Constraining a possible dependence of Newton's constant on the Earth's magnetic field

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August 10, 2004

Abstract

Some time ago Mbelek and Lachièze-Rey proposed that the discrepancy between the results of the various measurements of Newton's constant could be explained by introducing a gravielectric coupling between the Earth's gravitational and magnetic fields mediated by two scalar fields. A critical assessment of this model is performed. By calculating the static field configuration of the relevant scalar around a nucleus in the linearised theory and then folding this result with the mass density of the nucleus its effective gravitational mass is determined. Considering test bodies of different materials one finds violations of the weak equivalence principle for torsion-balance experiments which are four orders of magnitude beyond the current experimental limit, thus rendering the model non-viable. The method presented can be applied to generic theories with gravielectric coupling and seems to rule out in general the explanation of the discrepant measurements of Newton's constant by such couplings.

1 Introduction

The values of Newton's constant, obtained by various experiments, differ by the order of 10^{-3} (cf. [1] for an overview). This discrepancy is understandable from the field-theoretical point of view considering the extraordinary weakness of the gravitational interaction. Nevertheless the experimental situation is unsatisfactory because the discordance of measurements lies beyond the experimental errors given by the various groups. Hence it seems a legitimate hypothesis that the mismatch between the experimental values might be an indiction of "new physics". A proposal towards this direction has been put forward by Mbelek and Lachièze-Rey in [2] and has recently been further elaborated by them to include as a cosmological implication the variation of the fine-structure constant [3].

The model of [2] is based on an action which is claimed by the authors to consists of the action of dimensionally reduced five-dimensional Kaluza-Klein (KK) theory amended by matter fields and a second four-dimensional scalar field. The KK action provides the

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four-dimensional gravitational field, the electromagnetic field and a massless KK scalar. In contrast to conventional KK theory both the KK scalar as well as the second scalar are coupled to the electromagnetic field and the other matter fields in a way which cannot be reduced to a universal coupling.

The non-universal coupling between the electromagnetic field and the KK scalar leads to a dependence of the effective gravitational constant on the electromagnetic field. Mbelek and Lachièze-Rey investigated the effect of the Earth's magnetic field on the gravitational constant in view of this gravielectric coupling and, employing a dipole fit of the Earth's magnetic field, concluded that this new coupling could not only explain the discrepancy between the various measurements of Newton's constant but is also statistically preferred compared to the assumption of a constant gravitational coupling. In conclusion, the magnetic field of the Earth would lead to a modification of the gravitational coupling of the order of 10^{-3}. Conflict with the lack of gravitational anomalies in astrophysical sources, e. g. due to the Sun's magnetic field or in neutron stars, are avoided by assuming a temperature dependence of the gravielectric coupling leading to its vanishing at high temperatures. On the other hand, taking into account the extraordinary weakness of the Earth's magnetic field compared to the magnetic fields producable in laboratories, the result of [2] is not only of interest from the fundamental physics point of view but also seems to open the door to a whole new technology of gravity engineering. Considering, furthermore, the echo the model has found in popular science publications (see e. g. [4]) a thorough investigation of further implications of the model appears highly desirable.

Unfortunately, a closer analysis shows that the model is already ruled out by current experimental constraints. Due to the complicated coupling of both scalars (and the metric) to the electromagnetic field the model falls into the class of non-metric theories and will inevitably exhibit violations of the weak equivalence principle (WEP).[1] Since the model under consideration is a biscalar-tensor theory there are, however, no well established phenomenological constraints on its coupling parameters, like in the case of ordinary scalar-tensor theories.[2] Hence we resort to demonstrating the non-viability of the model for the special situation of an Eötvös experiment.

As could have been anticipated from the surprisingly strong gravielectric coupling, one finds a strong modification of the effective gravitational constant in the electric field of nucleus. The resulting modification of the ratio between the inertial and the effective gravitational mass is proportional to $(Z/A^{1/3})^2$ where Z is the proton number of the nucleus and A is its nucleon number. The resulting violations of the WEP are approximately four orders of magnitude beyond current experimental bounds from Eötvös experiments. Consequently, the model of [2] is not viable.

The layout of this treatise is the following: In the next section we discuss the underlying action of the model of [2]. We find that the action of model [2] is *not* as claimed the effective action of dimensionally reduced KK theory plus additional matter fields. The KK part of the action has an additional kinetic term for the scalar which is not obtainable by dimensional reduction of five-dimensional KK theory. Also the relative signs of the various terms in the action turn out not to be consistent. In particular the action of [2] has a negative stress-energy tensor for the electromagnetic field. We thus correct

[1] cf. [5] p. 22 ff. for the definition of the term "non-metric theory".
[2] cf. [6] for the constraints on scalar-tensor theories.

this inconsistency of the model by carrying out the necessary sign changes and take the resulting action as a starting point for our considerations. In Sec. 3 we derive the weak-field approximation of the model by linearising its action and deriving the linearised equations of motion. Starting from the static limit of the equations of motion we obtain the solution for the linearised KK scalar field in a nucleus in Sec. 4 by a semiclassical treatment of the nucleus. Inserting this solution in the static metric perturbation of a nucleus we determine the strength of violation of the WEP in the experiments [7, 8, 9]. In Sec. 5 we summarise our results and discuss the place of the model in the classification of test theories of gravity. We also comment on the generic incompatibility between a strong gravielectric coupling and the maintenance of WEP constraints. The appendix further discusses the inconsistency in the original action and spots part of its origin in the paper [10].

2 Inconsistency of the underlying action

Mbelek and Lachièze-Rey in [2] discuss properties of the action

$$S_{\mathrm{ML}} = S_{\text{``KK''}} + S_{\Psi} + S_{\mathrm{matter}} \,. \tag{1}$$

In Eq. (1) $S_{\text{``KK''}}$ (the meaning of the quotation marks will become clear below) denotes the action

$$S_{\text{``KK''}} = -\int d^4x\,\sqrt{-g}\left[\frac{c^4}{16\pi G_0}\Phi R + \frac{1}{4}\epsilon_0 \Phi^3 F_{\mu\nu}F^{\mu\nu} + \frac{c^4}{4\pi G_0}\frac{\partial_\mu \Phi\,\partial^\mu \Phi}{\Phi}\right], \tag{2}$$

where G_0 denotes the effective four-dimensional gravitational constant in the Einstein frame. Eq. (2) is claimed to be the reduced effective four-dimensional action of five-dimensional KK theory in the Jordan frame. The part of the action, Eq. (1), denoted S_Ψ is the action of a scalar field Ψ which is non-minimally coupled to other matter fields and also to the KK scalar Φ. The action for the second scalar Ψ is given by[3]

$$S_\Psi = \int d^4x\,\sqrt{-g}\,\Phi\,\left[\frac{1}{2}\partial_\mu\Psi\partial^\mu\Psi - W(\Psi,\Phi,\ldots)\right], \tag{3}$$

where the term W represents the non-minimal coupling of Ψ to other fields. Mbelek and Lachièze-Rey make the ansatz that the interaction term W can be expressed as a sum of interaction terms with different fields, all of which however also depend on the KK scalar Φ,

$$W(\Psi,\Phi,\ldots) = W(\Phi,\Psi) + W(\Phi,\Psi,A_\mu) + W(\Phi,\Psi,T_{\mathrm{matter}}), \tag{4}$$

In Eq. (4) A_μ denotes the electromagnetic vector potential and Φ is assumed to couple to conventional matter only via a coupling to the trace of its stress-energy tensor $T_{\mathrm{matter}} \equiv g_{\mu\nu}T^{\mu\nu}_{\mathrm{matter}}$. The interaction with the electromagnetic field is assumed to take the form

$$W(A_\mu,\Psi) = w_{\mathrm{EM}}(\Phi,\Psi)\,\epsilon_0 F_{\mu\nu}F^{\mu\nu}, \tag{5}$$

[3]The original action of [2] also included a self-interaction term of the $U(\Psi)$. This term is however later in [2] discarded as small. Thus we decide to neglect it in our considerations right from the beginning for conciseness.

where the coupling $w_{\rm EM}$ is a function of both scalar fields. The couplings of Φ to other fields remain unspecified but are assumed to be negligible in the weak-field approximation. The contribution $S_{\rm matter} = \int d^4x\sqrt{-g}\Phi\mathcal{L}_{\rm matter}$ in Eq. (1) represents the action of other matter fields with the usual minimal coupling to the effective metric $\Phi^2 g_{\mu\nu}$.

Surprisingly, the action $S_{\text{``KK''}}$, Eq. (2), features a kinetic term for the scalar which is usually absent in the effective KK action in Jordan frame (cf. e.g. [11, 12]),[4]

$$S_{\rm KK} = \int d^4x\,\sqrt{-g}\left[-\frac{c^4}{16\pi G_0}\Phi R + \frac{1}{4}\epsilon_0\Phi^3 F_{\mu\nu}F^{\mu\nu}\right] . \tag{6}$$

The scalar kinetic term in Eq. (2) is neither a total divergence nor can it be generated form Eq. (6) by a conformal transformation. Moreover varying the action $S_{\text{``KK''}}$, Eq. (2) with respect to Φ one obtains an equation of motion which is inconsistent with the extra-dimensional component of the five-dimensional equations of motion of KK theory in the Jordan frame,

$$R_{55} = \frac{8}{3}\pi G_5 g_{55} g^{MN}\delta^\mu_M\delta^\nu_N T_{\mu\nu} , \tag{7}$$

where G_5 is the five-dimensional Newton's constant, the 5 denotes the index of the compactified extra dimension, $M, N = 0, \ldots, 3, 5$, and $T_{\mu\nu}$ is the stress-energy tensor of four-dimensional matter including the scalar Ψ. The equation of motion for the KK scalar Φ, given in, Eq. (11) of [2], has the form one would obtain from the higher-dimensional component of the Ricci tensor amended by the terms one obtains by varying the action of the second scalar Ψ, Eq. (3), with respect to Φ. However, this would not be the correct equation of motion for Φ even if one would replace $S_{\text{``KK''}}$, Eq. (2), by Eq. (6).

Another problem with the action presented in [2] is that the kinetic terms of the KK scalar Φ and the other scalar Ψ enter with different signs (cf. Eqs. (2) and (3)). Actually, in [2] it is concluded that a *classical* instability is associated with the KK scalar Φ which is in contrast to all previous treatments of KK theory.[5] In order to locate the source of this misconception we analyse the conventions employed in [2]. By calculating the stress-energy tensor for both the electromagnetic and the scalar part of the action, Eq. (2), we find that the kinetic term of the KK scalar has the correct sign relative to the action of the electromagnetic field. Hence if the KK scalar in the action, Eq. (2) were a ghost, as claimed in [2], then also the energy of the electromagnetic field in the model would be negative.

The first two terms of the action, Eq. (2), just yield the usual KK action displayed for a metric with signature $(+,-,-,-)$ and negative sign of the Einstein tensor[6]

$$G_{\mu\nu} = -8\pi G T_{\mu\nu} , \tag{8}$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$ denotes the Einstein tensor. On the other hand, from the effective Einstein equations given in [2], there Eq. (4), and the kinetic term of the scalar Ψ we infer the conventions: metric signature $(+,-,-,-)$ and

$$G_{\mu\nu} = +8\pi G T_{\mu\nu} . \tag{9}$$

[4] Conventions are metric signature $(+,-,-,-)$ and positive sign of the Einstein tensor.

[5] For a didactic derivation and discussion of the KK action in various frames see e.g. [11].

[6] There is no way to deduce the sign of the Riemann tensor from Eq. (2).

Consequently the action of [2] is a mixture of contributions written in different conventions leading to the misinterpretation that KK theory is *classically* unstable.[7] In order to keep our reasoning as close as possible to the investigations of [2] but nevertheless avoid the inconsistency in sign we change the "KK" part of the action, (Eq. 2), to so-called Landau-Lifschitz timelike conventions,[8] i. e. metric signature $(+,-,-,-)$ and $G_{\mu\nu} = +8\pi G T_{\mu\nu}$. The action, Eq. (2), then becomes

$$S_{g\Phi F} = \int d^4x \sqrt{-g} \left[-\frac{c^4}{16\pi G_0} \Phi R + \frac{1}{4}\epsilon_0 \Phi^3 F_{\mu\nu} F^{\mu\nu} + \frac{c^4}{4\pi G_0} \frac{\partial_\mu \Phi\, \partial^\mu \Phi}{\Phi} \right] , \tag{10}$$

and the total modified action becomes

$$S = S_{g\Phi F} + S_\Psi + S_{\text{matter}} , \tag{11}$$

where S_Ψ is given by Eq. (3) and S_{matter} is the usual action of additional matter fields.

By conducting only the above minimal modification to render the action of [2] consistent we unfortunately have to drop the idea that the effective action under study can be derived by the KK mechanism from a five-dimensional theory. For an effective theory derived by KK reduction we would have to employ the effective action, Eq. (6) amended by the 55-component of the five-dimensional Einstein equations. In this case the 55-component of the Einstein-equations would take the role of the equations of motion for the KK scalar Φ,

$$\Box \Phi = \frac{1}{4}\epsilon_0 \Phi^3 F_{\mu\nu} F^{\mu\nu} + \frac{8}{3}\pi G_5 \Phi^3 g^{\mu\nu} T_{\mu\nu} . \tag{12}$$

In Eq. (12) the coupling of the KK scalar Φ to matter goes with the same power of Φ than that to the electromagnetic field. Consequently, in this case the contribution to the gravielectric coupling from the $F_{\mu\nu}F^{\mu\nu}$ term in S_{KK}, Eq. (6), will be of the same order of magnitude as that from the term, Eq. (5), and the effective theory will look very different from the model studied by Mbelek and Lachièze-Rey. In contrast to this the suggested sign changes in the action do only marginally affect the phenomenological discussion of [2] and render the action consistent at least as a four-dimensional theory.

Interestingly, an action differing from Eq. (2) only by a numerical coefficient has been presented as the action of KK theory in Jordan frame once before the works of Mbelek and Lachièze-Rey in [17]. However, this is an obvious misprint because [18], which is cited as the source of the action in [17], does not display it but is only concerned with the KK action in Einstein frame.

Obviously, the sign with which the kinetic term of the scalar appears in the KK action in Einstein frame is tied to the conventions employed. This is necessary because the relative sign of the Einstein tensor and the stress-energy tensor in the Einstein equations also depends on the conventions and one should not have the stability properties of the effective action changed by changing conventions. It is worth understanding the causes

[7] The classical stability properties of KK theory are discussed in a didactic way in [13] 298 ff. The alleged ghost in [2] is not related to the ghost appearing in KK theory with *more than one* extra dimension in [14]. The scalar ghost discussed in the latter article arrises similar to the conformal ghost in ordinary Einstein theory (cf. e. g. [15]) by singling out the conformal mode of the extra-dimensional hypersurface.

[8] The name referring to the book [16].

which in [2] led to the assumption of a *classical* instability of KK theory from this point of view. They are already rooted in Mbelek's and Lachièze-Rey's earlier article [10]. However, as this topic lies somewhat out of the main scope of our considerations we defer the analysis of [10] to the appendix.

In order to investigate the phenomenological viability of the minimally improved model based on [2] we derive in the following, the linearised equations of motion of the action, Eq. (11) which will enable us to study phenomenological aspects of the weak-field regime of the theory.

3 The linearised equations of motion

Already in [2] the equations of motion for the Lagrangian, Eq. (1), have been given and used to study the weak field regime of the model, in that case in particular the field configuration around the Earth. However, in [2] no consistent linearisation of the action was conducted but instead the full equations of motion where simplified by determining small terms from plausibility arguments and dropping these from the field equations. In order to put our analysis on a more systematic footing we linearise each term of the action in the fields $g_{\mu\nu}, \Phi, \Psi$ to lowest non-vanishing order and afterwards derive the equations of motion from this linearised action. This procedure seems especially desirable because below we will encounter an error in the equations of motion as displayed in [2] (and also in [3, 19]).

Following [2] we assume that the scalar Φ far from any matter sources takes the value $\Phi = 1$ and thus can be expanded as

$$\Phi = 1 + \varphi + O(\varphi^2)\,. \tag{13}$$

The field Ψ is assumed to asymptotically take the value v, thus having the expansion

$$\Psi = v + \psi + O(\psi^2)\,. \tag{14}$$

Around the Minkowski background the metric has the expansion

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{\sqrt{32\pi G_0}}{c^2}\left(1 - \frac{\varphi}{2}\right) h_{\mu\nu} + O(h^2, \varphi^2)\,, \tag{15}$$

where we have included a factor $\sqrt{32\pi G_0/\Phi}/c^2$ in order to ensure the symmetry of the linearised theory under time reversal — or in more practical terms the symmetry of the linearised gravitational interaction between two masses under the interchange of the masses — and we have made use of Eq. (13). Expanding each term to lowest non-vanishing order in φ *and* $h_{\mu\nu}$ we find for the KK part of the action

$$S_{g\Phi F,\text{lin}} = \int d^4x \left[\frac{c^4}{16\pi G_0}\mathcal{L}_{\text{FP}} + \frac{\sqrt{8\pi G_0}}{c^2}\left(1 - \frac{\varphi}{2}\right) h_{\mu\nu} T^{\mu\nu}_{\text{EM}} \right.$$
$$\left. +(1+3\varphi)\frac{\epsilon_0}{4} F_{\mu\nu}F^{\mu\nu} + \frac{c^4}{4\pi G_0}\partial_\mu\varphi\partial^\mu\varphi\right]\,. \tag{16}$$

where $\mathcal{L}_{\rm FP}$ denotes the usual Fierz-Pauli Lagrangian of the linearised gravitational field,

$$\mathcal{L}_{\rm FP} = \frac{32\pi G_0}{c^4}\left[\frac{1}{2}h_{\mu\nu,\sigma}h^{\mu\nu,\sigma} - h_{\mu\nu,\sigma}h^{\mu\sigma,\nu} + h_{\mu\sigma}{}^{,\sigma}h^{,\mu} - \frac{1}{2}h_{,\nu}h^{,\nu}\right], \tag{17}$$

with $h = \eta^{\mu\nu}h_{\mu\nu}$, and $T^{\mu\nu}_{\rm EM}$ in Eq. (16) denotes the stress-energy tensor of the electromagnetic field.

To determine the linearised action of the scalar fields Φ, Ψ we first note the expansion of $W_{\rm EM}(\Phi, \Psi, A_\mu)$,

$$\begin{aligned} W_{\rm EM}(\Psi,\Phi) &= W_{\rm EM}(v,1) + \left.\frac{\partial W_{\rm EM}}{\partial\psi}\right|_{\substack{\psi=0\\ \varphi=0}}\psi + \left.\frac{\partial W_{\rm EM}}{\partial\varphi}\right|_{\substack{\psi=0\\ \varphi=0}}\varphi + O(\phi^2,\varphi^2,\psi\varphi) \\ &= \epsilon_0 F_{\mu\nu}F^{\mu\nu}\left(w_{\rm EM}(v,1) + \left.\frac{\partial w_{\rm EM}}{\partial\psi}\right|_{\substack{\psi=0\\ \varphi=0}}\psi + \left.\frac{\partial w_{\rm EM}}{\partial\varphi}\right|_{\substack{\psi=0\\ \varphi=0}}\varphi\right) \\ &\qquad + O(\phi^2,\varphi^2,\psi\varphi)\,. \end{aligned} \tag{18}$$

Using Eq. (18) and assuming in accord with [2] that the couplings $W(\Phi, \Psi)$ and $W(T, \Phi, \Psi)$ in Eq. (4) are negligible we find as the action for the linearised scalar ψ

$$\begin{aligned} S_{\psi,\rm lin} = \int d^4x \Bigg[&\frac{1}{2}\partial_\mu\psi\partial^\mu\psi \\ &- \epsilon_0 F_{\mu\nu}F^{\mu\nu}\left(w_{\rm EM}(v,1) + \left.\frac{\partial w_{\rm EM}}{\partial\psi}\right|_{\substack{\psi=0\\ \varphi=0}}\psi + \left.\frac{\partial w_{\rm EM}}{\partial\varphi}\right|_{\substack{\psi=0\\ \varphi=0}}\varphi\right)\Bigg]. \end{aligned} \tag{19}$$

The total linearised action is composed from Eqs. (16), (19) and the linearised action of matter fields

$$S_{\rm lin} = S_{g\Phi F,\rm lin} + S_{\psi,\rm lin} + S_{\rm matter,lin}\,. \tag{20}$$

Due to the contribution, Eq. (18), the electromagnetic field couples non-minimally to ψ and the theory remains non-metric even in the linearised approximation in Φ and Ψ. As a particular phenomenological consequence we expect violations of the WEP. Since the WEP is tested to high accuracies a first question to the model of [2] should be if it respects current experimental bounds on violations of the WEP. We thus calculate the modifications to be expected from the action, Eq. (20), compared to four-dimensional Einstein theory for typical Eötvös experiments with test bodies of different materials.

In order to proceed in this direction we first obtain the equations of motion for φ and ψ by varying the linearised action, Eq. (20),

$$\Box\varphi + \frac{2\pi G_0\,\epsilon_0}{c^4}F_{\mu\nu}F^{\mu\nu}\left.\frac{\partial w_{\rm EM}}{\partial\varphi}\right|_{\substack{\psi=0\\ \varphi=0}} = 0\,, \tag{21}$$

$$\Box\psi + \epsilon_0 F_{\mu\nu}F^{\mu\nu}\left.\frac{\partial w_{\rm EM}}{\partial\psi}\right|_{\substack{\psi=0\\ \varphi=0}} = 0\,, \tag{22}$$

where

$$F_{\mu\nu}F^{\mu\nu} = 2\left(\frac{\mathbf{B}^2}{c^2} - \mathbf{E}^2\right) \tag{23}$$

with **B** and **E** being the 3-vectors of the electric and magnetic field, respectively and we have neglected a term of $O(h_{\mu\nu})$ in Eq. (21).

The equations of motion for the metric perturbation are also obtained from varying the action, Eq. (20). In harmonic gauge, $h^{\mu\nu}{}_{,\mu} - \frac{1}{2}h_{,\nu} = 0$, they simplify to

$$-\left(\eta_{\mu\lambda}\eta_{\nu\rho} + \eta_{\mu\rho}\eta_{\nu\lambda} - \eta_{\mu\nu}\eta_{\lambda\rho}\right)\Box h^{\lambda\rho}(x_\alpha) = \frac{\sqrt{8\pi G_0}}{c^2}\left[1 - \frac{\varphi(x_\alpha)}{2}\right]\left[T^{\mu\nu}_{\mathrm{EM}}(x_\alpha) + T^{\mu\nu}_{\mathrm{matter}}(x_\alpha)\right]. \tag{24}$$

In Eq. (24) the stress energy tensors of the scalars do not appear because they would enter the gravitational equations only at third order in the linearised fields $\varphi, \psi, h_{\mu\nu}$.

Taking the static limit of the Eqs. (21) and (22) we find

$$\triangle\varphi = \frac{4\pi G_0}{c^4}\left.\frac{\partial w_{\mathrm{EM}}}{\partial\varphi}\right|_{\substack{\psi=0\\ \varphi=0}} \epsilon_0\left(\frac{\mathbf{B}^2}{c^2} - \mathbf{E}^2\right), \tag{25}$$

$$\triangle\psi = \left.\frac{\partial w_{\mathrm{EM}}}{\partial\psi}\right|_{\substack{\psi=0\\ \varphi=0}} \epsilon_0\left(\frac{\mathbf{B}^2}{c^2} - \mathbf{E}^2\right). \tag{26}$$

Comparing our corrected equation of motion for the scalar φ with the dipole fit to the magnetic field of the Earth given in [2] we obtain the numerical value

$$\begin{aligned}\frac{2\pi G_0}{c^4}\left.\frac{\partial w_{\mathrm{EM}}}{\partial\varphi}\right|_{\substack{\psi=0\\ \varphi=0}} &= -(5.44 \pm 0.66)\,10^{-6}\,\frac{\mathrm{fm}}{\mathrm{TeV}}\\ &= -(3.40 \pm 0.41)\,10^{-8}\,\mathrm{m/J}.\end{aligned} \tag{27}$$

In the following section we will use this value of the coupling to analyse the strength of the violations of the WEP in Eötvös experiments in the model of [2].

Finally, we have for the gravitational field of a static source

$$\begin{aligned}&\triangle h^{00}(\mathbf{x}) = \frac{\sqrt{8\pi G_0}}{c^2}\left(1 - \frac{\varphi(\mathbf{x})}{2}\right)\left[T^{00}_{\mathrm{EM}}(\mathbf{x}) + T^{00}_{\mathrm{matter}}(\mathbf{x})\right],\\ &h^{ij}(\mathbf{x}) = \delta^{ij}h^{00}(\mathbf{x}), \quad i,j = 1,2,3, \qquad h^{\mu 0}(\mathbf{x}) = 0,\end{aligned} \tag{28}$$

where we have assumed that the pressure of the source can be neglected, $T^\mu{}_\mu = T^{00}$.

4 Violations of the WEP

As already emphasised above the theory based on the action of Eq. (11) or (20) cannot be cast into a form in which a combination of the metric and the Φ scalar couples universally to matter. As a consequence of this non-metricity the theory will feature violations of the WEP. In order to be a viable theory these violations must obey current experimental bounds. The violations of the WEP will manifest themselves when electromagnetic fields are present. Since a strong electric field is present in the nucleus of every atom already the data from conventional Eötvös experiments provide a first challenge to the theory.

In order to derive an estimate on the strength of violation of the WEP in an Eötvös experiment we employ a simple model of the atomic nucleus in which the mass density and the charge density of a specific element are assumed to be constant throughout the nucleus. Neglecting the effect of the electron sheath, the electric field of the nucleus is then given by

$$E(\mathbf{r}) = \frac{eZ\,\mathbf{e}_r}{4\pi\epsilon_0} \begin{cases} \dfrac{1}{r^2}\,, & r > R \\ \dfrac{r}{R^3}\,, & r \leq R\,. \end{cases} \tag{29}$$

Here $\mathbf{e}_r$ is the radially pointing unit vector, e is the charge of the proton, Z denotes the atomic number of the nucleus and R denotes the radius of the core which is related to the nucleon number, A, as

$$R = r_0 A^{1/3}\,, \tag{30}$$

with $r_0 = 1.3 \pm 0.1 \times 10^{-13}$ m being an empirical value (see e. g. [20]). In order to find the field configuration for the linearised scalar φ around a nucleus we note that the solution of the Poisson equation

$$\triangle\varphi(\mathbf{r}) = f(r) \tag{31}$$

for a spherically symmetric potential $f(\mathbf{r}) \equiv f(r)$, $r = |\mathbf{r}|$, extending to a radius $r = X$, is given by

$$\varphi(\mathbf{r}) = \int_0^X d^3r' \frac{f(r')}{|\mathbf{r} - \mathbf{r}'|}\,. \tag{32}$$

This can be brought into the form

$$\varphi(\mathbf{r}) = \frac{4\pi}{r} \begin{cases} \displaystyle\int_0^X dr' \left[r'^2 f(r')\right]\,, & \text{for } r > X \\ \displaystyle\int_0^r dr' \left[r'^2 f(r')\right] + \int_r^X dr' \left[r r' f(r')\right]\,, & \text{for } r \leq X\,, \end{cases} \tag{33}$$

In our case, inserting Eq. (29) into Eq. (25), the potential $f(r)$ under consideration is

$$f(r) = \frac{G_0\, e^2 Z^2}{4\pi\epsilon_0 c^4} \left.\frac{\partial w_{\mathrm{EM}}}{\partial\varphi}\right|_{\substack{\psi=0\\ \varphi=0}} \begin{cases} \theta(-X)\, 1/r^4\,, & r > R \\ r^2/R^6\,, & r \leq R\,, \end{cases} \tag{34}$$

where we have introduced a step function $\theta(X)$ as a primitive approximation for the shielding of the electric field outside of the nucleus by the electron sheath. Despite its assumed temperature dependence (cf. [2]) the coupling parameter $\partial w_{\mathrm{EM}}/\partial\varphi|_{\psi=0,\varphi=0}$ is a good approximation of the one obtained from the fit to the Earths magnetic field, because the temperature to be associated with a nucleus in its ground state is $T = 0$ and nearly all baryonic matter on Earth is in its ground state. For the time being we will not specify the radius X where the field drops to zero but instead keep it as a free parameter.

In the expression for the gravitational field of the nucleus (see below) the scalar field $\varphi(\mathbf{r})$ will be folded with the mass density of the nucleus $\rho(\mathbf{r})$ which is zero outside the core. Hence, only the case

$$r \leq R < X \tag{35}$$

is relevant for our considerations, i. e. we are only interested in the value of the scalar within the radius of the nucleus and the electric shielding by the electrons is assumed to take effect only outside of the core. Thus, the integral to solve becomes

$$\varphi(\mathbf{r}) = \frac{G_0\, e^2 Z^2}{\epsilon_0 c^4 r} \left.\frac{\partial w_{\mathrm{EM}}}{\partial \phi}\right|_0 \left(\int_0^r dr' \frac{r'^4}{R^6} + \int_r^X dr' \frac{r r'^3}{R^6} + \int_R^X dr' \frac{r}{r'^3} \right) . \tag{36}$$

Evaluating Eq. (36) one finds

$$\varphi(\mathbf{r}) = \frac{G_0\, e^2 Z^2}{\epsilon_0 c^4} \left.\frac{\partial w_{\mathrm{EM}}}{\partial \varphi}\right|_{\substack{\psi=0\\ \varphi=0}} \left(\frac{3}{4}\frac{1}{R^2} - \frac{1}{20}\frac{r^4}{R^6} - \frac{1}{2X^2} \right) . \tag{37}$$

In the following we can neglect the contribution from the last term in the sum because the shielding of the electric field by the electron sheath takes effect only far outside of the nucleus, i. e. we have $X \gg R$ for any element. Dropping the term $\sim X^{-2}$ we arrive at the final expression for the field of the linearised scalar φ around a nucleus of atomic number Z and nucleon number A,[9]

$$\varphi(r) = \varphi_0 \left(\frac{1}{20}\frac{Z^2 r^4}{r_0^4 A^2} - \frac{2}{4}\frac{Z^2}{A^{2/3}} \right) , \tag{38}$$

where we have used Eq. (30) and defined a constant

$$\varphi_0 \equiv \frac{G_0\, e^2}{\epsilon_0 c^4 r_0^2} \left.\frac{\partial w_{\mathrm{EM}}}{\partial \varphi}\right|_{\substack{\psi=0\\ \varphi=0}} . \tag{39}$$

As a next step we have to find the metric perturbation generated by a nucleus in the presence of the field Eq. (38). Using Eq. (32) on Eq. (28) we find for the 00-component of the metric-perturbation of a single nucleus

$$h^{00}(\mathbf{r}) = -\sqrt{8\pi G_0} \int_0^R d^3r' \left[\left(1 - \frac{\varphi(\mathbf{r}')}{2} \right) \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} \right] , \tag{40}$$

where we have used that the 00-component of the stress-energy tensor for the nucleus at rest is given by $T_{00} = \rho(\mathbf{r}')c^2$. As a reasonable approximation we can assume that the density of the nucleus is constant $\rho(\mathbf{r}') = \rho_0$. Hence, at a distance $r > R$ from the centre of the nucleus its effective gravitational mass is given by

$$M_G = r\,\rho_0 \int_0^R d^3r' \left[\left(1 - \frac{\varphi(\mathbf{r}')}{2} \right) \frac{1}{|\mathbf{r} - \mathbf{r}'|} \right] . \tag{41}$$

[9] There is no reason to expect the behaviour $\varphi \to \infty$ for $r \to 0$ as claimed in [21] because $r = 0$ is not an asymptotic point of the domain of φ. Hence nothing drives the solution of the differential equation Eq. (25) towards the solution of the homogeneous equation $\triangle\varphi = 0$.

Note that this expression is not related to the gravitational self-energy of the nucleus.[10] Instead we have simply absorbed the influence of φ on the interaction between gravitation and matter into the definition of an effective mass, Eq. (41). The back-reaction of φ, as well as of the metric field $h_{\mu\nu}$, on the mass of the nucleus is not considered as it is of higher order in the perturbation expansion. Hence a calculation of the self-energy contribution to the Eötvös experiment is beyond the scope of the linearised approach.[11]

Since we are going to check for the experimental limits of an Eötvös experiment using a torsion balance with different materials we are only interested in the metric perturbation outside of the material, i. e. only the case $r > R \geq r'$.

Violations of the equivalence principle are usually not formulated in terms of the metric perturbation generated by the test bodies but in terms of a dimensionless quotient of the effective gravitational and the inertial mass of the matter, M_G and M_I,[12]

$$a = M_G/M_I \,. \tag{42}$$

From our expression for the effective mass, Eq. (41), we easily deduce this ratio for our case

$$M_G = \frac{\displaystyle\int_0^R d^3r' \frac{1-\varphi(\mathbf{r}')/2}{|\mathbf{r}-\mathbf{r}'|}}{4\pi R^3/3r} \,, \tag{43}$$

where the constant density of the nucleus ρ_0 has cancelled out. Evaluating the integral with the help of Eq. (33) we find

$$a = 1 - \frac{51}{70}\frac{\varphi_0 Z^2}{A^{2/3}} \,. \tag{44}$$

The parameter a determined in this way is only valid for the nucleus without an electron sheath. Outside of the nucleus the shielding of the electric field by the electrons takes effect. Hence also the scalar φ which has the electric field as its source will be decaying (cf. Eq. (34)). Both fields will not change sign outside the nucleus. Therefore the change to the gravitational mass of the electron sheath will have the same sign as that of the nucleus. In order not to consider the complicated structure of the electric field outside the nucleus we resort to the limit of no violations of the WEP for the electron sheath, yielding the smallest violation of the WEP for the whole atom and thus yielding a lower bound on $|a-1|$ for the atom. Under this assumption a for the atom is given by

$$a = \frac{M_{G,\mathrm{nucl}} + M_e}{M_{I,\mathrm{nucl}} + M_e} \,, \tag{45}$$

where $M_{G,\mathrm{nucl}}$ and $M_{I,\mathrm{nucl}}$ are the gravitational and inertial mass of the nucleus, respectively, and M_e denotes the mass of the electrons of the sheath. Expanding this expression

[10] This was erroneously implied in [21].

[11] The self-energy contribution to a violation of the WEP is expected to be of the order of 10^{-25} and thus far beyond present experimental limits (cf. [22]).

[12] cf. [5] p. 24 ff. for an introduction to tests of the WEP and [6] for an overview on recent experimental results.

Element	Z	A	$(A/Z^{(1/3)})^2$	$a-1$
Be	4	9	3.70	$-1.3 \pm 0.2 \times 10^{-9}$
Al	13	27	18.78	$-6.3 \pm 1.0 \times 10^{-9}$
Cu	29	63.6	52.77	$-1.8 \pm 0.2 \times 10^{-8}$
Pt	78	195.1	180.86	$-5.9 \pm 1.0 \times 10^{-8}$
Au	79	197	184.34	$-6.0 \pm 1.0 \times 10^{-8}$

Table 1: Determination of the nucleonic contribution to the parameter a for beryllium, aluminium, cooper, platinum and gold. The average values for the nucleon numbers were calculated from the data in [23].

for M_e and using Eq. (44) we find to first order in M_e

$$a = 1 - \frac{51}{70}\frac{\varphi_0 Z^2}{A^{2/3}}\left[1 - \frac{Zm_e}{m_p}\right], \tag{46}$$

where m_e and m_p are the mass of the electron and the proton, respectively. An Eötvös experiment will test the parameter a not for a single atom but for a macroscopic body of a specific material. If the reaction of the torsion balance to the gravitational field of a distant object is studied the test bodies can be considered as point masses and the parameter a of the macroscopic body can be obtained by averaging over the parameters a of the single atoms. Hence for a macroscopic body consisting of one chemical element a is found by taking for A the average nucleon number of the element.

In a torsion-balance experiment the gravitational force exerted by two different materials is compared. The quantity extracted from the experiments is

$$\eta \equiv \frac{2|a_1 - a_2|}{|a_1 + a_2|}. \tag{47}$$

To make a comparison between our theoretical estimate and the two classic Eötvös experiments in Moscow [7] and Princeton [8] as well as the more recent one in Seattle [9] we calculate the coefficients a for the chemical elements used for the test bodies in these experiments. The results are given in Table 1.

From the values for a we obtain the parameter η for the material combinations of the Moscow [7] (Al and Au), Princeton [8] (Al and Pt) and the two Seattle experiments [9] (Be and Al,Cu), respectively,

$$\begin{aligned} \eta_{\text{Al–Au}} &= 5.7 \pm 0.9 \times 10^{-8}, \quad \eta_{\text{Al–Pt}} = 5.5 \pm 0.9 \times 10^{-8}, \\ \eta_{\text{Be–Al}} &= 5.2 \pm 0.8 \times 10^{-9}, \quad \eta_{\text{Be–Cu}} = 1.7 \pm 0.3 \times 10^{-8}. \end{aligned} \tag{48}$$

The errors for these results are dominated by the uncertainty in the radius of the nuclei i. e. by uncertainty of the parameter r_0. All other uncertainties are several orders of magnitude below this contribution. Thus, the value of the parameter η is rather sensitive to the nuclear model employed. Nevertheless, the above values give some reasonable impression of the size of η to be measured in Eötvös experiments.

Noting that the Moscow experiment has constrained the parameter $\eta_{\mathrm{Al-Au}}$ to 10^{-12} one finds that the violations to the WEP occurring in the model given by the action of Eq. (11) are four orders of magnitude above the experimental limits for the Moscow experiment. With this huge discrepancy we can safely conclude that even the most intricate model of the nucleus will non alter our basic conclusion about the non-viability of the theory described by the action, Eq. (11). This result also holds true for the original action of [2], including the contribution Eq. (2) instead of Eq. (10), because the sign change of the kinetic term of the scalar field only amounts to a sign change in the parameters $a - 1$, leaving unaffected the numerator of η and thus only slightly modifies the above results for η.

The model is also in conflict by several orders of magnitude, with the other two experiments which had obtained $\eta_{\mathrm{Al-Pt}} \lesssim 10^{-11}$ and $\eta_{\mathrm{Be-Al}} \lesssim 10^{-12}$, $\eta_{\mathrm{Be-Cu}} \lesssim 10^{-12}$. Thus, we can rule out the strong gravielectric coupling mediated by two scalar fields as suggested by Mbelek and Lachièze-Rey in [2] because it generates violations to the WEP far beyond the experimental bounds.

5 Conclusions

We have studied some aspects of the model proposed in [2]. The model is claimed to consists of five dimensional KK action which is reduced to four dimensions applying the usual cylinder condition. To this effective action which already includes electromagnetism and a KK scalar, the action of the other known matter fields and a further scalar are added. The second scalar couples to the KK scalar in a non-minimal way (in the five-dimensional theory). This coupling seems rather unnatural from the point of view of unification because it singles out the KK scalar compared to the other components of the five-dimensional metric. More importantly the coupling between both scalars and the electromagnetic field induces a non-metricity in the action (which cannot be removed by any conformal redefinition of the metric). The model proposed in [2] is thus not comprised in the set of biscalar-tensor theories investigated by [24] which is restricted to metric theories.

Unfortunately, we found that the action of the model as given in [2] cannot be deduced from five-dimensional gravity amended with additional fields by a KK compactification because it features a kinetic term for the KK scalar which is incompatible with the higher-dimensional Einstein equations of KK theory. It does not seem possible to change the model in a simple way to make it derivable from a five-dimensional theory by KK reduction without loosing the essential phenomenology of a gravielectric coupling which maintains astrophysical constraints.

In addition to this we realized that the structure of the action as given in [2] is inconsistent in the sense that it would yield a negative energy for the electromagnetic field. In order to study the phenomenology of a model which is self-consistent we thus have investigated a modified action with different signs for both the four-dimensional the electromagnetic and "KK"-scalar terms, Eq. (10), which eliminates all ghosts in the theory.

The non-metricity of the model directly leads to one test of its phenomenology: The violations to WEP induced by the non-metricity have to stay below current observational bounds. Since the model of [2] is required to exhibit a strong coupling between the gravitational and electromagnetic fields mediated by a scalar the model violates the experimental bounds and is thus ruled out. We have demonstrated this fact by estimating the static field of the scalar around a nucleus and then deriving from this result the effective gravitational mass of the nucleus. We found that the violations of the WEP generated in the model are ruled out from Eötvös experiments by several orders of magnitude. The strongest bound comes from the classic Moscow experiment by Braginsky and Panov [7] for which the violations of the WEP predicted by the model of [2] are found to be too large by four orders of magnitude. As a consequence of its violation of the WEP the model will also feature violations of the Einstein Equivalence principle: As the equations of the model are derivable from an action this directly follows from the proof of [25].

In conclusion it is not possible to explain the discrepancy of the various measurement of Newton's constant by assuming a sufficiently strong gravielectric coupling. This conclusion seems to be generic because the method to test the gravielectric coupling against constraints from torsion-balance experiments does not rely on the specific realisation of the coupling. It is always possible to determine the effective gravitational mass of test bodies from the linearised static configuration of the field, which mediates the gravielectric coupling, around the body. Since the baryonic matter of the test bodies in torsion-balance experiments is in the same state as the typical constituents of the Earth the test body will be exposed to a gravielectric coupling with the same coupling constant as experienced by the Earth as a total. A coupling of the gravitational field to the Earth's magnetic field of a strength of 10^{-3} will by the covariance of the electromagnetic field equations be accompanied by a strong coupling of the gravitational field to the electric field of atomic nuclei and is thus bound to generate experimentally non-viable violations of the WEP even if the model manages to escape astrophysical constraints.

Of course, the results found above do neither rule out scalar fields in general nor exclude that they may play an important role in the cosmological evolution. However, in the same sense as the binary-pulsar data yielded strong constraints on the Brans-Dicke parameter describing the coupling between gravitation and a scalar field (cf. e. g. [6]) our considerations put tight constraints on the possible coupling strength between a scalar field and the electromagnetic field. In particular they rule out that a scalar-mediated coupling of the electromagnetic and gravitational field can explain the discrepant measurements of Newton's constant.

Acknowledgement The author is grateful to J.-P. Mbelek for helpful discussions and explanations.

Appendix: No scalar ghost in KK theory

In their paper [10] Mbelek and Lachièze-Rey claim that the effective four-dimensional action of KK theory contains a scalar ghost in its spectrum. This claim is justified by an explicit decomposition of the five-dimensional Ricci scalar and thus the five dimensional Einstein-Hilbert action. In order to understand how their misconception arrises we take a quick tour through the appendix of [10] with special emphasis on the conventions employed.

The appendix of [10] uses a metric of signature $(+,-,-,-)$ and defines the (five-dimensional) Ricci scalar as

$$R = g^{AB}\left(\partial_N \Gamma^N_{AB} - \partial_B \Gamma^N_{AN} + \Gamma^N_{AB}\Gamma^M_{NM} - \Gamma^M_{AN}\Gamma^N_{BM}\right) \tag{49}$$

(Latin indices run from 0 to 4, Greek indices run from 0 to 3). From the definition of the Ricci scalar we see that their Einstein equations take the form

$$G_{MN} = +8\pi G_5 T_{MN} \tag{50}$$

and the gravitational and matter parts of their action thus must have opposite signs,

$$S = -\frac{1}{16\pi G_5}\int d^5x\sqrt{-g}R + \int d^5x\sqrt{-g}\mathcal{L}_{\text{matter}}\,. \tag{51}$$

Choosing a specific gauge and KK decomposing the Ricci scalar Mbelek and Lachièze-Rey find as the contribution of the KK scalar to the action[13]

$$^{(5)}R = {}^{(4)}R \ldots + \frac{1}{2}\frac{\partial_\mu \Phi \partial^\mu \Phi}{\Phi}\,. \tag{52}$$

Since the kinetic term for Φ has the inverse sign compared to that of a normal scalar field for the above metric signature it is concluded in [10] that the KK scalar is a ghost signalling a classical instability of the KK action.

However, due to the sign difference between the gravitational part and the matter part of the action, cf. Eq. (51), the KK scalar will appear in the action with the same sign as an ordinary scalar field. Thus, the KK scalar does not have a ghost nature and does not signify any instability of five-dimensional gravity. In conclusion, the scalar ghost occurred to Mbelek and Lachièze-Rey only because they did not keep track of their conventions and it is not present in the physical theory. It is needless to say that none of the articles [26, 27, 12], referred to in [21] as supporting the presence of an instability, mention any stability problem of the KK action.[14]

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[13] Actually, in the decomposition we encounter another error in the calculations of [10]: despite of choosing a gauge in which the components $g_{4\mu}$, are set to zero, they still have derivatives of these metric components present in their final decomposed Ricci scalar. In contrast to the claim in the footnote 5 of [10] there is of course no way of having a function vanishing everywhere with a non-vanishing derivative.

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