Optimal Trajectories for the Impulsive Deflection of Near Earth Objects

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Abstract

With the current technological level it is be possible to deflect an asteroid such as 99942 Apophis in case its orbit refinement reveals an unacceptable collision risk. Thanks to the solution of an ad hoc defined global optimisation problem we show that a pre-keyhole deflection is possible even with a very simple spacecraft and with a very late launch in 2026. We also give a mathematical description of the trajectory optimisation problems that have to be faced in the case of a more generic Asteroid Deflection missions defining analytically the objective function, that is the displacement in the encounter b-plane of the asteroid uncertainty ellipsoid.

Introduction

After its detection in December 2004, the asteroid labeled 2004 MN4 and later renamed "99942 Apophis" has attracted the interest of many astronomers and scientists due its unique characteristics. In the early days of its detection an astonishing 1/37 chance was predicted for the catastrophic event of an impact with the Earth during its 2029 close approach. Such an enormous risk focused the attention of many astronomers on the small rock (roughly 400m of diameter) and further data were made available and served to exclude, a few weeks after the asteroid discovery, the 2029 impact. With the current knowledge, it still may not be excluded though, that the close Earth encounter will change the asteroid orbit and make a resonant return in 2036, thus impacting the Earth. Although the chance currently foreseen is quite low and comparable to the background probability level of possible impacts with the NEO population, the case of Apophis is interesting as it may teach us a lot on how and when decisions should be taken to design and build an asteroid deflection mission. When 99942 Apophis was discovered many scientific studies were available on asteroid deflection methods and many results had already been published. Scheerers and Schweickart [1] studied the feasibility of a deflection strategy mission based on a long duration thrust applied to the asteroid by a spaceship powered by nuclear and able to land on its pole, reorient the asteroid spin axis and push with its high specific impulse plasma thrusters. Walker et al. [2] considered a similar concept using different advanced propulsion options, including a solar powered case, and performed an in depth system design of the resulting spacecraft. McInnes [3] proposed a solar sail mission able to optimally exploit a kinetic impactor deflection by inserting the spacecraft in a retrograde orbit with a cranking manoeuvre made possible by an advanced solar sail and a long permanence of the spacecraft at a close distance from the Sun. Izzo et al. [4] performed a trade-off between the kinetic impactor deflection and the long duration thrust deflection, showing that the former is able of achieving better performances in terms of deflection achieved with an optimised dynamic. This last result was obtained in the specific case of the asteroid 2003 GG21 whose orbit was modified to create an impact scenario and has not been extended for the general case even though the authors feel that it serves as a good indication on the different deflection strategies efficiency.

In this paper we discuss the trajectory optimisation of a generic asteroid deflection mission. Introducing an analytical expression for the objective function we are able to formulate the problem as a global optimisation problem, both in the case of a chemical propelled spacecraft and in the case of a low-thrust electric propulsion kinetic impactor. We are therefore able to optimise at the same time the impact geometry, the terminal relative velocity magnitude, the spacecraft mass and the impact time without performing any direct orbit propagation. We then consider, as a fictitious case, the pre-keyhole deflection of 99942 Apophis and we assess the performances of a small deflection mission inspired by the current baseline of the ESA Don Quijote spacecraft, as designed by ESA Concurrent Design Facility[5].

Resonant returns and keyholes

When an asteroid makes a close encounter with the Earth its orbit gets modified and, in some cases, it might come back and make a second encounter. Such events are known as resonant returns and are of particular interest as they may also result in the asteroid impacting the Earth during its second encounter. It is therefore important to locate the regions of the first encounter b-plane that, if crossed by the asteroid, result in a resonant return. Under the hypothesis of a keplerian motion between the two consecutive encounters, these regions may be described analytically, as it has been shown by Valsecchi et al. [6], and are called resonant circles. We introduce the (ξ, η, ζ) planetocentric reference frame used by Valsecchi et al. [6]. This coordinate system has the ξ, ζ coordinates on the b-plane and the η axis directed along the relative velocity vector at encounter. The axis ζ has the opposite direction of the projection on the b-plane of the planet's heliocentric velocity. In this coordinate system we read the asteroid Minimal Orbit Intersection Distance (MOID), (the minimal distance between the Earth orbit and the asteroid orbit) on the ξ axis and the asteroid phase (proportionally related to its mean anomaly) on the ζ axis. This is particularly convenient for our purposes: it is known that any optimal deflection strategy is aimed at modifying mainly the asteroid phase (see Izzo [7] for discussion) moving therefore the corresponding point in the b-plane mainly along its ζ axis.

We may regard the b-plane at the first encounter as a dart board and the incoming asteroid as the dart. A hit in its center (representing the Earth focused by its gravity) will result in an impact; a hit along a resonant circle will lead to a second encounter with the dart coming back and hitting the board again in the very same point. The zones of the board that lead the dart to return and hit the board in the center are, instead, called *keyholes*. Although an analytical theory has been proposed by Valsecchi et al. [6] a precise determination of their position and shape is more difficult and has to make use of precise and complex orbital propagators with accurate force models. As an example we reported in Figure 1 the b-plane of 99942 Apophis close encounter with the Earth in April 2029. In the Figure the "Dart Board" center is the Earth focused by its gravity and the keyhole corresponding to the 6/7 resonance is shown as evaluated by the analytical theory and by direct numerical simulation [8].

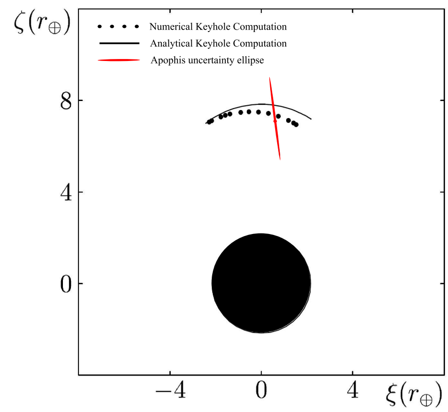

Figure 1: The b-plane of Apophis 2029 Earth encounter. The keyhole is shown as evaluated by the analytical theory and by a numerical situation. Apophis image on the b-plane is also shown as evaluated from the data that were available in June 2005. The image is a courtesy of Dr. Giovanni Valsecchi (INAF-IASF Italy)

B-plane representation of a deflection

When a new asteroid is discovered, the orbital parameters $\boldsymbol{\alpha}^*$ that are made available by the astronomers are the best fit of the available observations, in the sense of minimizing a certain function Q of the squared residuals. Writing Q as a function of the orbital parameters, we may write the following expansion:

$$Q(\boldsymbol{\alpha}) = Q(\boldsymbol{\alpha}^*) + \frac{1}{2}(\boldsymbol{\alpha} - \boldsymbol{\alpha}^*)\mathbf{H}(\boldsymbol{\alpha} - \boldsymbol{\alpha}^*) + ...$$

or its more useful statistical interpretation:

$$Q(\boldsymbol{\alpha}) = Q(\boldsymbol{\alpha}^*) + \frac{1}{m}(\boldsymbol{\alpha} - \boldsymbol{\alpha}^*)\boldsymbol{\Sigma}^{-1}(\boldsymbol{\alpha} - \boldsymbol{\alpha}^*) + ...$$

where $\boldsymbol{\Sigma}$ is the covariance matrix and m is the number of observations available. Hence we may define a neighborhood of the best fit $\boldsymbol{\alpha}^*$ by requiring that:

$$(\boldsymbol{\alpha} - \boldsymbol{\alpha}^*)\boldsymbol{\Sigma}^{-1}(\boldsymbol{\alpha} - \boldsymbol{\alpha}^*) < \chi^2$$

All the asteroid having elements in the ellipsoid defined by the above relation are called virtual asteroids [9]. The propagation of such an ellipsoid defines the set Θ of all the possible positions occupied by the virtual asteroids. A discussion on the accuracy of this definition of the virtual asteroids may be found in Milani and Valsecchi [10]. Consider now the encounter b-plane of the best fit $\boldsymbol{\alpha}^*$ and define the set $\Upsilon = \{\mathbf{X}|\mathbf{X} \in \Theta\}$ belonging to such a plane. In this section we study the effect on the set Υ of the application of an optimal deflection strategy to each of the virtual asteroids. It has been already shown, under certain general assumptions [7], that the phase shift of a given asteroid can be expressed as:

$$\Delta\tau = \frac{3a}{\mu}\int_0^{t_s}(t_s - t)\mathbf{v}\cdot\mathbf{A}(t)dt$$

where a is the asteroid orbit semimajor axis, μ is the Sun gravitational constant, t_s is the time-to-encounter at the deflection start $\mathbf{A}$ is the instantaneous momentum change applied per unit mass and $\mathbf{v}$ is the asteroid velocity. Such a phase shift corresponds, on the coordinate system defined on our b-plane to a change in the sole ζ coordinate given by:

$$\Delta\zeta = V_{ast}\Delta\tau\sin\psi = V_E\Delta\tau\sin\theta = \\ = \frac{3a}{\mu}V_E\sin\theta\int_0^{t_{end}}(t_s - t)\mathbf{v}\cdot\mathbf{A}(t)dt \quad (1)$$

where V_E is the module of the heliocentric velocity of the Earth, V_{ast} is the module of the heliocentric velocity of the asteroid, θ is the angle between the heliocentric velocity of the Earth and the relative velocity of the asteroid at encounter and ψ is the angle between the heliocentric velocity of the asteroid and its planetocentric velocity at encounter. Applying eq.(1) we find that, for reasonable values of χ^2 (all the virtual asteroids may approximated to have the same orbital parameters) all virtual asteroids have approximately the same $\Delta\zeta$. We may conclude that the effect of a deflection mission on the whole set Υ is a shift along the ζ direction of magnitude expressed by Eq.(1). At any stage of the asteroid orbit determination process we may determine the necessary amount we need to move its encounter b-plane projection Υ in order to obtain an empty intersection with keyholes and with the Earth image. Using Eq.(1) we may then determine whether we may achieve such a deflection with a given deflection strategy. Let us write Eq.(1) for two of the most researched deflection strategies: the kinetic impactor and the long duration thrust strategy:

- *Kinetic impactor case*: the instantaneous momentum change applied $\mathbf{A}$ may be written (see Izzo [4]) as $\mathbf{A}(t) = \eta\frac{m}{m+M}\mathbf{U}\delta(t)dt$ where η is a non dimensional constant that accounts for the effect of the ejecta materials and of the energy transferred into rotation, m is the mass of the impactor spacecraft, M is the asteroid mass, $\mathbf{U}$ is the velocity relative to the asteroid and δ is the Dirac's delta function. Substituting the expression back into Eq.(1) we get:

$$\Delta\zeta = \frac{3aV_E\eta\sin\theta}{\mu(m+M)}mt_s\mathbf{v}\cdot\mathbf{U} \quad (2)$$

- *Long duration thrust case*: the instantaneous momentum change applied $\mathbf{A}$ may be written as $\frac{\mathbf{T}}{m+M}$ where $\mathbf{T}$ is the thrust applied to the asteroid. Substituting the expression back into Eq.(1) we get:

$$\Delta\zeta = \frac{3a}{\mu} V_E \sin\theta \int_0^{t_{end}} (t_s - t)\mathbf{v} \cdot \frac{\mathbf{T}}{m+M} dt \quad (3)$$

Eq.(2) and Eq.(3) allow to define analytically the objective of a deflection mission.

Optimisation

Let us consider in more details the trajectory optimisation problem for the two asteroid deflection concepts outlined in the previous section.

The Kinetic Impactor

In the case of a mission that aims at deflecting an asteroid by sending a spacecraft to impact on its surface we must consider the maximization of the function given by Eq.(2). The quantities θ, a, μ and V_E are fixed and known from the impact geometry defined by the asteroid orbit determination. Once an impact is foreseen, the uncertainties related to the asteroid orbit do not influence significantly these quantities and we will therefore consider them as deterministic parameters. The greatest uncertainty is surely connected to the asteroid mass M, this important parameter has a linear effect on the deflection achieved and its uncertainty may therefore easily be dealt with.

The optimisation will aim at maximising the spacecraft mass at impact m (note that we now mean the impact between the spacecraft and the asteroid), the dot product $\mathbf{v} \cdot \mathbf{U}$, the impact time before the Earth encounter t_s, and the parameter η that tells us how precisely we manage to hit the asteroid center of mass and how large is the effect of the ejecta material expulsion.

The mass at impact m: This objective is quite common for the majority of space missions where a larger final mass allows for more sophisticated scientific payloads to be hosted on board. From the trajectory optimisation point of view it results in optimal discontinuous control laws for the low-thrust trajectory case.

The dot product $\mathbf{v} \cdot \mathbf{U}$: This objective makes transfers that “rendezvous” with the asteroid at its perihelion attractive and large final relative velocities aligned with $\mathbf{v}$ preferred. A retrograde impact, such as that suggested by McInnes [3] would be the best solution if considering this objective alone.

The impact time before Earth encounter t_s: This objective makes early launches and faster transfers preferred. In contrast with the previous objective the preferred “rendezvous” point will not be at the perihelion but at the earliest possible location. From the trajectory optimisation point of view it results in a continuous optimal thrusting whenever low-thrust is allowed.

The parameter η: This parameter is one of the crucial parameter that has to be estimated in such a mission. It largely depends on the highly uncertain composition of the asteroid and on the capability of the spacecraft of hitting the asteroid in its center of gravity. It is likely that even with the added information that missions like Deep Impact and Don Quijote will give us, a large uncertainty on this parameter will always remain, we here therefore use a very conservative estimate of $\eta = 1$.

It is in principle possible to consider both a chemical propulsion spacecraft and a low-thrust propulsion spacecraft whenever low ΔV are required for the deflection, such as in a pre-keyhole deflection. When larger deflection are needed, however, the large spacecraft mass required prevents the chemical propulsion to be considered at all.

The Long Duration Thrust

In the case of a deflection strategy that considers landing on the asteroid and pushing it continuously for a long period of time, issues like the asteroid rotation and the subsequent constraint on the possible push direction have to be addressed (see Scheeres and Schweickart [1] for a brief discussion). In this paper we do not go into the details of this kind of concept, more can be read also in Walker et al. [2], but we note how Eq.(3) may be used as the objective function of the spacecraft trajectory optimisation and that the problem of optimising the push direction may be solved separately taking into account design issues.

The MGA Kinetic Impactor Deflection

Let us consider a sequence of planets starting with the Earth and ending with the asteroid under consideration. We introduce the vector $\mathbf{x} = [t_{dep}, T_1, T_2, ..., T_{N+1}]$ where N is the number of planets we want to swing by before hitting the asteroid, t_{dep} is the departure epoch and the various T_i are the length of the ballistic arcs joining two consecutive planets. Given these definition we introduce the Multiple Gravity Assist (MGA) deflection optimisation in the following form:

$$\begin{aligned} \text{find:} \quad & \mathbf{x} \\ \text{to maximise:} \quad & \Delta\zeta(\mathbf{x}) = \frac{3aV_{ast}\eta\sin\theta}{\mu(m+M)} m t_s \mathbf{v}\cdot\mathbf{U} \\ \text{subject to:} \quad & C3_{max} \geq C3_{dep}(\mathbf{x}) \\ & \mathbf{r}_{p_{min}} \leq \mathbf{r}_p(\mathbf{x}) \end{aligned} \tag{4}$$

where the vector $\mathbf{r}_{p_{min}}$ contains the minimum values of the pericenter radii of the various hyperbole that define the swing bys and $C3_{max}$ is the square of the maximum escape velocity allowed by the launcher. Note that in the objective function the final mass m depends on the various ΔV that the spacecraft has to give during the swing by trajectories. Such a relation is given by the rocket equation $m = m_0 e^{-\frac{\sum \Delta V_i}{I_{sp} g_0}}$. The problem stated above has a dimension equal to $N+2$ and can be solved efficiently by the use of heuristic global optimisation techniques [11]. In this study an heuristic global optimiser named Differential Evolution [12] has been used to search the solution space.

Departure	Impact	$\|U\|$ km/sec	Deflection $\Delta\zeta$, km
16/4/2011	12/6/2012	7.93	87.8
25/4/2012	4/5/2013	8.14	63
25/9/2013	27/3/2014	8.29	43.5
3/6/2014	26/2/2015	10.9	18
16/4/2018	12/7/2019	7.32	57.58
17/4/2019	31/5/2020	8.02	43.37
29/12/2020	9/3/2021	7.15	47.4
8/9/2021	25/3/2022	9.01	17.23
2/3/2026	2/3/2027	7.09	10.05
1/4/2027	2/12/2027	7	3.96
5/2/2028	18/11/2028	5	0.45

Table 1: Possible Launches for the Don Quijote impactor (Hidalgo) as designed by the European Space Agency CDF [5]. Note that the keyhole dimension is .6 km and therefore deflection is possible even with a late launch.

Departure	Impact	$\|U\|$ km/sec	Deflection $\Delta\zeta$, km
6/9/2010	12/2/2012	15.04	90.40
23/4/2012	5/8/2014	18.30	93.95
14/11/2013	5/5/2014	9.68	49.10
11/6/2015	27/4/2017	19.04	83.96
31/12/2016	18/5/2018	13.87	47.30
5/1/2017	20/5/2018	13.80	48.30
3/9/2018	27/1/2020	14.94	49.03
20/4/2020	24/7/2022	18.07	43.36
2/11/2021	4/7/2022	8.55	-20.31
10/6/2023	14/4/2025	18.87	28.57
31/12/2024	3/5/2026	13.84	12.91
27/8/2026	8/1/2028	15.05	6.74

Table 2: Possible Launches for the chemical propelled spacecraft. $C3 = 3.5km^2/sec^2$, $M = 500kg$. All trajectories refer to a Earth-Venus-Apophis case which was found to be, in this case, consistently better than a direct transfer.

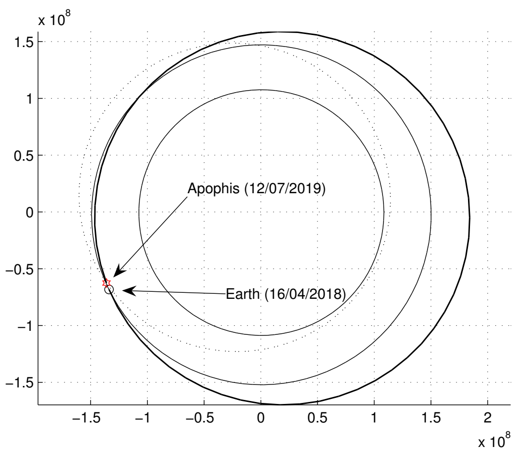

Figure 2: Globally optimal solution for the Don Quijote (Hidalgo) kinetic impactor. The maximum b-plane deflection achievable is $\Delta\zeta = 57.58km$, well above the keyhole thickness! The magnitude of the relative velocity at impact is $7.93\frac{km}{sec}$

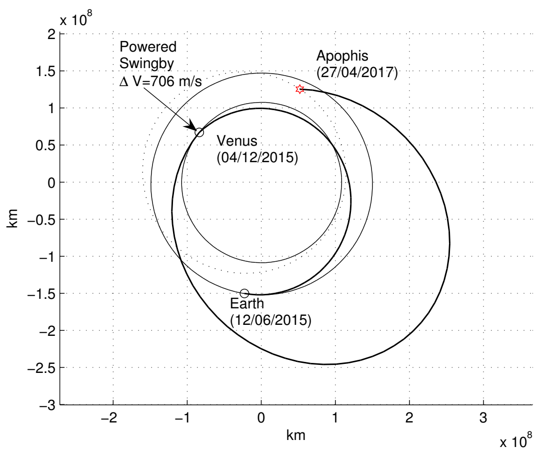

Figure 3: Globally optimal solution for a chemical propelled kinetic impactor launching after 2015. The maximum b-plane deflection achievable by such a spacecraft (500 kg with a launch C3 of 3.5 km^2/sec^2) is $\Delta\zeta = 84km$, well above the keyhole thickness! The magnitude of the relative velocity at impact is $19\frac{km}{sec}$

We take as example the case of the asteroid 99942 Apophis fixing its mass to $M = 4.6 \cdot 10^{10}kg$. Its April 2029 close encounter b-plane is shown in Figure 1. From the publicly available data available in the NeoDys web site (`newton.dm.unipi.it/cgi-bin/neodys/neoibo`) it may be estimated that the keyhole dimension in the ζ direction is of the order of .7km whereas the dimension of the Earth focused on the b-plane is between 2.14 and 2.15 Earth radii. A post encounter deflection would therefore be more challenging by a factor 38700. It makes therefore sense to investigate on the capability we currently have of deflecting an asteroid like Apophis before its next Earth encounter in case its refined orbit determination will gradually reveal the asteroid to be an impactor. We consider two different spacecraft designs. The first spacecraft is the Don Quijote spacecraft as designed by ESA Concurrent Design Facility (CDF) [5] with a mass of 790 kg and a C3 $2.26\frac{km}{sec}$ allowed by a Dnepr launcher. The second spacecraft has a mass of 500 kg, is allowed a $C3$ of $3.5\frac{km}{sec}$ and is equipped with a mono propellant propulsion module having an $I_{sp} = 225s$. We consider any date after 2010 as possible for launching the mission. The resulting global optimal trajectories for the two spacecrafts are shown in Figure 3 and Figure 2. In both cases, the deflection achievable is more than enough to prevent any risk of resonant return for the April 2029 encounter. The optimal solution results in a Venus swing and in a high final relative velocity for the second spacecraft option, whereas it results in a direct transfer and in a lower relative velocity for the first spacecraft option. The Venus swingby is infeasible for the second concept due to a low $C3$ allowed. If this could be increased to $2.5\frac{km}{sec}$ the globally optimal solution would also require a Venus swing and would be able to achieve 90 km of deflection.

Table 1 and Table 2 show, for the two spacecraft, the global optimal traqjectory for each year up to the year 2027. Unfeasible years are not reported. Don Quijote can achieve the deflection of the asteroid 99942 Apophis launching as late as 2026! In this case it is still possible to obtain a 10 km deflection along the ζ direction of the encounter b-plane even in the conservative hypothesis of $\eta = 1$.

The Low-Thrust deflection problem

If a large deflection is needed, if the asteroid is particularly massive or if the ΔV budget for the mission is too high, a massive kinetic impactor may achieve the necessary deflection using advanced propulsion systems capable of slowly accelerating the spacecraft for a long time (see Izzo et al.[4]). In the simpler case of a direct transfer (i.e. no swing by considered) the trajectory optimisation of such a problem may be mathematically formulated as follows:

$$\begin{array}{rl} \text{find:} & \mathbf{u}(t), \mathbf{v}_{dep}, t_{dep}, t_{arr} \\ \text{to maximise:} & \Delta\zeta = \frac{3aV_{ast}\eta\sin\theta}{\mu(m+M)} m t_s \mathbf{v}\cdot\mathbf{U} \\ \text{subject to:} & C3_{max} \geq C3_{dep}(\mathbf{v}_{dep}, t_{dep}) \\ & |\mathbf{u}| \leq T_{max} \\ & \dot{\mathbf{x}}_{s/c} = f(\mathbf{x}_{s/c}, \mathbf{u}) \\ & \mathbf{x}_{s/c}(t_{arr}) = \mathbf{x}_{tar}(t_{arr}) \end{array} \tag{5}$$

where $\mathbf{u}(t)$ is a piecewise continuous function representing the spacecraft thrust, $\mathbf{x}_{s/c}$ is the spacecraft position, $\mathbf{x}_{tar}$ is the target asteroid position and f is the function that describes the dynamic of the spacecraft in the solar system. If multiple swing by are considered we must add to the decision variables the swing by epochs and to the constraints the patching conditions on the incoming and outcoming trajectory arcs. Such a complex optimisation problem (it is an infinite dimension problem) may be tackled both with direct and indirect methods. We use a direct approach to the continuous optimisation problem, transcribing it into a Non Linear Programming (NLP) problem with a technique called Finite Elements in Time (FET)[13]. We then solve the NLP with a Newton method starting from some chosen initial guess. The global optimality of the solution is quite unlikely at this point and a multi-start method is used to improve the chances of converging to the global optimum. We take again the example of the asteroid 99942 Apophis and a 500 kg spacecraft equipped with two Qinetiq T5 Ion engines capable of a maximum thrust of 40 mN at 1 AU and with a specific impulse of 3100 sec. The departure C3 is constrained to be within 3.5 $\frac{km}{sec}$ whereas the launch date is forced to be later than 2011. Gradient descents from thirty different initial guess trajectories have been evaluated. The best result found is shown in Figure 4.

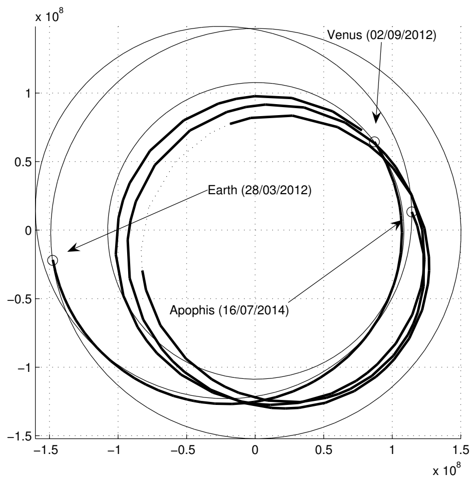

Figure 4: Optimal solution for a ion engine impactor spacecraft. The maximum b-plane deflection achievable is $\Delta\zeta = 97.58 km$, well above the keyhole thickness! The magnitude of the relative velocity at impact is $14.7 \frac{km}{sec}$

The spacecraft is able to achieve 97.58 km of deflection hitting the asteroid at a final relative velocity of 14.7 $\frac{km}{sec}$. With respect to the same spacecraft equipped with a simple mono-propellant chemical engine (see Table 2) there is an improvement in the deflection capabilities of the spacecraft. In this particular case these improvements are not as critical as it may be in other cases [4].

Conclusions

We developed a methodology to optimise trajectories for interplanetary asteroid deflection missions. The method relies upon an analytical definition of the objective function and allows to find the global optimal trajectory of a chemical propelled kinetic impactor within seconds. We also introduce the definition of two new mathematical global optimisation problems and we give an example of the possible solutions for some interesting cases inspired by ESA pre-phase A study Don Quijote and by the case of the asteroid 99942 Apophis. We demonstrate how the deflection of such an asteroid would be possible using such a small and simple spacecraft even with a very late launch in 2026.

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