Comments on Deployment and Control of Charged Space Webs

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***Abstract*— The deployment and control of rotating space webs (multi-tethered systems, two-dimensional tether systems, nets or similar large structures) is investigated. We discuss stability and controllability of different situations, in particular a new concept of using Lorentz interactions between charged parts of the space web with a planetary magnetic field is proposed. Further applications of Lorentz forces acting on two satellites connected by a tether are discussed.**

I. Introduction

Deployment and control of space webs (defined as a generic term for large structures, multi-tethered systems, two-dimensional tether systems, nets etc.) is a contemporary issue for advanced concepts in space exploration [1]–[5]. In this paper we investigate different strategies for the centrifugal deployment and control of such structures, putting emphasis on a new concept of using interactions of parts of the space web with a background magnetic field (planetary magnetic fields). We propose to charge parts of the web, which through the relative motion of the space web with respect to the magnetic field establishes a Lorentz force. If the web is oriented suitably in space, this force can be used for deployment and/or control.

Our paper is organised as follows: In section II centrifugal deployment and control without interaction with the magnetic field are considered. We prove that a free deployment is not controllable, though asymptotically stable. A symmetric system can be made controllable by acting with an external moment on the central hub. In section III the equations of motion for the case with Lorentz interaction are studied, which are then applied to two different cases in sections IV and V. In section IV the Lorentz force is used to cancel exactly the Coriolis force; we conclude that this system is stable and controllable. In section V a simplification to a rotating tether-system similar to the proposed BOLAS mission is considered and it is shown that the Lorentz interaction at reasonable charging levels could be used to compensate the damping of the rotation due to gravity gradient.

II. Deployment without magnetic field

In this section the centrifugal deployment of a symmetric space web is considered. Our analytic model is restricted to the situation in figure 1, a symmetric space web made by N tethers of length L reeled in the central hub and connected to N end masses m. Each end mass is connected to its neighbours by two tethers reeled inside the mass. We neglect the masses of the tethers. It should be emphasised that this simple situation shares many similarities and extends straightforwardly to numerous other configurations, such as distributed masses or space-webs of different shapes (e.g. split reflectors or solid reflectors) [1], [2]. The conclusion drawn here therefore apply up to minor changes to all these situations as well.

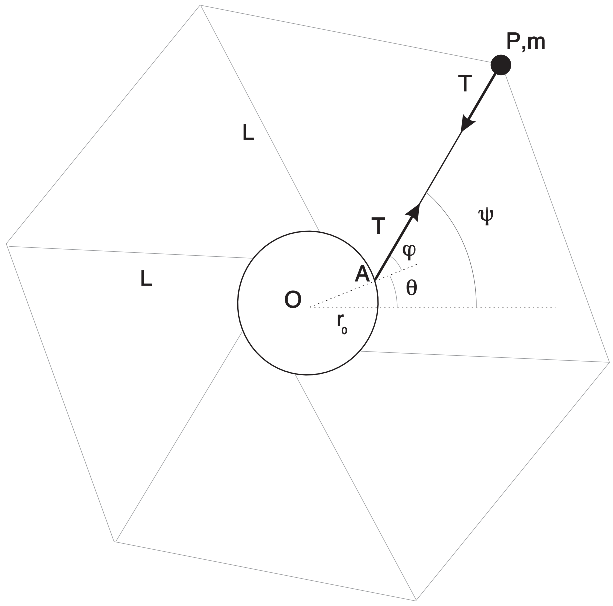

Fig. 1. Geometry of a symmetric deployment of a space web

Much of the simple modelling one can make on centrifugally deployed structures has been done using reduced degrees of freedom models [1]. As shown in a study performed by KTH in collaboration with the Advanced Concepts Team [2], also the deployment dynamic of a space web can be modelled using similar techniques. To derive the equations of motion in explicit coordinates as given in figure 1 we may follow Euler approach from the very beginning (as originally proposed by Melnikov and Koshelev [1]) by considering the conservation of angular momentum of the central hub and the conservation of linear momentum of the mass m:

$$I\ddot{\theta} = NTr_0 \sin\varphi \qquad \tilde{m}\ddot{\mathbf{r}}_P = N\mathbf{T} \tag{1}$$

Writing $\mathbf{r}_P = \mathbf{r}_A + \mathbf{r}_{AP} = r_0\hat{\mathbf{i}} + L\hat{\mathbf{b}}_1$ and deriving twice these expression by taking into account that $\dot{\hat{\mathbf{i}}} = \dot{\theta}\hat{\mathbf{k}} \times \hat{\mathbf{i}}$ and $\dot{\hat{\mathbf{b}}}_1 = \dot{\psi}\hat{\mathbf{k}} \times \hat{\mathbf{b}}_1$ we get

$$\ddot{\mathbf{r}}_p = (r_0\ddot{\theta}\sin\varphi - r_0\dot{\theta}^2\cos\varphi + \ddot{L} - L\dot{\psi}^2)\hat{\mathbf{b}}_1 + \\ + (r_0\ddot{\theta} + r_0\sin\varphi + 2\dot{L}\dot{\psi} + L\ddot{\psi})\hat{\mathbf{b}}_2 \ , \tag{2}$$

and thus, projecting the equations of motions along the $\hat{\mathbf{b}}_1, \hat{\mathbf{b}}_2$ axes, we obtain

$$\tilde{m}\ddot{L} + \tilde{m}r_0 \sin\varphi\ddot{\theta} - \tilde{m}L\dot{\psi}^2 - \tilde{m}r_0\cos\varphi\dot{\theta}^2 = -NT \ , \quad (3)$$

$$I\ddot{\theta} = r_0 NT \sin\varphi \ , \quad (4)$$

$$L\ddot{\psi} + r_0\cos\varphi\ddot{\theta} + r_0\sin\varphi\dot{\theta}^2 + 2\dot{\psi}\dot{L} = 0 \ , \quad (5)$$

with $\tilde{m} = Nm$. Again we notice that the analytical structure of the equations of motion is retained for many configurations of the structure different from the one in figure 1 (see chapters 3.2-3.5 in [1] and chapter 5.2 in [2].)

A. Controllability issues

Is the system of differential equations (3)-(5) controllable? That is, can we use the tension, or length, to control the system state? In order to answer these questions we here take the solution corresponding to a uniform rotation: $L = \bar{L}$, $\dot{\theta} = \bar{\omega}$, $\varphi = 0$. Substituting this in eq. (4) we find that $N\bar{T} = (\bar{L}+r_0)\omega^2\tilde{m}$. The tether tension is, in this reference motion, balancing exactly the centrifugal forces. Let us now introduce a small perturbation to this motion (we do not consider θ as a state variable, only its derivative ω as we are not interested in controlling the exact orientation of the central hub, only its angular velocity):

$$L = \bar{L} + \delta L \quad \omega = \bar{\omega} + \delta\omega \quad \varphi = \delta\varphi \quad T = \bar{T} + \delta T \quad (6)$$

If we neglect higher order terms in $\delta L, \delta\omega, \delta\varphi$ and in $\delta\dot{L}, \delta\dot{\theta}, \delta\dot{\varphi}$, we get the linear system of equations

$$\delta\ddot{L} = \bar{\omega}^2\delta L + 2\bar{\omega}(\bar{L}+r_0)\delta\omega + 2\bar{\omega}\bar{L}\delta\dot{\varphi} - \frac{N}{\tilde{m}}\delta T \ , \quad (7)$$

$$\delta\dot{\omega} = \frac{r0(\bar{L}+r_0)\bar{\omega}^2\tilde{m}}{I}\delta\varphi \ , \quad (8)$$

$$\bar{L}\delta\ddot{\varphi} = -r_0\bar{\omega}^2\left(\frac{(\bar{L}+r_0)^2\tilde{m}}{I} + 1\right)\delta\varphi - 2\bar{\omega}\delta\dot{L} \ . \quad (9)$$

This system can be transformed into the form $\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$, where $\mathbf{x} = [\delta\dot{L}, \delta\omega, \delta\dot{\varphi}, L, \delta\varphi]$ and $\mathbf{u} = \delta T$. Then

$$\mathbf{A} = \begin{bmatrix} 0 & a & b & c & 0 \\ 0 & 0 & 0 & 0 & d \\ e & 0 & 0 & 0 & f \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \ , \quad (10)$$

$$\mathbf{B} = \begin{bmatrix} u & 0 & 0 & 0 & 0 \end{bmatrix}^T \ , \quad (11)$$

where $a = 2\bar{\omega}(\bar{L}+r_0)$, $b = 2\bar{\omega}\bar{L}$, $c = \bar{\omega}^2$, $d = \frac{r0(\bar{L}+r_0)\bar{\omega}^2\tilde{m}}{I}$, $e = -2\bar{\omega}/\bar{L}$ $f = -\frac{r_0}{\bar{L}}\bar{\omega}^2\left(\frac{(\bar{L}+r_0)^2\tilde{m}}{I}+1\right)$ and $u = N/\tilde{m}$. With this notation the controllability matrix, defined as $C = [B, AB, A^2B, A^3B, A^4B]$ turns out to be

$$\mathbf{C} = \begin{bmatrix} u & 0 & (be+c)u & 0 & hu \\ 0 & 0 & 0 & deu & 0 \\ 0 & eu & 0 & gu & 0 \\ 0 & u & 0 & (be+c)u & 0 \\ 0 & 0 & eu & 0 & gu \end{bmatrix} \ , \quad (12)$$

with $g = (be+f+c)e$ and $h = (b^2e^2 + 2ebc + ade + ebf + c^2)$. Checking the rank of this matrix we get rank$(\mathbf{C}) = 4$ and we may therefore conclude that the linear system is not controllable which extends to the original non-linear system. This means that there is not any hope to restore a uniform rotation solution in a finite time by properly selecting δT. On the other hand this does not prevent us to try to damp all the oscillations as $t \to \infty$ obtaining an asymptotically stable feedback for the system.

Let us consider the following solution to (7)-(9): We set $\delta\dot{L} = k\delta\dot{\varphi}$. As a consequence (9) becomes a damped linear oscillator being critically damped if

$$k^2 = r_0\left(1 + \frac{(\bar{L}+r_0)^2\tilde{m}}{I}\right) = -\frac{f}{\bar{\omega}^2}\bar{L} \ . \quad (13)$$

Clearly, in a real application, the value of k would have to be selected according to the capabilities of the actuator. Equation (8) also turns into a damped linear oscillator, whereas (7) gives us the control law on the tension δT. As the three state variables are all damped linear oscillators, we can conclude that we have obtained a system that has in its origin as an asymptotically stable equilibrium position. A study on the eigenvalues of the resulting system matrix confirms that it is Hurwitz, thus granting its asymptotic stability. The conclusion is extended also to the non-linear case.

We may therefore conclude that the system is not controllable, but that we may stabilise asymptotically an equilibrium position by only using the tether tension. This is clearly a good news for the deployed system, but it also implies that we may not hope to control efficiently the deployment of the web without adding some control capabilities to our system.

B. Aiding the deployment with an external moment on the central hub

During the experiment ZNAMYA-2, as documented in the book by Melnikov and Koshelev [1], a DC motor with a drooping characteristic was used to impart a momentum to the central hub as to obtain a controlled deployment. The simplified equations of motion that describes this type of deployment are

$$\tilde{m}\ddot{L} + \tilde{m}r_0 \sin\varphi\ddot{\theta} - \tilde{m}L\dot{\psi}^2 - \tilde{m}r_0\cos\varphi\dot{\theta}^2 = -NT \ , \quad (14)$$

$$I\ddot{\theta} = r_0 NT\sin\varphi + M \ , \quad (15)$$

$$L\ddot{\psi} + r_0\cos\varphi\ddot{\theta} + r_0\sin\varphi\dot{\theta}^2 + 2\dot{\psi}\dot{L} = 0 \ , \quad (16)$$

which is the same as (3)-(5) considered in the previous sections with one control variable more: the moment M acting on the central hub. During the ZNAMYA-2 experiment a carefully chosen feedback law $M = M(\omega)$ was found to be an essential part in the successful deployment of the 20m reflector. Unfortunately, an analytical study of such a feedback is, in the full non-linear case, very difficult. If we repeat the linear study performed in the previous section we get $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}'\mathbf{u}'$, where $\mathbf{u}' = [\delta T, M/\bar{L}I]^T$ and $\mathbf{B}' = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & \bar{L}+r_0 & 0 & 0 \end{bmatrix}$. If we evaluate the controllability matrix, we now get that its rank is maximum and the system is thus controllable. Clearly, if we want to study the stability of some $M = M(\omega)$ feedback law during the deployment, the approach as defined in (6) is no longer useful.

A different approach may still allow to study the stability of a given feedback law within certain reasonable assumptions. We will consider that during the deployment, the system, at each instant, is not too far from the equilibrium motion that results from a given $\bar{L}(t)$, $\bar{\varphi} = 0$, and therefore set

$$T = \bar{T}(t) + \delta T \ , \qquad M = \bar{M}(t) + \delta M \ , \qquad L = \bar{L}(t) \ , \tag{17}$$

$$\omega = \bar{\omega}(t) + \delta\omega \ , \qquad \varphi = \delta\varphi \ . \tag{18}$$

Substituting in the equations of motion the barred reference solution we get, $N\bar{T}(t) = \tilde{m}(\bar{L}(t)+r_0)\bar{\omega}^2(t) - \tilde{m}\ddot{\bar{L}}$, $\bar{M} = I\dot{\bar{\omega}}$ and $\bar{\omega}(\bar{L}+r_0)^2 = const. = H_0$. Note that the third condition determines unequivocally the reference angular velocity law and corresponds to the conservation of the total angular momentum only to the limit $I \to 0$ or $\bar{L} \to \infty$, in which case we have the deployment of a multi-tether system. With these positions it is possible to study the deployment of a large structure with respect to a reference trajectory defined in terms of $\bar{L}(t)$ and having constantly $\bar{\varphi} = 0$. Note that we do not allow for any overshooting δL on the tether length as we assume this to be perfectly controllable.

Rewriting now (14)-(16) results in

$$2\bar{\omega}(r_0 + \bar{L})\delta\omega + 2\bar{L}\bar{\omega}\delta\dot{\varphi} - r\dot{\bar{\omega}}\delta\varphi = (N/\tilde{m})\delta T \ , \tag{19}$$

$$\delta\dot{\omega} = d\delta\varphi + \delta M/I \ , \tag{20}$$

$$\bar{L}\delta\ddot{\varphi} = -2\dot{\bar{L}}\delta\dot{\varphi} + f\bar{L}\delta\varphi - 2\dot{\bar{L}}\delta\omega - (\bar{L}+r_0)\delta M/I \ , \tag{21}$$

which is a set of non-autonomous linear equations. The time dependent coefficients complicate the study of the stability of a given $\delta M(\delta\omega)$ feedback. The time appears in the coefficients $\bar{L}$, $\dot{\bar{L}}$, $\bar{\omega}$, $\dot{\bar{\omega}}$, d and f. In the work by Rosenbrock [6], though, some criteria are given that ensure the stability also for non-autonomous systems and we will here use those results to prove the stability of a given feedback as done in a similar case by Natarajan et al. [7].

We start by rewriting the system in the form $\dot{\mathbf{x}} = \mathbf{A}(t)\mathbf{x} + \mathbf{B}(t)\mathbf{u}$, where $\mathbf{x} = [\delta\omega, \delta\dot{\varphi}, \delta\varphi]$, $\mathbf{u} = \delta M$ and

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & d \\ -2\frac{\dot{\bar{L}}}{\bar{L}} & -2\frac{\dot{\bar{L}}}{\bar{L}} & f \\ 0 & 1 & 0 \end{bmatrix} \ , \quad \mathbf{B} = \begin{bmatrix} 1/I \\ -\frac{\bar{L}+r_0}{\bar{L}I} \\ 0 \end{bmatrix} \ . \tag{22}$$

Note that we could get rid of the first equation that will give us the tension once the control is found. At this point we may define a state feedback $\mathbf{u} = \mathbf{u}(\mathbf{x})$ and try to study its stability. Here we take a generalisation of the feedback proposed by Melnikov and Koshelev [1] and used successfully during the ZNAMYA-2 reflector deployment. That feedback was

$$M = M_0(1 - \omega/\omega_0) \ , \tag{23}$$

where ω_0 was a reference angular velocity of the spinning system. Such a reference velocity could be changed during the deployment at certain given instants. This type of feedback law produces a dynamical response that is very difficult to study without having to perform numerical or real hardware experiments (which was in-fact the approach taken by Melnikov and Koshelev [1].) Our description is, in this sense, advantageous as we introduced a reference deployment in terms of the function $\bar{L}(t)$ and we may use the feedback

$$M = M_0(1 - \omega/\bar{\omega}) = -M_0\delta\omega = -k\delta\omega \tag{24}$$

where we have introduced k as a variable for the gain rather than M_0. We may see this feedback as the one proposed in [1] where we change the reference angular velocity ω_0 continuously. In our notation we then have $\mathbf{u} = -\mathbf{K}\mathbf{x}$, where $\mathbf{K} = [k, 0, 0]$, and the system of equations

$$\dot{\mathbf{x}} = (\mathbf{A}(t) - \mathbf{B}(t)\mathbf{K})\mathbf{x} = \tilde{\mathbf{A}} \tag{25}$$

is obtained. According to Rosenbrock [6], the stability of a linear time varying system is granted if at each instant the system is stable (considering time as fixed) and if the variation of the dynamical matrix is small enough. This translates directly on a careful selection of the reference deployment, that is, of the function $\bar{L}(t)$, which additionally has to fulfil the constraint of a positive tension during the complete deployment. To study the stability of the frozen system we consider here the Hurwitz-Routh criteria on the matrix $\mathbf{A} - \mathbf{BK}$. Its characteristic polynomial $a_0\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3$ has coefficients

$$a_0 = \bar{L}I \ , \tag{26}$$

$$a_1 = 2I\dot{\bar{L}} + k\bar{L} \ , \tag{27}$$

$$a_2 = r_0\bar{\omega}^2\left(I + m(\bar{L}+r_0)^2\right) + 2k\dot{\bar{L}} \ , \tag{28}$$

$$a_3 = r_0\bar{\omega}^2(k_0 + 2m(\bar{L}+r_0)\dot{\bar{L}}) \ . \tag{29}$$

The criteria essentially tells us that in order for $\mathbf{A} - \mathbf{BK}$ to be Hurwitz one needs to satisfy the condition $a_1a_2 > a_0a_3$ which translates to

$$2\dot{\bar{L}}I\bar{\omega}^2r_0(I + 2m\bar{L}r_0^2 + 2mr_0^2) + \\ + k\left(4I\dot{\bar{L}}^2 + m\bar{\omega}^2r_0\bar{L}(\bar{L}+r_0)^2 + 2k\bar{L}\dot{\bar{L}}\right) > 0 \ . \tag{30}$$

We immediately see that in case of a deployment $\dot{\bar{L}} > 0$ and therefore the above inequality is automatically satisfied at each time instant. This allows us to conclude (using Rosenbrock criteria) that for any reference deployment $\bar{L}(t)$ and $\bar{\omega}(t)$ there exist a positive real number τ for which the feedback considered results asymptotically stable using $\bar{L}(t/\tau)$ and $\bar{\omega}(t/\tau)$. In other words the proposed feedback allows us to track any given deployment trajectory as long as we take enough time.

Given some reference deployment $\bar{L}(t)$ and $\bar{\omega}(t)$ how can we locate the largest possible value of τ that still guarantee a feasible deployment? In other word how fast can we deploy a structure following a given reference deployment trajectory? In practical cases the reference trajectory is such that $\dot{\tilde{\mathbf{A}}} = 0$ after a finite time (the reference deployment duration). This means that the non-autonomous system after a finite time converges to a linear time independent system. Clearly if the latter is asymptotically stable (as in our case) then the non-autonomous system will eventually converge to the origin. It is of interest, though, that the solution does not explode during the non-autonomous phase as to avoid non-linearities to become important. Here we do not consider this issue and we rather focus on the condition on the tether tension $T(t) = \bar{T} + \delta T > 0$ that usually implies also that the state variable remain small while the reference trajectory vary, as confirmed by numerical simulations. Neglecting the small deviations from the reference trajectory, this condition brings to the inequality

$$\ddot{\bar{L}}(\bar{L} + r_0)^3 < H_0^2 \ , \tag{31}$$

which grants that there is enough centrifugal acceleration to actually deploy the tethers.

In the following we perform a numerical study on one particular case of the dynamic of the deployment. In the following we consider that $\bar{L}(\cdot)$ is a polynomial function defined in $[0, 1]$ that drives the deployment from L_0 to L_f. With this position only two relevant quantities—the deployment time τ and the feedback gain k—determine the behaviour of (25), once all the initial conditions are set. As an example we take the polynomial

$$\bar{L}(t) = -2\frac{L_f - L_0}{\tau^3}t^3 + 3\frac{L_f - L_0}{\tau^2}t^2 + L_0 \tag{32}$$

having zero derivative at $t = 0$ and $t = \tau$.

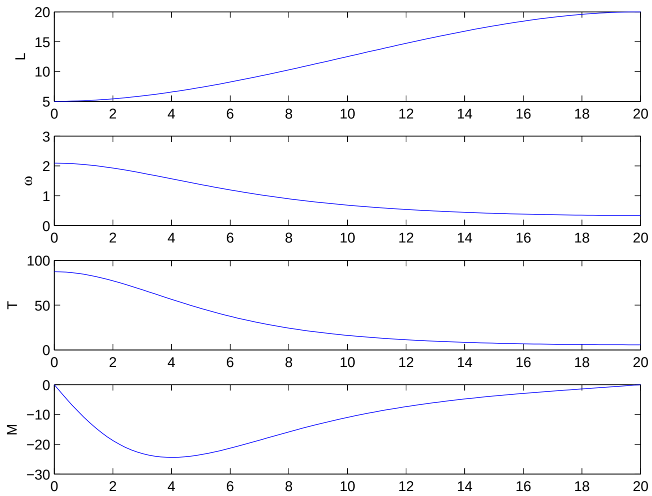

Fig. 2. Reference signals

We then simulate the deployment by integrating (25), where we set $L_0 = 5$m, $L_f = 20$m, $m = 2$kg, $r_0 = 5$m, $I = 125$ kg m^2, $\omega_0 = 20$rpm, $H_0 = \omega_0(r_0 + L_0)^2$ kg m^2 s^{-1}, $k = 100$, $\tau = 20$s. In figure 2 we show the corresponding reference solution. The system response to an initial condition $\mathbf{x}_0[0.1, 0.1, 0.1]$ is plotted in figure 3. In the simulations a gain matrix $\mathbf{K} = [k \ -k \ -k]$ has been used to improve the response. As shown in the pictures the feedback is able to stabilise the deployment quite efficiently. It has to be mentioned at this point that the performance of the controller depends significantly on the system considered. Large values of the central hub inertia, of the initial rotation speed, of the deployed mass m or of the deployment time τ can result in a solution with a very slow convergence. Short deployment

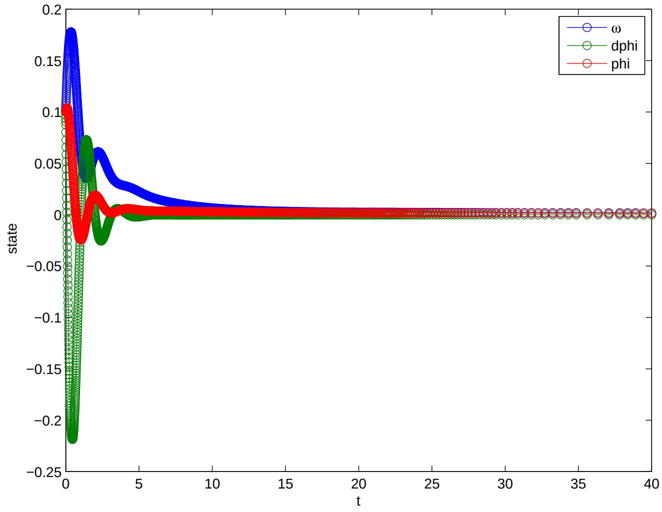

Fig. 3. System response to an initial condition $\mathbf{x} = [0.1, 0.1, 0.1]$

times result in larger torques M, so that in a real application the parameters have to be chosen carefully depending on the motor performance and on the system requirements.

III. Lorentz interaction with background magnetic field

We want to investigate, how electromagnetic interactions could help to deploy and control a space web along the lines of the previous section. There exist essentially two ways to establish such an interaction: either by means of currents (moving charges) or by charging parts of the structure. In the former case, a self-interaction is possible (as e.g. suggested for MagSail deployment in [3].) Here we exclusively consider charging parts of the structure and therefore rely on an external magnetic field. This concept is expected to be advantageous for control as it allows in a simple way to accelerate just parts of the space web. In practice we consider the same space web as in figure 1 but each mass m is seen as a capacitance that can be charged.

Charging some parts of the space web with a charge Q establishes the force

$$\mathbf{F} = Q\mathbf{v}_S \times \mathbf{B} = Q(\mathbf{v} - \boldsymbol{\omega}_e \times \mathbf{r}) \times \mathbf{B} \ , \tag{33}$$

where $\mathbf{B}$ is the external magnetic field (of the Earth or some other planet) and $\mathbf{v}_S = \mathbf{v} - \boldsymbol{\omega}_e \times \mathbf{r}$ the relative velocity of the space web with respect to the latter. For simplicity we will consider equatorial, circular orbits and we assume the magnetic field to be a dipole aligned with the rotational axes of the planet, which for Earth and Jupiter is taken as

$$B_{\text{Earth}}(r) = \frac{8.04 \times 10^6}{r^3[\text{km}^3]}\text{Tesla} \ , \quad B_{\text{Jupiter}}(r) = \frac{1.25 \times 10^{12}}{r^3[\text{km}^3]}T \ , \tag{34}$$

respectively. In this way the magnetic field always is perpendicular to the relative velocity and we encounter the situation depicted in figure 4. In principle, the rotational velocity of the masses around the central hub should be taken into account as well, but given the fact that this velocity remains much smaller than the relative velocity of the spacecraft with respect to the magnetic field this effect will be neglected. A few

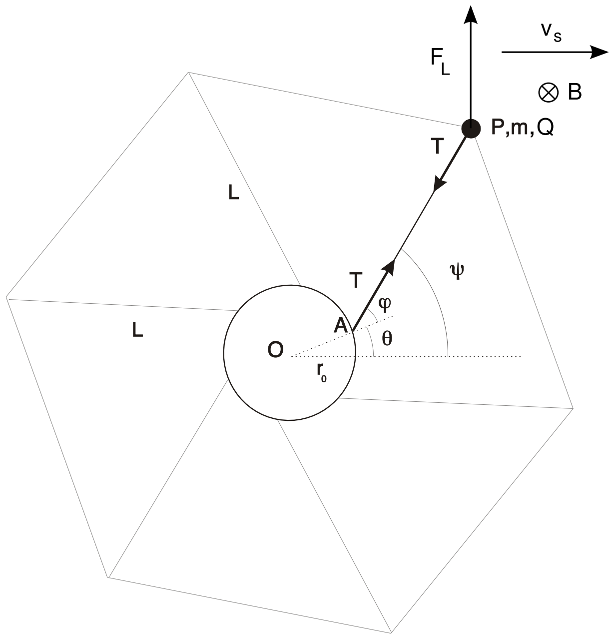

Fig. 4. Situation of a deployment assisted by the Lorentz force.

basic characteristics can be deduced immediately from figure 4: firstly the deployment of the space web can no longer be considered as symmetric as the relative direction of the Lorentz force depends on ψ, secondly one sees that the Lorentz force in general will contribute to torque as well as to the tension. Considering the first point we will assume a quasi-symmetrical case and we will neglect the interaction between the different tethers. Finally it should be noted that any type of interaction with a background magnetic field establishes a preferred plane of deployment due to the cross products appearing in (33); here we assume for simplicity that all angles are right angles.

One of the restrictions in centrifugal deployments is the Coriolis force, in practice the deployment speed v_D is restricted by the latter which should always be much smaller than the centrifugal force:

$$\frac{\omega(L+r_0)}{2v_D} \gg 1 \tag{35}$$

Charging of the masses can influence this relation in two ways, the Lorentz force can add to the centrifugal force and it can increase the angular momentum of the mass in order to diminish the Coriolis force. Adopting the simplifications defined above, the inequality (35) with the contributions from the Lorentz force becomes

$$m\omega^2(L+r_0) + QB_Ev_S\sin\psi \gg 2m\omega v_D - QB_Ev_S\cos\psi \ . \tag{36}$$

As can be seen from figure 4 only in the quadrants $\psi \in [0,\pi/2]$ and $\psi \in [\pi, 3\pi/2]$ the charge can be chosen in such a way that parallel and perpendicular component of the Lorentz force have a positive effect.

Assuming a maximal charging level of 1μC/kg the Lorentz force QB_Ev_S becomes

LEO	Jupiter@Io (non-retro)	Jupiter@Io (retro)
0.2μN/kg	0.95μN/kg	1.53μN/kg

(37)

Notice that the higher numbers in Jovian orbit mainly are due to the higher relative velocity induced by the fast rotation velocity of Jupiter. It is seen that the available forces cannot have a substantial influence in (36) but one still might think about using electromagnetic interactions for control. This conclusion changes if higher charging levels become feasible, cf. [8].

Let us now consider the equations of motion of the capacitances more in detail. Of course the structure remains similar to the one of eqs. (3)-(5), but as the situation can no longer be symmetrical for more than two masses m we should keep track of them separately. The complete system then reads

$$I\ddot{\theta} = r_0 \sum_i T_i \sin\varphi_i \ , \tag{38}$$

$$m\ddot{L} + mr_0\sin\varphi\ddot{\theta} - mL\dot{\psi}^2 - \\ -mr_0\cos\varphi\dot{\theta}^2 = -T \pm QB_Ev_S\sin\psi \ , \tag{39}$$

$$mL\ddot{\psi} + mr_0\cos\varphi\ddot{\theta} + \\ +mr_0\sin\varphi\dot{\theta}^2 + 2m\dot{\psi}\dot{L} = \pm QB_Ev_S\cos\psi \ , \tag{40}$$

Here (39) and (40) have to be considered for each mass separately. The sign in the Lorentz force depends on the relative orientation of the Earth magnetic field and the velocity, resp. Without loss of generality we will consider this to be positive in accordance with figure 1. For some simplified situations we will present an analytical treatment of this model.

IV. Synchronised deployment

One obvious application is a deployment where the angular velocity of the central hub and the capacitance remain synchronised due to the torque induced by the Lorentz force, i.e. $\varphi \equiv 0$ during the whole process. The relevant equation of motion then reduces to

$$2\dot{\theta}\dot{L} = \frac{QB_Ev_S}{m}\cos\theta \ . \tag{41}$$

Of course, this equation directly follows from the conservation of angular momentum of the capacitance

$$\frac{\partial}{\partial t}m\dot{\theta}r^2 = m\ddot{\theta}r^2 + 2m\omega r\dot{r} = rQB_Ev_S\cos\theta \ , \tag{42}$$

when a constant angular velocity is considered.

While (41) correctly describes the detailed dynamics it is not the ideal form to obtain realistic overall deployment velocities. As $\dot{L}$ does not depend on L it is natural to assume that $Q(t)$ in a deployment depends on time in the combination $\theta = \omega t$. Then by a suitable change of variables (41) can be written as

$$\frac{\partial L}{\partial\theta} = \frac{B_Ev_S}{2m\omega^2}Q(\theta)\cos\theta \tag{43}$$

and therefore the change of L per revolution becomes

$$\Delta L = \frac{B_Ev_S}{2m\omega^2}\int_0^{2\pi} Q(\theta)\cos(\theta)\mathrm{d}\theta \ . \tag{44}$$

For a given maximal charging level $Q_{\text{max.}}$ the maximal deployment speed accordingly is

$$\Delta L_{\text{max.}} = \frac{2B_E v_S}{m\omega^2} Q_{\text{max.}} \tag{45}$$

Assuming for the spinning space web 1 revolution/minute one obtains from the numbers in (37) a deployment speed of 31cm/d, 150cm/d and 240cm/d for LEO, non-retrograde Jovian and retrograde Jovian orbit, respectively.

A. Linearised equations of motion

A stability and controllability analysis similar to the one in II-B and II-A can be performed, whereby we take the solution (41) as our reference deployment, which as a special case also includes a web with fixed tether lenghts L. Thus we take the perturbation around a solution $L(t) = \bar{L}(t) + \delta L(t)$, $\theta(t) = \omega_0 + \bar{\omega}t + \delta\theta(t)$, $\varphi(t) = \delta\varphi(t)$. The synchronised deployment requires a specific choice of charges according to (41), which by introducing a variation of the Lorentz force takes the form

$$Q(t) B_E v_S \cos(\omega_0 + \bar{\omega}t) = 2m\dot{\bar{L}}\bar{\omega} + \delta F_L \cos(\omega_0 + \bar{\omega}t) \ . \tag{46}$$

The tension on each of the tethers has contributions from the Lorentz force as well as from the centrifugal force:

$$\bar{T} = m(\bar{L} + r_0)\bar{\omega}^2 - m\ddot{\bar{L}} + Q B_E v_S \sin(\omega_0 + \bar{\omega}t) \tag{47}$$

Plugging this solution into the equations of motion yields for the linear perturbations

$$\ddot{\delta\theta} = \frac{m r_0}{I} \sum_i \left((\bar{L}_i + r_0)\bar{\omega}^2 - \ddot{\bar{L}}_i \right) \delta\varphi_i \ , \tag{48}$$

$$\begin{aligned} \ddot{\delta L} &= 2(r_0 + \bar{L})\bar{\omega}\dot{\delta\theta} + 2\bar{L}\bar{\omega}\dot{\delta\varphi} + \bar{\omega}^2 \delta L \\ &\quad + 2\dot{\bar{L}}\bar{\omega}\,(\delta\theta + \delta\varphi) - \frac{\delta T}{m} + \frac{\delta F_L}{m} \sin(\omega_0 + \bar{\omega}t) \ , \end{aligned} \tag{49}$$

$$\begin{aligned} \bar{L}\ddot{\delta\varphi} &= -(\bar{L} + r_0)\ddot{\delta\theta} - r_0\bar{\omega}^2\delta\varphi - 2\dot{\bar{L}}(\dot{\delta\theta} + \dot{\delta\varphi}) \\ &\quad - 2\bar{\omega}\left(\dot{\delta L} + \dot{\bar{L}} \tan(\omega_0 + \bar{\omega}t)(\delta\theta + \delta\varphi) \right) \\ &\quad + \frac{\delta F_L}{m} \cos(\omega_0 + \bar{\omega}t) \ . \end{aligned} \tag{50}$$

The above equations of motion effectively are a system of $N + 1$ coupled differential equations for $2N$ tethers (where a symmetric situation for two opposite tethers is assumed.) An analytic treatment of this system is not possible due to the appearance of trigonometric functions on the right-hand-side of (48) as a consequence of (41). To proceed we therefore have to make the simplifying assumption that the influence of the Lorentz force on the central hub is negligible. Then (48) can be replaced by

$$\ddot{\delta\theta} = \frac{2Nmr_0}{I} \left((\bar{L} + r_0)\bar{\omega}^2 - \ddot{\bar{L}} \right) \delta\varphi \ , \tag{51}$$

for any of the $2N$ tethers, whereby we used the trick to add to the equation terms of the order that we have chosen to neglect, which however cannot spoil the solution to the order of accuracy considered. Furthermore it is assumed that we can control the length of the tether perfectly and therefore δL is set to zero. Stability of the system now can be analysed analogously to the case without magnetic field with $\mathbf{x} = (\dot{\delta\theta}, \dot{\delta\varphi}, \delta\theta, \delta\varphi)$ and the control $\mathbf{u} = \delta F_L$. Eq. (49) is obsolete in this case, as it just determines δT in terms of the remaining variables. The other two differential equations are rewritten as $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$ with

$$\mathbf{A} = \begin{pmatrix} 0 & 0 & 0 & a \\ b & b & c & c+d \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \ , \qquad \mathbf{B} = \begin{pmatrix} 0 \\ e \\ 0 \\ 0 \end{pmatrix} \tag{52}$$

and

$$a = \frac{2Nmr_0}{I} \left((\bar{L} + r_0)\bar{\omega}^2 - \ddot{\bar{L}} \right) \ , \qquad b = -2\frac{\dot{\bar{L}}}{\bar{L}} \ , \tag{53}$$

$$c = b\bar{\omega} \tan(\omega_0 + \bar{\omega}t) \ , \qquad d = -\frac{(\bar{L} + r_0)a + r_0\bar{\omega}^2}{\bar{L}} \ , \tag{54}$$

$$e = \frac{\cos(\omega_0 + \bar{\omega}t)}{\bar{L}m} \ . \tag{55}$$

As is easily seen the matrix $\mathbf{C} = (\mathbf{B}, \mathbf{AB}, \mathbf{A}^2\mathbf{B}, \mathbf{A}^3\mathbf{B})$ has determinant $\det \mathbf{C} = -a^2 e^4$, which ensures controllability almost everywhere. One exception is seen to be the case where the centrifugal part to the tension in (47) vanishes, which should not be of practical importance. The second exception is specific to the control with the Lorentz force and encounters in all points with $\cos(\omega_0 + \bar{\omega}t) = 0$, in other words where the Lorentz force exclusively contributes to the tension.

Let us look at the stability criterion according to Rosenbrock [6]. By introducing the feedback $\mathbf{u} = -\mathbf{K}\mathbf{x}$ it is seen that the matrix $\mathbf{A} - \mathbf{BK}$ has full rank only if $\delta\theta$ is among the feedback variables. On the other hand choosing $\delta\theta$ as only feedback variable cannot lead to linear stability of the deployed system. From a generic feedback $\mathbf{K} = (k_1, k_2, k_3, k_4)$ the characteristic polynomial

$$\begin{aligned} & t^4 + a_3 t^3 + a_2 t^2 + a_1 t + a_0 = \\ & = t^4 + (k_1 e - b)t^3 - (c + d - k_4 e)t^2 + a(k_2 e - b)t - a(c - k_3 e) \end{aligned} \tag{56}$$

is obtained. The Routh-Hurwitz criterion yields four conditions on the four feedback variables:

$$a_0 > 0 \qquad\qquad a_3 > 0 \tag{57}$$

$$a_3 a_2 - a_1 > 0 \qquad a_3 a_2 a_1 - a_4 a_1^2 - a_3^2 a_0 > 0 \tag{58}$$

From (53)-(55) $b \leq 0$ while together with (47) at least for small Lorentz forces $a > 0$ and $d < 0$. However, the signs of c and e change with t and therefore the conditions are not satisfied automatically. Going through the four conditions in (57) and (58) and by substiuting $\dot{\bar{L}}$ using (43) they translate into

$$\cos(\omega_0 + \bar{\omega}t) \left(\frac{Q B_E v_S}{\bar{\omega}} + k_2 \right) > 0 \ , \tag{59}$$

$$k_3 \cos(\omega_0 + \bar{\omega}t) > 0 \qquad |k_3| > |Q B_E v_s| \ , \tag{60}$$

$$k_4 \cos(\omega_0 + \bar{\omega}t) \geq 0 \qquad |k_4| \geq |Q B_E v_s| \tag{61}$$

and the condition to choose k_1 small enough to ensure that the left-hand-side of (58) is satisfied as well as k_2 small enough (eventually negative) such that right-hand-side of (58) is satisfied. Both conditions can be satisfied always, but they need some time-dependent fine tuning. Therefore we may conclude that the deployment can be made stable by an appropriate choice of $\bar{L}(t)$ and $\bar{\omega}$.

Asymptotically $\dot{\bar{L}} = 0$ and the stability analysis simplifies drastically. Now $k_4 = 0$ can be chosen consistently and the remaining parameters are constrained by

$$k_1 \cos(\omega_0 + \bar{\omega} t) > 0 \ , \qquad k_2 \cos(\omega_0 + \bar{\omega} t) > 0 \ , \tag{62}$$

$$k_3 \cos(\omega_0 + \bar{\omega} t) > 0 \ , \tag{63}$$

$$\frac{(\bar{L} + r_0)a + r_0\bar{\omega}^2}{a\bar{L}} |k_2| > |k_1| \ , \tag{64}$$

$$k_1 \left(\frac{(\bar{L} + r_0)a + r_0\bar{\omega}^2}{\bar{L}} k_2 - a k_1 \right) - k_2^2 k_3 \frac{\cos(\omega_0 + \bar{\omega} t)}{\bar{L} m} > 0 \ . \tag{65}$$

Not surprisingly the feedback depends on the orientation of the tether or in other words on the sign of the charging. However, as k_3 only appears in the last equation, it can be chosen in such a way that its absolute value remains constant.

B. *Concluding remarks*

In this section we have shown that a synchronised deployment of a space web relying on the electromagnetic interaction with a background magnetic field is possible in principle. However, as is seen from the numbers obtained, this interaction alone is no alternative to other proposed methods as long as the charging level is restricted to relatively small values as used here ($\approx 1\mu$C/kg). The concept could become interesting for this purpose if the charging level could be enlarged by about three orders of magnitude.

A slightly different scenario should be mentioned for completeness. It is possible to deploy the web by an external momentum applied on the central hub as discussed in section II-B and to use the Lorentz force just for control. Controllability has been shown for the special case of the synchronised deployment and the asymptotic stability is guaranteed as well. Within the approximation made here this new concept does not add any new possibilities, but one should keep in mind that we considered the web to be symmetric always. In contrast to the central hub, however, the Lorentz force can be chosen differently for each tether and thus this concept is able to damp non-symmetric perturbations as well. Of course the damping level is restricted by the maximal charge as well, the maximal (mean) power per kg that can be dissipated approximately is

$$\bar{P} = \frac{2}{\pi} Q B_E v_S R \omega \ , \tag{66}$$

which e.g. for a tether length of 30m and five revolutions per minute is between 2 and 15 μW/kg, for LEO and Jupiter, resp.

V. SPINNING UP AND ORBITAL TRANSFERS

A different regime where the equations of motion (38)-(40) simplify drastically are two satellites connected by a tether. Neglecting the mass of the tether this reduces to the special case $I = r_0 = \varphi = 0$:

$$m\ddot{L} - mL\dot{\theta}^2 = -T + Q B_E v_S \sin\theta \tag{67}$$

$$mL\ddot{\theta} + 2m\dot{\theta}\dot{L} = Q B_E v_S \cos\theta \tag{68}$$

Due to the trigonometric functions appearing on the right hand side of the equations, closed analytic solutions cannot be obtained. However, in a quasi-static situation, where the typical time scale for the change of the system is much larger than the revolution time, the equations can be approximated if a certain profile for the charge $Q(\theta)$ is chosen. By maximising the tangential force

$$Q(\theta) = \begin{cases} Q & \theta \in (-\frac{\pi}{2}, \frac{\pi}{2}) \\ -Q & \theta \in (\frac{\pi}{2}, \frac{3\pi}{2}) \end{cases} \tag{69}$$

one obtains

$$m\ddot{L} - mL\dot{\theta}^2 = -T \ , \tag{70}$$

$$mL\ddot{\theta} + 2m\dot{\theta}\dot{L} = \frac{2}{\pi} Q B_E v_S \ . \tag{71}$$

If we further assume $\dot{L} = 0$ during the process of spinning up (or slowing down) the simple solution

$$\omega(t) L = \omega_0 L + \frac{2}{\pi} \frac{Q B_E v_S}{m} t \tag{72}$$

is obtained.

In the following we consider two different concepts starting from the equation of motion (72): on the one hand two satellites connected by a tether shall be spun up in order to change the characteristics of their orbits after having cut the tether, on the other hand Lorentz force interactions could be used to compensate the damping of the rotational speed due to gravity gradient effects as has been reported for the BOLAS mission.

A. *Changing characteristics of spacecraft orbit*

One scenario for a system of two spinning satellites is to use their motion to change the characteristics of the orbit after cutting the tether between the two satellites. Equation (72) defines the change of velocity, the relevant quantity to be considered here. From (37) a change of about 1 to 10cm/(s·d) follows, clearly too small to have any relevant effect (of course this again might change if much higher charging levels become feasible.) However, there exists a simple process to enhance the effect, namely conservation of angular momentum. If we consider to start with a configuration of the web with length L_0 and angular velocity ω_0, deploy this to a new configuration $L_1 > L_0$, $\omega_1 = L_0^2 \omega_0 / L_1^2$, apply a mean force $\bar{F}$ during a time t and contract again to the length L_0 the new velocity of the satellite is

$$v_0' = L_0 \omega_0 + \frac{L_1}{L_0} \frac{\bar{F}}{m} t \ . \tag{73}$$

An extension and re-contraction by two orders of magnitude should be feasible, which results in accelerations of about 50cm/(s·h) at LEO, whereby the time to deploy and contract the system has not yet been taken into account.

We intend to use the resulting velocity to change the characteristics of the orbit of the satellites. One of the two satellites could be used as spare mass to allow the other an orbital transfer, however this scenario does not appear to be attractive due to the additional mass used. More promising is a scenario where the tether is cut while being aligned with the orbital motion, here the orbital angular momentum remains conserved, but the semimajor axes and the eccentricity change. In that case none of the two satellites is lost, but both of them enter similar new orbits. For simplicity, the original orbit is assumed to be circular with radius R (where $R \ll L$ such that the effects from the finite tether length can be neglected), the final orbit has semimajor axes a and eccentricity e (which is the same for both satellites). Notice that in this particular case the mutual node always will be π, which however will change if the original orbit has non-zero eccentricity as well. If the apogee of the final orbit shall be at a distance R_A then its semimajor axes and eccentricity are found as

$$a = \frac{R_A}{2 - \frac{R}{R_A}} \qquad e = 1 - \frac{R}{R_A} \; . \tag{74}$$

Alternatively, one might want to fix perigee and apogee which then determines the radius of the original orbit as

$$R = \frac{2R_A R_P}{R_A + R_P} \; . \tag{75}$$

If we bound in Earth orbit the perigee to be not below 6500km then the radius of the original orbit as a function of the apogee becomes

$$R[\text{km}] = \frac{2R_A[\text{km}]}{1 + R_A[\text{km}]/6500} \; . \tag{76}$$

In particular, if the apogee shall at GEO one obtains $R \approx$ 11200km.

The velocity needed for such a manoeuvre is

$$\Delta v^2 = \frac{\mu}{R} - \frac{\mu}{a} = \frac{\mu a}{R_A R_P} - \frac{\mu}{a} \; , \tag{77}$$

which for the example above results in $\Delta v \approx 4.4\text{km/s}$. Due to the higher orbit the Lorentz force is one order of magnitude smaller than given in (37). Therefore, interesting numbers can be achieved only if about four orders of magnitude can be gained either by a higher charging level or by a long tether length L_1 in equation (73).

B. Compensation of damping effects

The proposed BOLAS mission (Bistatic Observations with Low Altitude Satellites), a ionospheric science mission, is very similar to the design of possible missions proposed here. It consists of two satellites connected by a tether at a distance of 100m, spinning in its orbit plane. The only relevant difference is the high orbital inclination which is not compatible with the use of the Lorentz force as proposed here.

For the BOLAS mission a loss of its rotation has been reported due to gravity gradient effects [9], [10]. The original rotation speed is about 0.2rpm, the estimated loss about 4-5% in 30 days. Comparing with equation (72) the numerical values

$$\omega \approx 0.1\text{s}^{-1} \; , \qquad \frac{\partial \omega L}{\partial t} \approx 10^{-7}\text{s}^{-2} \tag{78}$$

can be deduced. From (37) it is therefore seen that the expected damping effect still is one order of magnitude smaller than the mean force due to the interaction with the magnetic field of the Earth. Therefore it should be possible to control the spinning of a tether system similar to the BOLAS mission and in particular to compensate for the damping effects caused by gravity gradient.

VI. Conclusions

We have investigated various aspects of deployment and control of space webs (large structures in space) with particular focus on the question how Lorentz forces could help in such a manoeuvre. As an example a symmetric configuration of N masses connected to a central hub as well as to its neighbours has been considered, however it is known that the ensuing analytical structure of the equations of motion extends immediately to many other situations, which also applies to the main conclusion drawn from this paper.

The use of Lorentz forces resulting from charging parts of the space web have been assessed as an alternative to an external torque applied to the central hub to balance the Coriolis force. Though deployment and control are possible in principle, the necessary charging levels are too high for any application in the near future. One should keep in mind, however, the distinct advantages of the Lorentz forces to stabilise non-symmetrical perturbations. Indeed, the concept of charging parts of the space web allows to damp such perturbations, while this is not possible with an external torque applied to the central hub. Therefore, the concept introduced here still might be seen as complementary

As a further application a two-satellite system connected by a tether has been considered. The Lorentz force might be used to spin up such a system, which eventually could provide additional kinetic energy to both of the satellites if the tether is cut. Here conservation of angular momentum allows to circumvent at least in parts the small forces available. In terms of applicability within the near future the compensation of damping effects of a rotating system due to gravity gradient appears to be most promising. From the numbers reported for the proposed BOLAS mission it can be seen that the necessary forces are within the possibilities of the proposed concept.

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