Problem Description for the 5^{th} Global Trajectories Optimization Competition

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Background

The Global Trajectory Optimisation Competition was initiated in 2005 by the Advanced Concepts Team of the European Space Agency. The Outer Planets Mission Analysis Group of the Jet Propulsion Laboratory, winner of GTOC1, organised the GTOC2 in 2006. The Aerospace Propulsion Group of the Dipartimento di Energetica of the Politecnico di Torino, winner of the GTOC2, organised the GTOC3 in 2008. The Interplanetary Mission Analysis team of the Centre National d'Etudes Spatieles de Toulouse, winner of the GTOC3, organised the GTOC4 in 2009. Finally, the team of Faculty of Mechanics and Mathematics of Lomonosov Moscow State Univercity, winner of the GTOC4, is very pleased to organise the GTOC5 this year.

The aim of this document is to reveal the problem to be solved for GTOC5.

Introduction

Traditionally the GTOC problems are kinds of global optimisation problems, that is to say complex optimisation problems characterised by a large number of local optima. Such problems can be solved either by means of local or global optimisation methods.

GTOC5 problem is a global optimisation problem and aims at fulfilling the following criteria:

• the design space is large and leads to an important number of local optima,

• the problem is complex but in any case it can be solved within the 4-weeks period allowed for the competition,

• its formulation is simple enough so that it can be solved by researchers not experienced in astrodynamics,

• even if some registered teams have already developed their own optimisation tools for interplanetary missions, the problem specificities make it new to all the teams,

• problem solutions can be easily verified.

Problem Description

Generalities

The mission proposed this year may be entitled: “ How to visit the greatest number of asteroids with revisiting ”.

Problem essence. The spacecraft starts from the Earth. The start moment should be chosen from the preliminarily set period of time. Spacecraft should visit asteroids from the presented list. For the first time spacecraft should rendezvous with an asteroid. For the second time the velocity of flyby should not be less than the set minimum value. The first rendezvous with an asteroid corresponds to delivery of the scientific equipment. The weight of scientific equipment makes 40 kg for each asteroid. The second asteroid flyby corresponds to delivery of 1 kg penetrator. Each mission is estimated by corresponding number of points: 0.2 for delivery of the equipment and then 0.8 for the penetrator. The spacecraft is equipped with a jet engine with low thrust. Duration of mission and final weight of the spacecraft is limited.

In honour of V. Beletskij 80th anniversary mission to Beletskij asteroid adds bonus points.

Dynamical model

The Earth and asteroids are assumed to follow Keplerian orbits around the Sun. The only forces acting on the spacecraft are the Sun’s gravity and the thrust produced by the engine (when this last one is on). The asteroid’s Keplerian orbital parameters in the J2000 heliocentric ecliptic frame are provided in the ASCII—file `ast-ephem-gtoc5.txt` that gives:

1. t_0 — epoch in modified Julian date (MJD),
2. a — semi major axis in AU,
3. e — eccentricity,
4. i — inclination, in degrees,
5. ω — argument of periapsis, in degrees,
6. Ω — longitude of the ascending node, in degrees,
7. M_0 — mean anomaly at epoch, in degrees.
8. j — asteroid number,
9. asteroid name.

Earth‘s and asteroids‘ orbital elements are given in the J2000 heliocentric ecliptic frame. Other required constants are given in Table 2.

Table 1: The Earth Keplerian orbital parameters.

t_0, MJD	54000
a, AU	0.999988049532578
e	$1.671681163160 \cdot 10^{-2}$
i, ^{o}	$0.8854353079654 \cdot 10^{-3}$
ω, ^{o}	287.61577546182
Ω, ^{o}	175.40647696473
M_0, ^{o}	257.60683707535
μ, km^3/s^2	398601.19

Other required constants are given in Table 2.

Table 2: Constants and conversion.

Sun's gravitational parameter μ_S, km^3/s^2	$1.32712440018 \cdot 10^{11}$
Astronomical Unit AU, km	$1.49597870691 \cdot 10^{8}$
Standard acceleration due to gravity, g_E, m/s^2	9.80665
Day, s	86400
Year, days	365.25
00:00 01 January 2015, MJD	57023
24:00 31 December 2025, MJD	61041

Spacecraft and Trajectory Constraints

The spacecraft is launched from the Earth, with hyperbolic excess velocity vector v_∞, $|v_\infty| \leqslant 5$ **km/s** and of unconstrained direction. The year of launch must lie in the range from 2015 to 2025, inclusive: $57023\ MJD \le t_s \le 61041\ MJD$.

The spacecraft has a constant specific impulse $I_{sp} = 3000$ s and its thrust level T is bounded. The thrust level can be modulate at will, that means that T can take any value between 0 and $T_{\max}$: $0 \le T \le T_{\max} = 0.3$ N. This maximum value $T_{\max}$ is constant and so does not depend on the distance between the spacecraft and the Sun. In addition, there is no constraint on the thrust direction. The spacecraft mass only varies during thrusting periods and is constant when the engine is off (coast periods).

The spacecraft has a fixed initial mass, i.e. wet mass, $m_i = 4000$ kg (that is not affected by the launch v_∞). We assume here that the spacecraft dry mass $m_d \ge 500$ kg, the propellant mass m_p and scientific mass m_s, i.e. $m_i = m_d + m_p + m_s$. Scientific mass m_s consists of scientific equipment mass and penetrators mass. For example, if the mission trajectory contains k rendezvous and m penetrations, $m_s = k \cdot (40\ kg) + m \cdot (1\ kg)$.

After launch, the spacecraft must provide a maximum number of asteroid missions. Asteroid mission means an asteroid rendezvous at first and then the same flyby asteroid with a velocity not less than $\Delta V^A_{\min} = 0.4$ km/s. Especially we notice that penetration before delivery of the scientific equipment is not considered and is not taken into account in performance index. The required models for rendezvous and flybys are given in the Appendix. A rendezvous requires the spacecraft position and velocity to be the same as those of the target asteroid. A flyby requires concurrence of position of spacecraft and a target asteroid. Velocity of spacecraft relating an asteroid should exceed the set minimum value $\Delta V^A_{\min}$.

List of asteroids is provided in ASCII—file `ast-ephem-gtoc5.txt`. The choice of asteroids is part of the optimization process. In addition, each asteroid's mission must be realized only once during the trajectories.

The flight time, measured from start to the end must not exceed 15 years:
$\mathcal{T} = t_f - t_s \leq 5478.75$ days.

Performance index

Index J equal to the number of spacecraft mission is maximized. An asteroid rendezvous and delivery of the scientific block is estimated by 0.2, and subsequent penetration by 0.8. For Beletskij asteroid the estimation raises 1.5 times: 0.3 for an unloading of the scientific block and 1.2 for the subsequent penetration.

As said before, when two solution yield the same value of J, we consider that the best one is solution that minimizes the following secondary performance index:

$$\mathcal{T} = t_f - t_s \to \min,$$

where $\mathcal{T}$ denotes the flight time that has to satisfy the following important constraint:

$$\mathcal{T} \leq 5478.75\ Day.$$

The formalized representation of index J and $\mathcal{T}$ is given in Appendix.

Solution format

Each team should return its best solution by email to gtoc5.msu@gmail.com on or before November 01, 2010. Two files must be returned. The first one should contain:

• a short description of the method used,

• a summary of the best solution found, at least: GTOC5 names of the visited asteroids, launch date, launch v_∞ , date and spacecraft mass at each flyby, date of the final rendezvous, thrust durations, total flight time T, value of the performance index J, value of the final mass mf,

• a visual representation of the trajectory, such as a projection of the trajectory onto the ecliptic plane.

This file should preferably be in PDF or PS format but Microsoft Word format should also be acceptable. The second file will be used to verify the solution returned. It must follow the format and units provided in the ASCII template file solution.txt. As can be seen in the file, trajectory data have to be provided at one-day increments for each inter-body phase of the trajectory. In addition, trajectory data have also to be provided at each time corresponding either to a flyby. So, when a flyby does not fall exactly on a one-day increment, the last time point for the phase should be reported using a partial-day increment from the previous time point. Moreover, the coordinate frame should be the J2000 heliocentric ecliptic frame.

Appendix

This appendix provides the equations describing the dynamics of the problem along with other background information.

Position and velocity in Keplerian orbits

The motion of the Earth and asteroids around the Sun is governed by the following equations:

$$\dot{x}^j = v_x^j, \quad \dot{y}^j = v_y^j, \quad \dot{z}^j = v_z^j,$$

$$\dot{v}_x^j = -\frac{\mu_S x^j}{(r^j)^3}, \quad \dot{v}_y^j = -\frac{\mu_S y^j}{(r^j)^3}, \quad \dot{v}_z^j = -\frac{\mu_S z^j}{(r^j)^3},$$

where j — number of the asteroid or symbol E for the Earth, x^j, y^j, z^j — position components, $r^j = \sqrt{(x^j)^2 + (y^j)^2 + (z^j)^2}$ — distance from the Sun, v_x^j, v_y^j, v_z^j — velocity components.

Due to t_0, a, e, i, ω, Ω, M_0 position and velocity in Keplerian orbits in the specified moment t can be determined by:

$$n = \sqrt{\mu_S / a^3},$$

$$p = a(1 - e^2),$$

mean anomaly M

$$M = n(t - t_0) + M_0,$$

$$M \to (-\pi, \pi], \quad \pi \approx 3.141592653589793238;$$

Kepler's equation:

$$E - e \sin E = M,$$

True anomaly θ:

$$\frac{\theta}{2} = \sqrt{\left(\frac{1+e}{1-e}\right)} \frac{E}{2},$$

$$r = \frac{p}{1 + e\cos\theta},$$

$$v_r = \sqrt{\frac{\mu}{p}} e \sin\theta, \quad v_n = \sqrt{\frac{\mu}{p}} (1 + e\cos\theta).$$

So:

$$
\begin{aligned}
x &= r[\cos(\theta+\omega)\cos\Omega - \sin(\theta+\omega)\cos i \sin\Omega],\\
y &= r[\cos(\theta+\omega)\sin\Omega + \sin(\theta+\omega)\cos i \cos\Omega],\\
z &= r\sin(\theta+\omega)\sin i,\\
v_x &= \frac{x}{r}v_r + (-\sin(\theta+\omega)\cos\Omega - \cos(\theta+\omega)\cos i \sin\Omega)v_n,\\
v_y &= \frac{y}{r}v_r + (-\sin(\theta+\omega)\sin\Omega + \cos(\theta+\omega)\cos i \cos\Omega)v_n,\\
v_z &= \frac{z}{r}v_r + \cos(\theta+\omega)\sin i \cdot v_n,
\end{aligned}
$$

Optimization problem

The motion of the spacecraft around the Sun is governed by the following equations:

$$\dot{x} = v_x, \quad \dot{y} = v_y, \quad \dot{z} = v_z, \quad \dot{m} = -T/c,$$

$$\dot{v}_x = -\frac{\mu_S x}{r^3} + \frac{T_x}{m} = F_x, \quad \dot{v}_y = -\frac{\mu_S y}{r^3} + \frac{T_y}{m} = F_y, \quad \dot{v}_z = -\frac{\mu_S z}{r^3} + \frac{T_z}{m} = F_z,$$

$$T \equiv \sqrt{T_x^2 + T_y^2 + T_z^2} \leqslant T_{\max} = 0.3 \text{ N}.$$

where x, y, z — spacecraft position components, v_x, v_y, v_z — spacecraft velocity components, T — thrust magnitude of the engine, $g_E = 9.80665\ m/s^2$ — standard acceleration due to gravity on the Earth surface, $I_{sp} = 3000$ s — specific impulse of the engine, $c = I_{sp} \cdot g_E$ — exhaust velocity, $r = \sqrt{x^2+y^2+z^2}$ — distance from the Sun.

Start from the Earth:

$$m(t_s) = m_i, \quad x(t_s) - x^E(t_s) = 0, \quad y(t_s) - y^E(t_s) = 0, \quad z(t_s) - z^E(t_s) = 0,$$

$$(v_x(t_s) - v_x^E(t_s))^2 + (v_y(t_s) - v_y^E(t_s))^2 + (v_z(t_s) - v_z^E(t_s))^2 \le v_\infty^2,$$

$$57023.0\ MJD \le t_s \le 61041.0\ MJD;$$

where $m_i = 4000$ kg — initial mass of spacecraft, $v_\infty \le 5$ km/s — hyperbolic excess velocity.

Delivery of scientific block to the j-asteroid:

$$m(t^{j-}) - m(t^{j+}) = 40\ kg,$$

$$x(t^j) - x^j(t^j) = 0, \quad y(t^j) - y^j(t^j) = 0, \quad z(t^j) - z^j(t^j) = 0,$$

$$v_x(t^j) - v_x^j(t^j) = 0, \quad v_y(t_f^j) - v_y^j(t^j) = 0, \quad v_z(t^j) - v_z^j(t^j) = 0,$$

where t^j — rendezvous moment j-asteroid.

Penetration:

$$x(t_p^j) - x^j(t_p^j) = 0, \quad y(t_p^j) - y^j(t_p^j) = 0, \quad z(t_p^j) - z^j(t_p^j) = 0,$$

$$m(t_p^{j-}) - m(t_p^{j+}) = 1\ kg,$$

$$\sqrt{(v_x(t_p^j) - v_x^j(t_p^j))^2 + (v_y(t_p^j) - v_y^j(t_p^j))^2 + (v_z(t_p^j) - v_z^j(t_p^j))^2} \geq \Delta V_{\min}^A,$$

$$t_p^j > t^j.$$

We should notify that the penetration takes place after the delivery of the scientific block, but it can be done any time after. The distance between the spacecraft and the asteroid at time t^j and t_p^j should not exceed 1000 km. Relative velocity at time t^j should not exceed 1 m/c in case of a rendezvous transfer and should be not less than 0.4 km/s in case of penetration.

The performance index:

$$J = \frac{3}{2}(\alpha_1 + \beta_1) + \sum_{j=2}^{n}(\alpha_j + \beta_j)$$

where n is the total number of asteroids in the list and where $\alpha_j \in \{0, 0.2\}$, $\beta_j \in \{0, 0.8\}$:

$$\alpha_j = \begin{cases} 0.2, & \text{— if rendezvous was fulfilled,} \\ 0, & \text{else.} \end{cases}$$

$$\beta_j = \begin{cases} 0.8, & \alpha_j > 0 \textit{ and } \exists t_p^j \in (t^j, t_f] \\ & t^j \text{— moments of } j\text{-asteroid rendezvous} \\ & t_p^j \text{— moments of } j\text{-asteroid flyby} \\ 0, & \text{else.} \end{cases}$$

The final moment of the mission is the moment of the last action: rendezvous or penetration:

$$t_f = \max_{\exists t_p^j, \exists t^j}(t_p^j, t^j),$$

$$\mathcal{T} = t_f - t_s \leq 5478.75\ ED, \quad m(t_f) \geq 500\ kg.$$

The second performance index:

$$\mathcal{T} = t_f - t_s \to \min.$$