

Deterministic Method for Space Trajectory Design with Mission Margin Constraints

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The problem of designing robust low-thrust transfer trajectories is considered. As the technology matures, the use of low-thrust propulsion for interplanetary missions becomes necessary. Recent experiences on low-thrust interplanetary missions show however that because of the very long thrust duration the propulsion system is prone to failure, even temporary. Also, the complex dynamics result in discrepancies between the flying trajectory and the desired one such that correction manoeuvres must be executed. An approach is thus to design the interplanetary low-thrust trajectory to account beforehand for these unexpected events and corrections. The concept of missed-thrust margin is thus introduced. The problem is posed as an optimisation problem with terminal constraint and maximum mass objective function. Missed-thrust margin constraints are placed at selected nodes. To reduce the computational complexity, the solution method proposed is based on the creation of a surrogate for the interior constraint evaluations.

I. INTRODUCTION

Thanks to advanced propulsion, such as electric propulsion systems, it has been made possible to design complex interplanetary space missions. At the cost of longer time of flight, it is possible to reach distant celestial objects with reasonable payload mass. Recent experiences on low-thrust interplanetary missions show however that because of the very long thrust duration the propulsion system is prone to failure, even temporary. Also, the complex dynamics result in discrepancies between the flying trajectory and the desired one such that correction manoeuvres must be executed. The problem of designing robust low-thrust transfer trajectories is thus considered. To some extent the question is whether there is sufficient time for (1) using a recovery trajectory that would satisfy the terminal constraints in case of failure, and (2) performing necessary corrections for missed-thrust.

An approach is thus to design the interplanetary low-thrust trajectory to account beforehand for these unexpected events and corrections. The concept of mission margin, or here missed thrust margin, has been introduced in Ref. [1] for the design of the Dawn mission. An optimisation problem is posed accounting for terminal constraints, maximum mass objective function, and intermediate constraints that defines the mission margins requirement. An issue is that the mission margin function is computationally heavy to compute, and thus it is not possible to place a constraint at any point along the trajectory. The solution method proposed is based on building a surrogate that can approximate accurately the missed-thrust margin function.

II. PROBLEM FORMULATION

II.1 Mission Margin

Mission margin, or in the present study missed-thrust margin, is defined in [1] and basically refers, in the present study, to missed-thrust margin. The missed-thrust margin can be seen as the amount of extra time we have, along a reference trajectory, to perform a correction manoeuvre in case of engine failure – failure to thrust or failure to thrust at the desired level. Indeed, in the present study, the missed-thrust margin is computed at several dates along a trajectory, as the maximum duration the spacecraft can stop thrusting momentarily while still being able to satisfy scientific objectives (e.g. terminal constraints).

This metric can be applied to any continuous thrust problem. In the present paper the mission margin function is denoted $M(\mathbf{x},t)$, where $(\mathbf{x},t)$ define the point of evaluation along a trajectory for given terminal constraints ψ.

So far, during the design phase, missed-thrust margins are modified by changing the system parameters such as date, time of flight or thrust amplitude. In the current study, the system design is not taken into account, and thus the robust trajectory is sought only by modifying the control and launch conditions.

II.2 Problem

The problem is formulated as an optimization problem with interior point constraints. The interior point constraints influence the robustness of the solution.

$$\begin{aligned} &\min_u J \\ &s.t. \\ &\psi\left(\mathbf{x}, t_f\right) = 0 \\ &\varphi_i\left(\mathbf{x}, t_i\right) \geq 0 \end{aligned} \quad [1]$$

The terminal constraints help us define the robustness and the missed-thrust margin. The intermediate constraints define the minimum value for the mission margin.

In this study, we focus on the robustness of the control, but the approach is quite general and can be applied to any optimal control problem where constraints are very expensive to compute.

II.3 Computation of the missed-thrust margin

Practically, thrust margins are computed at each date t along the trajectory. During thrust phases, the state $\boldsymbol{x}(t)$ is propagated ballistically during ΔT (t) and a new mass maximization optimal control problem is solved from the new state $\boldsymbol{x}(t + \Delta T\ (t))$. If the solver converges, the duration ΔT of the coast arc is increased, and the process restarts till reaching infeasibility. The maximum coast arc length is then the thrust margin value ΔT.

The process is similar for coast phases, although it can be simplified because, by definition, in this case the thrust margin decreases linearly to the thrust margin level of the next thrust point.

Computing the missed-thrust margin basically require the solving of several instances of the problem. Continuation can be used to improve the convergence speed. When changing ΔT, to find the length of the coast arc, we can also follow a dichotomy approach thus reducing the number of problem to solve.

An illustration of the missed-thrust margin is given on Figure (1).

By construction, we can derive some basic properties of the missed-thrust margin:

- The missed thrust margin is a positive quantity, in days.
- During coast period, the missed-thrust margin varies linearly with the remaining time to the next thrust period.
- Consequently, the missed-thrust can never be zero during a coast period.
- On the final thrust period, if optimal, the missed-thrust margin is zero.
- On a final optimal coast period, the missed-thrust margin decreases with rate less or equal to 1. The missed-thrust margin cannot exceed the remaining time of the mission.

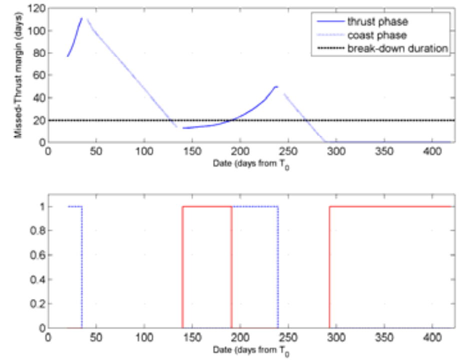

Figure 1: Missed-thrust margin and periods of success or failure of the mission.

One can immediately see that this metric is quite expensive to compute. Its implementation as a constraint in an optimisation algorithm can be computationally heavy or even impractical. In addition, derivatives with respect to the current spacecraft state are not readily available. Table 1 gives a brief overview of the computation time and the error for different optimal control methods, and different number of nodes when using direct transcription methods. The error is relative to the shooting method that is assumed accurate.

	Computation time (TU)
Shooting (ind.)	1
Pseudo Spectral (20)	0.81
Impulsive model (20)	0.58

Table 1 Comparison of the computation times for different optimisation methods.

The shooting method from indirect approach is the one giving the accurate results. Overall, direct transcription methods tend to slightly overestimate the mission margin value. That is because of the approximation made on the dynamics.

Consequently, two solutions are used to increase the computational speed. First, the mission margin is computed using an approximate low-thrust model. Second, we use a surrogate to have a fast evaluation of constraints based on the mission margin.

The impulsive low-thrust model [2, 3] is a direct transcription method which approximates low-thrust arcs as series of impulsive maneuvers (ΔV) connected by conics. Because the computation of the mission margin function does not involve any optimality criterion, but only the feasibility, a mid-fidelity model can be used. The advantage of the impulsive model is its fast convergence. We use 20 impulses in our experiment.

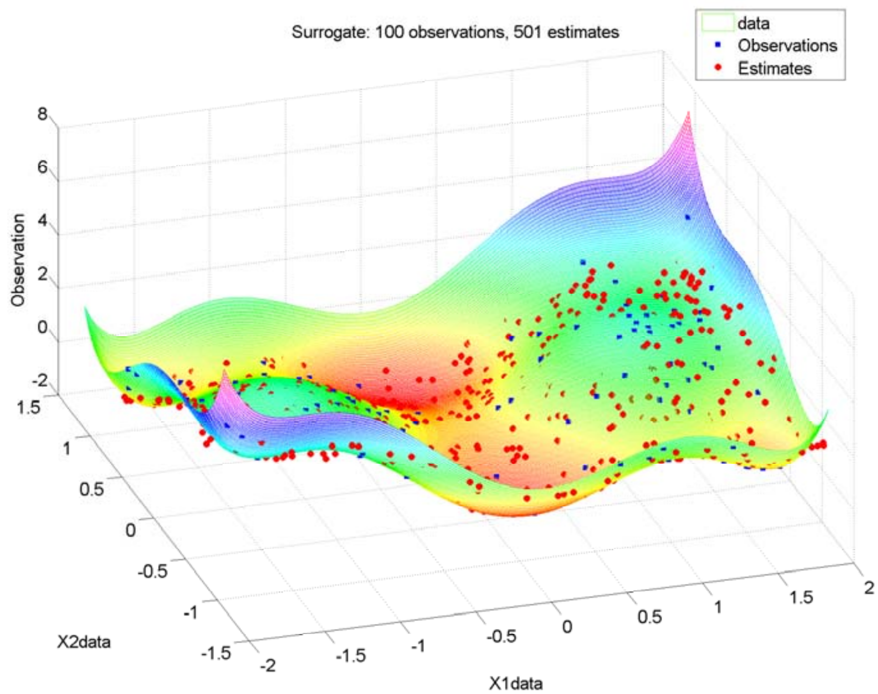

Figure 2. For the example, 100 observations have been made on a “six-hump camel back” function. From this sample a surrogate is made, from which we can predict a point on the surface, and compare it with the true value.

III. SURROGATE DESIGN

III.1 Surface Method

Because of the expensive evaluation of the mission margin function along a trajectory, for the intermediate constraints, it is proposed to use a surrogate for the mission margin function.

Similarly to a polynomial regression model, a surface is constructed from sampled evaluations of the thrust margin function, as shown on Figure 2. Each evaluation adds a new point on the surface to eventually provide a good interpolation scheme.

Reference [4] presents existing approaches for global optimization methods about objective function based surrogate. In addition, one can find tools for automatically generating surrogate models [5][6][7]. It seems that this method is particularly well suited for computational expensive simulation-based optimization [8].

Two considerations have to be made: the method for sampling (experiment) and the construction of the surrogate model (estimator). The sampling method and the number of sampled points are important as they affect the bias of the estimator. Among methods, we can cite the classic homogeneous grid sampling, the Latin hypercube sampling and the Orthogonal arrays sampling. Among available models for the surrogate, we can mention polynomial regression model, Kriging modelling, radial basis functions model. A full description of these models can be found in ref. [8].

Similar approaches for approximating expensive functions have been followed in global optimisation, using for instance neural networks [9]. Somehow, the construction of a surface onto which we interpolate to have an estimation of a function is similar to the training of a neural network.

There is however one difficulty. The variables we select for the interpolation shall be carefully selected. It seems convenient to use the variable that will later be used in the optimisation process.

On one hand, with an indirect method we would have a small decision vector (costate vector $\boldsymbol{\lambda}_0$ and few parameters) but a surface map that can be locally steep because of the sensitivity of the problem to the costate variables. Possibly, with an adjoint control transformation this sensitivity can be reduced. On the other hand, with a direct method the number of variables can become significant and thus a fair number of points is necessary for the best approximation.

In the current study, we shall use an indirect method. The reason is a matter of computational speed, accuracy and reduced state space. But, as shown before, the approach can still be followed with an approximate model when constructing the surrogate. The formulation of the optimal problem is presented in the next section.

III.2 Choice of the sampling method

Observations are made by evaluating the expensive function on a set of random points. The random points are generated in a hypercube around the optimal interior-constraints free solution.

We allow a maximum deviation on some variables to stay in a neighbourhood of the initial optimal solution.

III.3 Choice of the surrogate

According to literature, there is no general rule for choosing a surrogate, however it has been demonstrated that the most promising ones in term of accuracy and convergence to the original function are based on interpolating methods. We can cite for instance cubic splines, multiquadrics methods and kriging predictor.

Kriging is an interpolation method usually used in geostatistics applications [10]. In essence, in the kriging predictor, the function to model is expressed on a basis of simple functions φ and polynomials π, Ref. [11]. Considering n the number of sample points and d is the dimension of the state space, yields to the interpolation formula:

$$\tilde{\mathbf{y}} = a + \sum_{i=1}^{n} b_i \varphi(\mathbf{x} - \mathbf{x}_i) \quad [2]$$

$$\varphi(\mathbf{x}) = \exp\left(- \sum_{k=1}^{d} \theta_k |x_k|^{p_k} \right) \quad [3]$$

Where $a = \tilde{\mu}$, $b_i(\tilde{\mu}, \theta_i, p_i)$, θ_l and p_l are parameters. $\tilde{\mathbf{y}}$ is the estimate.

Kriging is known to out-perform other interpolation methods mainly because of its statistical derivation. Kriging models the function as a realisation of a stochastic process. A very clear and intuitive approach is presented by Jones [4]. Thus, parameters $a = \tilde{\mu}$, $b_i(\tilde{\mu}, \theta_i, p_i)$, θ_l and p_l are are selected to maximise a likelihood function of the observed data, which can be expressed as,

$$L(\{a, b, \theta, p\} | \mathbf{y}) = \frac{1}{(2\pi\boldsymbol{\sigma}^2)^{n/2}} \exp\left(- \frac{\sum_i (x_i - \mu)^2}{2\sigma^2} \right)$$

[4]

This assumes a normal distribution. The likelihood function is maximised for the parameters $\{\tilde{\mu}, \sigma^2, \theta_i, p_i\}$ Function φ is called the Kriging basis function. Other correlation model for φ can be used such as simple exponential, Gaussian, spherical. The choice should depend of the underlying process that is approximated. The choice made here seems to be quite general because of the embedded parameters.

An interesting feature of the Kriging predictor is the possibility to have an estimation of the error, thus having an indicator of the quality of the estimate.

Table 2 and Figure 2 give an example of application of the surrogate for estimating the value of a given function (here the six-hump camel back). The error is null, by construction, at the sampled points. There is a close agreement between the estimated and the reference values. In addition, the error estimation gives an indication on how reliable is the estimate.

	Reference	Surrogate	Est. Error
Point 1	2.299	2.239	0.1581
Point 2	0.2203	0.2203	0

Table 2. Example of surrogate output for six-hump camel back function. 100 sampled points.

A BFGS algorithm is used for the maximisation of the likelihood function, and the convergence is rather fast but depends of the size of the sample. In addition, the maximisation process involves a covariance matrix, which size depends of the number of observation. This matrix should be inversed during the process, and thus a large sample can lead to significant computation time and memory usage when seeking the best parameters of the surrogate. Some efficient algebra codes can be found for the inversion of large matrices, and some authors also address this issue for large date sets [12].

III.4 Reducing variations of the surface

By construction the missed-thrust margin surface is smooth, but we should assume that the function M defines a smooth hypersurface too in that hypercube. However, to avoid high variations of the surface owing to the sensibility awarded to the costate vector $\boldsymbol{\lambda}_0 = [\boldsymbol{\lambda}_R, \boldsymbol{\lambda}_V, \lambda_m]$, the adjoint control transformation (ACT) can be used,

$$\begin{bmatrix} \boldsymbol{\lambda}_R \\ \boldsymbol{\lambda}_V \end{bmatrix} \xrightarrow{ACT} \begin{bmatrix} \alpha \\ \beta \\ \dot{\alpha} \\ \dot{\beta} \\ u \\ v \end{bmatrix}$$

Where $\alpha, \beta, \dot{\alpha}, \dot{\beta}, u, v$ are new variables defining the orientation and amplitude of the costate vector (in the rest of the paper, for notation purposes we keep the notation $\boldsymbol{\lambda}$ even for the transformed costate) .

This change of mapping is usually used for finding meaningful initial guess to the optimal control problem. It is thus expected in the present case to reduce the variations of the surface.

IV. OPTIMISATION PROBLEM

IV.1 Low-Thrust Problem Formulation

The robustness improvement process starts from an optimal solution $(\overline{\mathbf{r}}, \overline{\mathbf{v}}, \overline{\mathbf{u}}, t)$ of the low-thrust trajectory problem. The process takes as initial guess the optimal solution and tries to increase the missed-thrust margin when required.

The low-thrust robustness process is based on an indirect method where the control structure is imposed, similarly to ref. [13], to follow the same structure as the optimal reference solution.

Considering a two-body problem, as for interplanetary trajectory design, the dynamics are defined as:

$$\mathbf{f}(\mathbf{r}, \mathbf{v}, m, \mathbf{u}; t) = \begin{bmatrix} \mathbf{v} \\ -\mu \dfrac{\mathbf{r}}{\|\mathbf{r}\|^3} + \dfrac{T}{m}\mathbf{u} \\ -q\|\mathbf{u}\| \end{bmatrix} \quad [5]$$

With ***r***, ***v***, *m* respectively the position, velocity and mass of the spacecraft in a given reference frame. The control ***u*** defines the thrust direction with amplitude *T*. (obviously, the dynamics must be the same as the one used for computing the reference solution).

The terminal constraints are defined as for a rendezvous problem, when the final position and velocity have to match the one of the optimal reference trajectory $(\overline{\mathbf{r}}, \overline{\mathbf{v}}, t)$, and thus,

$$\psi_f(\mathbf{x}, t_f) = \begin{bmatrix} \mathbf{r}(t_f) - \overline{\mathbf{r}}(t_f) \\ \mathbf{v}(t_f) - \overline{\mathbf{v}}(t_f) \end{bmatrix} \quad [6]$$

And the intermediate constraint on the missed-thrust margin:

$$\varphi_i(\mathbf{x}, t_i) \geq \Delta M \quad [7]$$

Where ΔM is the minimum missed-thrust margin we want to impose on the solution. A Kriging estimator is assigned for each interior constraint φ_i as an estimator of the mission margin function at that point.

Even though we are interested by a robust solution, we still want to maximise a given objective function J ,

$$\min_{\mathbf{z}} J$$

The time of flight and the launch date are fixed. The reason is to ensure that we are not changing the problem, and actually improving the robustness of the initially optimal solution.

With indirect method, a trajectory is uniquely defined with the costate vector $\boldsymbol{\lambda}$ and the switching structure definition. Consequently, the decision vector of the robust problem, but also the basis of the hypercube for interpolation and missed-thrust margin surrogate, is:

$$\mathbf{z} = [\mathbf{p}_0, \boldsymbol{\lambda}_0, \ldots, \boldsymbol{\lambda}_i, T_0, \ldots, T_j]$$

Where T_j is the duration of the thrust and coast periods, and $\mathbf{p}_0$ defines the initial conditions (e.g. launch energy, declination, …). A different costate vector $\boldsymbol{\lambda}_j$ can be assigned to each different thrust-coast segment, accounting for the fact that intermediate constraints on the state can result in jump conditions on the costate.

For good convergence, the gradient of the cost and the jacobian of the terminal constraints are provided. The jacobian of the intermediate mission margin constraint is computed by derivation of the interpolation rule, and provided to the optimizer.

IV.2 Interior constraints placement

Terminal constraints ψ_f are inexpensive to compute and thus do not require more consideration than usual. However, because the missed-thrust margin function is assumed continuous, with a known structure close to commutation points, it is not necessary to place intermediate constraints ψ_i everywhere along the trajectory. Considering previous comments on the basic property of the missed-thrust margin function, a missed-thrust constraint should only be placed at each coast-thrust commutation, and possibly in the middle of each thrust segment.

The first order derivative of the missed-thrust margin function can help decide if more points are necessary along the thrust segments. If points must be added, the optimization process is restarted with this new configuration. Although, this time considering the value of the mission margin at both ends of the thrust segment, a cubic spline interpolation provides a fair evaluation of the mission margin where necessary.

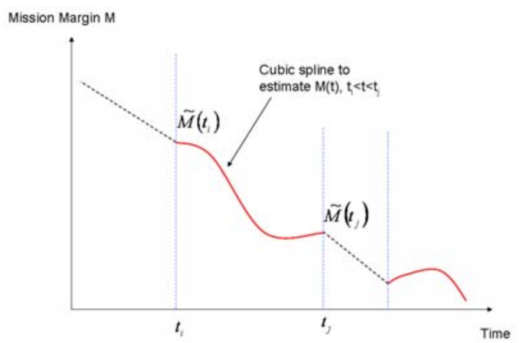

Figure 3. Cubic spline approach to guess the behaviour of the function M between nodes.

IV.3 Implementation

We propose a simple algorithm for designing a low-thrust interplanetary trajectory with mission margin constraints.

1. Construct a surrogate S for the mission margin function M
   a. Generate a set of initial points $\{\boldsymbol{x_i}\}$
   b. For each $\boldsymbol{x_i}$, compute the mission margin value M_i=M($\boldsymbol{x_i}$,t$_i$).
   c. Using the points $\boldsymbol{x_i}$ and the values M_i, maximise the likelihood function, to get the best Kriging predictor as surrogate
2. Find a feasible point on the surface for initial guess of the optimisation.
3. Optimize
   a. The intermediate constraints are evaluated using the surrogate S.
   b. The surrogate is updated with a given frequency. To do so, the current optimal point is used to evaluate the mission margin function, and the value is added onto the surface. This improves the surrogate fidelity to the original function.
   c. Using first derivative, we check if a minimum of the mission margin function may exist on a thrust interval.
4. Terminate.

The update of the surrogate can be made in parallel of the optimization with an efficient multi-thread process.

V. APPLICATION: SAFE MISSION TO AN ASTEROID

As a demonstration of the method, we consider the low-thrust transfer of a spacecraft to a near Earth asteroid (NEA). This could be the case for a manned mission. This type of mission is quite sensible since a failure can have catastrophic consequences. The purpose is thus to design a trajectory that, even though failure to thrust or discrepancy during the design and the operation happen, the mission can still be recovered to reach the target.

For simplification, we only consider the outbound leg, i.e. the transfer from Earth to the asteroid. The selected NEA is 2001 QJ142.

The orbital parameters are:

Semi-major axis a = 1.062 AU,
Eccentricity e = 0.086,
Inclination i = 3.106°,
Arg. of pericenter ω = 63.86°,
Long. Ascend. Node Ω = 184.47 °,
Mean anomaly M = 124.68°,
Epoch = 54000 in MJD

The electric propulsion system has a nominal thrust amplitude T=0.33N and a specific impulse Isp=2500s. The initial spacecraft mass is 2500kg. These values have been selected for the sake of the exercise and do not necessarily reflect the design of a future human transportation system into deep space.

The terminal constraint fixes the robustness of the solution. Usually, with a rendezvous constraint, the control ends with a thrust period, which as it has been mentioned before, has a 0 missed-thrust margin. A failure during this last period results in a high risk of failure of the mission (in practice changing the rendezvous date is sufficient to recover). The best way to increase the robustness is to split the final thrust arc in two arcs, as experienced in [13]. In that case, it is possible to design thrust arcs satisfying the minimum missed thrust margin, but for the final arc that should anyway be of very small duration thus small sensitivity on the constraints satisfaction. With a rendezvous in position only, if optimal, the solution has a final coast arc. This means that the approach to the asteroid is quite robust. One simply has to be sure that the duration of the coast period is sufficient for operational purposes (orbit determination and possible trajectory corrections with the EP system).

For the current example, a rendezvous constraint in position only is chosen. This is equivalent to a flyby. However, during the construction of the robust solution process, the terminal constraints are such that the terminal velocity vector always matches the one of the optimal solution. This is necessary to avoid radically different trajectory, and to be consistent with the robustness definition.

It is possible to influence the mission margins by simply modifying the parameters. For instance, we could have increased the thrust, thus shortened the thrust arc and increasing the missed-thrust margins (it seems the specific impulse do not influence the robustness much). However, in practice, these parameters are not only influenced by the engineering, but also consideration of system cost. It is though a good exercise to see which of the approaches produce the best solution trajectory: modifying the system parameters or modifying the control.

A preliminary global optimisation phase helps fixed the time of flight of 326 days and the launch date. The optimal control, the missed-thrust margin and the trajectory are plotted on Figure 4 and 5. As can be observed the mission margin decreases linearly during coast phase, and reaches about 19 days during the first coast period. This means that if the electric propulsion system fails to power on during the next 19 days, the mission is lost. However, even though the mission is lost for this current performance, it can most likely be recovered by increasing the time of flight and scheduling a different flyby date of the asteroid, if possible.

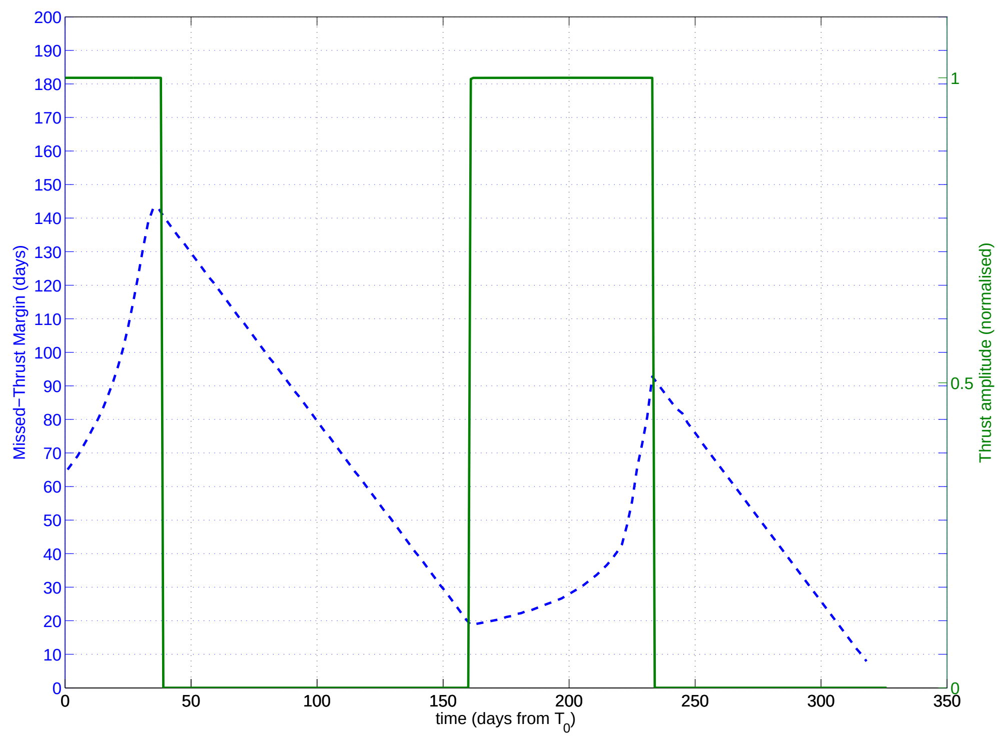

Figure 4. Optimal control of the problem is saturated. The missed-thrust margin (dotted line) is superimposed, and it gives an idea of the influence of the control over the margin.

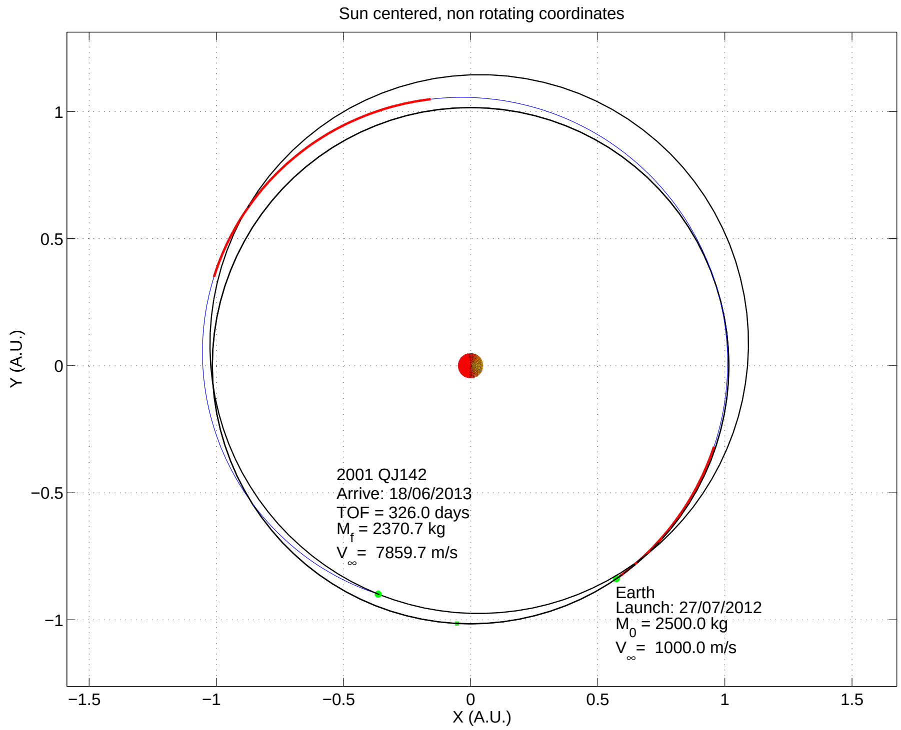

Figure 5. Optimal trajectory with two thrust periods and two coast periods.

	Optimal Solution	Robust Solution
Minimum Mission margin	18.9 days	26.7 days
Cost	2371 kg	2378.7 kg
Computation time	9.54 s	342 s

Table 3. Comparison between the optimal and robust solutions.

Considering the initial missed-thrust margins, the algorithm is applied to find the control that respects a mission margin of at least 25 days at the second off-on commutation. This is a very conservative and stringent quantity considering the actual reliability of electric propulsion system observed so far. However, some currently flying or future missions consider up to 28 days of margin. This depends on many factors, and for instance a multiple swing-by trajectory may require higher values. This time should account for many operational constraints.

We are looking for a more robust solution, with the same control structure (thrust – coast –thrust – coast). The initial switch times, from T0, are:

T1 = 38.4 days
T2 = 160.9 days
T3 = 233.6 days

Initial and terminal times are kept unchanged, thus we do not influence the launch window. That is for the sake of the exercise; however it may be possible to completely improve the robustness with a different launch time. Since we are only concerned by the second off-on commutation, the decision vector for constructing the surrogate is simply:

$$\mathbf{x} = \left[V_\infty, \alpha, \beta, \boldsymbol{\lambda}_0, T_0, T_1\right]$$

It is of dimension 11 (the costate vector does not need to account for the mass adjoint variable), and it is composed of the launch hyperbolic velocity, the costate vector, and the first thrust period duration and the first coast period duration. Initially, we took 1000 sample points to have an idea of the landscape of the missed-thrust margin function as depicted on Figures 6-8. But eventually, only 100 data points were used for the surrogate. In addition, we removed the points that result to 0 missed-thrust margins. A bigger sample set would result in significant computational time when setting up the surrogate.

The structure of the control is kept (thrust – coast –thrust – coast).

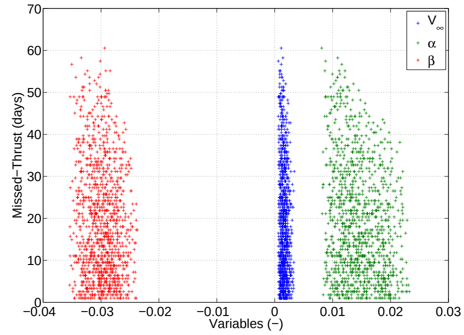

Figure 6 Projection of sampled points of the surrogate's hypersurface onto Vinf, alpha and beta axes.

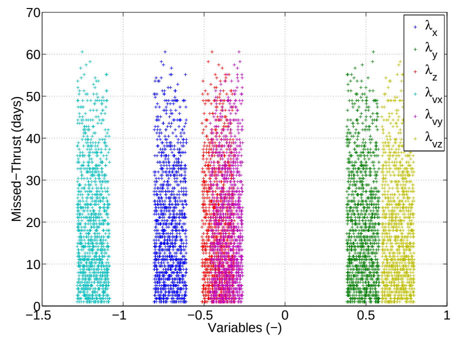

Figure 7 Projection of sampled points of the surrogate's hypersurface on the lambda axes

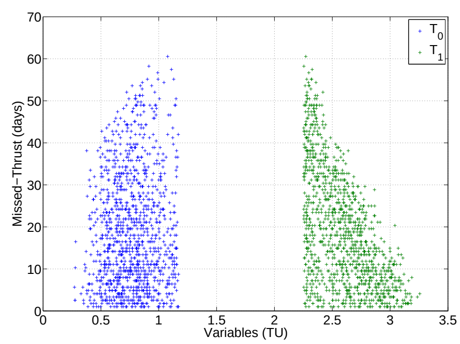

Figure 8 Projection of sampled points of the surrogate's hypersurface on the T0, T1 axes.

Some maximum dispersion has been used on the variables:

$\Delta T = 30$ days
$\Delta\alpha = 12$ deg
$\Delta\beta = 2$ deg
$\Delta V_\infty = 100$ m/s

As can be seen, there are trajectory solution that improve the original missed-thrust margin at the second coast-thrust commutation. For instance, we can reach as much as 60 days. A further investigation can help assessing the impact of such a change on the objective function value (e.g. terminal mass). The role of the optimisation process is now to find a point that is better than the original one robustness-wise and does not penalise the cost.

Results are summarized in Table 3. The mission margin reported after the optimisation was exactly 25 days, however this value comes from the surrogate, and a post-process analysis shows that the margin was indeed only 14 days. The process has been then restarted with additional and new sample points, in a small neighbourhood of the current solution, to eventually get a final solution that complies with the initial requirement. An alternative that could have been followed is to use the estimated error, and sum the estimated value with the estimated error for the intermediate constraint value. The program converged in 142 iterations with the optimiser SNOPT. The computational time reported for the robust solution includes the time required to construct the best estimator.

Figure 9 and 10 depict the robust control and trajectory respectively. As can be seen, the solution has a higher launch energy. It is intuitive that for such simple transfers, the best way to increase robustness with respect to missed-thrust margins to be as close as possible to a ballistic trajectory. Nonetheless, the approach followed here is a proof of concept and illustrates a simple method to compute low-thrust trajectories with imposed missed-thrust margin at particular point, with little computer power need.

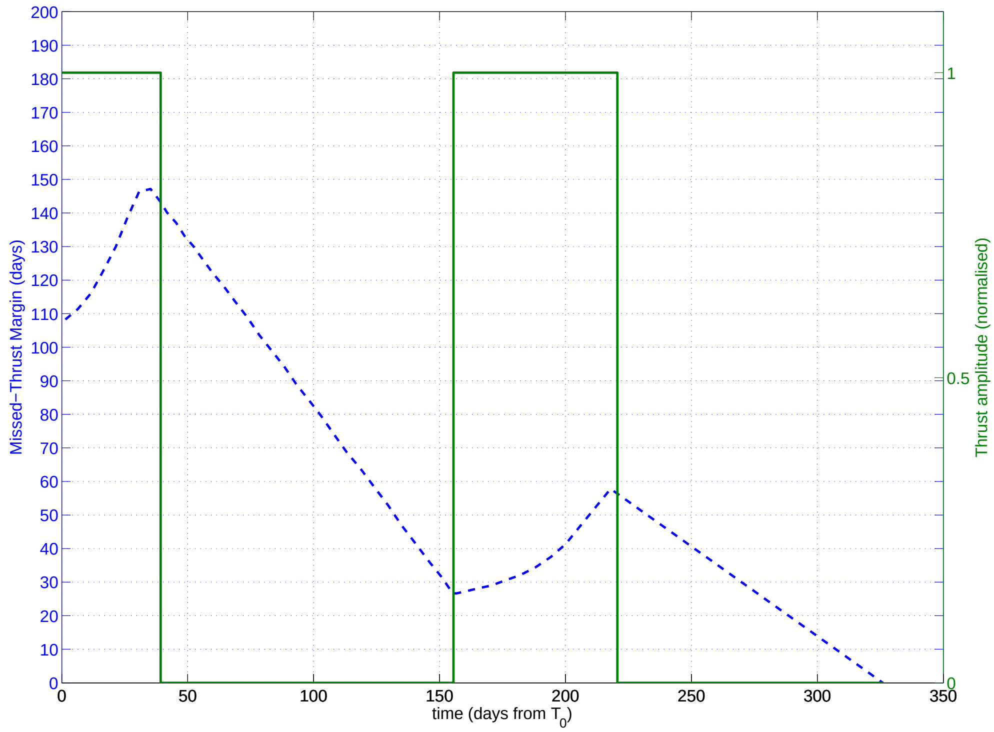

Figure 9 Robust control of the missed-thrust margin constraint problem. The missed-thrust margin (dotted line) is superimposed.

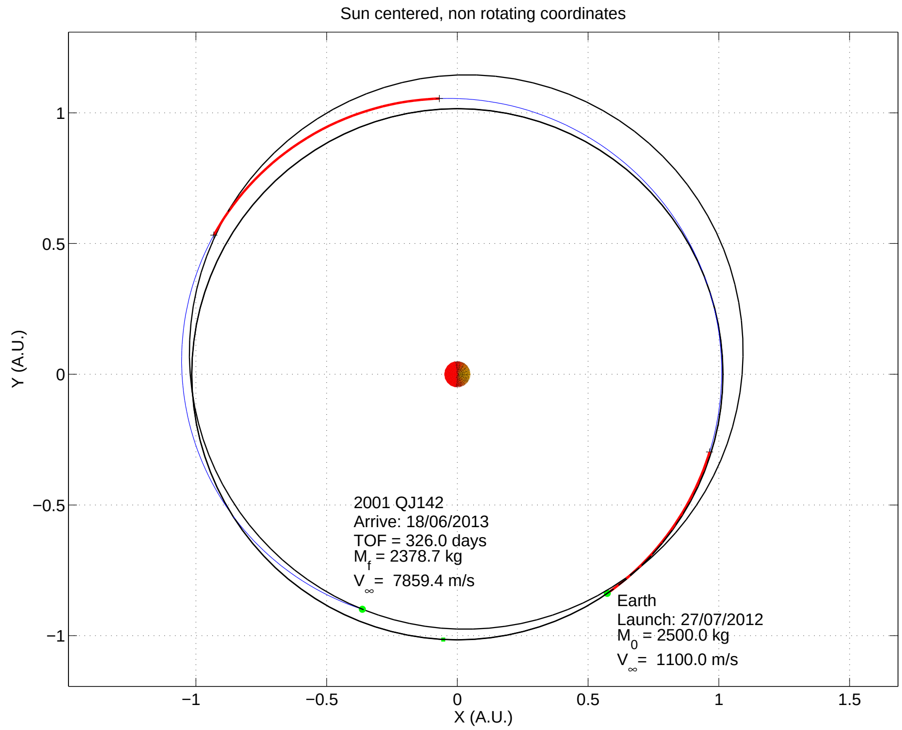

Figure 10 Robust trajectory, with the same boundary conditions as the optimal one. The control structure is kept during the process. Robustness slightly penalises the mass.

CONCLUSIONS

We tackled the designing low-thrust interplanetary trajectory subject to missed-thrust margin constraints. Because, these constraints are computationally expensive, a surrogate is created to provide a faster evaluation. The surrogate is constructed using observations and maximising a likelihood criterion. An optimal control solution is provided as initial guess to the robustness improvement procedure. The method is demonstrated on a low-thrust transfer to an asteroid, as a potential methodology to design low-thrust robust mission.

The approach is indeed quite general and can be applied to any optimisation problems where the objective function or the constraints may be computationally expensive to evaluate, as it has been done in other research fields. Other constraints can be thus considered, as could be the case to design robust human missions.

The surrogate approach has been very little explored in the scope of trajectory design. However, it can provide good insight of the launch opportunity, the time of flight, or simple an estimation of the global optimum value.

Further studies should focus on formulating the robust control problem without the use of missed-thrust margin.

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