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Dangerous NEO

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# ROBUST OPTIMISATION OF TRAJECTORIES INTERCEPTING DANGEROUS NEO

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## Abstract

In this paper the problem of designing optimal trajectory intercepting and deviating dangerous near earth objects have been solved with a direct optimisation approach. Deviation is obtained thrusting the designated target asteroid using a spacecraft equipped with a nuclear engine, which can provide an high level of thrust with an high specific impulse. Since the ephemeris and the mass of most near earth objects are characterised by a considerable uncertainty a method will be presented to evaluate the sensitivity with respect to poorly known optimisation parameters and to improve the robustness of the solution reducing the sensitivity of the solution with respect to uncertainties. A strategy to intercept and deviate some of the most dangerous Neo is then presented.

## Introduction

One of the issue related to the safety of our planet is the possible danger represented by relatively small celestial bodies like comets or asteroids that can eventually intercept the Earth causing a catastrophe as happen in Tombuska or even worse as what is supposed to be the cause of one of the most famous extinction in the history of the world.

So far, a number of scientist<sup>6</sup> has spent quite an effort to predict the orbit of some potentially dangerous objects, monitoring constantly this objects to ensure that no real danger is foreseen in the near future. However it is of interest to prevent such a catastrophic encounter thinking of a possible strategy of intervention in case a similar eventuality could happen.

One possibility is to deviate the trajectory of the incoming object using some sort of innovative propulsion system. A spacecraft made of a powerful engine with no payload but the engine in itself could

be launched to intercept the incoming object and deviate its trajectory. The engine taken into consideration here is the nuclear powered propulsion system though by Carlo Rubbia and known as 242 (from the atomic number of Americium)<sup>1</sup>.

Beside the actual applicability of this concept in the present or in the near future, intercepting an hazardous object represent an interesting problem from a trajectory design point of view. In particular, if, as in this case, some quantity are not well known, as the behaviour of the engine or the final conditions, due to the uncertainties on the orbital parameters of the incoming object.

Moreover the problem can be seen as a minimum mass problem before the rendezvous with the NEO and then a maximum deviation problem afterwards with an instantaneous increase in mass.

In this paper, the problem is solved with a direct transcription by finite elements in time and a multiphase approach. Furthermore, in order to take into account uncertainties on both the dynamics and boundary conditions the concept of *robustness* optimisation is applied to the resulting nonlinear programming problem.

In fact traditional nonlinear optimisation attempts to find a deterministic optimum of a cost function without taking into account random variation on the objective due to stochastic deviation from expected design parameters. The idea is to find a minimum (in this case the maximum deviation of the incoming object) which is robust with respect to random variations of the design parameters ensuring that the optimal control law found has to be changed slightly in case the dynamics or boundary conditions are effected by an error.

The idea is to trade off between optimality and robustness, in fact it is expected that the optimal solution found taking into account stochastic contribution is not as optimal as the deterministic one but still less sensitive to parameter variations.

The attempt is therefore to optimise at the same time the desired objective function and to minimise the deviation from the predicted value of such optimum.

In this paper two class of uncertainties have been taken into account: standard deviation from the nominal value of the thrust and the specific impulse

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and standard deviation from the predicted value of the orbital parameters and the mass of the incoming object. The resulting effect on the value of the maximum deviation of the trajectory is then minimised.

The proposed approach has been applied to the design of a trajectory intercepting a dangerous NEO. Then some other cases characterised by different kind of incoming objects and different strategies to reach them have been analysed to see the influence of different a different orbit and a different mass on the maximum deviation obtainable.

### Problem Formulation

#### Dynamic Model

A spacecraft is modelled as a point mass subject to the gravity attraction of the Sun and to the thrust provided by one or more low-thrust engines. The motion of the spacecraft is described in the J2000 reference frame centred in the Sun (Figure 1). The three components of the thrust vector  $\mathbf{u}$  represent the control:

$$\dot{\mathbf{r}} = \mathbf{v} \quad (1)$$

$$\dot{\mathbf{v}} = \nabla U(\mathbf{r}) + \nabla U_B(\mathbf{r}) + \frac{\mathbf{u}}{m}$$

$$\dot{m} = -\frac{u}{I_{sp}g_0} \quad (2)$$

where the gravity potential of the Sun is a function of the position vector  $\mathbf{r}$ :

$$U(\mathbf{r}) = \frac{\mu}{|\mathbf{r}|} \quad (3)$$

$$\mathbf{r} = \{r_x, r_y, r_z\}^T$$

and the perturbing effect of a third body is given by the following potential function:

$$U_B(\mathbf{r}) = \mu_B \left( \frac{1}{\mathbf{d}} - \frac{\langle \mathbf{d}, \boldsymbol{\rho} \rangle}{\rho^3} \right) \quad (4)$$

where  $\boldsymbol{\rho}$  is the position vector of the perturbing body with respect to the principal one,  $\mathbf{d} = \mathbf{r} - \boldsymbol{\rho}$  is the position vector of the spacecraft with respect to the perturbing body and  $\mu_B$  is the gravity constant of the perturbing body.

The state and the control vectors are then defined as follows:

$$\mathbf{x} = \{r_x, r_y, r_z, v_x, v_y, v_z, m_p\}^T; \quad (5)$$

$$\mathbf{u} = \{u_x, u_y, u_z\}^T$$

The mass of the spacecraft is divided into propellant mass  $m_p$  and dry mass  $m_D$ . An upper bound  $T_{max}$  and a lower bound  $T_{min}$  was put on the thrust magnitude:

$$T_{min} \leq u = \sqrt{u_x^2 + u_y^2 + u_z^2} \leq T_{max} \quad (6)$$

The upper bound is the maximum level of thrust provided by the selected engine, the lower was taken

$1 \times 10^{-5}$  times  $T_{max}$  to avoid singularities in the Hessian matrix when minimum mass problems are solved.  $I_{sp}$  is the specific impulse of the engine and  $g_0$  the gravity constant on Earth surface.

For the following analyses the maximum thrust foreseen for the Rubbia's engine has been taken equal to 1700 N with an  $I_{sp}$  of 3200s. The resulting dry mass of the spacecraft has been taken to be 30000kg. Now deviation manoeuvre ends when all the propellant has been used and only the dry mass of the spacecraft plus the mass of the asteroid are left, therefore a terminal constraint must be put on final mass.

A further boundary constraint must be put on the mass after interception since a sudden increase of mass due to the rendezvous with the asteroid must be taken into consideration.

Therefore, in general, the trajectory is considered divided in two legs: a transfer leg from the Earth to the asteroid, a deviation leg after interception. Then the two legs, corresponding each one to a phase, are assembled together as explained below.

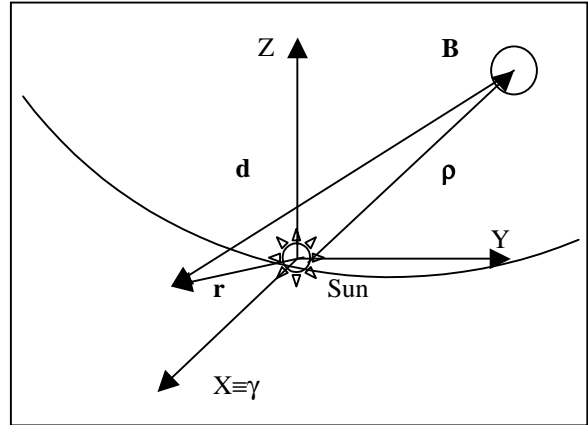


Figure 1. Inertial reference frame centred in the Sun: the xy plane is the ecliptic plane and x axis points toward the 2000 mean vernal equinox.

#### Swing-by Model

The simplest way to model a gravity assist manoeuvre is to resort to link-conic approximation: the sphere of influence of a planet is assumed to have zero radius and the gravity manoeuvre is considered instantaneous. Therefore the instantaneous position vector is not affected by the swing-by:

$$\mathbf{r}_i = \mathbf{r}_o = \mathbf{r}_p \quad (7)$$

where  $\mathbf{r}_i$  is the incoming heliocentric position,  $\mathbf{r}_o$  is the outgoing heliocentric position vector and  $\mathbf{r}_p$  is the planet position vector, all taken at the epoch of the encounter. For an ideal hyperbolic orbit, not subject to perturbations or  $\Delta v$  manoeuvres, the modulus of the incoming relative velocity must be equal to the modulus of the outgoing relative velocity:

$$\tilde{v}_i = \tilde{v}_o \quad (8)$$

Furthermore the outgoing relative velocity vector is rotated, due to gravity, of an angle  $\beta$  with respect to the incoming velocity vector and therefore the following relation must hold:

$$\tilde{\mathbf{v}}_o^T \tilde{\mathbf{v}}_i = -\cos(2\beta) \tilde{v}_i^2 \quad (9)$$

where the angle of rotation of the velocity is defined as:

$$\beta = a \cos\left(\frac{\mu}{\tilde{v}_i^2 \tilde{r}_p + \mu}\right) \quad (10)$$

All quantities with a tilde are relative to the swing-by planet and  $\tilde{r}_p$  is the periapsis radius of the swing-by hyperbola.

### NEO Model

NEO are considered as point masses and no special assumption has been made on their attitude, mass distribution and gravity field. In particular the gravity field of the asteroids selected for the present analysis is so weak that it can be reasonably considered negligible. Ephemeris are computed on the base of mean orbital parameters taken from the state of the art observation of dangerous NEO. The mass has been estimated on the base of the relation between the apparent magnitude of the NEO and the radius, considering mean density of 2000 kg/m<sup>3</sup> and an albedo of 0.5. As shown in figure the uncertainty on the mass is considerable. For present analysis a limited number of NEO among the most dangerous according to the estimated number of probable encounters with the Earth have been taken into consideration.

In table 1 the three selected dangerous NEO are shown along with the most probable values for their mass and mean orbital elements. In table 2 the uncertainties on the orbital elements are reported along with the range of possible diameters that can be computed from the apparent magnitude. From the diameter the corresponding uncertainties on the mass will be computed considering the asteroid as perfect sphere.

It should be noticed that, apart from asteroid SG344 which is the less massive, the orbital parameters of the other asteroids present quite a remarkable uncertainties in particular in the mean anomaly. Since this will cause a significant variation in the state at interception point, terminal conditions for the intercepting leg of the trajectory will be taken as one of the most important source of error for robustness analysis.

On the other hand the wide range of diameters considered for each asteroid implies that the variation in mass can not be considered as a small perturbation in the neighbourhood of the mean expected value. For this reason a parametric analysis of the effect of the mass of the asteroid has been performed using the mass of the asteroid as parameter.

Table 1. List of considered dangerous NEO

ID	SG344	GK	UG
m (kg)	7.1e7	1.5e8	8.8e9
a (AU)	0.978	1.926	1.229
E	0.067	0.598	0.258
i (deg)	0.109	5.608	4.682
$\Omega$ (deg)	192.54	15.22	12.72
$\omega$ (deg)	274.61	111.73	225.84
M (deg)	144.44	27.39	326.58

Table 2. Uncertainties (1- $\sigma$ ) on selected NEO

ID	SG344	GK	UG
diameter (m)	30-70	40-100	190-420
a (AU)	7.62e-7	0.0283	0.00919
e	3.96e-6	0.0031	0.03702
i (deg)	8.65e-6	0.0175	0.5779
$\Omega$ (deg)	0.0016	0.0332	1.177
$\omega$ (deg)	0.0017	1.032	1.596
M (deg)	7.39e-4	10.22	25.56

### Optimisation Approach

#### Transcription Method

A general trajectory design problem can be decomposed in  $M$  phases, each one characterised by a time domain  $D^j$ , with  $j=1, \dots, M$ , a set of  $m$  dynamic variables  $\mathbf{x}$ , a set of  $n$  control variables  $\mathbf{u}$  and a set of  $l$  parameters  $\mathbf{p}$ . Furthermore, each phase  $j$  may have an objective function

$$J^j = \phi^j(\mathbf{x}_0^b, \mathbf{x}_f^b, t_f, \mathbf{p}) + \int_{t_i}^{t_f} L^j(\mathbf{x}, \mathbf{u}, \mathbf{p}) dt \quad (11)$$

a set of dynamic equations

$$\dot{\mathbf{x}} - \mathbf{F}^j(\mathbf{x}, \mathbf{u}, \mathbf{p}, t) = 0 \quad (12)$$

a set of algebraic constraints on states and controls

$$\mathbf{G}^j(\mathbf{x}, \mathbf{u}, \mathbf{p}, t) \geq \mathbf{0} \quad (13)$$

and a set of boundary constraints

$$\psi^j(\mathbf{x}_0^b, \mathbf{x}_f^b, \mathbf{p}, t) \Big|_{t_0}^{t_f} \geq 0 \quad (14)$$

Among boundary constraints a set of inter-phase link constraints exist that are used to assemble all phases together

$$\psi^j(\mathbf{x}_f^b, \mathbf{x}_{j-1}^b, \mathbf{p}, t) \geq 0 \quad (15)$$

This optimal control problem can be efficiently transcribed into a general nonlinear programming problem (NLP)<sup>3,8</sup>, using direct transcription by finite elements in time (DFET)

The general form of a NLP problem can be stated as follow:

$$\min F(\mathbf{y}) \quad (16)$$

subject to

$$\begin{aligned} \mathbf{c}(\mathbf{y}) &\geq 0 \\ \mathbf{b}_l &\leq \mathbf{y} \leq \mathbf{b}_u \end{aligned} \quad (17)$$

where,  $\mathbf{y}=[\mathbf{x},\mathbf{u},t_p,t_f]^T$  is the vector of non-linear variables containing states  $\mathbf{x}$  and controls  $\mathbf{u}$ ,  $F(\mathbf{y})$  is the discrete form of the objective function (11) and  $\mathbf{c}(\mathbf{y})$  are the discrete forms of equations and boundary conditions and

Differential equations governing the dynamics of the spacecraft are transcribed in the following weight residual form

$$\int_{t_i}^{t_f} \left\{ -\dot{\mathbf{w}}^T \mathbf{x} - \mathbf{w}^T \mathbf{f} \right\} dt + \mathbf{w}_f^T \mathbf{x}_f^b - \mathbf{w}_i^T \mathbf{x}_i^b = 0 \quad (18)$$

where  $\mathbf{w}(t)$  are weight functions and  $\mathbf{f}$  contains the right hand side of equation and. Now the time domain is decomposed into finite intervals and on each of them a base of piecewise polynomials  $f$  of order  $p-1$  is used to represent states and controls

$$\begin{Bmatrix} \mathbf{x} \\ \mathbf{u} \end{Bmatrix} = \sum_{s=1}^p f_s(t) \begin{Bmatrix} \mathbf{x}_s \\ \mathbf{u}_s \end{Bmatrix} \quad (19)$$

where  $\mathbf{x}_s$  and  $\mathbf{u}_s$  are collocation points.

Continuous integrals are then developed into Gauss quadrature sum. As a results differential constraints and are transcribed into a set of algebraic constraints of the form:

$$\sum_{i=1}^p \sigma_i \left[ \dot{\mathbf{w}}(\xi_i)^T \mathbf{x}(\xi_i) + \mathbf{w}(\xi_i)^T \mathbf{f}(\xi_i) \frac{\Delta t}{2} \right] - \mathbf{w}_f^T \mathbf{x}_f^b + \mathbf{w}_i^T \mathbf{x}_i^b = 0 \quad (20)$$

where  $\sigma$  are Gauss weights and  $\xi$  are Gauss points and  $\Delta t$  is the length of a time element. Objective function (11) and path constraints are transcribed in the following way:

$$F = \phi(\mathbf{x}^b, t_i, t_f) + \sum_{i=1}^p \sigma_i L[\mathbf{x}(\xi_i), \mathbf{u}(\xi_i), \xi_i] \quad (21)$$

$$G(\mathbf{x}_s, \mathbf{u}_s, t_s) \geq 0 \quad (22)$$

It should be noted that, for the objective function and for path constraints an actual integration is not really needed, this is due to the proper choice of the polynomials, that vanish at integration points. The integration is then reduced to a simple collocation process.

The Jacobian and Hessian matrices of the resulting NLP problem are highly sparse.

In order to introduce swingbys the trajectory is split into phases each one with its own set of constraints and its own objective function. All the phases are then linked together to form a unique NLP problem. Interphase link can either be swingby equations and or a simple set of link constraints:

$$\begin{aligned} \mathbf{x}^b(t_i^{j+1}) &= \mathbf{x}^b(t_f^j) \\ t_i^{j+1} &= t_f^j \end{aligned} \quad (23)$$

which state simply that boundary values at the end of phase  $j$  must be equal to boundary values at the beginning of phase  $j+1$ .

### Sensitivity and Robustness

There are several possible ways to include information about error in the states or in the design parameters in the optimisation approach and different quantities can be minimised to enhance the robustness of the solution found.

In reference 7 it is stated that the robust design requires that the objective function value is in a neighbourhood of the optimal value for all nearby values of the  $x^*$ , where  $x^*$  is the value of the state for which the objective is optimal. This requires the computation of the expectation:

$$\min_x \bar{R}(x) = \int_D f(x + \xi) w(\xi) d\xi \quad (24)$$

where  $D$  is the domain of all possible stochastic aberrations  $\xi$ , and  $w$  is a multidimensional probability density function of the stochastic aberration.

The idea is now to perform a so-called 'proactive' design that is to say to design a trajectory including uncertainties and minimising their effect on the objective function. A way to do it is to solve a bi-objective problem trading between optimality and robustness. Once an index of robustness is defined, this trading-off can be realised minimising the original objective function and an index  $R$  of robustness:

$$\min_x \bar{J} = \beta_1 J + \beta_2 R \quad (25)$$

where  $\beta_1$  and  $\beta_2$  are weighting factors.

Now if the above mentioned definition of robustness is considered and the expectation is used as index of robustness then objective (25) requires the minimisation of both the nominal unperturbed objective function and the expectation.

This approach, however, requires the computation of the multidimensional integral (24) and the introduction of stochastic quantities.

Furthermore since the objective function can be a function of the controls only, a definition which makes use of the aberration in the controls too, and not just of the states, would be good.

In this paper two definitions of robustness are considered. Both of them try to minimise the influence of uncertainties by minimising the derivative of the objective function with respect to an uncertain quantity.

The first definition requires that the objective function remains in a neighbourhood of the optimum for all nearby values of the controls. This can be read in another way saying that for any nearby variation of the quantity that are uncertain the corresponding variation on the controls do not affect the objective function.

Therefore according to this first definition the attempt is to minimise the nominal value of the objective function along with a weighted sum of the derivatives of the objective function with respect to the controls:

$$F_r^1 = F(x, u, p) + \sum_i^n \beta_i \left( \frac{\Delta F}{\Delta u_i} \right)^2 \quad (26)$$

Anyway this does not include explicitly the influence of any uncertain parameters.

Now since, as can be read in Table 2, for present problem most of the uncertainties are related to the final state and since even the objective function is a function of the final state the idea is to minimise the nominal value of the objective function and a weighted sum of the influence of the final state on the controls:

$$F_r^1 = F(x, u, p) + \sum_j^m \sum_i^n \beta_i \left( \frac{\Delta u_i}{\Delta p_j} \right)^2 \quad (27)$$

the resulting solution is expected to be less sensitive to any variation of any of the parameters  $p_j$  since the controls result to be less sensitive or better a small variation of the controls yields a meaningful correction of the uncertain state.

Now the influence of the control parameters on the objective function can be expressed as:

$$\frac{\Delta F(x(u, p), u, p)}{\Delta u} = \frac{\partial F}{\partial u} + \frac{\partial F}{\partial x} \frac{\partial x}{\partial u} \quad (28)$$

The problem now reduces to compute the influence of the each control parameter onto each state but this can be derived from the Jacobian matrix, which is already available at each step from the SQP optimisation.

In fact the set of constraint functions  $\mathbf{C}$  is the implicit relationship that link the controls to any of the states. Differentiating the constraints functions yields:

$$\nabla_u C + \nabla_x C \nabla_u x = 0 \quad (29)$$

Now the gradient of the constraints with respect to the controls can be derived from the Jacobian matrix generated by the DFET transcription, which can be generally expressed as:

$$\mathbf{J} = \begin{bmatrix} \mathbf{U} & \mathbf{X}_i & \mathbf{S} \\ \mathbf{0} & \mathbf{B}_i & \mathbf{B}_f \\ \mathbf{A} & \mathbf{0} & \mathbf{0} \end{bmatrix} \quad (30)$$

where matrix  $\mathbf{U}$  contains the partial derivatives of the controls, matrix  $\mathbf{S}$  contains partial derivatives of the states including terminal values,  $\mathbf{X}_i$  is the matrix containing partial derivatives of initial states while matrix  $\mathbf{A}$  contains partials of inequality constraints on controls and  $\mathbf{B}_i$  and  $\mathbf{B}_f$  are boundary constraints on initial and final states.

Now the variation of the states can be expressed as a function of the variation of the controls:

$$\begin{aligned} \mathbf{S} \Delta \mathbf{x} &= \mathbf{U} \Delta \mathbf{u} \\ \frac{\Delta x_i}{\Delta u_j} &= k_{ij} \end{aligned} \quad (31)$$

where  $k_{ij}$  is an element of the matrix  $\mathbf{K}$  obtained from:

$$\mathbf{K} = -\mathbf{S}^{-1} \mathbf{U} \quad (32)$$

The second definition of robustness does not take into account the controls but just the dynamics considering

the controls frozen. This means that looking at the dynamics only the objective function must remain in a neighbourhood of the optimum for any nearby variation of the states. This translates into the minimisation of the nominal objective function and of a weighted sum of the derivatives of the objective with respect to uncertain parameters:

$$F_r^1 = F(x, u, p) + \sum_j^m \beta_j \left( \frac{\Delta F}{\Delta p_j} \right)^2 \quad (33)$$

the influence of a general parameter  $p$  on the objective function can be evaluated as:

$$\frac{\Delta F(x(u, p), u, p)}{\Delta p} = \frac{\partial F}{\partial p} + \frac{\partial F}{\partial x} \frac{\partial x}{\partial p} \quad (34)$$

where the influence of the state from a general parameter  $p$  can be extracted again from the Jacobian matrix:

$$\nabla_p C + \nabla_x C \nabla_p x = 0 \quad (35)$$

however in this way only the explicit influence of  $p$  onto a given state  $x$  can be computed.

It should be noticed that if  $p$  is an initial state Eq. 35 can be used to compute the influence of the initial state on the final state.

Now starting from the partition of the Jacobian matrix, the variation of state on initial conditions can be computed as:

$$\mathbf{S} \Delta \mathbf{x} = \mathbf{X}_i \Delta \mathbf{x}_i \quad (36)$$

where the vector  $\Delta \mathbf{X}$  is defined as follows:

$$\Delta \mathbf{x} = \{ \Delta \mathbf{x}^i, \Delta \mathbf{x}_f \}^T \quad (37)$$

the two vectors  $\Delta \mathbf{x}_i$  and  $\Delta \mathbf{x}_f$  are perturbations respectively of the initial and final boundary values while  $\Delta \mathbf{x}^i$  are the variations of the internal nodes.

Inverting the matrix  $\mathbf{S}$  (which is an upper triangular matrix) we get:

$$\{ \Delta x_i, \Delta x_f \}^T = \mathbf{S}^{-1} \mathbf{X} \Delta x_i \quad (38)$$

Finally the transition matrix  $\mathbf{T}$  between the initial and the final state can be derived<sup>5</sup>:

$$\Delta \mathbf{x}_f = \mathbf{T} \Delta \mathbf{x}_i \quad (39)$$

Both of the definition given above, basically aim to move the optimal point far away from regions with an high slope, toward regions, of the solutions space, which are smoother.

As a preliminary test case, the two concepts of optimality, above mentioned, have been applied to a simple test case of a point mass moving, in a fixed normalised time equal to 1, from a point A with cartesian coordinates (0,0,0) and velocity (0,0,0) to a point B with cartesian coordinates (1,1,1) and zero terminal velocity.

The dynamics is governed by the following simple differential system:

$$\dot{\mathbf{r}} = \mathbf{v} \quad (40)$$

$$\dot{\mathbf{v}} = \frac{\mathbf{u}}{m} \quad (40)$$

$$\dot{m} = -\frac{u}{I_{sp}g_0} \quad (41)$$

and two different objective functions have been considered:

$$J_1 = -m_f \quad (42)$$

$$J_2 = \int_0^1 u^2 dt \quad (43)$$

where  $m_f$  is the final mass that has to be minimised, while the second objective requires the minimisation of the control.

Now if the first definition of robustness is applied the problem is to minimise either of the two objective functions written above and the weighted sum of the influence of the controls on the terminal state. The result is reported in Fig. 2 for the minimum mass problem and in Fig. 3 for the minimum control problem.

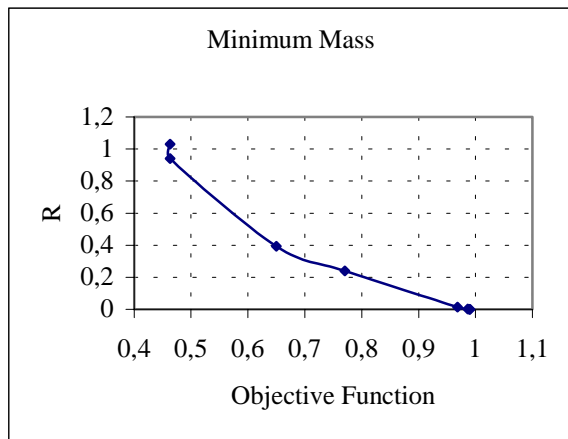


Figure 2. Pareto's optimal region for minimum mass problem

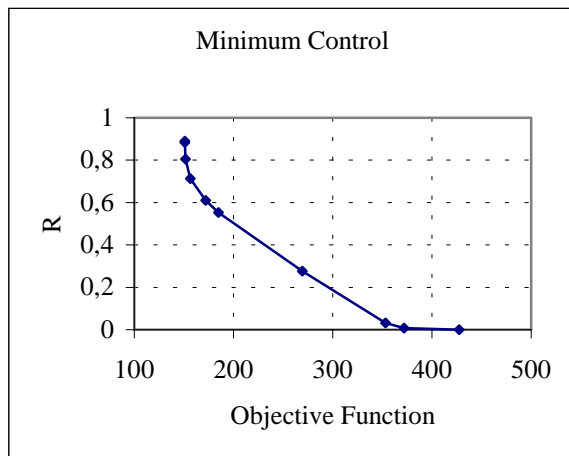


Figure 3. Pareto's optimal region for minimum control problem

From this preliminary analysis on a simple test case it is clear that according to the given definition of robustness the point on the Pareto's curve<sup>14</sup> closest to the utopia point can be considered as a good compromise between robustness and optimality.

It should be noticed that for the minimum mass problem, the solution obtained applying proactive design minimising the influence of the controls on the objective function, corresponds basically to the nominal minimum mass solution.

The same applies conceptually even for the second definition of robustness, therefore in the following both robustness approaches will be applied to analyse the trajectory designed to intercept and deviate the asteroids.

When the value obtained for the two index of robustness allows for a proactive design of the trajectory a new solution will be computed.

### NEO Interception

#### Mission to asteroid 1994GK

The first asteroid selected is the 1994 GK, which is characterised by an apparent magnitude of 24.2, which correspond to an estimated mass  $1.5e8$  kg. However due to the uncertainties in the diameter (as can be seen in Table 2, related to the apparent magnitude the mass is expected to range between  $6.7e7$  and  $10.5e9$  kg. For this asteroid 7 possible impacts are foreseen with the first probable impact foreseen for 2051 at 0.32 Earth radii.

Its high aphelion and low perihelion with the small inclination of 5.6 deg on the ecliptic make it interesting to analyse the impact of the strategy selected to deviate and intercept the asteroid on the maximum deviation obtainable.

First of all a favourable launch date has been searched that allows for different ways to intercept the asteroid maintaining a relative low cost of the transfer since most of the propellant has to be used to deviate the asteroid.

At first a direct transfer that intercept the asteroid close to its aphelion has been investigated. The trajectory depart from a circular orbit around the Earth at 20000 km with an inclination of 52 degrees. Since the acceleration provided by the nuclear engine is quite high just few revolutions are enough to inject the spacecraft into an escape trajectory. The resulting trajectory is represented in Fig. 4 where thrust arc are represented with a solid line while coast arcs are represented with a dashed line.

Propellant consumption is about 18% percent of the initial mass and the obtained escape velocity is of 4.4 km/s. The transfer trajectory, which correspond to a second phase and has been linked to the escape trajectory, has been represented in Fig. 5.

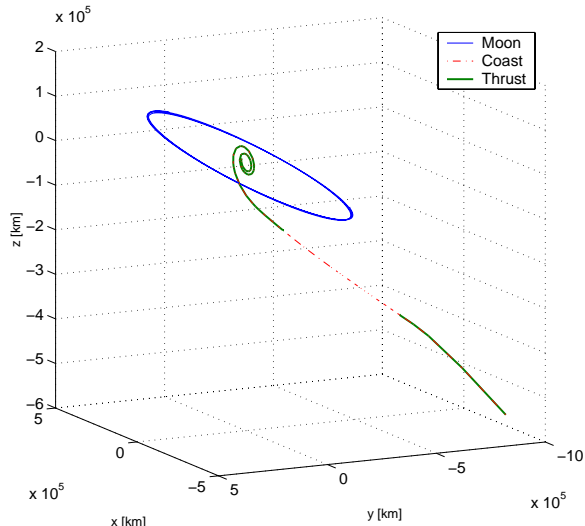


Figure 4. Escape trajectory from the Earth

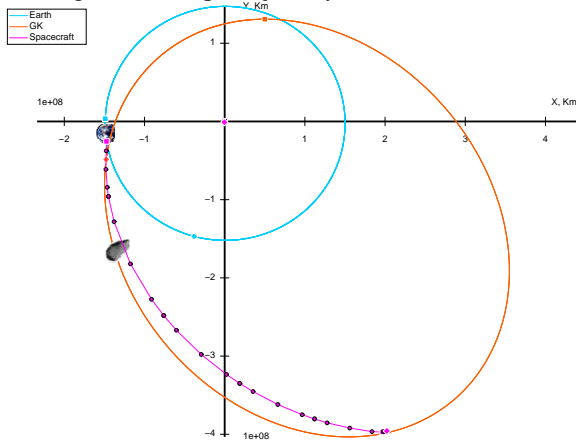


Figure 5. Intercepting Trajectory distant encounter: direct transfer.

In Fig. 6 and 7 the two options via the Earth and via Venus are represented while the close encounter option has been plotted in Fig. 8. In table 3 the four options are summarised and compared.

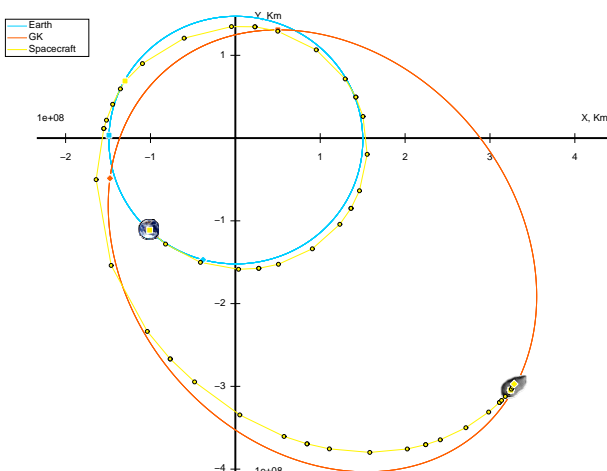


Figure 6. Intercepting trajectory distant encounter via Earth swing-by

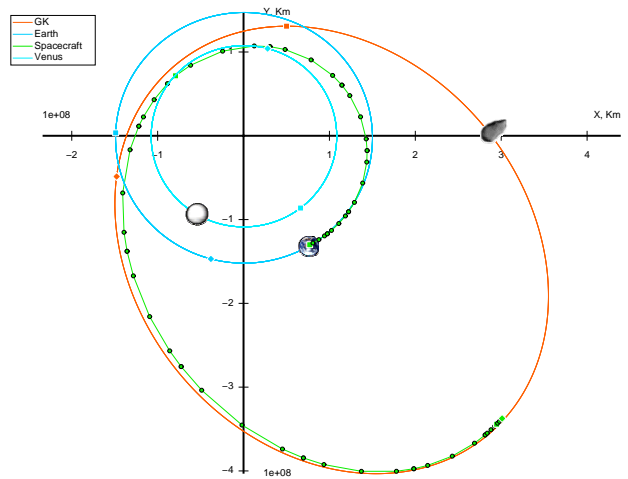


Figure 7. Intercepting Trajectory distant encounter via Venus swing-by

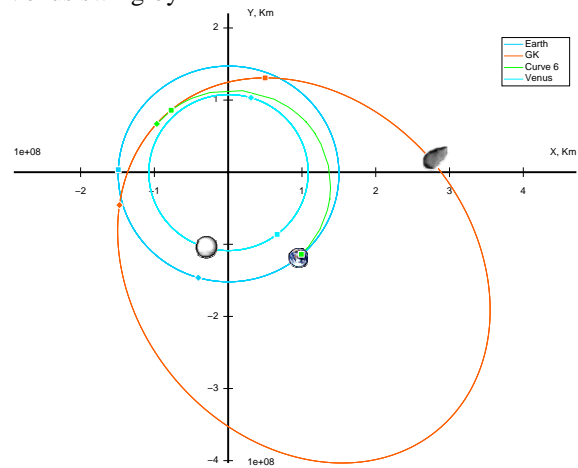


Figure 8 Intercepting Trajectory close encounter: departure

Table 3 Comparison among different options

STRATEGY/VALUE	$\Delta$ (KM)	$\Delta V$ (M/S)	DEPARTURE
Gk direct	459648	5.5	24-03-2011
Gk close	106750	5.421	9-07-2010
Gk Venus	524700	9.792	23-07-2010
Gk Earth	572710	8.364	8-05-2010

Now if the first definition of robustness is applied to the direct transfer, close encounter, information about the sensitivity of the controls with respect to terminal conditions can be derived. Computing the associated index of robustness  $R$  from the Jacobian matrix leads to an interesting result: the partial derivatives of final conditions with respect to controls are remarkably high and not surprisingly tends to decrease to zero as the associated control approaches the terminal state. That is to say that final conditions are greatly affected by a small change in the thrust at the beginning of the transfer and poorly affected by a

change at the end. This holds for position and velocity while final mass results to be poorly affected by all the controls essentially because the Isp of the reduce the influence of a change in the thrust on the final value of the mass. The obtained solution therefore result intrinsically robust to a variation in final conditions with respect to the mass and no proactive design is applied.

In figure 10.the index R of robustness computed using derivatives of terminal displacement, velocity and mass (normalised using transfer period for time and astronomical units for lengths) with respect to controls is represented against each control. As can be seen a small change in the control cause a sensitive change in final position and velocity but not in the mass. On the other hand, as shown in figure 11 if a low thrust engine with the same  $I_{sp}$ , is used for the transfer (providing an acceleration of  $6e-5 \text{ m/s}^2$  against the  $1.7e-2 \text{ m/s}^2$  of Rubbia's engine) the sensitivity of the terminal state to the controls is significantly reduced.

Now if a swingby is introduced along the transfer leg the gradient of final states with respect to the controls before the swing-by can be computed as follows:

$$\frac{\partial x_f(x_o(x_i(u)))}{\partial u} = \frac{\partial x_f}{\partial x_o} \frac{\partial x_o}{\partial x_i} \frac{\partial x_i}{\partial u} \quad (44)$$

where  $x_o$  is the outgoing state vector after the swing-by and  $x_i$  is the incoming state vector before swing-by,i.e. final state of the phase preceding the swing-by. The resulting robustness index R has been plot in figure 12. As expected the swing-by amplifies all the errors in the controls preceding the gravity assist manoeuvre.

Now initial conditions of the following leg of the trajectory starting from the interception and ending with the shut off of the engine, can influence final deviation.

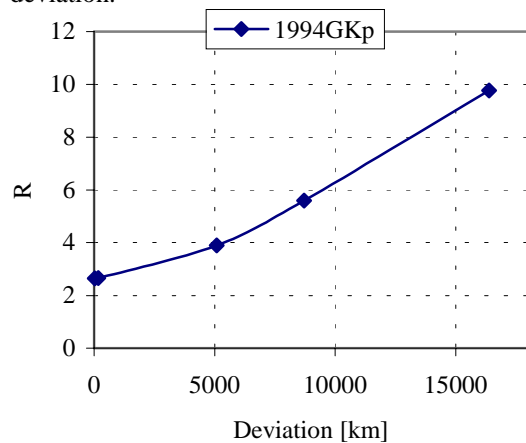


Figure 9. Robustness vs. optimality for the worst case of the GK deviation

Therefore the second definition of robustness has been applied and proactive design used to modified

the trajectory in order to minimise the influence of errors in the initial conditions on final deviation. The result is the trade-off curve plotted in Fig. 9. As can be noticed robustness increase as R decrease leading to a corresponding decrease in the maximum deviation obtainable

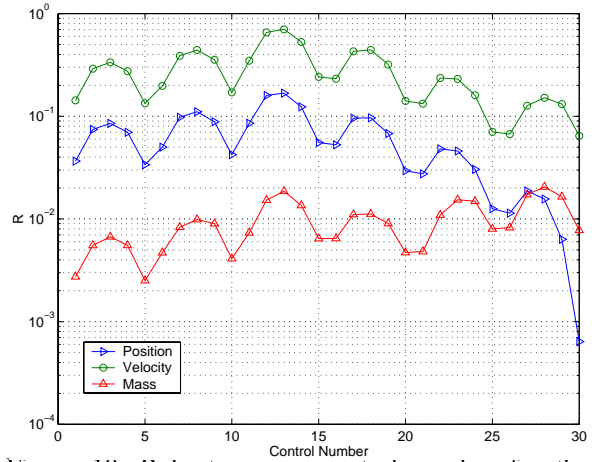


Figure 10. Robustness vs. control number for the direct transfer: GKp case

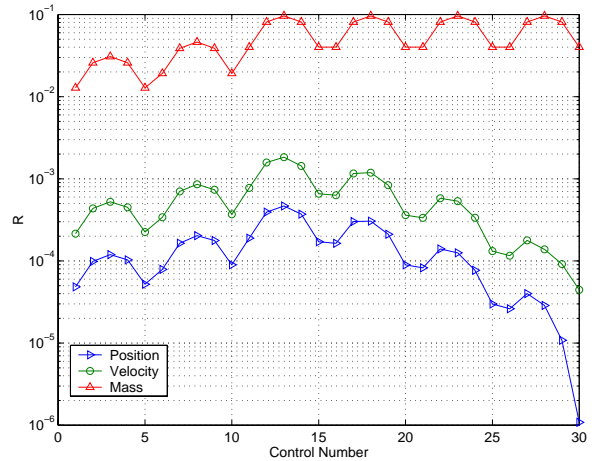


Figure 11. Robustness vs. control number for the direct transfer with a low thrust engine.

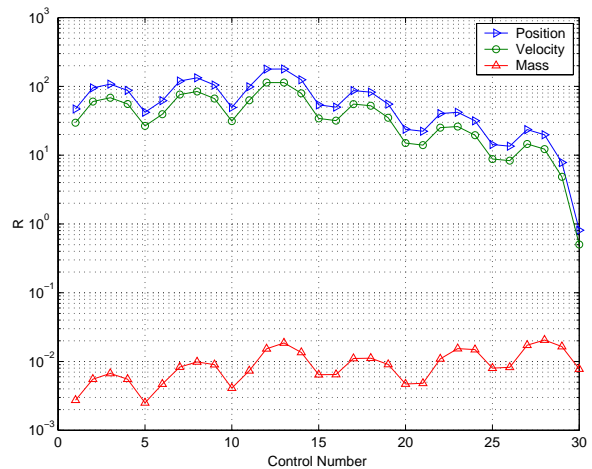


Figure 12. Robustness vs. control number: indirect transfer with Venus swing-by

The improvement in robustness is however not so significant to justify a different design of the trajectory. It is however true that the initial mass was not included in the design parameters, therefore it is expected that and since the reduction in deviation due to an increase in robustness is essentially due to a reduction in the mass available for the deviation the proactive design suggest simply to ad a considerable margin to the propellant mass.

**Mission to Asteroid 1994UG**

The asteroid 1994UG is more massive than the 1994GK according to its absolute magnitude  $H=21.134$ . For the 1994 UG 85 impacts are foreseen with the first one at 0.7 Earth radii the first of April 2091. Its orbit is less eccentric and less inclined than the 1994 GK therefore no special strategies have been considered. However the combination of mass and low perihelion are interesting to investigate the maximum capability of deviating a dangerous asteroid.

A direct transfer has been designed with an encounter in the vicinity of the aphelion. The resulting trajectory has been represented in Fig. 13 for a launch the 9/06/2010. Now due to the ration between Earth revolution period and asteroid revolution period the next favourable conditions for a launch occur after 11 years.

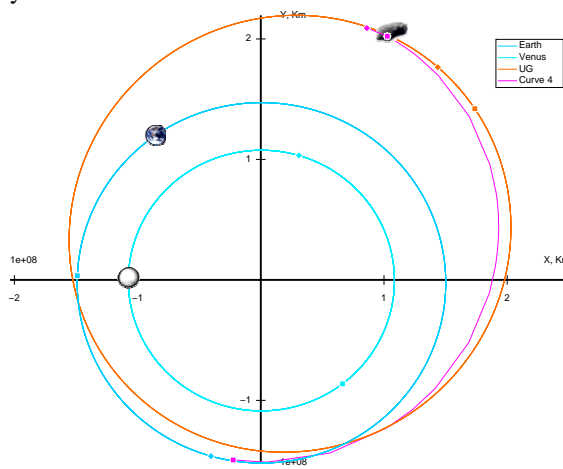


Figure 13. Asteroid 1994UG intercepting Trajectory

**Mission to Asteroid 2000SG344**

The asteroid 2000 SG344 is the less massive of the three selected with its apparent magnitude  $H=24.8$  but its orbit is quite interesting because is remarkably close to Earth's orbit. It is therefore expected that the cost to intercept the 2000SG344 is quite low. This combined with the small mass ranging between  $2.8274e7$  kg and  $3.5919e8$  kg should allow for a huge deviation. For the SG344 69 potential impacts are foresee and the first one should be the 19-09-2068 at

0.93 Earth radii. In figure 14 an intercepting trajectory has been plot for a launch on the 28/11/2016.

It should be noticed that due to the ratio between the revolution period of the asteroid and the revolution period of the Earth the same favourable conditions will occur again after 31 years.

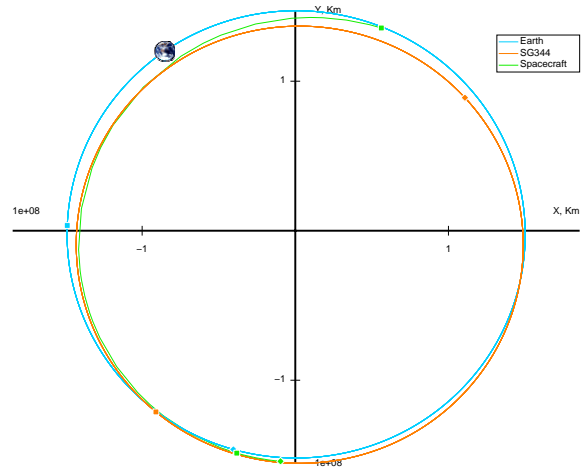


Figure 14. Asteroid SG344, intercepting Trajectory.

**Asteroid Deviation**

Deviation has been defined as the distance between the point of intersection of the orbit, computed after deviation, with a sphere of radius equal to one astronomical unit and the point of intersection of the orbit, before deviation, with the same sphere of radius equal to one astronomical unit.

Since the deviation is a function of the  $\Delta v$  given to the asteroid in Fig. 15 the deviation obtained for all the asteroid considered, for different strategies, has been plotted versus  $\Delta v$ .

Furthermore the value of deviation as function of the mass at launch and of the mass of the asteroid have been reported in Figs. 16 and 17. In particular in Fig. 17 for each one of the asteroids considered in this paper, the maximum value of deviation has been computed for each mass computed with the available range of diameters.

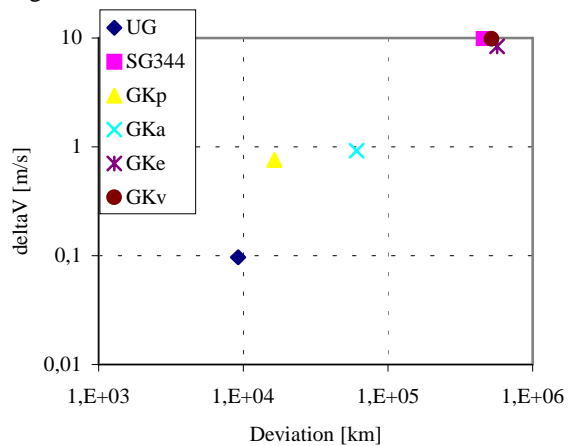


Figure 15. Deviation vs. delta-v.

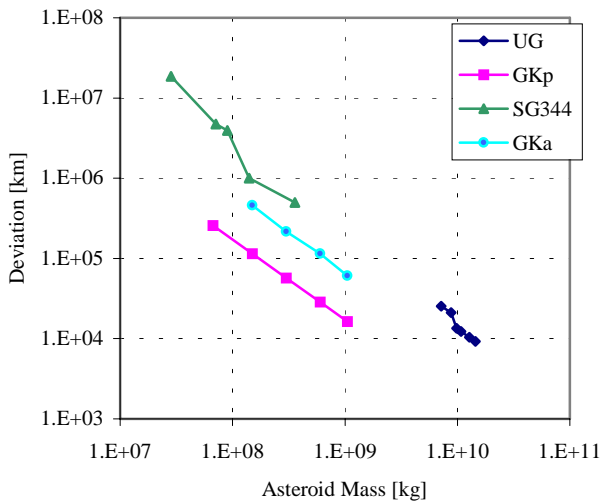


Figure16. Deviation vs. mass at launch

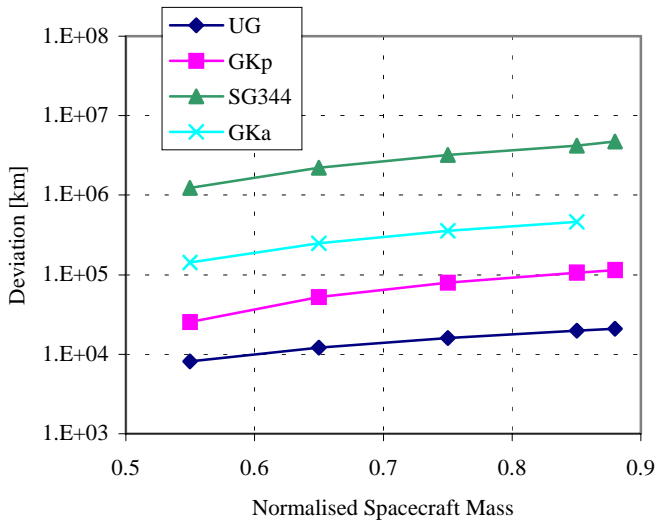


Figure17. Deviation vs. mass of the NEO

### Final Remarks

In this paper the problem of intercepting and deviating dangerous near earth objects has been investigated. Different trajectories have been designed using a direct optimisation approach based on a transcription technique based on finite elements in time. Due to the uncertainties in both the ephemeris and the mass of the asteroids a procedure has been presented to analyse the robustness of the solution with respect to perturbations in the expected value either of the controls or of boundary conditions. The idea is to derive from the Jacobian matrix of the associated nonlinear programming problem the sensitivity of the controls and of boundary conditions on relevant uncertain parameters.

When possible, robustness can be improved minimising the derivative of the objective function with respect to uncertain parameters. Although this

appears to be interesting not just to analyse but even to design a trajectory trading-off between optimality and robustness, a great deal of attention must be put in the computations of the partials. In particular if no second order derivatives of the constraints are available the overall process may result computationally expensive and may introduce a considerable noise in the computation of the gradients.

About asteroid deviation it is remarkable how a limited  $\Delta v$  can produce a consistent deviation when properly applied, however the required mass of the spacecraft increases rapidly with the decrease of the magnitude making the deviation of asteroids with  $H$  lower than 21 quite difficult if their orbit has a low perihelion.

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