A comparison between models of flexible spacecrafts[1]

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Abstract

Flexibility plays an important role in the design of space missions. Algorithms able to derive the dynamical equations for a generic chain of flexible and rigid bodies have been developed in the past decades so that accurate dynamic simulations of large multi-body chains are possible. On the other hand control devices are getting more and more sophisticated so that the guidance of satellite platforms equipped with last generation fly-wheels is a quite complex task. In order to develop control laws, restrictive hypotheses are commonly introduced to make the equation of motion as simple as possible: rigid body dynamic, small angular velocities, small deformations, symmetric appendages bending, reduced degrees of freedom and many other hypotheses lead to as many different mathematical models. Simple modeling leads to fast simulations and allows an easier design of attitude steering laws or vibration suppression controls. In this work a mathematical model describing the full non-linear dynamic of a flexible satellite platform equipped with a system of rigid fly-wheels is developed in an explicit form. The equations are than compared to those obtained by applying standard multi-body analysis. The advantages of having an explicit form of the equations in term of the sole state of the central platform (the state of the wheels is considered as a control) are particularly attractive for control design. The set of Ordinary Differential Equations written returns widely used model when restrictive hypotheses are introduced and has to be used in connection to a preliminary Finite Element Method analysis in order to evaluate the structural invariants.

Introduction

In the future space missions the flexibility of structural elements will certainly become an increasingly important issue. Large solar arrays are being considered for a number of classical and advanced mission concepts. Flexible booms have are already used in interplanetary missions (having RTGs, magnetometers or other payloads on their tip) and for stabilization purposes. Large space structures are continuously proposed in advanced concepts regarding the exploration strategy of our solar system

[1]Professor Chiara Valente died before she could see the final release of this work . We would like to dedicate our work in this paper to her, being a little thank to her invaluable support.

(interplanetary gateways concepts) and beyond. As a consequence multi-body dynamic has attracted the attention of many researchers in the past years. Methods to obtain the set of decoupled non linear equations governing a generic holonomic system made of rigid and deformable bodies have been researched. Both assumed modes methods and finite element methods have been used to model the flexible degrees of freedom these systems. Newton-Euler approach, Lagrange approach and the more recent Kane approach have been used to write the final equations governing the dynamic of the holonomic system. Depending on the complexity of the problem these equations may result to be rather complicated and heavily coupled. Methods and algorithms for the formulation and solution of the equation of motion have been proposed during the last decade and are heavily based on a computer approach. The great interest that Kane method has recently arisen in many researcher is greatly due to the fact that it does not require any awkward method or trick to compute the component of the generalized forces, its whole procedure is quite systematic and therefore well designed for a straight forward computer implementation. Banerjee et al. [1] used a recursive approach and Kane's method to derive the equation of motion of a chain of both flexible and rigid bodies interconnected with hinges. Some results on non linear elasticity behaviour are also available. Recent works by Bajodah et al. [2] and by Meghdari and Fahimi [3] showed how, by properly defining the generalized accelerations, the final system of equations of a generic multi-body system has to be uncoupled. On the other hand, in recent years, many works focused their attention on the design and test of control steering laws for satellites and vibration suppression of flexible modes. These designs require an explicit model representing the dynamical system to control. In all these works a somewhat incomplete model of a spacecraft had therefore to be considered. A satellite equipped with any system of fly-wheels is a good example of a multi-body system for which efficient control laws are extremely desirable. The platform may be considered flexible and the control devices rigid bodies constrained to the main platform. Such a multi-body system is particularly interesting as new fly wheels systems such as Control Moment Gyroscopes (CMG) and Variable Speed Control Moment Gyroscopes (VSCMG) are becoming more and more studied as actuators in aerospace applications. The influence of structural vibrations on the attitude dynamic will be more and more meaningful as these new control devices will appear on the modern satellites. In fact, since these new actuators are able to produce greater torques (and therefore fastyer manoeuvres), they increase the effects due to the nonlinear coupling between the flexible balance and the angular momentum balance. In such a situation it is important to develop an explicit model that can describe in a precise way these effects, keeping the formulation as easy as possible for control design purposes.

Equations of motion

In this section a set of equations able to describe the motion of a satellite platform equipped with flexible appendages and a cluster of Variable Speed Control Moment Gyroscopes is presented. The model applies also to Control Moment Gyroscopes and

Reaction Wheels controlled satellites. The model is mainly taken from [4] and the set of equations is here presented again to correct some typos that affected the quoted paper. The equations of motion for the system under consideration are:

$$\left\{\begin{array}{l} \mathbf{f} = M\mathbf{a} + \dot{\omega}^x \sum \mathbf{L}_i \varepsilon_i + \sum_i \mathbf{L}_i \ddot{\varepsilon}_i + \\ + \omega^x(\omega^x \sum \mathbf{L}_i \varepsilon_i) + 2\omega^x \sum \mathbf{L}_i \dot{\varepsilon}_i \\ \hline \mathbf{J}_T \dot{\omega} + \sum_k Y_{g_k} \ddot{\gamma}_k \mathbf{g}_k + \sum_k I^w_{s_k} \dot{\Omega}_k \mathbf{s}_k + \sum_k I^w_{s_k} \Omega_k \dot{\gamma}_k \mathbf{t}_k + \\ + \sum_i \left(\mathbf{G}_i + \sum_j (\mathbf{H}_{ij} + \mathbf{H}_{ji}) \varepsilon_j \right) \dot{\varepsilon}_i \omega + \\ + \sum_k (Y_{s_k} - Y_{t_k}) \dot{\gamma}_k (\mathbf{t}_k \mathbf{s}_k^T + \mathbf{s}_k \mathbf{t}_k^T) \omega + \omega^x \mathbf{J}_T \omega + \\ + \sum_k Y_{g_k} \dot{\gamma}_k \omega^x \mathbf{g}_k + \sum_k I^w_{s_k} \Omega_k \omega^x \mathbf{s}_k + \sum_i (\mathbf{S}_i - \sum_j \mathbf{P}_{ij} \varepsilon_j) \ddot{\varepsilon}_i + \\ + \omega^x \sum_i (\mathbf{S}_i - \sum_j \mathbf{P}_{ij} \varepsilon_j) \dot{\varepsilon}_i + \sum_i \mathbf{L}_i \varepsilon_i^x \mathbf{a} = \mathbf{g} \\ \hline \rho \ddot{\varepsilon}_i + c_i \dot{\varepsilon}_i + k_i \varepsilon_i = \\ = -\mathbf{L}_i^T \mathbf{a} - (\mathbf{S}_i - \sum \mathbf{P}_{ij} \varepsilon_j)^T \dot{\omega} + 2\omega^T \sum \mathbf{P}_{ij} \dot{\varepsilon}_j \\ + \frac{1}{2} \omega^T \left(\mathbf{G}_i + \sum (\mathbf{H}_{ij} + \mathbf{H}_{ji}) \varepsilon_j \right) \omega - \frac{\mu}{R^3_{\oplus T}} \mathbf{R}_{\oplus T} \cdot \mathbf{L}_i \end{array}\right. \tag{1}$$

to which the dynamical relations for each V.S.C.M.G. (see [5]) may be added:

$$\left\{\begin{array}{l} K_{g_k} = Y_{g_k} (\omega \cdot \mathbf{g}_k + \dot{\gamma}_k) \\ \dot{K}_{g_k} + [(Y_{t_k} - Y_{s_k})(\omega \cdot \mathbf{s}_k) - I_{w_{s_k}} \Omega_k](\omega \cdot \mathbf{t}_k) = G_k \end{array}\right. \tag{2}$$

$$\left\{\begin{array}{l} K_{s_k} = Y_{w_{s_k}} (\omega \cdot \mathbf{s}_k + \Omega_k) \\ \dot{K}_{s_k} = S_k. \end{array}\right. \tag{3}$$

An explication of all the symbols appearing in this model is given in table 1.

The kinematics relations are, in terms of the quaternion:

$$\dot{\mathbf{q}} = \mathbf{G}(\mathbf{q}) \mathbf{q}$$

and

$$\dot{\mathbf{R}}_{\oplus T} = \mathbf{v}_{\oplus T}$$

These complete the total system of equations that describe the motion of the spacecraft in our case.

The decoupled set of equation

We now consider eq.(1), this is a set of coupled 6+2N ODE (where N is the number of assumed modes used). The coupling leads to a greater complexity of the numerical algorithm aimed at solving the system. It is therefore important to decouple the equations in order to get a new set of equation ready to be efficiently solved numerically. In order to do so we write again the complete set of equation in the following form:

$$
\begin{aligned}
& M\mathbf{a} - \mathbf{LE}^x\dot{\omega} + \sum \mathbf{L}_i\ddot{\varepsilon}_i = \Phi \\
& \mathbf{J}_T\dot{\omega} + \mathbf{LE}^x\mathbf{a} + \sum \tilde{\mathbf{S}}_i\ddot{\varepsilon}_i = \Psi \\
& \mathbf{L}_k \cdot \mathbf{a} + \tilde{\mathbf{S}}_\mathbf{k} \cdot \dot{\omega} + \rho_k\ddot{\varepsilon}_k = \Theta_k
\end{aligned}
$$

where $\mathbf{LE}^x$ is the skew symmetric matrix defined by $\mathbf{LE}^x\dot{\omega} = \sum \mathbf{L}_i\varepsilon_i \times \dot{\omega}$ and $\tilde{\mathbf{S}}_\mathbf{k} = \mathbf{S}_k - \sum \mathbf{P}_{kj}\varepsilon_j$. The definition of the remaining new symbols (Φ, Ψ, Θ_k) are as follows:

$$
\begin{aligned}
& \Phi = \mathbf{f} - \omega \times (\omega \times \sum \mathbf{L}_i\varepsilon_i) - 2\omega \times \sum \mathbf{L}_i\dot{\varepsilon}_i \\
& \Psi = \mathbf{g} - \sum Y_{g_j}\ddot{\gamma}_j\mathbf{g}_j - \sum I^w_{s_j}\dot{\Omega}_j\mathbf{s}_j - \sum I^w_{s_j}\Omega_j\dot{\gamma}_j\mathbf{t}_j - \sum(Y_{s_j} - Y_{t_j})\dot{\gamma}_j(\mathbf{t}_j\mathbf{s}_j^T + \mathbf{s}_j\mathbf{t}_j^T)\omega - \\
& - \omega^x\mathbf{J}_T\omega - \sum Y_{g_j}\dot{\gamma}_j\omega^x\mathbf{g}_j - \sum I^w_{s_j}\Omega_j\omega^x\mathbf{s}_j + \\
& - \omega^x \textstyle\sum_i(\mathbf{S}_i - \sum_k \mathbf{P}_{ik}\varepsilon_k)\dot{\varepsilon}_i - \sum_i(\mathbf{G}_i + \sum_k(\mathbf{H}_{ik} + \mathbf{H}_{ki})\varepsilon_k)\varepsilon_i \\
& \Theta_k = -c_k\dot{\varepsilon}_k - k_k\varepsilon_k + 2\omega \cdot \sum \mathbf{P}_{kj}\dot{\varepsilon}_j + \tfrac{1}{2}\omega \cdot \left(\mathbf{G}_k + \sum(\mathbf{H}_{kj} + \mathbf{H}_{jk})\varepsilon_j\right)\omega) + Q_k
\end{aligned}
$$

Inverting this last system we arrive at the final set of explicit ordinary differential equations:

$$
\begin{aligned}
& \mathbf{a} = \left([\mathbf{LL}] + [\mathbf{L\tilde{S}}][\mathbf{SS}][\mathbf{\tilde{S}L}]\right)^{-1} \cdot \\
& \cdot \left\{ [\mathbf{L\tilde{S}}][\mathbf{SS}]\left(\Psi - \sum \frac{\tilde{\mathbf{S}}_i}{\rho_i}\Theta_i\right) + \Phi - \sum \frac{\mathbf{L}_i}{\rho_i}\Theta_i \right\} \\
& \dot{\omega} = [\mathbf{SS}]\left(\Psi - \sum \frac{\tilde{\mathbf{S}}_i}{\rho_i}\Theta_i - [\mathbf{\tilde{S}L}]\mathbf{a}\right) \\
& \ddot{\varepsilon}_k = \frac{1}{\rho_k}\left(\Theta_k - \mathbf{L}_k \cdot \mathbf{a} - \tilde{\mathbf{S}}_k \cdot \dot{\omega}\right)
\end{aligned}
\tag{4}
$$

where:

$$
\begin{aligned}
& [\mathbf{LL}] = M\mathbf{u} - \sum \frac{\mathbf{L}_i\mathbf{L}_i^T}{\rho_i} \\
& [\mathbf{SS}] = \left(\mathbf{J}_T - \sum \frac{\tilde{\mathbf{S}}_i\tilde{\mathbf{S}}_i^T}{\rho_i}\right)^{-1} \\
& [\mathbf{L\tilde{S}}] = \mathbf{LE}^x + \sum \frac{\mathbf{L}_i\tilde{\mathbf{S}}_i^T}{\rho_i} \\
& [\mathbf{\tilde{S}L}] = \mathbf{LE}^x - \sum \frac{\tilde{\mathbf{S}}_i\mathbf{L}_i^T}{\rho_i}
\end{aligned}
\tag{5}
$$

Notwithstanding the complex notation the decoupled set of equation eq.(4) is a simple set of ODE ready to be efficiently solved numerically. Many interesting situations may be studied with the aid of this system, from the control problems of a simple satellite equipped with a three axis control system to the dynamic simulation of a deformable satellite with a complex system of fly-wheels. According to the case, the terms in eq.(1) and therefore in eq.(4) may be considered or neglected.

Comparisons with other models

The problem of writing the equations for a flexible spacecraft has been widely researched in the past. A fundamental contribution has been given by Meirovitch and Quinn [6] who wrote a correct form of these equations. Their solution, though, introduces some integral operators that makes it impossible for the system to be decoupled.

A perturbative technique had therefore to be used by the authors in order to obtain some results. One of the most complete works on multi body dynamic is that of Banerjee et al. [1]. In this paper a set of equations is obtained for multi body chains that takes into account also the terms related to the so-called geometric stiffening effect. These terms are introduced to correct the error that occurres if the equations of motion are derived from a prematurely linearized expression of the velocity and are relevant when non linear elasticity has to be accounted for. The numerical stability of the problem is also increased when these terms are accounted for. In the equations here presented non linear elasticity is not accounted for, good numerical stability properties are though obtained by considering all the higher terms in ε. We now show how equations (1) are equivalent to the algorithm used by Msc ADAMS (a commercially used code in multi-body analysis, see [7]). Let us consider the vector of generalized coordinates ξ for the flexible body as

$$\xi = \begin{bmatrix} x \\ y \\ z \\ \psi \\ \theta \\ \varphi \\ \varepsilon_i, i = 1...N \end{bmatrix}$$

where ψ, θ, φ are the Euler angles used to determine flexible body orientation, x, y, z are the components in a Newtonian reference frame of spacecraft's center of mass and ε_i are the N flexible coordinates. From the velocity of a generic particle belonging to the body it is possible to write the expression of kinetic energy as follows

$$T = \frac{1}{2}\dot{\xi}\mathbf{M}(\xi)\dot{\xi} \tag{6}$$

where $\mathbf{M}(\xi)$ is the generalized $(6+N) \times (6+N)$ mass matrix written below

$$\mathbf{M}(\xi) = \left[\begin{array}{c|c|c} M_{tot}\mathbf{I} & -\mathbf{A}\sum \mathbf{L}_i \varepsilon_i^x \mathbf{B} & \mathbf{L}_i \\ \hline -\mathbf{A}\sum \mathbf{L}_i \varepsilon_i^x \mathbf{B} & -\mathbf{B}^T[\mathbf{J}_{\mathcal{R}} - \sum \mathbf{G}_i \varepsilon_i - \sum\sum \mathbf{H}_{ij}\varepsilon_i\varepsilon_j & -\mathbf{B}^T[\mathbf{S}_i - \sum \mathbf{P}_{ij}\varepsilon_j] \\ \hline \mathbf{L}_i & -\mathbf{B}^T[\mathbf{S}_i - \sum \mathbf{P}_{ij}\varepsilon_j] & \rho\mathbf{I} \end{array}\right] \tag{7}$$

In this last equation $\mathbf{A}$ is the rotation matrix from the body reference frame to the inertial reference frame, $\mathbf{B}$ is the kinematic relation matrix and all the other quantities are those defined in table (1). Substituting $\mathbf{M}(\xi)$ in eq. (6) and recalling the definitions of potential energy it is possible to obtain the equation of motion from a Lagrangian approach:

$$\mathbf{M}\ddot{\xi} + \dot{\mathbf{M}}\dot{\xi} - \frac{1}{2}[\frac{\partial \mathbf{M}}{\partial \xi}\dot{\xi}]\dot{\xi} + \mathbf{K}\xi + \mathbf{f}_g + \mathbf{C}\dot{\xi} = \mathbf{Q}.$$

These last equations are the ones used by Msc Adams [7]. Eq. (1) and therefore eq.(4) are equivalent to this relation if the generalized mass matrix is written for the case of a flexible spacecraft equipped with a generic system of fly-wheels.

We now compare our model with the ones used to study the steering laws of next generation fly-wheels. If the hypotheses of rigid spacecraft is introduced in equations (1) one may obtain the simpler equation:

$$\begin{aligned}&\mathbf{J}_T\dot{\omega}+\sum_k Y_{g_k}\ddot{\gamma}_k\mathbf{g}_k+\sum_k I^w_{s_k}\dot{\Omega}_k\mathbf{s}_k+\sum_k I^w_{s_k}\Omega_k\dot{\gamma}_k\mathbf{t}_k+\\&+\sum_k(Y_{s_k}-Y_{t_k})\dot{\gamma}_k(\mathbf{t}_k\mathbf{s}_k^T+\mathbf{s}_k\mathbf{t}_k^T)\omega+\omega^x\mathbf{J}_T\omega+\\&+\sum_k Y_{g_k}\dot{\gamma}_k\omega^x\mathbf{g}_k+\sum_k I^w_{s_k}\Omega_k\omega^x\mathbf{s}_k=\mathbf{g}.\end{aligned}\tag{8}$$

This equation describes the dynamic of a rigid platform equipped with a system of fly-wheels and may therefore be used for RW systems, CMG systems and VSCMG systems. The equation developed may be compared to the model developed by Tsiotras [8] for a rigid VSCMG system, or equivalently by Shaub [9] or Izzo and Valente [5]. Notational differences apart, those equations are identical to the above one.

Two special flexible spacecraft configurations (see Figure (1)) that have been studied by several authors [10, 11] are now considered. A rigid platform is considered to have one degree of freedom and a number of flexible appendages symmetrically and rigidly mounted with a uniform spacing between them. No fly wheels are mounted on board. If the appendages are supposed to deform in the sole direction orthogonal

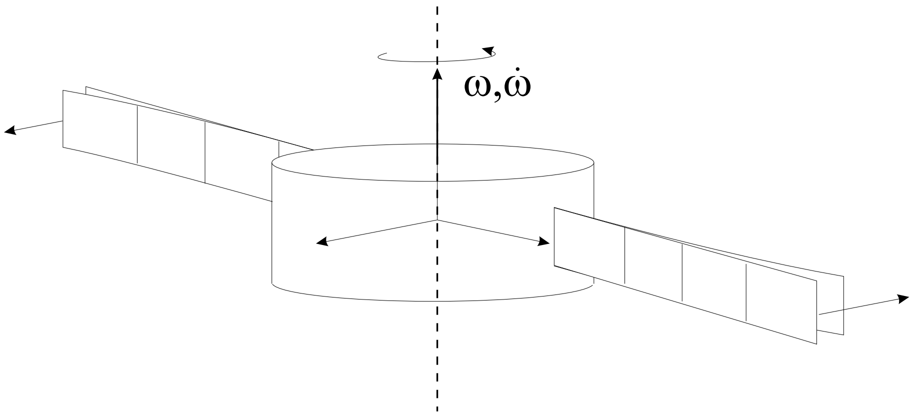

Figure 1: One degree of freedom flexible satellite.

to the angular velocity of the platform and anti-symmetrically the center of mass of the system will always remain located in the same point and therefore $\mathbf{a}=\mathbf{v}=\mathbf{0}$. An immediate effect is that the first equation of 1 may not be considered any more. Assuming normalized modes and constant density ρ the generalized mass density ρ_k coincides numerically with ρ (in the case of beams or shells a linear density or a density per surface unit have to be introduced). Furthermore, if we look at the definition of the vectors $\mathbf{P}_{ij}$, as the appendages are supposed to deform in one sole direction, it is easy to see that $\mathbf{P}_{ij}=\mathbf{0}$. Also the terms in $\mathbf{G}_j\omega$ and $\mathbf{G}_j\dot{\omega}$ are all zero as can be easily verified by direct calculation. If we observe that $\mathbf{S}_j$ is, in this case, parallel to ω and $\dot{\omega}$ we may write again Eq.(1):

$$\begin{array}{l}\mathbf{J}\dot{\omega}+\sum_i \mathbf{S}_i\ddot{\varepsilon}_i=\mathbf{g}\\ \rho\ddot{\varepsilon}_k+c_k\dot{\varepsilon}_k+k_k\varepsilon_k=-\mathbf{S}_k\cdot\dot{\omega}+\frac{1}{2}\omega\cdot\sum(\mathbf{H}_{kj}+\mathbf{H}_{jk})\omega\varepsilon_j\end{array} \tag{9}$$

We have obtained again the model derived in Singh et al.[11] that, introducing the hypothesis of small angular velocities of the platform, becomes the linear system of equations that appeared in the work by Larson and Likins [12] and that has been widely used in subsequent researches:

$$\begin{array}{l}J\ddot{\theta}+\sum_i S_i\ddot{\varepsilon}_i=u\\ \rho\ddot{\varepsilon}_k+c_k\dot{\varepsilon}_k+k_k\varepsilon_k=-S_k\ddot{\theta}\end{array} \tag{10}$$

where u is the external momentum applied along the $\hat{\mathbf{e}}_3$ axis, S_i is the sole component of $\mathbf{S}_i$ and θ is the rotation angle of the platform.

An interesting case is that considered by Di Gennaro [13]. The center of mass dynamic is neglected as if the satellite rigid platform was constrained in a spherical motion around its center of mass in the undeformed configuration. We therefore have to set again $\mathbf{a}=\mathbf{v}=0$. A single reaction wheel is mounted on board, the satellite and its spin velocity will be indicated with Ω while its inertia around the spin axis $\mathcal{F}_b^T\mathbf{s}$ will be indicated with I^w. The angular velocity ω and its derivative is considered to be small. Under these hypothesis the second and the third of Eq.(1) become immediately:

$$\begin{array}{l}\mathbf{J}\dot{\omega}+I^w\dot{\Omega}_j\mathbf{s}+I^w\Omega\omega^x\mathbf{s}+\sum_i \mathbf{S}_i\ddot{\varepsilon}_i=\mathbf{g}\\ \rho\ddot{\varepsilon}_k+c_k\dot{\varepsilon}_k+k_k\varepsilon_k=-\mathbf{S}_k\cdot\dot{\omega}\end{array} \tag{11}$$

These equations, if compared to those used in Ref.[13] present some differences. In particular the terms $\omega^x\mathbf{J}\omega$ and $\sum\omega^x\mathbf{S}_i\dot{\varepsilon}_i$ appear in that work even if, for example, $\frac{1}{2}\omega\mathbf{G}_k\omega$ and $\sum\mathbf{G}_i\dot{\omega}\varepsilon_i$ are neglected.

H. Bang [14] bases his model upon the hypotheses of antisymmetric bending of flexible beams and upon the absence of any external force. Thus he can exclude momentum balance because the coupling between this equation with angular momentum balance and flexible balance disappears. Even though this procedure seems to be attractive, it allows to derive all the structural invariants defined in table 1 only for systems that satisfy the aforementioned hypotheses. The last N equations of eqq.(1) can be easily shown to be coincident to Bang's model when these hypotheses are introduced. To show this we write position of each particle belonging to the flexible spatial domain as:

$$\vec{\mathbf{r}}=\vec{\mathbf{r}}^r+\vec{\mathbf{r}}^d$$

where $\vec{\mathbf{r}}^d(t,\vec{\mathbf{r}}^r)=y(x\vec{\mathbf{b}}_1,t)\vec{\mathbf{b}}_2$ is the beam deflection. The force equilibrium equation for a a beam constrained to bend in a direction only is

$$\rho\mathbf{a}+EI\frac{\partial^4 y}{\partial x^4}=0$$

Substituting in this equation the body axes components of acceleration for each appendage $\vec{\mathbf{a}} = \vec{\mathbf{a}}_{\oplus_T} + \ddot{\vec{\mathbf{r}}}^r + \ddot{\vec{\mathbf{r}}}^d(\vec{\mathbf{r}}^r, t)$ one can obtain:

$$\begin{aligned} &\rho\left(\ddot{\mathbf{r}}^d + \dot{\omega}^x \mathbf{r}^d + 2\omega^x \dot{\mathbf{r}}^d + \omega^x(\omega^x \mathbf{r}^d)\right) = \\ &= \mathbf{f} - \rho\left(\ddot{\mathbf{r}}^r + \dot{\omega}^x \mathbf{r}^r + 2\omega^x \dot{\mathbf{r}}^r + \omega^x(\omega^x \mathbf{r}^r)\right) - \rho \mathbf{a}_{\oplus_T}. \end{aligned} \tag{12}$$

Equation 12 is a Partial Differential Equation that it is time consuming to solve. The hypothesis of assumed modes, not used in Bang's work, would permit to project the flexible balance on the complete basis formed by the the shape functions, obtaining, for this particular case, the flexible balance presented in Eq. (1).

Ford and Hall [15] consider the dynamic of a spacecraft equipped with gimballed momentum wheels and having *N* Euler-Bernoulli beams attached. This appendages are allowed to flex only in a plane which has normal component $\hat{\mathbf{n}}$. The addition of an Euler-Bernoulli appendage to the rigid spacecraft results in the addition of a new set of differential equation to the ones representing the dynamical balances of a rigid spacecraft. Their differential equations, though, require to invert, at each time step, a high dimension matrix which slows down the computation considerably.

Conclusions

An explicit set of equations has been presented to study the dynamic of a flexible satellite platform equipped with a generic system of fly-wheels. A generic spacecraft platform is described through its mass properties, through the geometric disposition of the fly wheels and through some structural invariants. The formulation proposed in this paper introduces the advantage of expressing in a direct way the dependence of spacecraft angular velocity on the spin axis and gimbal axis input torques. The model here presented is shown to be equivalent, though fully explicited, to the one used by commercially available codes such as Msc ADAMS. The model can be considered a trait d'union between two different research fields, one concerning the study of multibody dynamic and one dealing with the design of steering laws for satellite platforms.

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$\mathbf{J}$:	Combined inertia matrix of the satellite platform and the point masses m_j placed in $\oplus_{\mathcal{W}_j}$. The frame to which this matrix is referred is $\mathcal{F}_b$ centered in $\oplus_T$.
$\mathbf{C}_j$:	By definition equals $\mathcal{F}_b \mathcal{F}_j^T$ and is therefore a matrix in which the columns represent the components of gimbal, spin and transverse axis of the j-th VSCMG in the $\mathcal{F}_b$ frame. In a VSCMG system this matrix is time varying, being its components related to the gimbal angles.
$\mathbf{Y}_j$:	Inertia matrix of the j-th gimbal-plus-wheel-structure in the $\mathcal{F}_j$ frame centered in $\oplus_{\mathcal{W}_j}$.
$\mathbf{J}_T$:	$\mathbf{J} + \sum \mathbf{C}_j \mathbf{Y}_j \mathbf{C}_j^T + \sum \mathbf{G}_i \varepsilon_i + \sum_i \sum_k \mathbf{H}_{ik} \varepsilon_i \varepsilon_k$ Total inertia matrix of the system (body axis).
ω:	Components of the angular velocity of $\mathcal{F}_b$ with respect to $\mathcal{F}_i$ in the $\mathcal{F}_b$ frame. This is the quantity we would like to control, being related to the attitude of the satellite.
Y_{g_j}:	Moment of inertia around the gimbal axis of the j-th gimbal-plus-wheel-structure.
$I_{s_j}^w$:	Moment of inertia around the spin axis of the j-th wheel.
γ_j:	Gimbal angle of the j-th VSCMG. This is the angle of rotation of the gimbal with respect to its initial position.
Ω_j:	Spin velocity of the j-th wheel.
$\mathbf{s}_j, \mathbf{g}_j, \mathbf{t}_j$:	Components in the $\mathcal{F}_b$ frame of the vectors $\hat{\mathbf{a}}_s, \hat{\mathbf{a}}_g, \hat{\mathbf{a}}_t$.
$\mathbf{L}_i$:	This is the translational participation factor of the $i-th$ mode projected in the $\mathcal{F}_b$ frame.
$\mathbf{S}_i$:	This is the rotational participation factor of the $i-th$ mode projected in the $\mathcal{F}_b$ frame.
$\mathbf{G}_i$:	$\int_{\mathcal{R}} \rho(2(\vec{\phi}_i \cdot \vec{\mathbf{r}}^r)\vec{\mathbf{1}} - \vec{\phi}_i \vec{\mathbf{r}}^r - \vec{\mathbf{r}}^r \vec{\phi}_i) dV$
$\mathbf{P}_{ik}$:	$\int_{\mathcal{R}} \rho(\vec{\phi}_i \times \vec{\phi}_k) dV$
$\mathbf{H}_{ik}$:	$\int_{\mathcal{R}} \rho((\vec{\phi}_i \cdot \vec{\phi}_k) - \vec{\phi}_i \vec{\phi}_k) dV$
ε_i	This is the $i-th$ flexible coordinate.
$\mathbf{g}$:	Components of the sum of all external torques acting on the system in the $\mathcal{F}_b$ frame.

Table 1: Significance of all the quantities appearing in Eq.(1).