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Optimization of Interplanetary Trajectories for Impulsive and Continuous Asteroid Deflection

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The chaotic dynamic of near earth objects and our limited knowledge on the asteroid population makes it impossible to predict the next impact of an asteroid with our planet. The collision at Jupiter of the comet Shoemaker–Levy 9, the more recent discovery of the asteroid 99442 Apophis, and other similar events, have stimulated the discussion on our capabilities of deflecting an asteroid in collision route with Earth. We propose a simple expression to evaluate the merit of a given deflection strategy. We then specialize the derived analytical expression in the special cases of a kinetic impactor and of a long duration thrust deflection. We then study two examples. The first deals with a massive spacecraft equipped with an advanced low-thrust propulsion system. We show, fictitiously, that it is convenient to send such a spacecraft to impact the asteroid rather than to rendezvous and push it. In the second example, we solve the optimal interplanetary trajectory problem to deflect the asteroid 99942 Apophis. We find that several optimal interplanetary trajectories exist that allow a 750-Kg spacecraft to be launched from 2015–2026, and impact 99942 Apophis allowing prekeyhole deflections of orders of magnitude larger than the likely required amount.

Nomenclature

$\boldsymbol{A}(t)$	=	deflection strategy
a	=	asteroid semimajor axis
f	=	focusing factor
$\boldsymbol{g}(t)$	=	gravitational and disturbance forces acting on the asteroid
$\boldsymbol{i}_v$	=	velocity unit vector
$\boldsymbol{i}_p$	=	unit vector inertially fixed in the heliocentric frame
M	=	mean anomaly
m	=	mass
m_c	=	fuel mass available at asteroid rendezvous
N	=	number of available observations
n	=	mean motion
$\boldsymbol{p}$	=	asteroid momentum
Q	=	function of the squared residuals used to define the virtual asteroid
R_E	=	Earth orbit radius
$\tilde{t}$	=	encounter time
t_c	=	coast time
t_p	=	deflection duration or push time
t_s	=	time to encounter when the deflection starts
t_1	=	deflection start
t_2	=	deflection end
$\boldsymbol{U}$	=	relative encounter velocity
$\boldsymbol{X}$	=	generic point of the encounter b-plane
$\boldsymbol{\alpha}$	=	orbital parameters vector
Γ	=	encounter b-plane image of the virtual asteroid
$\Delta\xi,\Delta\zeta$	=	deflection achieved on the b-plane
$\boldsymbol{\Delta V}$	=	impulsive velocity change of the asteroid
δ	=	Dirac's delta function
Θ	=	set of all the possible positions occupied by a virtual asteroid
θ	=	angle between the asteroid relative velocity and the Earth velocity
ξ,η,ζ	=	reference frame on the encounter b-plane
Π	=	encounter b-plane
Σ	=	covariance matrix
σ	=	time to encounter
τ	=	time from t_1
ψ	=	angle between the asteroid relative velocity and the asteroid velocity
ω	=	argument of perihelion
ϵ	=	orbital energy

Subscripts

ast	=	asteroid
E	=	Earth

Superscript

*	=	best fit

I. Introduction

POTENTIALLY hazardous objects (PHOs) are comets and asteroids that have been pulled by the complex gravitational interaction with the other planets into orbits that allow them to have close encounters with the Earth. A commonly accepted definition of these objects is based on their orbital parameters and on their apparent magnitude. The minimal orbit interception distance (MOID), that is the minimum distance between the asteroid orbit and the Earth orbit, is an important criteria that distinguish PHOs from the rest of the near Earth objects (NEO) population. The dynamic of PHOs is largely dominated by their gravitational interaction with the sun, even though perturbations are continuously changing their heliocentric orbits, and complex dynamical phenomena may bring them in a collision route with our planet. Scientists agree that these events, fortunately rare, have occurred in the past with different consequences. It is now commonly accepted that the Cretaceous-Tertiary mass-extinction was due to such a collision [1]. The magnitude of the damage created by an impact would depend on a number of factors related to the asteroid nature and from its relative velocity at impact. Because this information is often not available, the asteroid apparent magnitude, and therefore its dimension, are

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frequently taken as a metric. Typical values for the size of a PHO range from 50-m-diam to kilometers.

Because of the uncertainties and errors connected with orbit determination and with long term numerical propagation of asteroid trajectories, particularly affected by the Yarkowsky effect [2], impact events have to be described using probabilistic approaches. The concept of "virtual impactors" introduced by Milani et al.[3] is particularly fruitful in this respect. These are essentially virtual asteroids whose impact with the Earth is compatible with the available observations and with the modeling uncertainties of their dynamics. One such case is the recently discovered asteroid 99942 Apophis, whose impact with the Earth in 2036, though extremely unlikely, is still compatible with our current knowledge of the object's orbit and physical properties. Looking at the vast number of objects that are continuously being discovered and defined as PHOs, we find many virtual impactors. It is not clear whether we will eventually have to deflect one such asteroid in the next century. In the past decades there has been quite an effort to conceive methods able to deflect an asteroid having an unacceptable risk of impact with the Earth. The feasibility of any of these methods has certainly to do with the technological challenges they pose, but also to their general efficiency in terms of time required to achieve a given deflection. A classification of different deflection methods can be read in the paper by Hall and Ross [4]. Scheeres and Schweickart [5] studied more in depth the feasibility of a deflection strategy mission based on a long duration thrust applied to the asteroid by a spaceship powered by a nuclear reactor and able to land on its pole, reorient the asteroid spin axis, and push with its high specific impulse plasma thrusters. McInnes [6] proposed a solar sail mission able to optimally exploit a kinetic impactor deflection by inserting the spacecraft in a retrograde orbit with a cranking maneuver made possible by an advanced solar sail and a long permanence of the spacecraft at a close distance from the sun. More recently, Lu and Love [7] proposed to use gravity to gently push the asteroid out of its collision route. Though all these concepts are all theoretically sound, the related spacecraft design and mission analysis considerations have rarely been discussed in detail. In the paper of Walker et al. [8], a deflection mission using a long duration thrust provided by different advanced propulsion options, including a solar powered case, is studied taking into account preliminary system design issues and interplanetary trajectory optimization. In terms of precursor deflection missions, an important action being undertaken at the moment is the currently ongoing industrial study of a mission to demonstrate our capability to alter the course of an asteroid. The mission, named Don Quijote [9,10], is essentially a projectile aiming to impact some NEO with a large MOID and whose impact would be observed by a second spacecraft orbiting the asteroid. The orbiter would allow for the asteroid orbit change to be precisely measured and for some added science to be performed.

In this paper, we focus on the interplanetary trajectory optimization of a generic deflection mission. The fine structure of the $\boldsymbol{\Delta V}$ required for deflection, mainly associated with the orbital period of the asteroid, was already discovered by Park and Ross [11] in a simple two-dimensional case and their results were later refined to include third-body effects and study the asteroid orbit influence on the mission [12,13]. In another paper, Conway [14] showed how the determination of the optimal direction in which an impulse should be applied may be found without making use of an explicit optimization. His paper considered last minute deflections where a huge momentum transfer is needed. In their recent paper, Dachwald and Wie [15] optimize a solar sail trajectory for an impactor mission evaluating the objective function by means of a forward integration in time. In this paper, we introduce an analytical expression that relates the deflection achieved in the b-plane to the trajectory flown and that is valid when the time between the deflection start and the Earth close approach is large. Based on this finding, we are able to define the objective function in the trajectory optimization using a simple expression. We show how to specialize our equations for the case of a kinetic impactor and of a long duration thrust deflection. In the first case, we show how we may optimize, at the same time, the impact geometry, the terminal relative velocity magnitude, the spacecraft final mass, and the impact time. We then consider, as a fictitious case, the prekeyhole deflection of 99942 Apophis and we assess the performances of a small deflection mission inspired by the current baseline of the Don Quijote spacecraft [9,10].

II. Asteroid Deflection Formula

Consider an asteroid having a close encounter with the Earth. In the encounter b-plane, we define the (ξ, η, ζ) planetocentric reference frame already used by Valsecchi et al. [16]. This coordinate system has the ξ, ζ coordinates on Π and the η-axis directed along the relative velocity vector at encounter. The axis ζ has the opposite direction of the projection on the b-plane of the planet's heliocentric velocity. In this coordinate system, we may read the asteroid MOID on the ξ-axis and the asteroid phase on the ζ-axis [16]. A generic strategy aims at deflecting the asteroid orbit by changing its momentum according to the equation

$$\frac{\mathrm{d}\boldsymbol{p}(t)}{\mathrm{d}t} = \boldsymbol{g}(t) + m_{\mathrm{ast}}\boldsymbol{A}(t)$$

This can be a sudden change of the asteroid momentum or a slow and gentle push given by some devices brought to the asteroid; in any case, we will consider it to be defined in $[t_1, t_2]$. In this section, we derive an analytical relationship between the deflection strategy $\boldsymbol{A}(t)$ and the displacement of the asteroid image on the encounter b-plane $\Delta\zeta$, $\Delta\xi$. We assume that only secular effects are significant. This means that we discard all those contributions to the deflection that would not be greater had they been given some time in advance. We will later discuss in detail what the effects are of such an assumption. We here note that, in general terms, secular terms dominate when the time between deflection and Earth close approach is large, whereas they are not as significant in last minute deflections. Let $\tilde{t}$ be the encounter epoch: at a generic t the encounter will take place after $\sigma = \tilde{t} - t = \Delta M/n$ where ΔM is the mean anomaly difference between the asteroid position at t and at $\tilde{t}$, and n is the asteroid mean motion. The infinitesimal variation of the time to encounter due to some external action is

$$\mathrm{d}\sigma = -\frac{\Delta M}{n^2}\,\mathrm{d}n + \frac{\mathrm{d}\Delta M}{n}$$

where we may identify two different contributions, the first due to a phase shift and the second due to the other effects (eccentricity, inclination, etc.). By disregarding this second contribution (as it does not introduce any secular effects), we get

$$\mathrm{d}\sigma \cong -\frac{\Delta M}{n^2}\,\mathrm{d}n$$

We write the above expression again in terms of the orbital energy ϵ defined as $\epsilon = -\mu/2a$, where μ is the gravitational constant of the sun and a is the semimajor axis of the asteroid orbit

$$\mathrm{d}\sigma = -\frac{\Delta M}{n^2}\,\mathrm{d}n = \frac{3}{2}\Delta M\sqrt{\frac{a}{\mu}}\mathrm{d}a = 3\Delta M\sqrt{\frac{a^3}{\mu}}\frac{a}{\mu}\,\mathrm{d}\epsilon$$

The mean anomaly difference can be expressed as $\Delta M = n(\tilde{t} - t)$ and the orbital energy change may be related to the deflection strategy by the expression $\mathrm{d}\epsilon = \boldsymbol{v}_{\mathrm{ast}} \cdot \boldsymbol{A}\,\mathrm{d}t$. We have

$$\mathrm{d}\sigma = \frac{3a(\tilde{t} - t)}{\mu}\boldsymbol{v}_{\mathrm{ast}} \cdot \boldsymbol{A}\,\mathrm{d}t$$

and integrating over time

$$\Delta\sigma = \frac{3a}{\mu}\int_{t_1}^{t_2}(\tilde{t} - t)\boldsymbol{v}_{\mathrm{ast}} \cdot \boldsymbol{A}\,\mathrm{d}t$$

that expresses the variation of the time to encounter induced by the deflection strategy $\boldsymbol{A}(t)$. We reset the origin of the time axis to t_1 introducing the new time variable $\tau = t - t_1$:

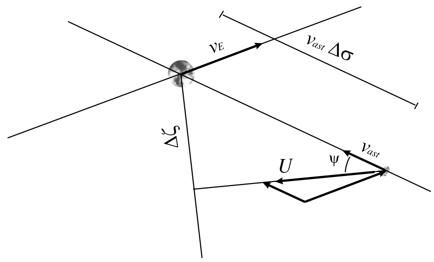

Fig. 1 Encounter geometry with orbits approximated to straight lines.

$$\Delta\sigma = \frac{3a}{\mu}\int_0^{t_p}(t_s - \tau)\boldsymbol{v}_{\text{ast}}\cdot\boldsymbol{A}\,\mathrm{d}\tau \tag{1}$$

where $t_p = t_2 - t_1$ is the push time and $t_s = \tilde{t} - t_1$ the time before encounter the deflection starts. To relate $\Delta\sigma$ to the displacement of b-plane image of an asteroid, we consider the orbits as perfectly intersecting straight lines.

In this case, simple geometric relations (see Fig. 1) show that

$$\Delta\zeta = v_{\text{ast}}\Delta\sigma\sin\psi = v_E\Delta\sigma\sin\theta$$

where v_E is the magnitude of the heliocentric velocity of the Earth, v_{ast} is the module of the heliocentric velocity of the asteroid, θ is the angle between the heliocentric velocity of the Earth and the relative velocity of the asteroid at encounter, and ψ is the angle between the heliocentric velocity of the asteroid and its planetocentric velocity at encounter. Note that no change in the MOID is possible if we take into account only the secular variations induced by $\boldsymbol{A}$ and, therefore, we take $\Delta\xi = 0$. We have found an analytical relation between the deflection strategy and the b-plane displacement of an asteroid image:

$$\Delta\zeta = \frac{3a}{\mu}v_E\sin\theta\int_0^{t_p}(t_s - \tau)\boldsymbol{v}_{\text{ast}}(\tau)\cdot\boldsymbol{A}(\tau)\,\mathrm{d}\tau \tag{2}$$

We will often refer to this formula as to the asteroid deflection formula.

Consider now Eq. (2) under the hypothesis made by Scheeres and Schweickart [5]. In their study of a deflection mission the asteroid semimajor axis was set to be equal to the Earth orbit radius $a = R_E$ and the deflection strategy was $A(t) = A\boldsymbol{i}_v$ where $\boldsymbol{i}_v$ is the asteroid velocity unit vector. The asteroid eccentricity was considered to be zero. Under these assumptions and introducing the coasting time $t_c = t_s - t_p$, Eq. (2) becomes

$$\Delta\zeta = -\frac{3R_E}{\mu}\sqrt{\frac{\mu}{R_E}}\sin\theta\int_0^{t_p}(t_c + t_p - \tau)A\sqrt{\frac{\mu}{R_E}}\,\mathrm{d}\tau$$

simplifying and integrating

$$\Delta\zeta = \frac{3}{2}At_p(t_p + 2t_c)\sin\theta$$

which modifies the expression derived by Scheeres and Schweickart [5] including the factor $\sin\theta$ which accounts for the effect of the encounter geometry. We may consider the preceding expression useful for asteroids on eccentric orbits and with $a = R_E$. In which case, $\sin\theta$ can be evaluated as a function of the eccentricity as $\sin 2\theta = -e$ and the preceding equation may still be used as a first estimate. The only approximation being that of the asteroid velocity being considered as uniformly equal to its circular velocity. For small eccentricities (and $a = R_E$) we get

$$\Delta\zeta = -\frac{3e}{4}At_p(t_p + 2t_c)$$

showing that for these asteroids the phase shifting strategy is not very effective.

In the case of an impulsive deflection strategy, we may again apply Eq. (2). The deflection strategy is, in this case, $\boldsymbol{A} = \boldsymbol{\Delta V}\delta(0)$

$$\Delta\zeta = \frac{3a}{\mu}v_E\sin\theta\int_0^{t_p}(t_s - \tau)\boldsymbol{v}_{\text{ast}}\cdot\boldsymbol{\Delta V}\delta(0)\,\mathrm{d}\tau \tag{3}$$

Exploiting the Dirac delta function to evaluate the integral [17], we get

$$\Delta V_{\parallel} = \frac{\sqrt{\mu R_E}\Delta\zeta}{3t_s v_E\sin\theta\sqrt{a(2a - R_E)}}$$

This expression gives us the $\Delta V_{\parallel}$ that has to be applied in the direction of the asteroid velocity to shift the asteroid image on the b-plane by an amount $\Delta\zeta$. Thus, Eq. (2) may be used also for impulsive deflections and, in particular, for the kinetic impactor deflection concept. If the velocity change is a consequence of the impact of a spacecraft (kinetic impactor) with the asteroid, we may relate $\boldsymbol{\Delta V}$ to the impact geometry exploiting the conservation of total momentum before and after the impact

$$K(m_{\text{sc}}\boldsymbol{v}_{ac} + m_{\text{ast}}\boldsymbol{v}_{\text{ast}}) = (m_{\text{sc}} + m_{\text{ast}})(\boldsymbol{v}_{\text{ast}} + \boldsymbol{\Delta V})$$

where K accounts for a nonperfectly inelastic impact. We observe that a value $K = 1$ corresponds to a quite conservative assumption, as the effect of ejecta materials during an impact is currently thought to allow for values of K much greater than one. Hence, we get the following expression

$$\boldsymbol{\Delta V} = K\frac{m_{\text{sc}}}{m_{\text{sc}} + m_{\text{ast}}}\boldsymbol{U}$$

that substituted into Eq. (3) holds the final expression

$$\Delta\zeta = K\frac{3av_E\sin\theta}{\mu}\frac{m_{\text{sc}}t_s}{m_{\text{sc}} + m_{\text{ast}}}\boldsymbol{U}\cdot\boldsymbol{v}_{\text{ast}} \tag{4}$$

that relates the impact geometry to the deflection magnitude obtained.

III. Accuracy of the New Expression

The asteroid deflection formula Eq. (2) is based upon the assumption that secular changes on $\Delta\zeta$ are only to be taken into account. In this section, we verify the accuracy of this assumption by comparing the results returned by the asteroid deflection formula with numerical simulations. A campaign was performed on different fictitious impact scenarios created from a number of PHOs. The orbital elements of the tested PHOs were taken from online available databases. For each asteroid considered, the impact scenario was created by changing the asteroid argument of perigee as to make its MOID equal to zero. The numerical evaluation of the MOID was based upon the algorithm developed by Bonanno [18]. The intersection created between the two orbits was then considered to be an impact point, changing the asteroid mean anomaly at epoch accordingly. Different deflection strategies were applied for different push times and at different start times. The outcome, in terms of the $\Delta\zeta$ obtained, was compared with that evaluated by Eq. (2).

In the simulations, the sole effect of the sun gravity was accounted for. In the scenarios considered, the initial asteroid image on the b-plane was $\zeta = 0$, $\xi = 0$, and $\Delta\zeta$ could therefore be related to the minimum distance between the asteroid and the Earth (the pericenter of the planetocentric hyperbola), simply multiplying it by a focusing factor [5] $f = (1 + 2m_E/U^2R_E)^{-1/2}$. All the simulations showed a good agreement between the numerical simulation and the asteroid deflection formula whenever the obtained deflection was significant. As an example, we consider the highly eccentric asteroid 2003 GG21 and two different deflection strategies: 1) thrust aligned with the asteroid velocity $\boldsymbol{A}(t) = A\boldsymbol{i}_v$, and 2) thrust fixed in the heliocentric frame (the direction chosen is that perpendicular to the orbital plane and to the eccentricity vector) $\boldsymbol{A}(t) = A\boldsymbol{i}_p$.

To create a zero MOID object, the argument of perihelion of 2003 GG21 was set to be $\omega = 95.33$ deg so that the orbits intersect at the asteroid descending node. We also took $A = 1.5710^{-13}$ km/s^2 as

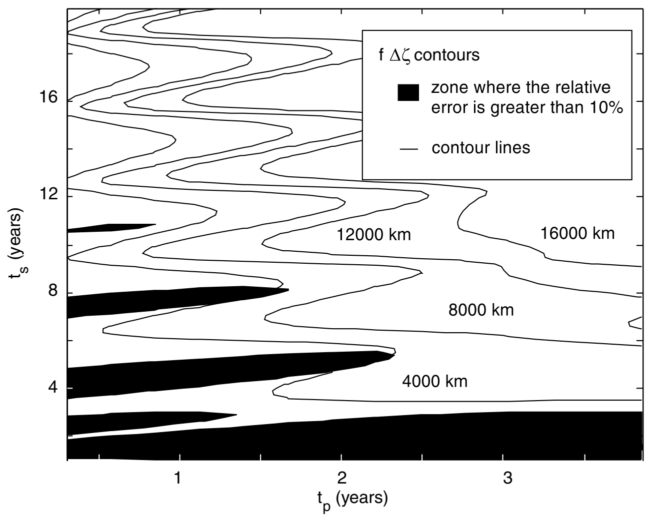

Fig. 2 Contour lines and relative error on $f\Delta\zeta$: thrust aligned with velocity.

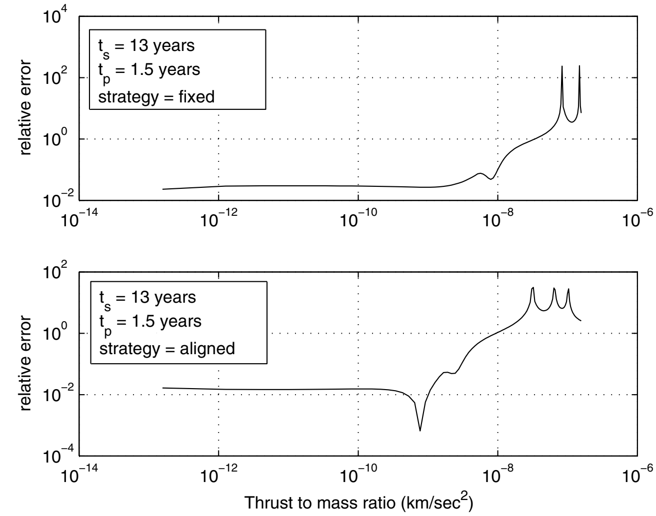

Fig. 4 Comparison for different thrust to mass levels.

recently considered by Walker et al. [8]. Figure 2 refers to case 1 and shows the contour plots of $f\Delta\zeta$ as obtained by Eq. (2) together with those zones in which the relative error was evaluated to be greater than 10% with respect to the numerical simulation. In this zone, the hypothesis under which we derived Eq. (2) is too simplistic and the optimal deflection strategy is not to change the asteroid phase leaving its MOID unaltered; it may actually be very different, as found, for example, by Conway [14]. Nevertheless, this zone is also less interesting as it is associated with optimal deflections that are not practical. Figure 3 is related to case 2 and shows the same representation of $f\Delta\zeta$ and its relative error. A 20% relative error is here considered rather than 10%, to enhance the figure readability. We note how in this second case the relative error is generally larger. In the interesting parts of the plot though, when the achieved deflection is maximal, the relative error drops significantly. The results, confirmed by all the other simulations performed, show a relative error that is small around the extremal points of $f\Delta\zeta$. For late deflections, that is for small values of t_s, the accuracy of the expression becomes less and less reliable as the secular effects do not have time to accumulate. Equation (2) is an accurate approximation of the asteroid b-plane image displacement whenever t_s is large enough, that is where we actually can hope to realize a real case deflection [8]. In Fig. 4, the comparison is extended for different values of the thrust to mass ratio showing that the range of validity of Eq. (2) extends also to larger values of A.

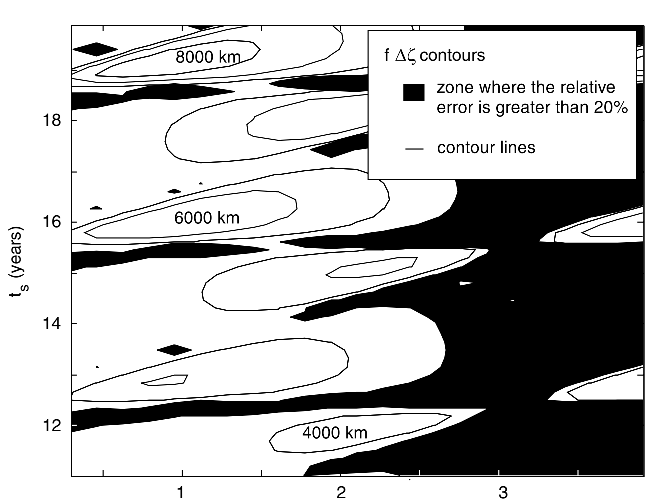

Fig. 3 Contour lines and relative error on $f\Delta\zeta$: thrust inertially fixed.

IV. Encounter B-Plane

The study of the encounter b-plane gives us the magnitude of $\Delta\zeta$ necessary to achieve a deflection. When a new asteroid is discovered, the orbital parameters $\boldsymbol{\alpha}^*$ that are made available by the astronomers are the best fit of the available observations, in the sense of minimizing a certain function Q of the squared residuals [3]. Deterministic information on the asteroid orbit is practically never available and one has to deal with this source of uncertainty when planning a deflection mission. In this section, we introduce some relevant subsets of the b-plane Π that are useful when planning a deflection mission.

A. Virtual Asteroids

Writing Q as a function of the orbital parameters, we write

$$Q(\boldsymbol{\alpha}) = Q(\boldsymbol{\alpha}^*) + \frac{1}{2}(\boldsymbol{\alpha} - \boldsymbol{\alpha}^*)\boldsymbol{H}(\boldsymbol{\alpha} - \boldsymbol{\alpha}^*) + \dots$$

or its more useful statistical interpretation

$$Q(\boldsymbol{\alpha}) = Q(\boldsymbol{\alpha}^*) + \frac{1}{N}(\boldsymbol{\alpha} - \boldsymbol{\alpha}^*)\Sigma^{-1}(\boldsymbol{\alpha} - \boldsymbol{\alpha}^*) + \dots$$

Hence, we may define a neighborhood of the best-fit $\boldsymbol{\alpha}^*$ by requiring that

$$(\boldsymbol{\alpha} - \boldsymbol{\alpha}^*)\Sigma^{-1}(\boldsymbol{\alpha} - \boldsymbol{\alpha}^*) < \chi^2$$

where χ is a real number defining the allowed deviation from the best fit. All the asteroids having elements in the ellipsoid defined by the preceding relation are called virtual asteroids [3]. The propagation of such an ellipsoid, accounting for the uncertainties related to the asteroid dynamic, defines the set Θ of all the possible positions occupied by the virtual asteroids. A discussion on the accuracy of this definition of virtual asteroids may be found in [19]. On the encounter b-plane of the best-fit $\boldsymbol{\alpha}^*$, we define the set $\Gamma = \{\boldsymbol{X} | \boldsymbol{X} \in \Theta\}$ containing the b-plane images of all the virtual asteroids.

B. Keyholes and Focused Earth

The set Γ gives us information on where the asteroid could hit the b-plane, the information on where the asteroid should not hit the b-plane is given by two other subsets of Π. The first one is trivially the locus of all the points that if hit result in a direct impact of the Earth as corresponds to the asteroid being put in a planetocentric hyperbola intersecting the Earth. We may approximate such a set by a circle centered in the origin of our reference frame and having radius $f^{-1}R_E$. The second subsets contain the so-called keyholes. These are those zones of the b-plane that when hit put the asteroid into a new

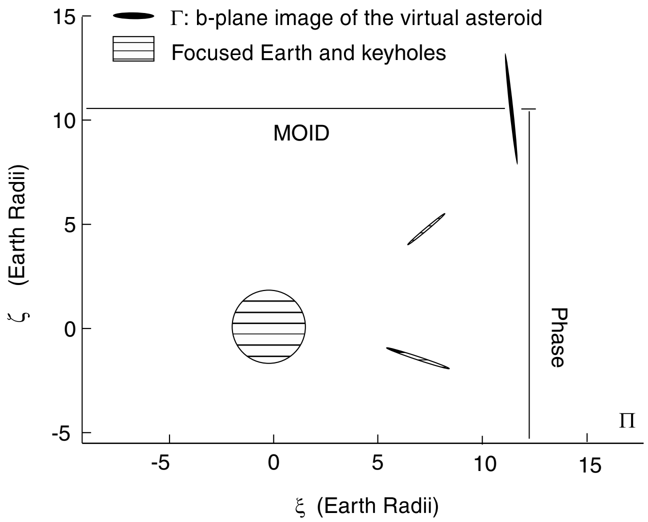

Fig. 5 Visualization of an hypothetical encounter b-plane Π.

orbit resonant with the Earth and leading to an impact at the following encounter. Although an analytical theory has been proposed by Valsecchi et al. [16] a precise determination of the keyhole position and shape is more difficult and has to make use of precise and complex orbital propagators and accurate force models.

In Fig. 5, we have visualized an example of what the b-plane of an asteroid encounter may look like. If the intersection between Γ and the other sets is not empty, the collision chance is nonzero. We may either wait for a refinement of the asteroid orbit determination and a shrinking of the set Γ or, if the risk is judged to be not acceptable, we may start to plan a deflection mission which would then have as an objective to place Γ in a safe part of the b-plane. We may therefore apply Eq. (2) to each virtual asteroid. This results in a translation of the whole set Γ along the ζ direction of an amount predicted by the asteroid deflection formula applied to the best-fit $\boldsymbol{\alpha}^*$.

V. Interplanetary Trajectory Optimization for Asteroid Deflection

The design of the interplanetary trajectory for an asteroid deflection mission has the objective of maximizing the miss-distance between the Earth and the asteroid. We could formalize this problem as an optimal control problem in which the objective function is evaluated by forward integration in time [4]. This approach has the disadvantage of having a high computational cost for each objective function evaluation and of hiding under a complex mathematical structure the basic principles of an optimal deflection. At the cost of accepting the approximations discussed in preceding sections, Eq. (2) may be taken as a valid substitute of forward time integration. Taking this approach, we use it to deepen our understanding of two largely debated deflection strategies: the kinetic impactor and the long duration thrust.

A. Kinetic Impactor

The idea of sending a spacecraft to impact against an asteroid has been found as one of the most efficient methods for deflection [20,21]. The design of the interplanetary trajectory in this case will aim at obtaining the maximum performance index $J(\boldsymbol{x}_f, t_f) = \Delta\zeta$ as given by Eq. (4):

$$J(\boldsymbol{x}_f, t_f) = K\frac{3av_E\sin\theta}{\mu}\frac{m_{\mathrm{sc}}t_s}{m_{\mathrm{sc}}+m_{\mathrm{ast}}}\boldsymbol{U}\cdot\boldsymbol{v}_{\mathrm{ast}} \tag{5}$$

We therefore aim at maximizing the spacecraft mass at impact m_{sc}, the dot product $\boldsymbol{v}_{\mathrm{ast}}\cdot\boldsymbol{U}$, the impact time before the Earth encounter t_s, and the parameter K.

1) The mass at impact m_{sc}: This objective is quite common in many interplanetary trajectory optimizations. As a result, the control structure of the optimal solution is typically discontinuous.

2) The impact geometry $\boldsymbol{v}\cdot\boldsymbol{U}$: Transfers that impact with the asteroid at its perihelion are attractive with respect to this objective and large final relative velocities aligned with $\boldsymbol{v}$ are preferred. A retrograde impact, such as that suggested by McInnes [6], would be the best solution if considering this objective alone. A low-thrust engine gives much more flexibility to change, optimizes the impact geometry, and constitutes an attractive option for a spacecraft that has to achieve large deflections, considering also the large spacecraft masses that would be needed.

3) The impact time t_s: This objective makes early launches and faster transfers preferred. As a consequence, the impact point along the asteroid orbit will be shifted as early as possible.

4) The parameter K: This parameter is a crucial quantity that is likely to carry the largest uncertainty in such a mission. It depends on the composition of the target asteroid and on the capability of the spacecraft to hit the asteroid precisely and thus avoid transferring energy into the asteroid rotation. It is likely that even with the added information that missions like NASA Deep Impact or ESA Don Quijote will give us, a large uncertainty on this parameter will remain.

For low spacecraft masses, a chemical propulsion system may be envisaged, whereas for higher masses it is likely that a low-thrust propulsion system will first spiral the spacecraft out of the Earth sphere of influence and then will push it to impact against the asteroid.

B. Long Duration Thrust

The application of a slow and gentle push $\boldsymbol{T}$ over a long period of time has also been given a fair attention as a possible deflection mechanism. The push may be given by the gravitational interaction between the asteroid and a spacecraft continuously hovering over the asteroid, as recently proposed by Lu and Love [7], or directly by the spacecraft engines after the spacecraft has attached itself on the asteroid surface. A discussion on the effect of the asteroid rotational motion in this case may be found in Scheeres and Schweickart [5]. In both cases, the spacecraft will use its thrust to fly an interplanetary trajectory and rendezvous with the asteroid. Once the asteroid is reached, its deflection is achieved by $\tilde{\boldsymbol{A}}(t) = \tilde{\boldsymbol{T}}(t)/m_{\mathrm{ast}}$, that is the optimal thrusting strategy maximizing $\Delta\zeta$. The objective function $J(\boldsymbol{x}_f, t_f)$ of the interplanetary trajectory optimization will therefore be $J = \Delta\zeta[\tilde{\boldsymbol{T}}(\boldsymbol{x}_f, t_f)]$ or, in explicit terms

$$J(\boldsymbol{x}_f, t_f) = \begin{cases} \max_{\boldsymbol{T}\epsilon\mathcal{F}} & \int_0^{t_p}(t_s-\tau)\boldsymbol{v}_{\mathrm{ast}}\cdot\boldsymbol{T}\,\mathrm{d}\tau \\ \text{subject to:} & \int_0^{t_p}\boldsymbol{T}\,\mathrm{d}\tau < c \end{cases}$$

where $\mathcal{F}$ is the functional space $\boldsymbol{T}$ belongs to (for example, the space of piecewise continuous functions subject to a $T_{\max}$ limit on their norm). The solution to this problem is, in general terms, complicated as it may involve a switching structure of the type bang-off-bang. To simplify the problem, one can get rid of the possible bang-off-bang structure by assuming that the optimal thrusting strategy is $\tilde{\boldsymbol{T}}(\tau) = T_{\max}\boldsymbol{i}_v(\tau)$ defined in $\tau\in[0, g_0m_cI_{\mathrm{sp}}/T_{\max}]$. This corresponds to assuming that the optimal strategy is to thrust continuously with the maximum available level $T_{\max}$ along the asteroid velocity vector and up to when all the remaining fuel mass m_c has been consumed:

$$J(\boldsymbol{x}_f, t_f) = T_{\max}\int_0^{I_{\mathrm{sp}}g_0m_c/T_{\max}}(t_s-\tau)v_{\mathrm{ast}}(\tau)\,\mathrm{d}\tau \tag{6}$$

The interplanetary trajectory will therefore have to carry the spacecraft to rendezvous with the asteroid maximizing the spacecraft mass at rendezvous m_{sc} (directly proportional to the fuel left for pushing m_c, which in turn is directly proportional with the push time t_p), the impact time before the Earth encounter t_s, and adjusting the rendezvous point along the asteroid orbit so that the effect of its velocity is exploited optimally. Note that in the concept of the gravitational tug by Lu and Love [7], the use of Eq. (6) introduces no approximation as in that case no switching structure is allowed being the gravitational attraction always active.

VI. Example 1: Deflection Strategies Trade-off

Here, we use Eq. (5) and (6) to trade-off the long duration thrust deflection concept with the kinetic impactor in the case of the advanced spacecraft considered by Walker et al. [8]. We select the orbit of asteroid 2003 GG21 and we assume its mass to be 10^{10} kg, corresponding to a 200-m-diam asteroid with an average 2.4 $\mathrm{g/cm^3}$ density. The asteroid parameters were modified as to create an impact with the Earth at epoch 9224 MJD2000 so that $a = 2.14$ AU, $e = 0.709$, $i = 10.12$ deg, $\Omega = 13.23$ deg, $\omega = 95.34$ deg, $M = 41.2$ deg at Epoch 60768 MJD. We take the wet mass of the spacecraft to be 18,000 kg, we assume advanced nuclear electric propulsion (NEP) capable to deliver $2N$ of thrust with a specific impulse of 6700 s [8]. We also assume a zero departure $C3$ reached after a spiral out phase common to both mission profiles and requiring 2000 kg of fuel mass [8]. From that point, we perform the optimization of a direct transfer constraining the departure date to be after 5500 MJD2000 (January 2005). The continuous optimal control problem is approached with a direct method [22] and Eq. (5) and (6) are used as objective functions. We set $K = 1$ to be conservative with respect to the result claimed later.

Some results are reported in Table 1 and the trajectories are visualized in Fig. 6. Note that the impact trajectory is almost perpendicular to the asteroid orbit, forming an angle of 81.26 deg, and yet the impact geometry is such that $\boldsymbol{U} \cdot \boldsymbol{v}_{\mathrm{ast}} = 1898\ \mathrm{km^2/s^2}$ and the change in the asteroid orbit energy is significant. The impact point is quite near to the asteroid perihelion and the trajectory never gets nearer than 0.6 AU from the sun (the z motion, not shown in Fig. 6, is quite significant). In the case considered, the kinetic impactor strategy is shown to outperform the long duration thrust. The same spacecraft is able to achieve a much larger deflection when it uses its high specific impulse engines to accelerate toward a maximum momentum exchange impact, rather than rendezvousing with the asteroid and pushing. Here the trade-off is really between using each kilogram of mass available to expel it at high speed via an advanced propulsion system or to accelerate the spacecraft toward a high energy impact. We may think of the impact case as being an advanced propulsion system able to expel at once the entire final spacecraft mass with an exhaust velocity of $-\boldsymbol{U}$. Following this idea, we may evaluate the equivalent effective specific impulse of the push in the kinetic impactor scenario, here defined as $g_0 \bar{I}_{\mathrm{sp}} = \boldsymbol{U} \cdot \boldsymbol{v}_{\mathrm{ast}} / v_{\mathrm{ast}}$, and that in the simulation performed was evaluated to be 3985 s. This value is only about 60% of the specific impulse assumed for the actual advanced propulsion system, but the chance to impart the impulse all at once and near the perihelion makes, in the case considered, the kinetic impactor strategy more efficient. In the asteroid deflection formula this is obvious looking at the integrand $(t_s - \tau)\boldsymbol{v}_{\mathrm{ast}} \cdot \boldsymbol{A}$: any mass expelled after a time τ from the deflection start t_s contributes increasingly less to the miss-distance. Besides, in the kinetic impactor scenario, all the mass available is used as propellant and the factor K would be greater than one, whereas in the long thrust duration case only a part of the final spacecraft mass could be used, depending on the spacecraft design.

VII. Example 2: Case of 99942 Apophis

After its detection in December 2004, the asteroid labeled 2004 MN4 and later renamed 99942 Apophis attracted the interest of many astronomers and scientists due to its unique characteristics. In the early days of its detection, an astonishing 1/37 chance was predicted for the catastrophic event of an impact with the Earth during its 2029 close approach. Such an enormous risk focused the attention of many astronomers on the small rock (roughly 400 m of diameter) and further data were made available and served to exclude, a few weeks after the asteroid discovery, the 2029 impact. With the current knowledge, it still may not be excluded that the close Earth encounter will change the asteroid orbit and make a resonant return in 2036, thus impacting the Earth. Although the chance currently foreseen is quite low and comparable to the background probability level of possible impacts with the NEO population, the case of Apophis is interesting to test the techniques here developed.

The encounter b-plane Π for the 99942 Apophis close encounter in 2029 is shown in Fig. 7 together with the asteroid image Γ (as evaluated with the data available in June 2005 and graciously provided by Giovanni Valsecchi, Istituto di Astrofisica Spaziale e Fisica Cosmica, Italy), the focused Earth and the position of a keyhole that intersects Γ. The keyhole position is shown as evaluated both via a numerical computation and using the analytical theory developed by Valsecchi et al. [16]. A deflection mission would have to move the uncertainty ellipsoid so that its intersection with the keyhole and with the focused Earth is the empty set. In the case of 99942 Apophis, further observation and data available will most probably shrink the uncertainty ellipsoid to few kilometers by 2015. The order of magnitude of the deflection needed would consequently be of few kilometers. This small amount makes it possible to consider very simple spacecraft designs to achieve the deflection before the close encounter in 2029. We consider the Don Quijote spacecraft as designed by the Concurrent Design Facility (CDF) [10] with a mass of 790 kg and a C3 2.26 km/s allowed by a Dnepr launcher. The interplanetary trajectory optimization problem is, in this case, quite simple and may be stated in a compact mathematical form:

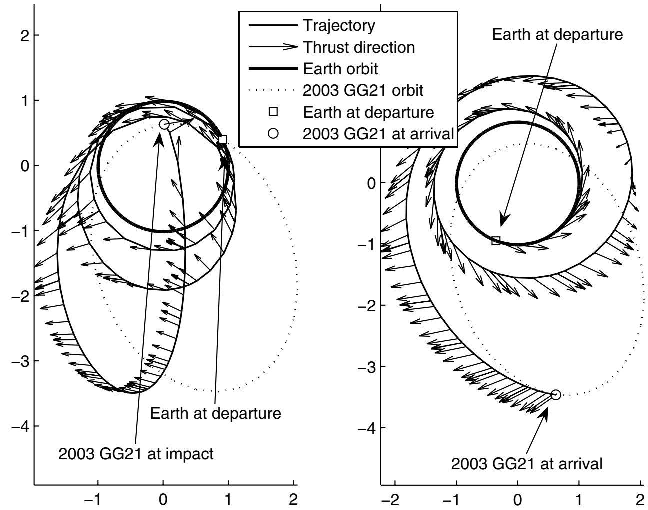

Fig. 6 Optimal trajectories found in the two cases.

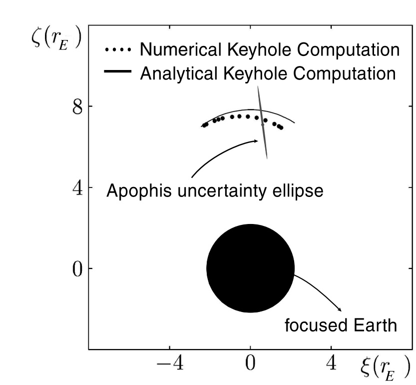

Fig. 7 B-plane of 99942 Apophis Earth encounter in 2029.

Table 1 Optimization results

	Kinetic impactor	Long duration thrust
Departure epoch, MJD2000	5768	5629
Arrival epoch, MJD2000	7938	7264
Flight time, yrs	5.94	4.47
Mass at arrival, kg	10,262	11,675
$\Delta\zeta$, km	46,728	9567
Asteroid ΔV, m/s	6.37 10^{-2}	N/A
$\Delta\sigma$, s	1569	392

Table 2 Possible launches for Don Quijote impactor (Hidalgo) as designed by CDF [10]

Departure	Impact	$\lvert U \rvert$ km/s	Deflection $\Delta\zeta$, km
16 April 2018	12 July 2019	7.32	57.58
17 April 2019	31 May 2020	8.02	43.37
29 Dec. 2020	09 March 2021	7.15	47.4
08 Sept. 2021	25 March 2022	9.01	17.23
02 March 2026	02 March 2027	7.09	10.05
01 April 2027	02 Dec. 2027	7	3.96
05 Feb. 2028	18 Nov. 2028	5	0.45

find:

$$t_{\mathrm{dep}}, t_{\mathrm{arr}} \in \mathcal{I}$$

to maximize:

$$\Delta\zeta(t_{\mathrm{dep}}, t_{\mathrm{arr}}) = \frac{3 a v_{\mathrm{ast}} K \sin\theta}{\mu(m_{\mathrm{ast}} + m_{\mathrm{sc}})} m_{\mathrm{sc}} t_s \boldsymbol{v}_{\mathrm{ast}} \cdot \boldsymbol{U} \tag{7}$$

subject to:

$$C3_{\max} \geq C3_{\mathrm{dep}}(\boldsymbol{x})$$

where $C3_{\max}$ is the maximum C3 allowed by the launcher. The problem has, in this case, a finite dimension equal to two and can be solved very efficiently both by the use of heuristic global optimization techniques and by using deterministic global optimizers [23]. Here we apply differential evolution [24] to search the solution space defined as $\mathcal{I} = [2015, 2029] \times [2015, 2029]$.

In Table 2, the optimal launches are shown for each year, and in Fig. 8 the globally optimal trajectory (in 2018) is visualized. The maximum b-plane deflection achievable is $\Delta\zeta = 57.58$ km, well above the keyhole thickness. The magnitude of the relative velocity at impact is 7.93 km/s. These results give us an idea on the capability of a simple mission such as Don Quijote to achieve a prekeyhole deflection. Several optimized trajectories exist after 2015 that could achieve the deflection of 99942 Apophis.

VIII. Conclusions

We have introduced an analytical expression relating the amount of asteroid deflection achievable by a generic space mission to the characteristic of the interplanetary trajectory flown. The numerical accuracy of the expression is tested for different strategies and thrust to mass ratios revealing good accuracy with respect to the results returned by direct numerical integrations. We find that the expression, named asteroid deflection formula, may be conveniently used as objective function in the optimization of the interplanetary trajectory of asteroid deflection missions, regardless of the type of deflection mechanism one intends to use. In the case of kinetic impactors, the formula takes an even simpler form and no numerical quadratures are needed for its evaluation. We show and explain why the kinetic impactor strategy outperforms the long duration thrust in the case selected and we give some hints on how such a result could be generalized. Using the results developed, we perform an interplanetary trajectory global optimization for the Don Quijote spacecraft in the case of a putative deflection mission for the asteroid 99942 Apophis, showing how its deflection is well within the reach of such a spacecraft.

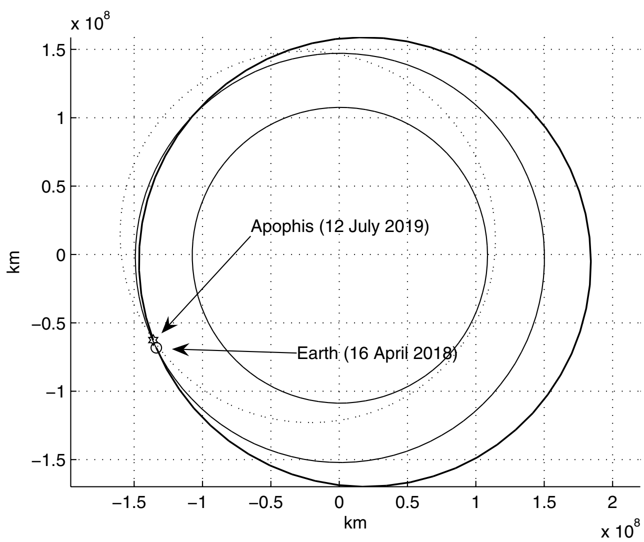

Fig. 8 Globally optimal solution for the Don Quijote (Hidalgo) kinetic impactor.

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References

[1] Alvarez, L., Alvarez, W., Asaro, F., and Michel, H., "Extra-Terrestrial Cause for the Cretaceous-Tertiary Extinction," *Science*, Vol. 208, No. 4448, 1980, pp. 1095–1108.

[2] Vokrouhlicky, D., Milani, A., and Chesley, S., "Yarkowsky Effect on Small Near-Earth Asteroids: Mathematical Formulation and Examples," *Icarus*, Vol. 148, No. 1, Nov. 2000, pp. 118–138.

[3] Milani, A., Chesley, S., Boattini, A., and Valsecchi, G., "Virtual Impactors: Search and Destroy," *Icarus*, Vol. 145, No. 1, May 2000, pp. 12–24.

[4] Hall, C. D., and Ross, I. M., "Dynamics and Control Problems in the Deflection of Near-Earth Objects," Proceedings of the 1997 AIAA/AAS Astrodynamics Conference, *Advances in the Astronautical Sciences*, Vol. 97, Univelt, San Diego, CA, 1997, pp. 613–632; also American Astronomical Society Paper 97-640.

[5] Scheeres, D., and Schweickart, R., "The Mechanics of Moving Asteroids," *Proceeding of the 2004 Planetary Defense Conference*, AIAA Paper 2004-1446, 2004.

[6] McInnes, C., "Deflection of Near-Earth Asteroids by Kinetic Energy Impacts from Retrograde Orbits," *Planetary and Space Science*, Vol. 52, No. 7, June 2004, pp. 587–590.

[7] Lu, E., and Love, S., "A Gravitational Tractor for Towing Asteroids," *Nature (London)*, Vol. 438, Nov. 2005, pp. 177–178.

[8] Walker, R., Izzo, D., de Negueruela, C., Summerer, L., Ayre, M., and Vasile, M., "Concepts for Near Earth Asteroid Deflection using Spacecraft with Advanced Nuclear and Solar Electric Propulsion Systems," *Journal of the British Interplanetary Society*, Vol. 59, July–Aug. 2005, pp. 268–278; also International Astronautical Congress Paper 04-Q.5.08 presented at the 55th International Astronautical Congress in Vancouver.

[9] Galvez, A., "Paving the Way for an Effective Deflection Mission: State of the Art NEO Precursor Missions," *Planetary Defense Conference: Protecting Earth from Asteroids*, AIAA Paper 2004-1425, 2004.

[10] CDF Team, "NEO2 CDF Study Report," ESA, TR CDF-39(A), 2005.

[11] Park, S., and Ross, I., "Two-Body Optimization for Deflecting Earth-Crossing Asteroids," *Journal of Guidance, Control, and Dynamics*, Vol. 22, No. 3, 1999, pp. 415–420.

[12] Ross, I., Park, S., and Porter, S., "Gravitational Effects of Earth in Optimizing DV for Deflecting Earth-Crossing Asteroids," *Journal of Spacecraft and Rockets*, Vol. 38, No. 5, 2001, pp. 759–764.

[13] Park, S., and Choi, K., "Optimal Low-Thrust Intercept/Rendezvous Trajectories to Earth-Crossing Objects," *Journal of Guidance, Control, and Dynamics*, Vol. 28, No. 5, 2005, pp. 1049–1055.

[14] Conway, B., "Near-Optimal Deflection of Earth-Approaching Asteroids," *Journal of Guidance, Control, and Dynamics*, Vol. 24, No. 5, 2001, pp. 1035–1037.

[15] Dachwald, B., and Wie, B., "Solar Sail Trajectory Optimization for Intercepting, Impacting, and Deflecting Near-Earth Asteroids," *AIAA Guidance, Navigation, and Control Conference and Exhibit*, AIAA, Paper 2005-6176, 2005.

[16] Valsecchi, G., Milani, A., Gronchi, G., and Chesley, S., "Resonant Returns to Close Approaches: Analytical Theory," *Astronomy & Astrophysics*, Vol. 408, 2003, pp. 1179–1196.

[17] Arfken, G., *Mathematical Methods for Physicist*, Academic Press, Orlando, F, 1985.

[18] Bonanno, C., "An Analytical Approximation for the MOID and its Consequences," *Astronomy and Astrophysics*, Vol. 360, 2000, pp. 411–416.

[19] Milani, A., "Virtual Asteroids and Virtual Impactors," *Proceedings of the International Astronomical Union Colloquium N.197 Dynamics of Populations of Planetary Systems*, edited by Z. Knezevic, and A. Milani, 2005, pp. 219–228.

[20] Ivashkin, V., and Smirnov, V., "An Analysis of Some Methods of Asteroid Hazard Mitigation for the Earth," *Planetary and Space Science*, Vol. 43, No. 6, June 1995, pp. 821–825.

[21] Izzo, D., de Negueruela, C., Ongaro, F., and Walker, R., "Strategies for Near Earth Object Impact Hazard Mitigation," *Proceedings of the 2005 AAS-AIAA Space Flight Mechanics Conference*, Advances In Space Research, Vol. 121, Univelt, San Diego, CA, 2005, pp. 699–708; also American Astronomical Society Paper 05-147.

[22] Vasile, M., and Bernelli-Zazzera, F., "Optimizing Low-Thrust and Gravity Assist Maneuver to Design Interplanetary Trajectories," *Journal of the Astronautical Sciences*, Vol. 51, No. 1, 2003.

[23] Izzo, D., Becerra, V., Myatt, D., Nasuto, S., and Bishop, J., "Search Space Pruning and Global Optimization of Multiple Gravity Assist Trajectories," *Journal of Global Optimization*, 2006 (to be published).

[24] Storn, R., and Price, K., "Differential Evolution: A Simple and Efficient Heuristic for Global Optimization over Continuous Space," *Journal of Global Optimization*, Vol. 11, No. 4, 1997, pp. 341–359.