On the Solution of Interplanetary Trajectory Design Problems by Global Optimisation Methods

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Abstract A study of global optimisation methods in the field of interplanetary trajectory has been performed. From the No Free Lunch Theorem, it is impossible that an algorithm outperforms all others in all the possible applications, therefore the aim of this work was two fold: to identify a suitable global optimisation algorithm that outperforms all others in a particular transfer typology; to identify a suitable global optimisation algorithm family that outperforms all others in all mission analysis transfer problems. At first a characterisation of the different transfer families, depending on propulsion system and number of planetary bodies involved was conducted. The model characterisation was performed within the search space to describe the morphological features of the objective function, and within the objective function to identify continuity and convexity. Once the optimisation problem has been fully defined, an exhaustive and systematic analysis of the resulting objective function structure has been performed in order to identify typical features which would mostly affect the global search.

Keywords: mission analysis, trajectory optimisation, global optimisation tools

1. Introduction

In the last two decades, global optimisation approaches have been extensively used towards the solution of complex interplanetary transfers. As operational costs have been increasingly reduced, space systems engineers have been facing the challenging task of maximising the payload-launch mass ratio while still achieving the primary mission goals. Methods ranging from genetic algorithms [1] to neurocontrollers [2], from shooting methods [3] to collocation methods [4] have been used with varying effectiveness. Unfortunately the efficiency, both computational and performance-wise, of these approaches are strongly linked to the type of problem that has to be solved. It would therefore be hugely beneficial if mission designers could rely on a limited number of global optimisation methods depending on the type of trajectory design, which has to be accomplished.

To achieve this ambitious goal, initially, a thorough identification and modelling of the main types of orbital transfers has to be performed. The orbital transfer typologies will be identified both on the basis of the propulsive system (impulsive or low thrust) and on the number of planetary bodies contributing to the dynamics of the system. The models identified previously will then have to be characterised, in order to hopefully identify some common features and recognize different transfer families within the same transfer typology as a function of the parameters of the problem. This will be performed through a two-fold analysis: within

the search space, by means of a topological analysis aiming to identify variables which are useful in the description of the morphological structure of the objective function; within the objective function aiming to identify its structure and evaluating its continuity and convexity characteristics.
Systematic and/or probabilistic methodologies will be used: we first proceed by analysing the main characteristics of common trajectory design problems in mission analysis. In doing this we make use of two simple and basic algorithms: a random start search with SQP local optimisation and a grid search with regular sampling of the objective function. This analysis will contain the seed for the development of the appropriate solution algorithm since the complexity of the problem is intrinsically associated to the solving algorithm.

2. Optimisation Algorithms

Algorithms for global optimisation can be mainly classified in three classes: *stochastic algorithms*, which involve a suitably chosen random sample of points and subsequent manipulation of the sample to find good local minima; *guaranteed algorithms*, which are deterministic algorithms that guarantee the identification of a global optimum with a required accuracy; *metamodel algorithms* that exploit the construction of metamodels, and do not perform the global search on the real objective function, but on a metamodel of it.
Further, stochastic algorithms two main subclasses have been analysed: *Simulated Annealing (SA)*, which performs the global search based on successive update steps, where the update step length is proportional to an arbitrarily set parameter which can play the role of a temperature; *Evolutionary Algorithms (EAs)*, which globally search the solution space by simulating the self-optimising natural process of evolution. *Evolutionary Algorithms (EAs)* can be further divided in three main branches: *Genetic Algorithms (GAs)*, where a wide exploration of the search space and the exploitation of promising areas are ensured by means of the mutation, crossover and selection operators; *Evolutionary Programming (EP)*, whose classical scheme makes use of the only mutation operator and simulate the natural evolution at phenotypic level; *Evolutionary Strategies (ESs)*, which simulate the natural evolution at a phenotypic level, but also make use of recombination operators.
The set chosen embraces classical genetic algorithms including different genetic operators for performing the global search (GAOT and GATBX), genetic algorithms with sharing and migration operators (GAOT-shared and GATBX-migr respectively), evolutionary programming (Fast Evolutionary Programming, FEP), differential evolution (DE), an improved simulated annealing (Adaptive Simulated Annealing, ASA), branching methods (glbSolve and MCS), response surface based optimisation algorithms (rbfSolve) and, an innovative hybrid systematic-heuristic method combing branching techniques and evolutionary programming (EPIC).

3. 2-Impulse Transfers

As an example of a 2-impulse transfer, let us consider a direct transfer from Earth to Mars. The objective function has been taken as the overall impulsive ΔV: the sum of the magnitudes of the relative velocities at the beginning, ΔV_I, and the end, ΔV_F, of the interplanetary transfer phase. The search space is characterized by two design variables: date of departure from Earth, t_0, and Earth-Mars transfer time, tt.
A systematic analysis of the objective function structure allowed the identification of a remarkable quasi-periodicity with respect to the date of departure from Earth, which can be related to the synodic period of the Earth-Mars system.
The best identified solution is characterized by a ΔV value of 5678.904 m/s, corresponding to a transfer time of 203.541 d starting from the 7^{th} June 2003.

Method	ΔV [m/s]	Function evaluations	Runtime [STU]
GAOT	5688.424 (σ = 10.352)	1011.200 (σ = 5.903)	$7.645 \cdot 10^{-3}$ ($\sigma = 2.082 \cdot 10^{-3}$)
GAOT-shared	5993.229 (σ = 246.338)	1011.200 (σ = 9.259)	$7.424 \cdot 10^{-3}$ ($\sigma = 1.706 \cdot 10^{-3}$)
GATBX	5912.194 (σ = 468.795)	1010 (σ = 0)	$5.491 \cdot 10^{-3}$ ($\sigma = 9.684 \cdot 10^{-4}$)
GATBX-migr	5750.769 (σ = 186.124)	1010 (σ = 0)	$4.669 \cdot 10^{-3}$ ($\sigma = 1.664 \cdot 10^{-3}$)
FEP	5751.066 (σ = 185.936)	1027.900 (σ = 16.045)	$9.640 \cdot 10^{-3}$ ($\sigma = 1.156 \cdot 10^{-3}$)
DE	5825.500 (σ = 183.588)	1013.600 (σ = 10.384)	$3.429 \cdot 10^{-3}$ ($\sigma = 2.601 \cdot 10^{-4}$)
ASA	5892.491 (σ = 500.108)	1001 (s = 0)	$2.994 \cdot 10^{-3}$ ($\sigma = 1.543 \cdot 10^{-4}$)
GlbSolve	5931.243	1005	$1.149 \cdot 10^{-3}$
MCS	5678.904	1010	$1.261 \cdot 10^{-2}$
EPIC	5679.100 (σ = 0.579)	1040 (σ = 11)	N/A

Table 1. Summary of results for the two impulse direct planet-to-planet transfer problem.

The performances of each global optimization tool in solving the 2-impulse direct planet-to-planet transfer are now reported. The evaluation criteria is based on the analysis of the optimal solution reached with a fixed number of model function evaluations. Due to the presence of not optimized codes among the tested ones, timing will not be considered as a main evaluation criterion. Table 1 shows the summary of results. It can be seen that all the algorithms reach the main basin of attraction, corresponding to the global minimum. It is interesting to observe the improvement gained by the MCS algorithm compared with the performances of the more classic globSolve tool: MCS and globSolve algorithms have been both inspired by DIRECT method for global optimization [6]; however, unlike the globSolve, MCS uses a branching method which allows for a more irregular splitting procedure. The MCS approach leads to obvious improvements in the effectiveness at identifying the basin of attraction of the best known solution in the 2-impulse direct planet-to-planet transfer problem, making the algorithm performances less dependent on the upper lower bounds, especially referring to design variables associate to objective function periodicities. It is also worth highlighting the effects of the sharing operator on the GAOT performances: by promoting the diversity of the individuals in the population, the GAOT-shared algorithm hinders the concentration of the individuals around the optimal solutions; this can lead to low accuracy at describing the optimum solutions. We can then conclude that, in the simple case of 2-impulse direct planet-to-planet transfer problem, the MCS algorithm have shown to be the best performing one. The same result has been obtained, independently, in a parallel study performed at the University of Reading [8].

4. Low-Thrust Transfers

As an example of a low-thrust transfer, let us consider a direct transfer from Earth to Mars. The thrust level has been supposed to be constant throughout the whole transfer and bounded in intervals corresponding to real thrusters values. The thrust direction during the transfer trajectory is however a design variable and is evaluated by means of azimuth and elevation angles defined in the local horizontal plane. The objective function is assumed to be a weighted sum of several terms: the magnitude of the spacecraft relative position with respect to Mars at the end of the integration of motion, R_F; the magnitude of the spacecraft relative velocity with respect to Mars at the end of the integration of motion, V_F; the propellant mass required by the thrusters for the interplanetary transfer only, m_{prop}. The search space is therefore characterized by sixteen design variables: date of departure from Earth t_0, Earth-Mars transfer time, tt, magnitude of Earth escape velocity, V_E, thrust level u and six parametrization values of the thrust azimuth and elevation during the transfer.
In order to further analyse the structure of the objective function, the distribution of the local minima over the whole search domain has been studied. The analysis of the normalized mean

Method	Objective function	Function evaluations	Runtime [STU]
GAOT	140.301 (σ = 105.016)	80011.500 (σ = 6.451)	20.157 (σ = 2.013)
GAOT-shared	321.426 (σ = 79.997)	80007.600 (σ = 5.379)	38.441 (σ = 2.937)
GATBX	145.060 (σ = 52.383)	80020 (σ = 0)	39.279 (σ = 7.476)
GATBX-migr	119.016 (σ = 74.373)	80020 (σ = 0)	14.934 (σ = 2.1802)
FEP	160.067 (σ = 89.894)	80062.800 (σ = 29.001)	20.888 (σ = 2.096)
DE	87.454 (σ = 15.589)	80020.800 (σ = 11.153)	12.676 (σ = 0.091)
ASA	199.431 (σ = 37.710)	80001 (σ = 0)	12.993 (σ = 1.090)
GlbSolve	154.175	80069	31.902
MCS	330.712	80000	17.7233
RbfSolve	352.787	1000	77.754
EPIC	10.24 (σ = 11.33)	80799 (σ = 16952)	N/A

Table 2. Summary of results for the Earth-Mars low thrust transfer problem.

distance of the local minima and the corresponding objective function values [5] lead to the identification of a big-valley structure, mainly related to the quasi-periodicity with respect to the date of departure from Earth.

The best local minimum found, whose main features can be resumed with a transfer time of 203.541 d, a thrust level of 0.151 N, a propellant mass of 124.59 kg and an Earth escape velocity of 2739.5 m/s, corresponds to an objective function value of 6.37.

The performances of each global optimization tool in solving the low thrust Earth-Mars transfer are reported in Table 2. Results seem to indicate that GAOT and GAOT-shared tools, as well as the non randomized algorithms glbSolve, MCS and rbfSolve are not suitable for global optimisation of low-thrust direct planet-to-planet transfer problems using the mathematical models here employed. Among the remaining tools, DE, and GATBX-migr showed good performances in a Pareto optimal sense: in particular, DE and GATBX-migr resulted in similar, even though low, rate of success in identifying the basin of attraction of the global minimum. However, by considering that the rate of success is evaluated by performing local optimisation processes requiring similar further objective function evaluations and by noting that DE reaches a lower, and less fluctuating, value for the objective function, the DE tool seems to be preferable with respect to GATBX-migr.

5. Low-Energy Transfers

The possibility of designing low energy lunar space trajectories exploiting more than one gravitational attraction is now investigated. In particular, the framework of the Restricted Three-Body Problem (R3BP) is here analysed and lunar transfers are studied which take advantage of the dynamic of the corresponding libration points. The interior stable manifold associated to the libration point L1 in the Earth-Moon system, W^s_{L1} , is propagated backward for an interval of time t_W. Corresponding to W^S_{L1}, the exterior unstable manifold, W^U_{L1} , can be evaluated. The manifolds W^S_{L1} and W^U_{L1} constitute in fact a transit orbit between the forbidden region through the corresponding thin transit region. However, the manifold W^S_{L1} does not reach low distances from Earth. To solve this problem, starting from a circular orbit around the Earth, an arc resulting from the solution of a Lambert's three-body problem is used for targeting a point on the stable manifold. It is worth noting that such an approach leads to a final unstable orbit around the Moon with mean altitude equal to 21600 km.

As a consequence of the previously described formulation, a first impulsive manoeuvre, ΔV_1, is used to put the spacecraft in the Lambert's three-body arc from the initial circular orbit around the Earth. A second impulsive manoeuvre, ΔV_2, is performed to inject the spacecraft on the capture trajectory W^S_{L1}. Hence, the sum of the two impulsive manoeuvres, ΔV, is chosen to be the objective function for the optimisation processes. As a consequence the search

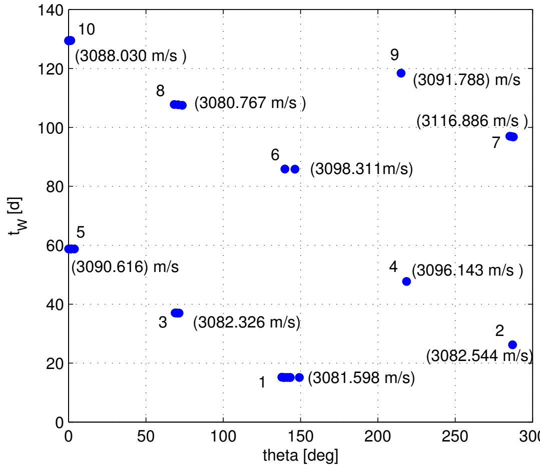

Figure 1. Families of Lunar transfers comparable with the best identified one (subgroup 8).

space is characterized by the following design variables: the angle identifying the starting point over the initial circular orbit, θ; the time of the backward propagation of the stable manifold, t_W, and the transfer time corresponding to the Lambert's three-body arc, t_L.
The best solution identified is characterized by an overall transfer time of 108.943 d and corresponds to an objective function value of 3080.767 m/s.
A careful analysis of the distribution of the local minima over the search space lead to the identification of several sets of local optimal solutions, comparable with the best identified one in terms of objective function values, which are characterized by similar values of the time spent on the three-body Lambert's arc, t_L. The presence of such subgroups can be related to the presence of big valley structures deriving from the periodicity of the objective function on t_W. We can state that such subgroups describe a set of different families of Lunar transfers where the term "family" is referred to solutions lying on different niches on the search space, as defined in [7]. In particular ten local minima groupings can be isolated over the analysed search space, as shown in Figure 1.

The performances of each global optimization tool in solving the low-energy transfer problem are reported in Table 3. The results allow us to infer the following. The stochastic algorithms GAOT-shared, GATBX-migr and DE and the non randomized codes glbSolve and rbfSolve, cannot be considered as suitable for solving the previously identified problem, due to their inability in identifying basin of attractions corresponding to either the best known solution or the comparable ones. GAOT, GATBX and ASA identify the basin of attraction of good solutions but present relatively large standard deviations, implying that not always the attraction basins are identified successfully. As a consequence MCS and FEP turn out to be the best performing tools for the problem of lunar transfer using libration points.

6. Summary

The aim of this study was an investigation of the effectiveness of some global optimisation techniques at solving practical problems related to space trajectory design. Following a complete and comprehensive objective function structure analysis a set of global optimisation tools has been selected for testing purposes. By considering the objective function value reached at the end of the optimisation process, the number of objective function evaluations performed required and the effectiveness at identifying the basin of attraction of the best known solution as well as of good solutions comparable to the best known one, results of

Method	ΔV	Function evaluations	Runtime [STU]
GAOT	3160.307 (σ = 74.987)	5012.200 (σ = 4.638)	24.788 (σ = 2.093)
GAOT-shared	3321.952 (σ = 101.235)	5010.900 (σ = 6.657)	20.197 (σ = 2.488)
GATBX	3158.640 (σ = 63.572)	5010 (σ = 0)	38.556 (σ = 4.180)
GATBX-migr	3218.439 (σ = 127.349)	5010 (σ = 0)	35.895 (σ = 5.012)
FEP	3134.626 (σ = 60.928)	5017.200 (σ = 15.640)	31.578 (σ = 5.232)
DE	3233.064 (σ = 94.020)	5019.600 (σ = 10.276)	16.106 (σ = 0.428)
ASA	3194.447 (σ = 109.524)	4783.600 (s = 58.971)	37.544 (σ = 5.110)
GlbSolve	3343.104	5025	21.640
MCS	3148.107	5010	32.575
RbfSolve	3579.249	474	6.128

Table 3. Summary of results for the low-energy transfer problem.

the test phase can be resumed as follows.

Two impulse direct planet-to-planet transfer problem: due to its deterministic features, the success at reaching the best known solution and the corresponding relatively low number of required objective function evaluations, Multilevel Coordinate Search (MCS) turned out to outperform all the remaining algorithms, thus resulting as the best performing one.

Low thrust direct planet-to-planet transfer problem: due to the highly complex nature of the search space, low rate of success characterized all the tested algorithms at identifying the basins of attraction of both the best known solution and solutions comparable to it. In such an environment in particular, DE and GATBX-migr resulted in similar, even though low, rate of success in identifying the basin of attraction of the global minimum. However, by noting that DE reaches a lower, and less fluctuating, value for the objective function, the DE tool seems to be preferable with respect to GATBX-migr.

Low energy Lunar transfer problem: the structure of the objective function and the search space results in comparable performances for the majority of global optimisation algorithms used. However, by taking into account the minimum value of the objective function reached, and the standard deviation, we can identify MCS and FEP as the best performing tools for the problem of lunar transfer using libration points.

It is worth noting that limitations affects the achieved results. First of all, each mission analysis class has been investigated by selecting a particular transfer problem and by facing it with proper, but anyway particular, mathematical models. Further analyses should be performed, including additional transfer problems, alternative mathematical models and search space definitions. Secondly, it is widely known that optimisation algorithms can be suitably tuned to enhance their performances. However, as already occurred in remarkable existing comparative studies [7], due to the comparative purposes of this work, the large scale of comparisons performed, the available devices and the high time required by some optimisation case, it was impossible to do such tuning.

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