Using Newton's Method to Search for Quasi-Periodic Relative Satellite Motion Based on Nonlinear Hamiltonian Models

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Abstract

In this paper, the monodromy variant of Newton's method is applied to locate periodic or quasi-periodic relative satellite motion. Advantages of using Newton's method to search for periodic or quasi-periodic relative satellite motion include simplicity of implementation, ability to deal with nonlinear dynamics, repeatability of the solutions due to its non-random nature, and fast convergence. A nonlinear Hamiltonian model is derived which incorporates eccentricity of the reference orbit, nonlinear gravitational terms and the J_2 perturbation. Closed-loop control of relative satellite motion is simulated with the aid of a discrete LQR controller with impulsive actuation.

Introduction

The need to minimise fuel consumption in formation flying spacecraft has motivated significant research aimed at improving our understanding of the long term relative motion of satellites. A classical method for analysing the relative motion of satellites was proposed by Clohessy and Wiltshire in the 1960s [1], who linearised the two-body problem about a circular orbit and solved the resulting linear system. It should be noted that dynamic modelling deficiencies will lead to unexpected relative satellite motion, while any inaccuracy in the prediction of the motion will lead to unnecessary manoeuvring to eliminate possibly benign perturbations. Neglecting gravitational perturbations (particularly the J_2 term) and other modelling simplifications, such as the assumption of linear dynamics and circular orbits, leads to reference orbits that require excessive fuel for tracking purposes. Thus, mission design would benefit significantly from relative orbits generated with more detailed models that include both gravitational perturbations and nonlinearities.

A number of methods have been proposed which in different ways incorporate the effects of perturbations, but most of these methods linearise the dynamics prior to finding conditions for periodic relative motion [2, 3]. In this paper, the monodromy variant of Newton's method [4] is applied to locate periodic or quasi-periodic relative satellite motion. The method is capable of dealing with nonlinear dynamics, so there is no need to linearise the equations of motion.

Hamiltonian Formulation of Relative Motion

The Hamiltonian formulation of relative satellite motion allows for additional conservative forces to be added to the Hamiltonian, while nonconservative forces can be added in the momenta equations of motion. Inspired by [5] we derive the Hamiltonian function that yields the Hill-Clohessy-Wiltshire (HCW) [1] dynamics, and then derive the effect of perturbations on the Hamiltonian. The coordinate system (a rotating Cartesian Euler-Hill system shown in Figure 1), denoted by $\Re$, is defined by the unit

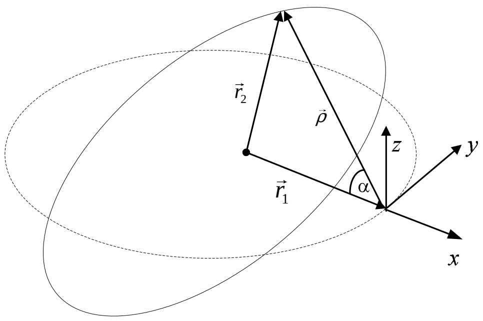

Figure 1: Euler-Hill reference frame for relative motion

vectors $\hat{x},\hat{y},\hat{z}$. In [5] the equations of motion are derived on a circular reference orbit of radius a about the Earth. The co-ordinate system is rotating with frequency $\omega = \sqrt{\mu/a^3}$, where μ is the gravitational constant. However, in the following derivation, the reference orbit is not restricted to a circle in order to account for the effect of eccentricity. The distance from the center of earth to the master satellite is denoted r, and the rate of change of the true anomaly is denoted $\dot{\theta}$. The reference orbit plane is the fundamental plane, the positive $\hat{x}-$axis points radially outward, the $\hat{y}-$axis is in the along track direction, and the $\hat{z}-$axis is orthogonal to both $\hat{x}$ and $\hat{y}$ and is out of the leader satellite plane. The velocity of the follower satellite is given by:

$$\vec{v} = {}^{\Im}\vec{\omega}^{\Re} \times \vec{r}_1 + \frac{d^{\Re}}{dt}\vec{\rho} + {}^{\Im}\vec{\omega}^{\Re} \times \vec{\rho} \tag{1}$$

where $\vec{r}_1 \in \mathbb{R}^3$ is the inertial position vector of the leader satellite along the reference orbit, $\vec{\rho} = [x,y,z]^T$ is the relative position vector in the rotating frame, and ${}^{\Im}\vec{\omega}^{\Re} = [0,0,\dot{\theta}]^T$ is the angular velocity of the rotating frame $\Re$ with respect to the inertial frame $\Im$. Denoting $\|\vec{r}_1\| = r$ and substituting into (1) gives:

$$\vec{v} = \begin{bmatrix} \dot{x} - \dot{\theta}y + \dot{r} \\ \dot{y} + \dot{\theta}x + \dot{\theta}r \\ \dot{z} \end{bmatrix} \tag{2}$$

The kinetic energy per unit mass is given by $K = \frac{1}{2}\|\vec{v}\|^2$. Initially assuming a spherical attracting Earth, the potential energy of the follower satellite, whose position vector is $\vec{r}_2$, is the usual gravitational potential written in terms of $\rho = \|\vec{\rho}\|$ and expanded using Legendre polynomials.

$$U = -\frac{\mu}{\|\vec{r}_2\|} = -\frac{\mu}{\|\vec{r}_1 + \vec{\rho}\|} = -\frac{\mu}{r\left[1 + 2\frac{\vec{r}_1 \cdot \vec{\rho}}{r^2} + \left(\frac{\rho}{r}\right)^2\right]^{1/2}} = -\frac{\mu}{r}\sum_{k=0}^{\infty} P_k(\cos\alpha)\left(\frac{\rho}{r}\right)^k \tag{3}$$

where the $P_k(\cos\alpha)$ are the Legendre polynomials,

$$\cos\alpha = \frac{\vec{\rho}\cdot\vec{r}_1}{r\rho} = \frac{-x}{\rho} \tag{4}$$

and α is the angle between $\vec{r}_1$ and the relative position vector $\vec{\rho}$. The Lagrangian $\mathfrak{L}$ is:

$$\mathfrak{L}^{(0)} = K - U = \frac{1}{2}\{\dot{x} - \dot{\theta}y + \dot{r})^2 + (r\dot{\theta} + \dot{\theta}x + \dot{y})^2 + \dot{z}\} + \frac{\mu}{r}\sum_{k=0}^{\infty} P_k(\cos\alpha)\left(\frac{\rho}{r}\right)^k \tag{5}$$

Using Legendre polynomials up to $k = 3$ and substituting (4) into (5) gives

$$\mathfrak{L}^{(0)} = \frac{1}{2}((\dot{x} - \dot{\theta}y + \dot{r})^2 + (r\dot{\theta} + \dot{\theta}x + \dot{y})^2 + \dot{z} + \frac{\mu}{r} - \frac{\mu x}{r^2} + \frac{3\mu x^2}{2r^3} - \frac{\mu(x^2 + y^2 + z^2)}{2r^3}) \tag{6}$$

In [5], Kasdin and Gurfil proceed using simplifying calculations by taking a normalization of $\dot{\theta}$ and r; here we proceed and calculate the Hamiltonian without normalization. To calculate the Hamiltonian first derive the canonical momenta:

$$\begin{aligned} p_x &= \frac{\partial \mathfrak{L}^{(0)}}{\partial \dot{x}} = \dot{x} - \dot{\theta}y + \dot{r} \\ p_y &= \frac{\partial \mathfrak{L}^{(0)}}{\partial \dot{y}} = \dot{y} + \dot{\theta}(r + x) \\ p_z &= \frac{\partial \mathfrak{L}^{(0)}}{\partial \dot{z}} = \dot{z} \end{aligned} \tag{7}$$

then using the Legendre transformation $H = \sum \dot{q}_i p_i - \mathfrak{L}$, the Hamiltonian corresponding to the HCW equations, henceforth called the unperturbed Hamiltonian, is given by:

$$\begin{aligned} H^{(0)} =& \frac{1}{2}(-p_x^2 - p_y^2 + p_z^2 + 2p_y(p_y - r\dot{\theta} - \dot{\theta}x) + 2p_x(p_x - \dot{r} + \dot{\theta}y) \\ &+ \frac{2\mu}{r} + \frac{2\mu x}{r^2} - \frac{3\mu x^2}{r^3} + \frac{\mu(x^2 + y^2 + z^2)}{r^3}) \end{aligned} \tag{8}$$

From the Hamiltonian function the vector fields can be calculated using Hamilton's canonical equations:

$$\begin{aligned} \dot{q}_i &= \frac{\partial H}{\partial p_i} \\ \dot{p}_i &= -\frac{\partial H}{\partial q_i} \end{aligned} \tag{9}$$

where $q \in \mathbb{R}^3$ are the degrees of freedom in the configuration space and $p \in \mathbb{R}^3$ are the conjugate momenta. Calculating the corresponding Hamiltonian vector fields yields dynamical equations which are equivalent to the well known HCW equations (with eccentricity of the reference orbit). If the orbit of the leading satellite is circular, the constraint for periodic motion is $p_{y_0} = \omega(r - x_0)$. Unlike the classical HCW formulation the equations can account for non Keplerian forces by adding perturbations to the Hamiltonian.

The nonlinear Hamiltonian and perturbations

Nonlinear gravitational potential

Instead of computing the Legendre expansion in the gravitational potential equation (3), the nonlinear equations are obtained by writing:

$$\|\vec{r}_2\| = ((r+x)^2 + y^2 + z^2)^{1/2} \tag{10}$$

Then the Hamiltonian function is calculated as

$$H^{(0)} = -\frac{p_x^2}{2} - \frac{p_y^2}{2} - \frac{p_z^2}{2} + p_y(p_y - \dot{\theta}r - \dot{\theta}x) + p_x(p_x - \dot{r} + \dot{\theta}y) - \frac{\mu}{((r+x)^2 + y^2 + z^2)^{1/2}} \tag{11}$$

The corresponding Hamiltonian equations can be derived using (9).

Earth oblateness perturbations

It is also important to account for the zonal harmonics in the gravitational potential. The J_2 harmonic is known to be the most significant. Assuming that the earth is oblate and axially symmetric, the external potential is given by [6]:

$$U = \sum_{k=2}^{\infty} U_k = -\frac{\mu}{\|\vec{r}_2\|} \sum_{k=2}^{\infty} J_k \left(\frac{R_e}{\|\vec{r}_2\|}\right)^k P_k(\cos\phi) \tag{12}$$

where ϕ is the follower spacecraft colatitude angle ($\cos\phi = Z/\|\vec{r}_2\|$), Z is the normal deflection in an inertial, geocentric-equatorial reference frame and $J_k(k = 2, 3..)$ are constants of the zonal harmonics. Assuming that the reference orbit is not inclined relative to the equatorial plane, so that $Z = z$, and given (10) it is easy to find the following Hamiltonian perturbation (to be added to $H^{(0)}$) arising from the effect of the J_2 harmonic:

$$H^{(1)} = \frac{J_2 R_e^2 u(-(r+x)^2 - y^2 + 2z^2)}{2((r+x)^2 + y^2 + z^2)^{5/2}} \tag{13}$$

Eccentricity is accommodated into the model as r, $\dot{r}$ and $\dot{\theta}$ depend on the reference orbit, which can be defined with a particular eccentricity. This analysis can be extended to higher order zonal harmonics. When simulating the motion, it is necessary to define $\dot{\theta}$, r and $\dot{r}$ independently of the Hamiltonian system and treat them as inputs to the simulation model. A simple Keplerian model was employed in this work to find the reference trajectory of the master satellite for cases with and without eccentricity, but without any other perturbation. For other perturbations to the reference orbit such as J_2, the nonlinear propagator STK was used to compute the time dependent variables of the reference orbit.

Newton's method for locating periodic or quasi-periodic orbits

In the Hamiltonian formulation of the HCW equations, there are explicit initial conditions that give periodic relative motion. As perturbations are added, the same initial conditions will no longer yield closed orbits. However, a numerical method proposed by Marcinek and Pollak [4], based on a variant of Newton's method, may locate periodic or quasi-periodic orbits. Newton's method has previously been applied to two degrees of freedom continuous systems in [7]. However, the relative satellite motion model has three degrees of freedom and the increased dimensionality makes the search more complex. The HCW initial conditions provide a good initial guess and, as complexity is added, new initial conditions may be obtained in a stepwise manner. The purpose of the method is to find a periodic orbit which has period T^*, i.e. it satisfies the condition $\mathbf{X}^*(T^*) = \mathbf{X}^*(0)$, where $\mathbf{X} = [x, p_x, y, p_y, z, p_z]^T$ describes a point in the phase space. The "$*$" notation is used to signify points on the periodic orbit. Newton's method starts at a point $\mathbf{X}(t)$ initiated at $t = 0$ on a surface of section, e.g. $y = 100$ km, and the trajectory returns to the surface of section at some later time T. An assumption of this method is that the initial trajectory is close to the periodic orbit $\mathbf{X}^*(t)$. This means that the trajectory almost closes in upon itself at time T, which is itself approximately T^*. As the approximate orbit is close to the assumed periodic orbit, the separation between the two can be calculated using the equations of motion linearized about the approximate orbit $\mathbf{X}(t)$ [7]. The method represents an iterative improvement to the choice of initial conditions for the periodic orbit:

$$\mathbf{X}^{(k+1)}(0) = \mathbf{X}^{(k)}(0) + (\mathbf{I} - \mathbf{M})^{-1}[\mathbf{X}^{(k)}(T) - \mathbf{X}^{(k)}(0)] \tag{14}$$

where k is an iteration index, and the approximate monodromy matrix is defined as

$$\mathbf{M} = \mathrm{e}^{\mathbf{H}''(T)T} \tag{15}$$

where $H''_{ij}(t) = \partial^2 H/\partial X_j \partial X_i$ is the 6×6 Hessian matrix of the Hamiltonian with respect to the coordinates and momenta evaluated along the approximate trajectory.

In practice, one encounters a problem because there are at least two trivial directions in phase space which lead to unit eigenvalues of $\mathbf{M}$. One is along the trajectory, and the other one is perpendicular to the energy surface. The matrix $(\mathbf{I} - \mathbf{M})$ is thus singular. A constraint that we used to increase the rank of $(\mathbf{I} - \mathbf{M})$ is energy conservation $\delta H = 0$. Therefore,

$$\left(\frac{\partial H}{\partial x}, \frac{\partial H}{\partial p_x}, \frac{\partial H}{\partial y}, \frac{\partial H}{\partial p_y}, \frac{\partial H}{\partial z}, \frac{\partial H}{\partial p_z}\right) . (\delta x, \delta p_x, \delta y, \delta p_y, \delta z, \delta p_z) = 0 \tag{16}$$

The left hand vector from (16) can replace a row in the $(\mathbf{I}-\mathbf{M})$ matrix and the corresponding row of the vector $(\mathbf{X}(T)-\mathbf{X}(0))$ is set to zero. In addition, to avoid any further problems due to the rank deficiency of $(\mathbf{I}-\mathbf{M})$, we have used the Moore-Penrose pseudo-inverse to implement (14).

Principal Component Analysis

Given that the use of bounded or drifting trajectories as control references carries practical difficulties over long-term missions, a method based on principal component analysis [10] was developed to project a bounded or slowly drifting trajectory found using Newton's method, to a plane defined by the first two principal components. In this way a planar trajectory can be produced which is almost closed. The state variables on this projected trajectories can in turn be fitted to sinusoidal functions to provide closed periodic trajectories that preserve useful information about the original bounded or slowly drifting trajectory. It is also necessary to project the corresponding canonical momenta, as the method used for closed-loop control requires full state feedback. Given projected position and time it is simple to calculate the velocity components and therefore calculate the projected canonical momenta.

Closed Loop Control

In order to evaluate the effect of the quality of the model used to generate the periodic reference trajectory, a study involving closed loop control of a simulated master/follower formation was performed.

Linearization of the relative motion dynamics

The nonlinear relative motion dynamics with actuation can be written as follows:

$$\dot{\mathbf{X}}(t) = \mathbf{f}(\mathbf{X}(t)) + \bar{\mathbf{B}}\mathbf{u}(t) \tag{17}$$

where $\mathbf{X} = [x,\ y,\ z,\ p_x,\ p_y,\ p_z]^T$ is the state vector consisting of relative positions and canonical momenta, and $\mathbf{u} = [u_x,\ u_y,\ u_z]^T$ is a vector of actuations (in units of acceleration), $\mathbf{f}(\mathbf{X})$ is a smooth vector field, and $\bar{\mathbf{B}}$ is given by:

$$\bar{\mathbf{B}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{18}$$

It is assumed that the relative motion is controlled by adjusting the motion of the follower satellite only. Define a reference trajectory $\mathbf{X}_r(t) \in \Re^6$ over a time interval $t \in [t_0, t_f]$, which approximately satisfies $\dot{\mathbf{X}}_r(t) \approx \mathbf{f}(\mathbf{X}_r(t))$. Notice that it is reasonable to make this approximation as the reference trajectory to be used approximates a solution of the relative motion dynamics given by (17) with $\mathbf{u} = 0$. Consider small deviations $\mathbf{x}(t)$ from $\mathbf{X}_r(t)$, such that $\mathbf{x}(t) = \mathbf{X}(t) - \mathbf{X}_r(t)$. Using a first order Taylor series expansion, it is possible to write the following expression for the deviations from the reference:

$$\dot{\mathbf{x}}(t) \approx \bar{\mathbf{A}}(t)\mathbf{x}(t) + \bar{\mathbf{B}}\mathbf{u}(t) \tag{19}$$

where

$$\bar{\mathbf{A}}(t) = \left[\frac{\partial \mathbf{f}}{\partial \mathbf{X}}\right]\Bigg|_{\mathbf{X}_r(t)} \tag{20}$$

To simplify the control design, it is assumed that $\bar{\mathbf{A}}(t)$ does not change much along the reference trajectory so that, for the purposes of control design, the following time invariant model is assumed:

$$\dot{\mathbf{x}} = \bar{\mathbf{A}}\mathbf{x}(t) + \bar{\mathbf{B}}\mathbf{u}(\mathbf{t}) \tag{21}$$

where

$$\bar{\mathbf{A}} = \left[\frac{\partial \mathbf{f}}{\partial \mathbf{X}}\right]\Bigg|_{\mathbf{X}_0} \tag{22}$$

and $\mathbf{X}_0$ is a suitable point that belongs to the reference trajectory $\mathbf{X}_r(t)$. For the purposes of this study, $\mathbf{X}_0$ was defined as the initial point of the trajectory $\mathbf{X}_r(t_0)$. It should be noted that in this application tighter control should be possible by considering the changes in $\bar{\mathbf{A}}(t)$ along the reference trajectory.

Model discretisation

Consider that the actuation is performed using thrusters which provide impulsive thrust with a sampling interval T_s and a duration d, such that it is reasonable to assume that the control vector $\mathbf{u}$ is defined as follows:

$$\mathbf{u}(t) = \begin{cases} \mathbf{u}_k/d & t_k \leq t \leq t_k + d \\ \mathbf{0} & t_k + d < t < t_k + T_s \end{cases} \tag{23}$$

where k is a sampling index, t_k is a sampling instant, such that $t_k = kT_s$, and $\mathbf{u}_k = [\Delta v_x, \Delta v_y, \Delta v_z]^T$ is the control signal (in velocity units) provided by a discrete-time controller. The magnitude $||\mathbf{u}_k|| = (\Delta v_x^2 + \Delta v_y^2 + \Delta v_z^2)^{\frac{1}{2}}$ will henceforth be called DeltaV, while the sum $\sum_{k=0}^{N} ||\mathbf{u}_k||$ will be called accumulated DeltaV per orbit, where $N = \text{floor}(T/T_s)$. Under these assumptions, the discrete time model can be expressed as follows:

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{u}_k \tag{24}$$

where

$$\mathbf{A} = e^{\bar{\mathbf{A}}T_s}, \ \mathbf{B} = e^{\bar{\mathbf{A}}T_s} \int_0^d e^{-\bar{\mathbf{A}}r} dr \ \bar{\mathbf{B}}/d \tag{25}$$

Discrete Linear Quadratic Regulator with impulsive actuation

The control technique employed was the discrete linear quadratic regulator (LQR) [9] with impulsive actuation. The LQR technique was chosen due to its ability to handle the problem of formation flying control, as has already been shown in the literature [8], and also due to its simplicity for practical implementation purposes. Impulsive control was chosen to approximate the way actual satellite thrusters work. The following quadratic performance index is minimised:

$$J = \frac{1}{2} \sum_{k=0}^{\infty} \left[\mathbf{x}_k^T \mathbf{Q} \mathbf{x}_k + \mathbf{u}_k^T \mathbf{R} \mathbf{u}_k \right] \tag{26}$$

The minimisation of (26) subject to the linear discrete-time dynamics (24) can be achieved by the well know Ricatti solution [9], so that the optimal control law is a simple state feedback given by:

$$\mathbf{u}_k = -\mathbf{K}(\mathbf{X}(t_k) - \mathbf{X}_r(t_k)) \tag{27}$$

Results

For the model with eccentricity, nonlinear gravitational terms and the J2 perturbation, using the initial conditions given by the HCW conditions and applying Newton's method found the initial conditions below that give a bounded, quasi-periodic orbit. Figure 2 shows the relative trajectory for the nonlinear study model, which includes eccentricity of 0.005, nonlinear gravitational terms and J_2: $x_0 = 9.6556$, $y_0 = 10.0036$, $z_0 = 9.9681$, $p_{x_0} = 0$, $p_{y_0} = 7.6636$, $p_{z_0} = -0.0012$. In cartesian coordinates, the initial conditions found are: $x_0 = 9.6556$, $y_0 = 10.0036$, $z_0 = 9.9681$, $\dot{x}_0 = 0.0114$, $\dot{y}_0 = -0.0221$, $\dot{z}_0 = -0.0012$. Relative satellite motion has been simulated in closed loop using the impulsive LQR controller described above. The model used to simulate the relative motion with actuation included gravitational nonlinearities, eccentricity and J_2 perturbations. The simulations were performed in the Matlab/Simulink environment.

In order to implement the reference trajectories for control as functions of time in non-trivial cases, each state variable trajectory projected by PCA was fitted to a sinusoidal function of time of the form $r_i(t) = a_i \sin(\omega_i t + \phi_i) + b_i, i = 1, \ldots, 6$. This simple function structure provided a good fit, with appropriate values for the parameters $(a_i, \omega_i, \phi_i, b_i)$, in all cases considered. The controller parameters used are: $T_s = 4$ s, $d = 1$ s, $Q = I_{[6\times6]}$, $R = 10^9 I_{[3\times3]}$. Figure 3 shows the relative trajectory under closed loop LQR control for the case of using a reference and controller generated with a model that considers eccentricity, nonlinear gravitational potential, and J_2. In this

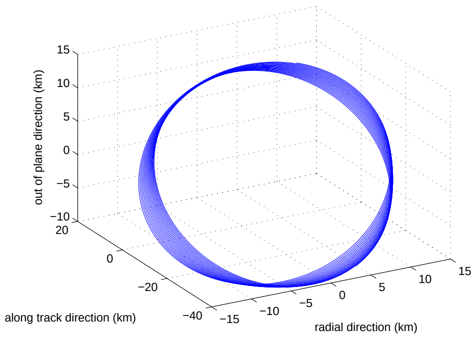

Figure 2: Trajectory found by Newton's method using a model with eccentricity, nonlinear gravitational terms and J_2 - the trajectory is bounded.

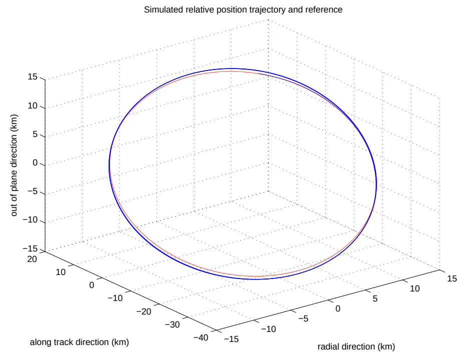

Figure 3: Relative trajectory under LQR control. Reference and controller were obtained from model with eccentricity, nonlinearities and J_2.

case, the DeltaV per orbit was 0.00599 km/s. For comparison, when the model used to generate the reference trajectory and controller considers the same eccentricity value, but ignores nonlinear gravitational perturbations and J_2, the required DeltaV per orbit to yield a similar tracking accuracy was 0.015758 km/s.

The authors ran several cases with different levels of model complexity, and the results obtained indicate that the quality of the model employed to generate the reference trajectory used for control purposes has an important influence on the resulting amount of DeltaV required to track the reference trajectory [11]. The model used to generate LQR controller gains also has an effect on the efficiency of the controller.

Conclusions

The monodromy variant of Newton's method has been applied to locate periodic or quasi-periodic relative satellite motion. Advantages of using Newton's method to search for periodic or quasi-periodic relative satellite motion include simplicity of implementation, ability to deal with nonlinear dynamics, repeatability of solutions due to its non-random nature, and fast convergence. A nonlinear Hamiltonian model of relative satellite motion has been derived which incorporates eccentricity of the reference orbit, nonlinear gravitational terms and the J_2 perturbation. A method based on principal component analysis was developed to project a bounded or slowly drifting trajectory found using Newton's method to a plane. These projected periodic trajectories were used as reference trajectories in closed loop LQR control simulations. The results obtained indicate that the quality of the model employed for generating the reference trajectory used for control purposes has an important influence on the resulting amount of fuel required to track the reference trajectory.

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