Automated asteroid selection for a 'grand tour' mission

Dario Izzo†, Tamás Vinkó†, Claudio Bombardelli†, Stefan Brendelberger†, Simone Centuori†

†ESA, Advanced Concepts Team, ESTEC Keplerlaan 1, Postbus 299,2200 AG, Noordwijk

contact: dario.izzo@esa.int

Abstract

In designing a mission to visit multiple asteroids and comets finding the optimum choice of the asteroid sequence and computing optimum trajectories linking different small bodies represent a brand new problem in the field of astrodynamics and trajectory optimisation. After introducing a general description of the multiple asteroid rendezvous problem using low thrust propulsion we propose a possible solution approach divided in three phases: a combinatorial optimisation phase, a global optimisation and local optimisation phase. This solution approach is applied to the particular case of the asteroid grand tour problem assigned at the 2nd edition of the Global Trajectory Optimisation Competition. The results obtained are presented and discussed.

1 Introduction

Missions to asteroids are currently at the centre of the attention of national space agencies, academic communities and media. Deep Impact, Hayabusa, Dawn are just famous and successful names among a larger number of ongoing efforts aimed at improving our understanding of the potential benefits and threats asteroids offer to human kind. A number of missions aimed at obtaining the largest possible set of information on asteroids are being studied at different detail levels, from conceptual (e.g. SWARM and ANTS) to phase-A design (e.g. Don-Quijote) to operations (e.g. Dawn). From a mission design point of view, asteroid mission design is strongly driven by the initial asteroid choice, often constrained by scientific rationales, but otherwise left free to the mission designer. Given the large number of asteroids and comets known so far when this additional degree of freedom is added to the trajectory optimisation process the dimension of the search space increases considerably. Furthermore, when the mission objective is to rendezvous with more than one asteroid the problem dimensions grows enormously.

Past studies were focused on the problem of multiple asteroid fly-bys. Missions like Rosetta and Cassini were designed in order to obtain fast flybys of a number of small bodies as secondary science mission goals, while low-cost missions whose primary objective was to fly-by with the highest number of asteroids were studied by different authors. Brooks and Hampshire [1], for instance, derived multiple-asteroid fly-by impulsive trajectories based on geometric distance between known asteroids and randomly generated keplerian trajectories. Bender and Bourke [2] later analysed a similar problem for the case of solar electric propulsion. Perret et al. [3] performed a systematic search for multiple asteroid fly-bys using a branch-and-prune criterion for limiting the maximum deltaV. These studies did not consider the possibility of randezvous with multiple asteroids due to the much higher deltaV requirements. On the other hand later advances of electric propulsion technology made this option feasible and a number of authors started addressing different aspects of multiple asteroid randezvous missions. For example Bender and Friedlander [4]... Morimoto et al. [5] considered a multiple rendezvous mission in which the sequence of asteroids to be visited was not selected a priori and employed a genetic algorithm to the trajectory design and optimisation. On the other hand the number of candidate asteroids considered was rather low (21 asteroids in total) making it possible, in principle, to analyse all possible sequences in a reasonable amount of time. In this article we deal with the most general case of optimising a multiple asteroid rendezvous mission (using either chemical or electric propulsion) in which N asteroids have to be visited starting from a relatively large sample of n asteroids while maximising a given performance index J. For this case the large number of possible asteroid combinations makes enumerative searches unreasonable and a method has to be devised in order to quickly select the most promising sequences. After defining the optimisation problem in a rather general way we propose a solution strategy aimed at providing a number of optimised chemical trajectories in a fully automated way. Thereafter we describe the process from which low-thrust optimised trajectory can be obtained. Finally we apply our solution strategy to multiple asteroid rendezvous problem proposed by the Jet Propulsion Laboratories (JPL) for the 2nd edition of the Global Trajectory Optimisation Competition. The method and results obtained are discussed and suggestions are given for future improvements.

2 Problem Definition

The multiple asteroid rendezvous problem (or ásteroid grand tour' problem) can be formulated as follows. Given n sets $\mathcal{A}_i$, $i = 1, \ldots, n$ of asteroids not necessarily disjointed, each of them containing m_j asteroids, consider a generic trajectory which launches from Earth and subsequently performs a rendezvous of duration T_j with one (not already visited) asteroid from each of the N sets, in general without a preferred sequential order. The propulsion system employed (chemical, low-thrust, solar sail, etc.) will be specified by the problem and gravity assist with solar system planets may or may not be permitted. After defining a suitable performance index J a trajectory is sought which maximises J while subject to a number of given constraints (e.g. launch window, maximum thrust, etc.). The reason for grouping different asteroids into different sets comes from the need to maximise the scientific return of a space mission. For example one can choose to place asteroids of different taxonomic classes in different sets so that each class of asteroid is visited.

3 The Solution Approach

A solution approach for the asteroid grand tour optimisation problem is now presented. We make here the assumption that no planetary fly-bys are employed which considerably reduces the complexity of an already difficult problem. The method is general enough to accommodate both impulsive and low thrust propulsion.

The overall optimisation strategy proposed can be summarised in 3 steps.

First, a combinatorial optimisation is performed based on a generalised distance between asteroids orbits and providing a number of candidate sequences. Second, for each candidate sequence an optimised impulsive trajectory is derived based on a global optimisation method. Lastly, if electric propulsion is considered, the best trajectory obtained are taken as initial guess for the local optimisation phase. One of the reasons to select this particular strategy was to create a tool that can quickly provide a relatively large number of sequences in a (ideally) fully automated way without resorting to astrodynamics considerations to direct the search for promising trajectories.

1. Combinatorial Optimisation
2. Global Optimisation
3. Local Optimisation

During the combinatorial optimisation a number of 'interesting' asteroid sequences are selected. For each given sequence, a global optimisation problem is solved in order to locate a good phasing between the asteroids assuming impulsive deltaV maneuvers. Finally, if a low-thrust propulsion startegy is requested, a final optimisation step is performed whereby the most promising solutions found on step 2 are used as initial guess to be refined with a local optimisation technique in order to meet the constraints imposed by the problem description. Note that if low-thrust propulsion is considered the global optimisation phase has to be tailored in order to provide suitable solutions for carrying out the local optimisation step.

3.1 Combinatorial Optimisation

Let us introduce n sets $\mathcal{A}_i$, $i = 1, \ldots, n$ containing the different asteroid groups member and the set $\mathcal{A}_0$ containing only one element, the Earth. Let us then introduce a *generalised distance* $d : \mathcal{A}_i \times \mathcal{A}_j \to \mathbb{R}$, $i, j = 0, \ldots, n$ between each pair of asteroids[1]. At this point the following problem is considered. Choose one element $A_i \in \mathcal{A}_i$ in each set and a permutation s of $\{1, 2, .., n\}$ so that

$$J_{comb} := d(A_0, A_{s(1)}) + \sum_{i=1}^{n-1} d(A_{s(i)}, A_{s(i+1)})$$

is minimised.

This problem is a combinatorial optimisation problem that can be tackled, in general, with a number of different techniques both stochastic and deterministic. When the order of visit for the different groups is left free the number of all possible combinations is given by:

$$N_c := n! \prod_{i=1}^{n} m_j$$

It is easy to see that when the number of asteroids to be visited is ≥ 4 already with a number of asteroids of the order of 100 for each group the resulting combinations makes in general enumerative search not attractive and a more efficient strategy is desired. We choose to implement a deterministic algorithm, in particular a Branch and Prune based technique. The algorithm is able to return the list of all the solutions to the problem that are below a given bound. Sorting through this list would then give a range of candidate sequences which are to be studied in the subsequent optimisation steps.

Clearly, the choice of the *generalised distance function* $d(A_i, A_j)$ is a crucial step in the process. Ideally $d(A_i, A_j)$, which is a function of the 6 orbital elements of each of the two asteroids and possibly a number of additional parameters to accomodate the problem constraints, should be tailored in such a way to maximise its correlation with the partial cost of a trajectory linking the two asteroids A_i and A_j while satisfying the constraints dictated by the problem (e.g. low thrust transfer with maximum thrust $F_{\max}$). Many choices of $d(A_i, A_j)$ are possible and generally leading to different sequence ranking and the quest for the best formulation is the subject of an ongoing research effort by the authors.

[1] Strictly speaking it is not necessary that $d(A_i, A_j) = d(A_j, A_i)$ holds.

3.2 Global Optimisation

The definition of the *generalised distance* between asteroid pairs used to select a first sample of candidate sequences is not at all sufficient to guarantee that the latter will lead to good trajectories. This is because it cannot account for the phasing between subsequent asteroid pairs. As a matter of fact the influence of the phasing can have a large impact on the final merit function as it will be clear from the numerical example reported later on. One possible approach to address the phasing issue is to introduce simplified dynamical models representing impulsive transfer trajectories for each asteroid sequence and optimise a given trajectory decision vector using global optimisation techniques. This approach not only returns asteroid sequences which have good phasing characteristics but can provide, when properly and carefully tuned, good initial guesses for a subsequent low-thrust optimisation. For this purpose, different kinds of dynamical model are hereby considered. The first model considered includes Lambert arcs between all the asteroids and can be represented by the following formal transcription:

$$\begin{aligned} \text{find:} \quad & \mathbf{x} \in \mathbb{I}^{2n} \\ \text{to minimise:} \quad & J(\mathbf{x}) := J_{chem} \end{aligned}$$

where the decision vector $\mathbf{x}$ contains the Earth departure epoch, the transfer time to the first asteroid, the waiting time on the first asteroid, the transfer time to the second asteroid and so on.

The second model considered is obtained from the first model by replacing a number of Lambert arcs with exponential sinusoid arcs. This variation provides trajectories that are closer to the low thrust trajectory type and can therefore be used as a better initial guess for the last optimisation phase. This advantage comes at the price of a model with higher complexity and more difficult to optimise. The problem can be described by the following transcription:

$$\begin{aligned} \text{find:} \quad & \mathbf{x} \in \mathbb{I}^{2n+m} \\ \text{to minimise:} \quad & J(\mathbf{x}) := J_{exp} \end{aligned}$$

where the decision vector $\mathbf{x}$ contains the Earth departure epoch, the transfer time to the first asteroid, the waiting time on the first asteroid, the transfer time to the second asteroid and so on. The last m components of the decision vector contain the shape parameters k_i of the exponential sinusoid arcs.

3.3 Local Optimisation

Starting from the best solutions provided by in the global optimisation phase a Non-Linear-Programming (NLP) problem is created and solved for each asteroid-to-asteroid transfer phase and the optimal solution in terms of the controls can then be reconstructed. This process serves the purpose of fixing the infeasibilities coming from the simplified models used in the global optimisation phase and delivering a truly feasible trajectory.

4 The GTOC2 Competition Problem

A particular case of Multiple Asteroid Rendezvous Problem can be found in the second Global Trajectory Optimisation Competition (GTOC2) organised by the Jet Propulsion Laboratory. The problem consisted of optimising a low thrust trajectory performing a rendezvous with 4 of 910 asteroids grouped in 4 sets of 96, 176, 300 and 338 elements. The 1500-kg spacecraft was launched from Earth with a free launch deltaV of a maximum of 3.5 km/s, was equipped with an electric propulsion system with a maximum thrust of 0.1 N, 1000kg of propellant and 4000 s of specific impulse. It was requested to spend at least 90 days on each of the first 3 asteroids. The objective function to maximise was defined as:

$$J := m_f/t_f$$

where m_f is the final spacecraft mass and t_f the total mission time both measured at the point of rendezvous with the 4th asteroid.

4.1 Combinatorial Optimisation

Following the combinatorial optimisation scheme described earlier it is necessary to define a generalised distance between asteroids. Clearly we want to penalise those orbit pairs that would result in an expensive transfer. In this case 'expensive' means both a high fuel consumption and a high transfer time. For the present analysis we have chosen d as the ΔV consumption along a three impulse transfer connecting the two asteroid orbits. The two impulses are given at the pericenter of the orbit having the smallest apocenter (apocenter raise) and at the apocenter of the other orbit (pericenter raise and inclination change). All in all the definition of $d(A_i, A_j)$ may be given as:

$$d(A_i, A_j) = \Delta V_1 + \Delta V_2$$

where

$$\Delta V_1 = \sqrt{\mu} * \left(\sqrt{2/r_{p1} - 2/(r_{p1} + r_{a1})} - \sqrt{2/r_{p1} - 2/(r_{p1} + r_{a2})} \right)$$

$$\Delta V_2 = \sqrt{V_i^2 + V_f^2 - 2V_i V_f \cos i_{rel}}$$

$$V_i = \sqrt{\mu}\sqrt{2/r_{a2} - 2/(r_{p1} + r_{a2})};$$

$$V_f = \sqrt{\mu}\sqrt{2/ra2 - 1/a_2};$$

$$cosi_{rel} = \cos i_1 \cos i_2 + \sin i_1 \sin i_2 \cos \Omega_1 \cos \Omega_2 + \sin i_1 \sin i_2 \sin \Omega_1 \sin \Omega_2$$

Note that the orbital parameters $r_{a1}, r_{p1}, i_1, a_1, \Omega_1, \omega_1$ are those of the asteroid having, in the pair, the smallest apocenter. Using this definition of d our algorithm returned in five minutes (using Matlab implementation in a PC with P4 1.8Ghz), with a pruning bound $B = 15.2$ km/sec, a list of 13132 asteroid sequences. Most of the sequences returned belong to the permutations 4321, 4231 suggesting that a good trajectory would visit first the Athens, then the S,C type asteroids and then the Jupiter comets.

It is not possible, at this point, to be sure that the best sequence has not been pruned out. We may though be confident that the selected sequences will lead to a small mass consumption in the sense outlined above.

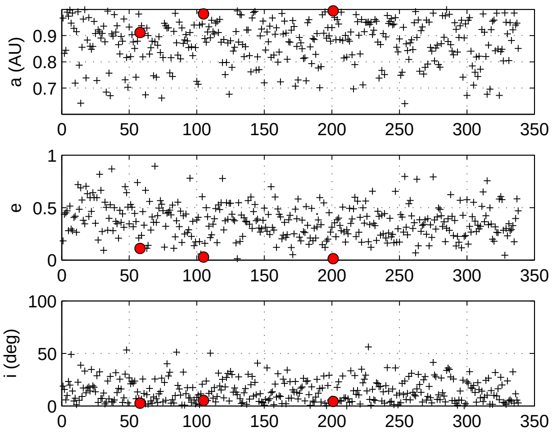

Figure 1: Asteroids belonging to group 4 and appearing in the first 13132 sequences

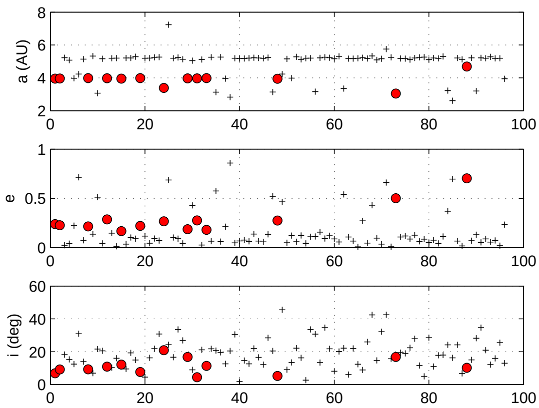

Figure 2: Asteroids belonging to group 1 and appearing in the first 13132 sequences

4.2 Global Optimisation

The global optimisation phase was done based on the two models previously described. For the first model the global optimisation problem was transcribed as follows:

$$\begin{aligned} \text{find:} \quad & \mathbf{x} \in \mathbb{I}^8 \\ \text{to minimise:} \quad & J(\mathbf{x}) := m_f \end{aligned}$$

The reason for choosing final mass m_f of the spacecraft as the cost function without accounting for the time was dictated by the need to find trajectories which were more likely to be suitable for low-thrust propulsion. The bounds on the solution space were set as $\mathbb{I}^8 = [5479, 12784] \times [150, 500] \times [90, 500] \times [150, 2000] \times [1500, 2000] \times [150, 3000] \times [90, 500] \times [150, 2000]$. Again, the choice of the bounds was driven by the same reason mentioned above. Note for instance the relatively high value of the lower bound on the waiting time on the second asteroid (1500 days) in order to accommodate the spiralling out time of a low-thrust trajectory. This global optimisation problem was fed to a distributed computing environment [**?**] and solved using a combination of three type of Differential Evolution. In Figure 3 the objective function $J = m_f/t_f$ is plotted for the first optimised 12800 sequences. Each sequence has been optimised with respect to the final mass using the Differential Evolution algorithm six times. The trajectory of our selected asteroids combination given by this model is visualized in Figure 5.

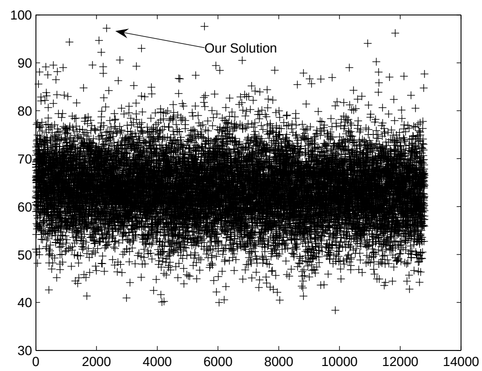

Figure 3: Objective function for the first 12800 sequences.

The best solutions found with this first model were then re-optimised with an exponential sinusoid model. For this second model it was decided to employ only one exponential sinusoid arc describing the transfer between the first asteroid and the second. For this transfer it was in fact evident that a multi-revolution spiralling trajectory was needed. The problem can be described by the following transcription:

$$\begin{aligned} \text{find:} \quad & \mathbf{x} \in \mathbb{I}^9 \\ \text{to minimise:} \quad & J(\mathbf{x}) := m_f/t_f \end{aligned}$$

where the decision vector $\mathbf{x}$ contains the Earth departure epoch, the transfer time to the first asteroid, the waiting time on the first asteroid, the transfer time to the second asteroid and so on. Its last component is the shape parameter k_2 of the exponential sinusoid. The trajectory of our selected asteroids combination given by this model is visualized in Figure 4.

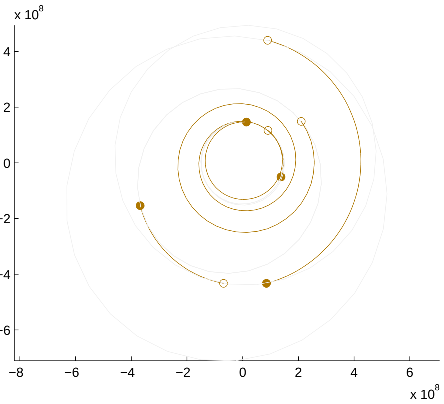

Figure 4: Refinement of trajectory in Figure 5 using exponential sinusoids.

5 Final Trajectory

The details of the best solution found by our team is described in Table 1. Figure 10 is a visual representation of our solution trajectory, where

- the Total flight time: $t = 9.523$ years,
- and the Final Objective Function Value: J=**87.05** kg/year.

The thrust profile for the different phases is plotted in Figures 6, 7, 8 and 9.

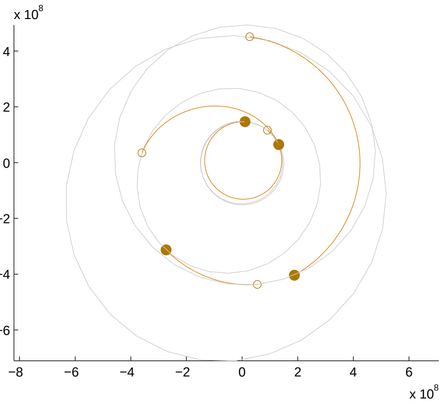

Figure 5: Result of the first global optimisation model (using Lambert arcs).

Table 1: Final Trajectory Data.

Solar system object	group	arrival date (MJD)	stay (days)	departure date (MJD)	$\Delta V \infty$ (km/s)	mass (kg)
Earth	–	–	–	57371.94	2.58	1500
3170221	4	57747.45	101.32	57848.77	0	1444.8
2000574	3	59485.22	101.56	59586.78	0	1084.4
2000209	2	60034.50	104.26	60138.76	0	985.79
2011542	1	60850.62	–	–	–	829.00

Conclusions

A possible approach to solve the complex multiple asteroid rendezvous problem has been presented. The method has been proven capable of returning good asteroid sequences in a semi-automatic fashion. Nevertheless many improvements are needed in order to produce sequences which are closer to the global optimum. Future work should be focused on exploring different way of defining the asteroid generalised distance and assessing its impact on the preliminary sequence selection. Also the process of returning a good initial guess for the local optimisation phase deserves further improvement as only a few among the best globally optimised trajectories were suitable as initial guess trajectories for the local optimiser.

References

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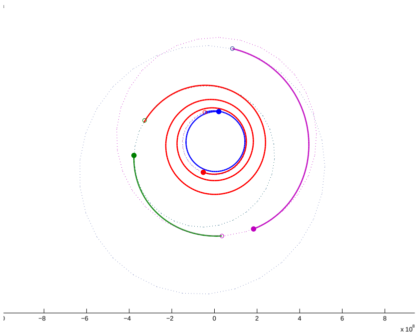

Figure 6: Final Optimised Trajectory.

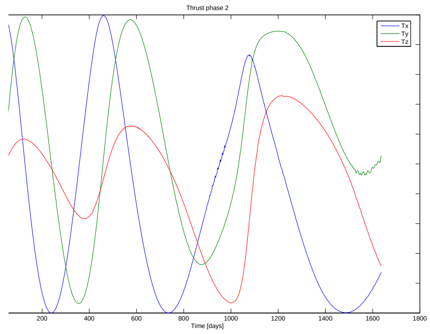

Figure 8: Phase 2 Thrust Profile.

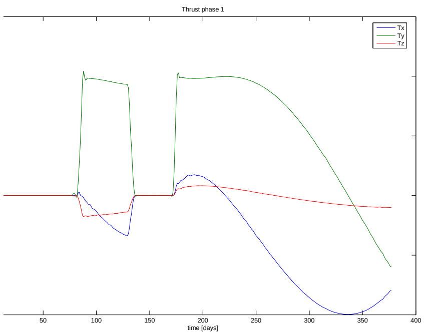

Figure 7: Phase 1 Thrust Profile.

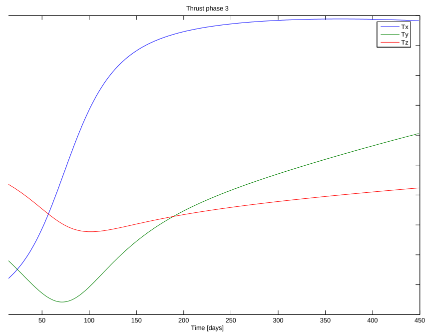

Figure 9: Phase 3 Thrust Profile.

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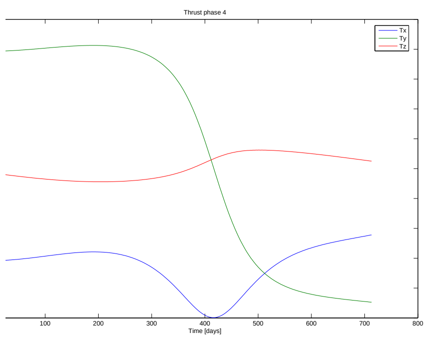

Figure 10: Phase 4 Thrust Profile.