a genetic algorithm, where constraints are imposed by means of penalty functions. To meet the final accuracy requirements, a final, local optimisation is performed using Matlab's `fminsearch` function.

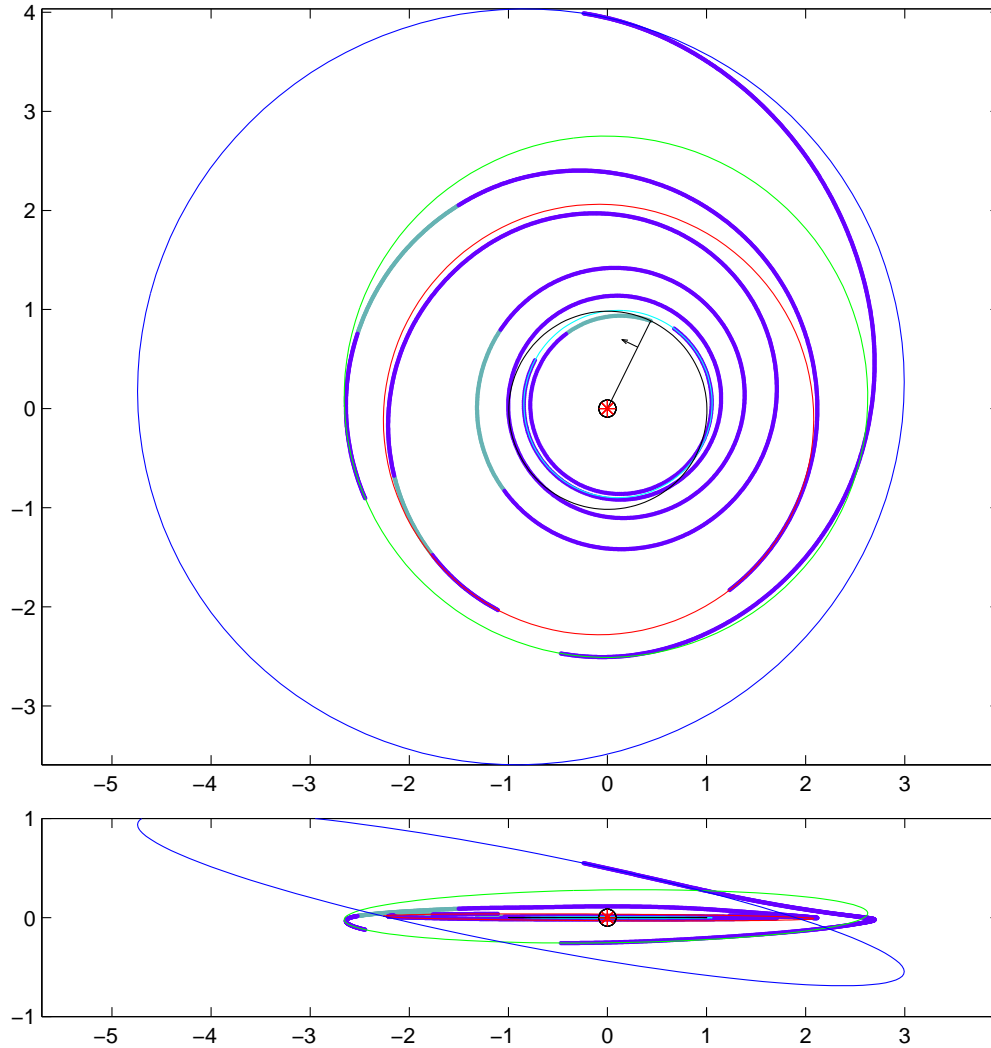


Figure 10: trajectory, xy and xz projections

Rank 11: Team 18, Carnegie Mellon (27.94 kg/yr)

Candidate asteroid sequences were determined by screening out first those requiring large plane changes or large rotations of the line of nodes, and second, those with large changes in semimajor axis. Within this reduced group, candidate dates of departure were found based on when the asteroids were least separated in space. Finally, local optimisation was performed using a direct transcription method in modified equinoctial elements.

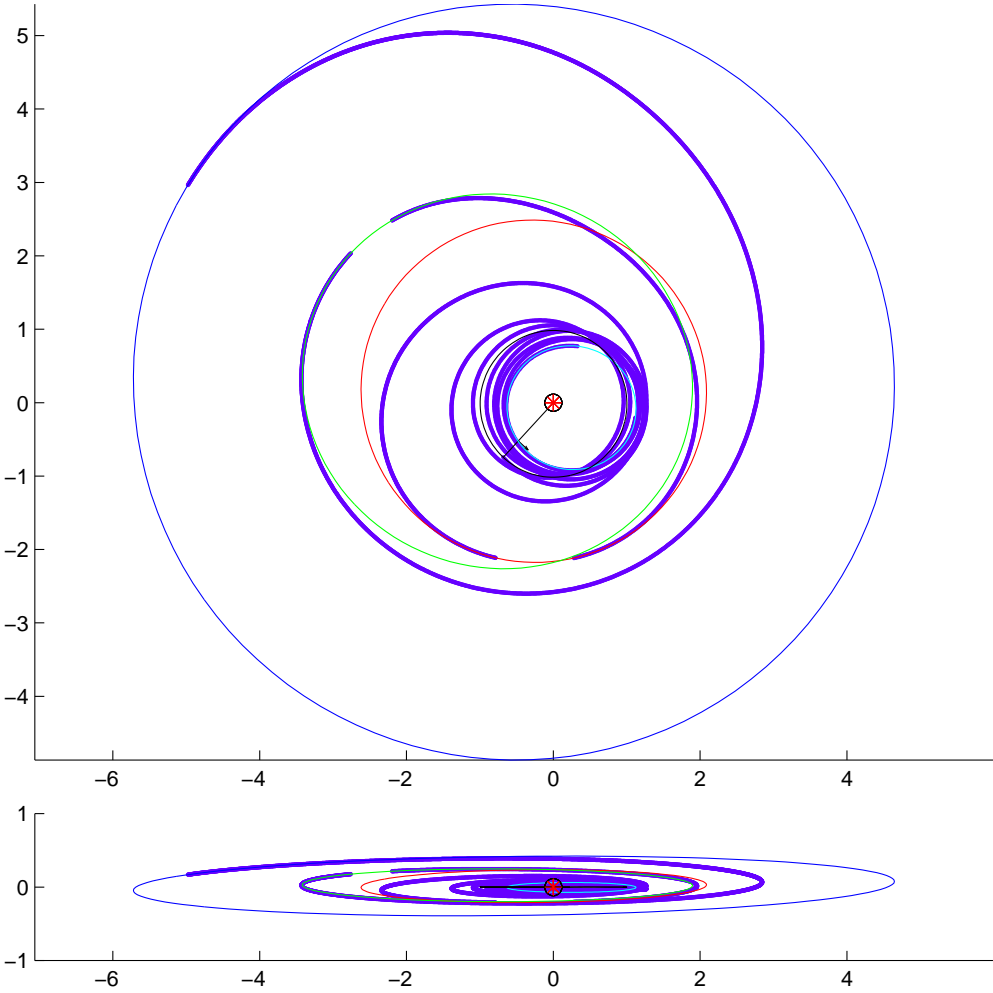


Figure 11: Carnegie Mellon trajectory, xy and xz projections

Team 17, University of Glasgow, et al. (73.87 kg/yr)

Initial asteroid screening was based on the orbit elements. Then, two different optimisation approaches were taken. In the first one, the dynamical models were coded as “black-box” functions that were to be optimised by various global and local optimisation packages. In the second approach, which yielded the final solution, candidate solutions were first found using Lambert arcs and shape-based low-thrust arcs, selected by an evolutionary branching algorithm. These candidates were then refined using a direct optimisation method. Large constraint violations in the final solution occurred because of insufficient time to sufficiently refine the solution.

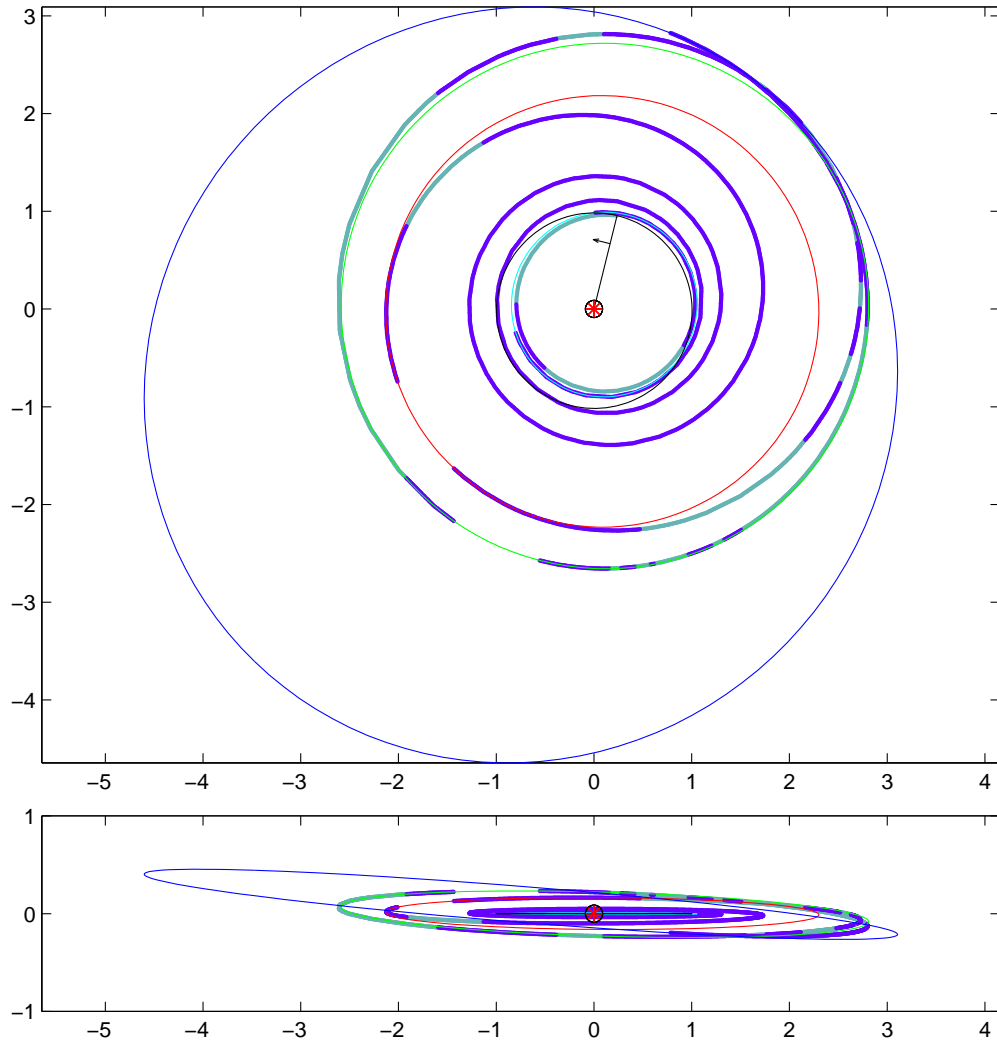


Figure 12: Glasgow *et al.* trajectory, xy and xz projections

Team 21, TU Delft and Dutch Space (15.95 kg/yr)

The initial asteroid screening was based on the estimated ΔV to change only the ascending node, or only the inclination, or to perform a Hohmann transfer. Promising sequences were then examined more closely, first using exponential sinusoids, and second, since the shape method violated the constraints too much, with evolutionary algorithms. The independent variables were taken as the various times and launch v_∞ , and the parameters of a low-dimensional parametrisation of the thrust profile.

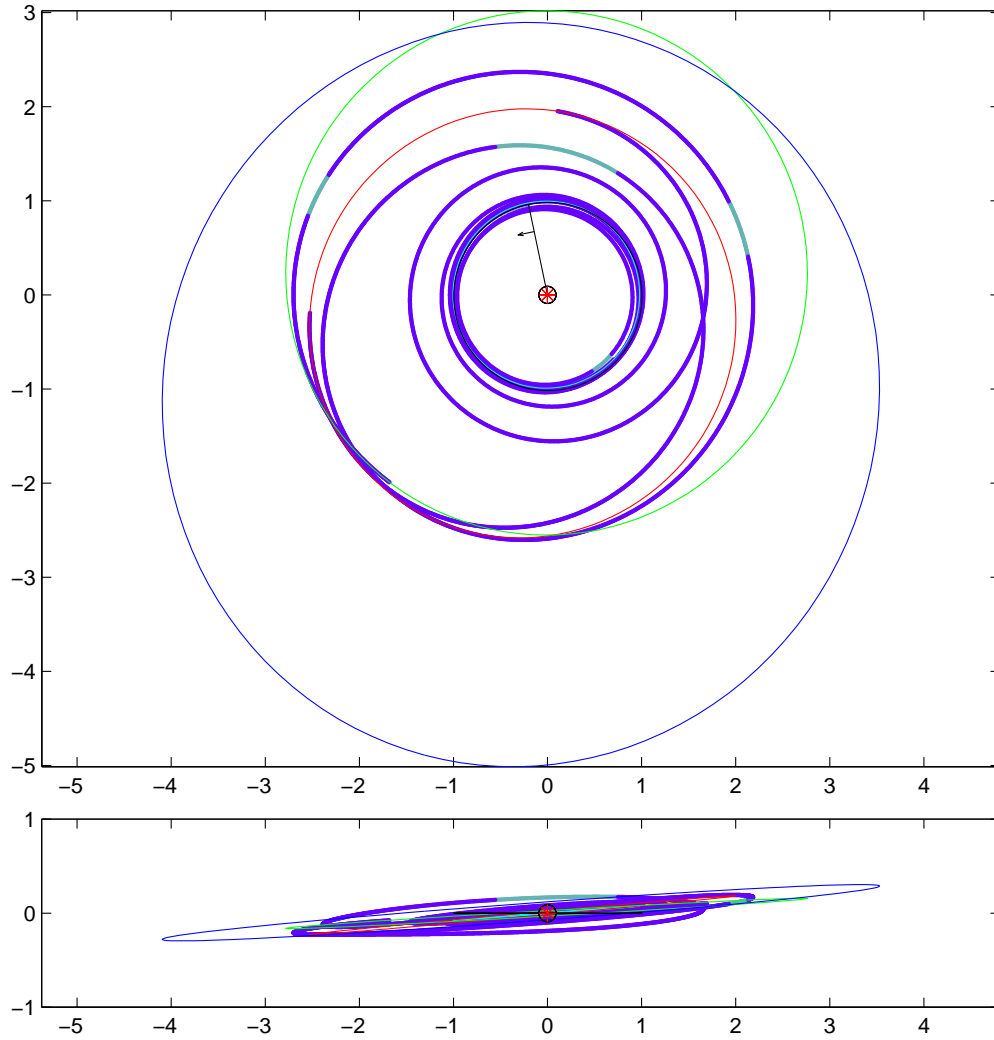


Figure 13: TU Delft - Dutch Space trajectory, xy and xz projections (last leg not plotted).

Team 23, FUNDP

Even as newcomers to the field of astrodynamics, Team 23 quickly realised that Gauss's variational equations would be useful. By taking a pseudo-inverse of these equations in a least-squares sense, they were able to determine a thrust profile to reach any given asteroid by choosing suitable dates. Due to the short time available for the competition, a trajectory reaching only an asteroid in Group 4 was computed. The flight time and spacecraft mass at rendezvous with the asteroid were 5.708 years and 1168.9 kg, yielding an "asteroid-1 objective function" of 204.8 kg/yr.

Team 26, University of Maribor

A graph theory approach to the problem is proposed. Assuming that methods are available for easily computing trajectories from one point in the four-dimensional space-time to a neighbouring point in space-time, the proposed graph-theory method would scan these points in polynomial time to find the sequence of points that best joins an initial point with a desired final point in the graph.