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Dario Izzo

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ON THE DEFLECTION OF POTENTIALLY HAZARDOUS OBJECTS*

Dario Izzo†

More than half a thousand asteroids have already been identified that pose a potential hazard to our planet having orbits with a relatively small distance with respect to the Earth's orbit. Potentially Hazardous Objects are responsible for a number of catastrophic events that have taken place during the history of our planet. The question whether our society has a sufficiently developed technology level to deflect one of such objects is a question that has no easy answer. Many concepts have been proposed to deflect hazardous asteroids, ranging from the exploitation of Yarkovsky effect to attaching ion engines on the asteroid to push it far enough from the Earth, to impacting the asteroid at hypervelocity with a large enough mass. In order to assess the feasibility of all these possibilities, a simple way to evaluate the asteroid miss-distance after some action has been transferred to the asteroid itself is greatly beneficial. In this paper, such an expression is derived and discussed. The expression is found in the framework of simple two-body dynamics and accounts for the eccentricity of the asteroid orbit as well as for a generic deflection strategy. A comparison with a full numerical simulation of the asteroid deflection mechanism reveals that the formula has a high degree of accuracy for a wide range of asteroid orbits and acceleration values.

INTRODUCTION

Near Earth Objects (NEOs) are comets and asteroids that have been pulled by the complex gravitational interaction with the other planets into orbits that allow them to have close encounters with the Earth. The dynamic of these objects is largely dominated by their gravitational interaction with the Sun, even though perturbations are continuously changing their orbital parameters and complex dynamical phenomena may sometimes bring these asteroids to collide with our planet. These events, fortunately rare, have occurred in the past in different locations. The magnitude of the damages created is largely dependant on the asteroid size that, typically for NEOs, is 50m diameter and larger. Due to the uncertainties and errors connected with long term numerical propagation of the trajectories, it is not possible to have deterministic simulations of the asteroid-Earth close encounters and these events have therefore to be described with probabilistic approaches. As a consequence, in the past decade there has been quite an effort to derive concepts and missions to deflect an asteroid foreseen to have an unacceptable impact risk with the Earth. Many of these concepts rely on imparting some ΔV to the asteroid in order to make its orbit change by the necessary amount. The feasibility of any of

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[*] This article contains the theoretical background of the Earth-asteroid miss-distance evaluation used in the ESA Advanced Concepts Team study[1] "Concepts for Near-Earth Asteroid deflection using spacecraft with advanced nuclear and solar electric propulsion systems" presented at the Vancouver International Astronautical Congress in October 2004.

[†] Advanced Concepts Team, ESA\ESTEC, Noordwijk, The Netherlands.

these concepts is heavily related to the “warning time”, that is the time before impact that the risk is predicted and to the asteroid physical properties.

Near Earth Objects are not a discovery of our century, they have indeed always been there and our civilization thrived even when it was not aware of them, it would nevertheless be a major achievement to be able to protect our planet from the unlikely event of a catastrophic collision. The development of the necessary theoretical background needed to study the controlled deflection of massive objects is faced in this work and the insights gained are discussed.

THEORETICAL BACKGROUND

The description of the dynamic of an asteroid to which some deflection strategy is applied is an astrodynamical issue that might be classified in the “extremely low-thrust” class of problems. If we take into account that the mass of an average 150m diameter asteroid with a 2 g/cm^3 density is roughly $3\cdot 10^{10}$ kg we immediately realize that any acceleration artificially induced on the asteroid would be of a very small magnitude and so would be the resulting changes on the orbital parameters. For example if we plan to land on the asteroid and push it with some high specific impulse thrusters, the thrust-to-mass ratio would be of an order of magnitude of 10^{-9} m/s^2 even with an overoptimistic 10 N thrusting capability. For larger asteroids, these values decrease rapidly making it very difficult to think about some deflection strategy. This simple observation is the key to write an analytical expression that solves the asteroid deflection problem. The problem may be formulated as follows. Take two objects in keplerian orbits and suppose that at t_i, the impact time, they occupy the same position in space. Suppose now that one of the objects (from now on the asteroid) is, prior to impact, pushed by a generic external force $\vec{T}(t)$ whose time history, the deflection strategy, is known. As a consequence the minimal distance s between the two objects, the miss-distance, may no longer be zero. We will here determine the relation between the deflection strategy and the miss-distance. The non linearity of the problem, and in particular the gravitational pull of the Sun, prevents the asteroid deflection problem to have a general solution. In order to be able to establish some general relation we will, hence, need to exploit the small magnitude of the action that we may impart to the asteroid and the possibility of imparting this action a long time before the impact takes place. Given these constraints, we may think to change the orbital period of the asteroid. Suppose in-fact that we designed the deflection strategy in order to change other parameters, but not the period, the miss-distance gained would not depend on how in advance we plan to impart the change. On the other hand, if we could change the period of the object, even by a small amount, with a large time in advance, we would still get a finite change in the miss-distance. This basic reasoning convinces us that any small change in the period gets amplified by the time-before-impact we apply it, thus allowing us to conclude that it is, in a realistic asteroid deflection scenario, our main goal. Other strategies are of course viable if we planned our deflection late. In this case, though, it is likely that we could not avoid the collision due to the high ΔV required. Having this in mind we start developing our new expression. We make use of the variation of parameters perturbative technique, and in particular we start from the expression of the energy ε of the orbit of the object we desire to deflect:

$$\varepsilon = -\frac{\mu}{2a} \quad \Rightarrow \quad d\varepsilon = \frac{\mu}{2a^2}da$$

where μ is the gravitational parameter of the central body and a the semi-major axis of the asteroid orbit. We fix the origin of our time axis in t_i and we introduce τ as the time taken by the asteroid to reach the minimum distance with the impact point along its motion. We therefore write, in general:

$$\tau = \frac{\Delta M}{n}$$

where ΔM is the mean anomaly difference between the asteroid position and the minimum distance point, and $n = \sqrt{\frac{\mu}{a^3}}$ is the asteroid orbital mean motion. By differentiating the above relation, we obtain an expression for the change in τ due to some external action:

$$d\tau = -\frac{\Delta M}{n^2} dn + \frac{1}{n} d\Delta M$$

The first term in the above expression takes into account the change that the deflection strategy gives to the orbital mean motion, whereas the second term accounts for changes in the orbit geometry. By doing so we consider the asteroid orbit as a fixed railway on which the asteroid is only allowed to accelerate or decelerate. This hypothesis holds only if the overall real change in the orbital parameters is very small, as small as to move the minimum distance point of a negligible amount compared to overall gained miss-distance due to the mean motion change. This is, as we will show later, the case for what we refer to as a “very low-thrust regime”. We continue writing:

$$d\tau \cong -\frac{\Delta M}{n^2} dn = -\frac{3}{2} \Delta M \sqrt{\frac{a}{\mu}} da = 3 \Delta M \sqrt{\frac{a^3}{\mu}} \frac{a}{\mu} d\varepsilon$$

The expression describes now the effect of a change in the orbital energy on the time that the asteroid will take to reach the intersection between the two orbits. Taking into account that $\Delta M = \tau n \cong (t_s - t)n$ and $d\varepsilon = \vec{v} \cdot \vec{A} dt$ the above expression becomes:

$$d\tau = \frac{3a(t_s - t)}{\mu} \vec{v} \cdot \vec{A} dt$$

where we introduced $\vec{A} = \frac{\vec{T}}{m}$ as the action imparted on the asteroid, t_s as the time prior to the impact we start applying the action, and t as the time-to-impact abscissa. Integrating the above expression between zero and t_p (the push time) we get the finite change in τ:

$$\Delta \tau = \frac{3a}{\mu} \int_0^{t_p} (t_s - t) \vec{v} \cdot \vec{A} dt$$

which tells us how much late, or in advance, the asteroid will reach the impact point. At that point the Earth will no longer be there as it will have moved by an amount equal to:

$$s = \frac{3aV_{Earth}}{\mu}\int_0^{t_p}(t_s - t)\vec{v}\cdot\vec{A}dt \tag{1}$$

We might assume a circular Earth orbit in which case:

$$s = \frac{3a}{\sqrt{\mu R_{Earth}}}\int_0^{t_p}(t_s - t)\vec{v}\cdot\vec{A}dt \tag{2}$$

The distance s we have thus evaluated is visualized in Figure 1, where a zoom of the close encounter is sketched. We now show that this quantity is related to the miss-distance and we write such a relationship.

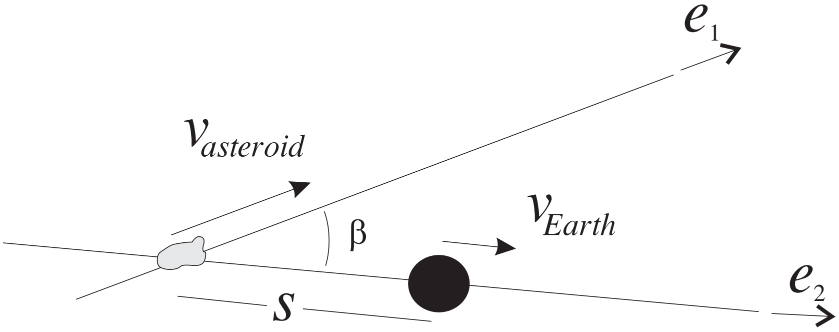

Figure 1: Close encounter geometry

Assume that both the Earth and the asteroid move on rectilinear paths, such as those visualized in Figure 1, in proximity of the close encounter. Introduce the two unit vectors $\hat{e}_1$ and $\hat{e}_2$ and write the position of the asteroid $\vec{r}_{ast}$ and of the Earth $\vec{r}_E$ in the form:

$$\begin{cases} \vec{r}_{ast} = v_{ast}t\hat{e}_1 \\ \vec{r}_E = (v_E t + s)\hat{e}_2 \end{cases}$$

The relative distance between the two massive bodies may be written:

$$d^2 = v_{ast}^2 t^2 + (s + v_E t)^2 - 2v_{ast}t(s + v_E t)\cos\beta$$

where $\cos\beta = \hat{e}_1 \cdot \hat{e}_2$. If we take the derivative and we equal it to zero we find that the value of t in which the distance has a minimum is:

$$t^* = \frac{v_{ast}\cos\beta - v_E}{v_{ast}^2 + v_E^2 - 2v_{ast}v_E\cos\beta}\,s = \varphi\, s$$

Substituting back into the distance expression we get:

$$d_{\min} = \gamma\, s \tag{3}$$

where $\gamma = \sqrt{v_{ast}^2\varphi^2 + (1+v_E\varphi)^2 - 2v_{ast}\varphi(1+\varphi v_E)\cos\beta}$. The value of the parameter γ is given in Table 1 for some PHOs whose argument of perigee has been modified in order to obtain a zero Minimal Orbital Intersection Distance (MOID). As it results from the above expressions it depends on the impact angle β and on the asteroid velocity at the impact. Equation (3), referred to as formula for asteroid deflection, constitute the fundamental result discussed in this paper. This new expression establishes the wanted relation between the miss-distance and the deflection strategy. The assumption on which the expression is based is that the contribution to the variation of the time-to-impact and therefore on the miss-distance is mainly given by a change in the asteroid mean motion. This hypothesis will be verified numerically in the following paragraph when the accuracy of the expression found will be discussed.

Asteroid	γ
(2004 GG21)	.9997
(2004 JA27)	.7416
(2004 HF12)	.9908
(2004 VA)	.9572
Nereus	.7981
Icarus	.653
Apollo	1

Table 1: Some values of γ

As a last remark note that the miss distance here defined does not take into account the final part of the asteroid trajectory, which will be immersed entirely in the Earth gravitational field. This may be considered with a patched conic approximation where the $d_{\min}$ has to be considered as the semi-minor axis of the final hyperbolae. The real closest approach would then take place at the perigee of such an hyperbolae introducing a lensing effect of the planet on the miss distance, see Scheeres and Schweickart[3].

THE ACCURACY OF THE NEW EXPRESSION

Some past work has already been done on asteroid deflection strategies and, in particular, in the work by Scheeres and Schweickart[3],some preliminary considerations upon possible deflection missions are made. These are based upon a miss-distance formula that is there derived for the simpler case of near circular asteroid orbits and tangential thrust. We here show how that expression is a particular case of eq.(2). We consider an asteroid orbit whose semi-major axis equals the Earth orbit radius $a = R_{Earth}$ and a deflection strategy $\vec{A}$ constantly aligned with the velocity vector so that $\vec{v} \cdot \vec{A} = vA$. To be consistent to the notation used in the quoted work we also introduce the coasting time t_c so that $t_s = t_c + t_p$. Writing again eq.(2) we get:

$$s = -\frac{3R_{Earth}}{\sqrt{\mu R_{Earth}}} \int_0^{t_p} (t_c + t_p - t) A \sqrt{\frac{\mu}{R_{Earth}}} d(t_c + t_p - t)$$

and:

$$s = 3A \left| \frac{(t_c + t_p - t)^2}{2} \right|_0^{t_p} = \frac{3}{2} A t_p (t_p + 2t_c)$$

which is the expression derived in the work by Scheeres and Schweickart and there used as a first estimate of the miss distance. We note here that in that work no parameter γ was introduced and that the miss distance was therefore overestimated even in those cases in which the restrictive hypothesis above introduced could be considered to be valid.

In order to assess the accuracy of the obtained results, a numerical campaign was performed on a number of different PHOs impact scenarios. The orbital elements of all PHOs were taken from the Jet Propulsion Laboratories database. For each asteroid an impact scenario was created by changing the asteroid argument of perigee of an amount sufficient to make the MOID vanish. The numerical evaluation of the MOID was based upon the algorithm developed by Bonanno[4]. The intersection created between the two orbits was considered to be an impact point, and the Right Ascension of the Ascending Node was also changed, without loss of generality, in order to make the impact take place in $z = 0, \quad y = 0$. Different thrust-to-mass ratios were tried and different strategies implemented. In particular, a thrust constantly aligned with the velocity vector and a thrust constantly aligned with the $\hat{e}_p$ direction of the orbital frame (the semilatus rectum unit vector) was considered. Simulations were also made accounting for the different power available at different distances from the Sun so that an action imparted by a putative high specific impulse engine powered by solar panels was simulated. The numerical integration was performed taking into account the sole gravity of the Sun and the minimum distance between the Earth and the Asteroid was evaluated during the obtained trajectories. These lengthy simulations were then compared to those obtained by applying the formula for asteroid deflection (3). Some examples of the results obtained are shown in Figure 3. In these particular simulations, the magnitude of the action was set to be $A = 1.57 \cdot 10^{-10} m/s^2$ and the miss distance was evaluated for a wide range of push times t_p and start times t_s. This thrust level has been recently used, in an Advanced Concepts Team study[1], for the design of an asteroid deflection mission. In the first two graphs, the asteroid 2004 GG21 was considered and an aligned with velocity deflection strategy was implemented. The miss-distance difference between the numerical simulation and the asteroid deflection formula is always smaller than 400km. As a result the contour lines are almost overlapped. The accuracy retuned by the novel expression is particularly good if we think to a real deflection scenario in which the achievement of a miss-distance of at least 10000km would be desired. The error with respect to the achievement of this goal would then be within a few percent if the formula for asteroid deflection was used instead of a numerical simulation. The following two graphs refer to the same asteroid but to an inertially fixed deflection strategy. The error is still quite low in the interesting parts of the graph and anyway never greater than 800km. The case of the asteroid 2004 JA27 (γ far from unity) is shown in the last two graphs. In this case, the action considered is aligned with the velocity vector. Investigations on the accuracy of the asteroid deflection formula with higher thrust levels have also been performed revealing that in the range of extremely low thrust-to-mass ratios, i.e. small imparted ΔV, the formula for asteroid deflection is an extremely good approximation. Some of the results returned by this analysis are shown in figure 2, where the deflection of asteroids 2004 GG21 (thrusting aligned to velocity) and 2004 JA27 (thrusting fixed in orbital frame) is considered again for some fixed values of t_s, t_p and for increasingly high thrust-to-mass ratios.

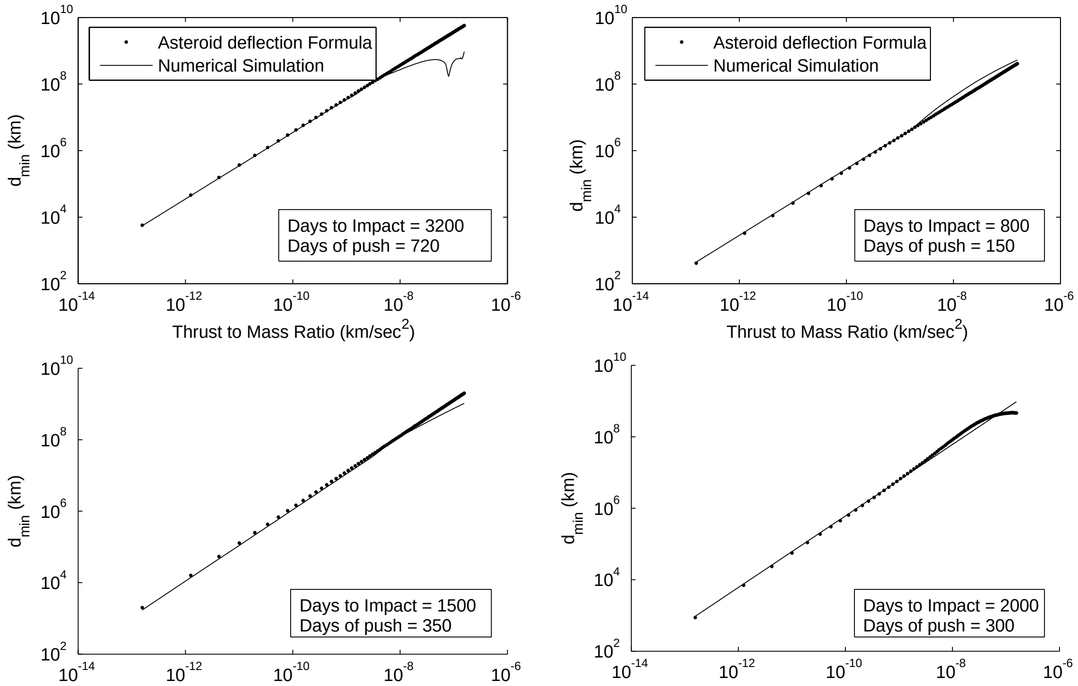

Figure 2:In the left 2004 GG21 asteroid is considered with an aligned to velocity vector strategy, whereas in the right 2004 JA27 is considered with an inertially fixed strategy. The graph show how the asteroid deflection formula returns very good results in the extremely low-thrust-to-mass ratio regime.

The examples here shown are, again, just samples of a more complete testing that showed how Eq.(3) represents the analytical solution to the asteroid deflection problem in the low-thrust-to-mass ratio regime.

DISCUSSION

We here draw some general conclusion on the asteroid deflection using the obtained analytical result.

1. If any deflection strategy is applied N orbital period before ($t_s = \bar{t}_s + NT$), the effect on the miss distance is easily evaluated and it is given by the expression $\Delta d_{\min} = N\gamma \frac{6\pi\sqrt{a^5}}{\mu\sqrt{R_{Earth}}}\Delta E$. The change is therefore proportional to the energy change of the asteroid.

2. Given that any deflection strategy foresee to exert on the asteroid an action $\vec{A}$ for a given duration, it is convenient to exert such an action so that the term $\int_0^{t_p}(t_s - t)\vec{v}\cdot\vec{A}dt$ is maximised. This happens during each orbit, the sooner the better, around the perihelion with a small advance offset due to the term $(t_s - t)$ in the integral.

3. Asteroids with small values of the parameter γ are particularly difficult to deflect. Such a parameter is far from unity whenever the velocity difference between the Earth and the asteroid is high and the angle β between the orbits at the intersection point is small. Thus, an asteroid having its orbit tangent to the Earth's orbit at the impact point ($\gamma = 0$) is very difficult to deflect as the change in mean motion would not be effective and other more expensive strategies would have to be implemented.

4. The thrust components perpendicular to the asteroid velocity have a minor effect on the deflection strategy whenever $\gamma \neq 0$. As a consequence the thrust should therefore be aligned as much as possible to the velocity vector in these cases. In the given assumptions this is in-fact the optimal deflection strategy.

5. The relation between the thrust magnitude and the miss distance gained is linear so that doubling the thrust level result in doubling the obtained miss-distance. This is one of the consequences of the extremely low-thrust regime of the asteroid deflection.

NOTE ON THE YARKOVSKY EFFECT CONCEPT

The temperature difference between the two sides of an orbiting object (asteroids, spacecraft and natural satellites) is the cause of a disturbance called after the Polish scientist who discovered it at the beginning of this century: the Yarkovsky effect. There have been some recent debate on weather this could be used to deflect an incoming asteroid. Some concepts have been proposed to enhance its magnitude as to perturb the asteroid motion of the required amount. We here want to perform a quick assessment of this idea by using our newly discovered eq.(3). We consider a best case scenario in which, after some putative intervention, the asteroid acquires a perfect reflectivity. In this case, the sum between the solar radiation pressure force and the Yarkovsky effect is maximised. We therefore evaluate this force in the case of a spherical asteroid of 150m of diameter getting an overestimate order of magnitude of $\overline{A} = 10^{-14}$ km/sec^2 at the Earth distance. The small magnitude of this force allow us to use eq.(3) to evaluate the effect on the gained miss-distance. We get:

$$d_{\min} = \frac{3a\gamma}{\sqrt{\mu R_{Earth}}} \int_0^{t_s} (t_s - t)\vec{v} \cdot \vec{A} dt$$

being the push time equal to the start time. Given that the direction of the perturbance is aligned with the Earth-Sun vector we now write $\vec{v} \cdot \vec{A} = vA\sin\beta = v\overline{A}\left(\frac{R_{Earth}}{r}\right)^2 \sin\beta$, hence:

$$d_{\min} = 3a\gamma\overline{A}\sqrt{\frac{R_{Earth}^3}{\mu}} \int_0^{t_s} (t_s - t) \frac{v\sin\beta}{r^2} dt$$

After some manipulations, it is possible to show that the above expression might be written in the form:

$$d_{\min} = kN$$

where k is the change in the miss-distance per asteroid orbit and N is the number of revolution along the asteroid orbit we have to start this deflection. The expression for k is:

$$k = 3a\gamma\overline{A}\sqrt{\frac{R_{Earth}^3}{\mu}} \int_0^{T} t \frac{v\sin\beta}{r^2} dt$$

where T is the orbital period of the asteroid.

Asteroid	k (km)
(2004 GG21)	17.05
(2004 JA27)	5.05
(2004 HF12)	16.71
(2004 VA)	11.03
Nereus	3.36
Icarus	14.75
Apollo	4

Table 2: Some values of k

In Table 2 some values of k are given for some asteroids. These numbers allow us to conclude that an exploitation of the Yarkovsky effect in an asteroid deflection scenario is definitely something that would be outperformed by other deflection methods. The explanation is all in the poor phase shift that such a force is able to give to an asteroid. The main player in an asteroid deflection is therefore inhibited by this strategy and the time required to achieve a given miss-distance would thus be unacceptable. In order to make this concept work it would be necessary to move the force direction away from the radial one and bend it towards the velocity vector.

CONCLUSIONS

A general analytical expression to evaluate the relation between an asteroid deflection strategy and the resulting miss-distance is derived and its consequences discussed. The results suggest that the principal effect one would exploit to obtain a deflection by means of some small acceleration continuously imparted to the asteroid is that of maximising the mean motion change in the asteroid orbit. Numerical simulations show that the solution found has a great accuracy in the extremely low thrust to mass ratio regime, which is the regime an asteroid deflection would be likely to be in due to the extremely high asteroids mass.

ACKNOLEDGMENTS

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REFERENCES

1. R.Walker, D.Izzo, C.de Negueruela, L.Summerer, M. Ayre, M. Vasile: “**Concepts for Near-Earth Asteroid deflection using spacecraft with advanced nuclear and solar electric propulsion systems**”, Paper IAC-04-Q.5.08, 55th International Astronautical Federation Congress, Vancouver, Oct.4-8, 2004.

2. D.Izzo, A.Galvez, F.Ongaro, R.Walker: “**Strategies for Near Earth Object Impact Hazard Mitigation**”, Paper AAS 05-147, Copper Mountain, Colorado, January 23-27, 2005.

3. D.J.Scheeres, R.L.Schweickart: “**The Mechanics of Moving Asteroids**”, Paper AIAA 2004-1446, Planetary Defense Conference, Orange County, California, February 24-26, 2004.

4. C.Bonanno: “**An analytical approximation for the MOID and its consequences**”, Astronomy and Astrophysics, 360, 411-416, 2000.

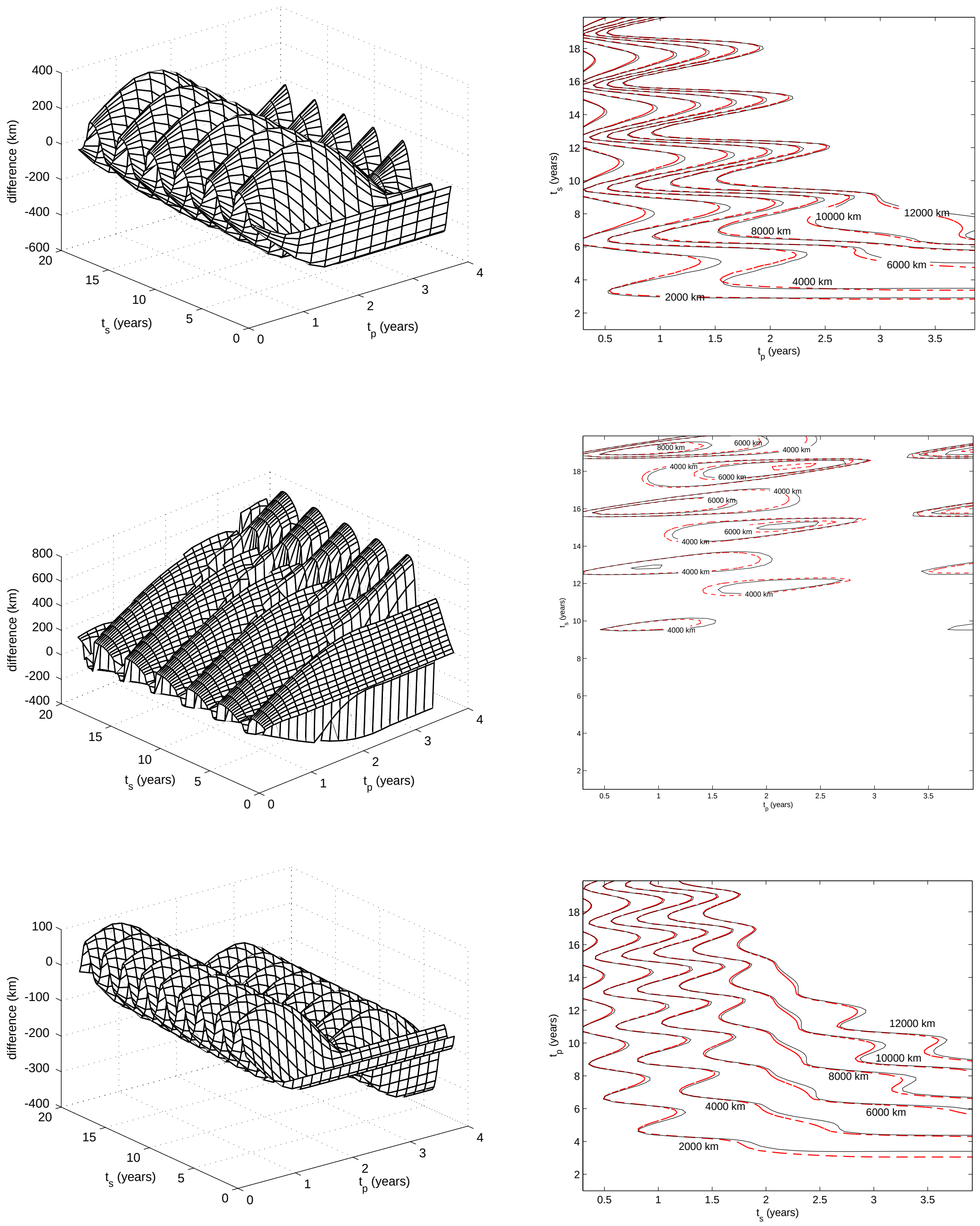

Figure 3: The difference between a full numerical simulation and the use of the asteroid deflection formula. From the top the case of 2004 GG21 (thrust aligned with velocity vector), GG22 (thrust fixed in the inertial space with direction $\hat{e}_p$) and 2004 JA27 (thrust aligned with the velocity vector). The dotted lines in the right graphs (almost overlapped with the continuous ones) represent the analytical expression.