

Physics-to-gauge conversion at black hole horizons

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Abstract

Requiring the presence of a horizon imposes constraints on the physical phase space. After a careful analysis of dilaton gravity in 2D with boundaries (including the Schwarzschild and Witten black holes as prominent examples), it is shown that the classical physical phase space is smaller as compared to the generic case if horizon constraints are imposed. Conversely, the number of gauge symmetries is larger for the horizon scenario. In agreement with a recent conjecture by 't Hooft, we thus find that physical degrees of freedom are converted into gauge degrees of freedom at a horizon.

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1. Introduction

The standard derivation of the Euler–Lagrange equations of motion (EOM) requires boundary conditions on the variation of fields. By introducing appropriate boundary terms in the action one can employ different variational principles. The careful treatment of such boundary terms has become a focus of interest in several branches of modern field theory and string theory. In particular, in the context of black holes (BHs) edge states have been investigated from various angles (cf e.g. [1]); the basic idea of these papers goes back to the seminal one by Witten [2].

The most familiar example where such issues can be addressed is the Hilbert action of Einstein gravity (EH) in D dimensions in the presence of a (in general non-smooth) boundary, which for Lorentzian signature reads (setting the gravitational coupling $\kappa = 1$)

$$S_D^{\text{EH}} = \frac{1}{2} \int_{\mathcal{M}_D} d^D x \sqrt{|g|} R + \int_{\partial\mathcal{M}_{(D-1)}} d^{(D-1)}x \sqrt{|h|} K + \int_{\partial\partial\mathcal{M}_{(D-2)}} d^{(D-2)}x \sqrt{|\sigma|} \alpha. \quad (1.1)$$

Here R is the curvature scalar, K is the trace of the extrinsic curvature, h is the determinant of the induced metric at the boundary (the piecewise smooth part being denoted by $\partial\mathcal{M}$), σ is the determinant of the induced metric at each corner and α is the local rapidity (which is the Lorentzian equivalent of a deficit angle) associated with a given corner. The action (1.1) has been used for instance in [3–6]; its boundary terms correspond to the York–Gibbons–Hawking (YGH) boundary term [7], the presence of which can be derived, e.g. from the consistency of the variational principle, i.e. by requiring functional differentiability, because the EH action contains second derivatives of the metric.

A fertile field for investigation of gravity always has been its spherical reduction to 2D. In this context, the frequently used second-order action [8] of a scalar–tensor theory

$$S^{(2)} = \frac{1}{2} \int_{\mathcal{M}} d^2x \sqrt{-g} [XR + U(X)(\nabla X)^2 - 2V(X)], \quad (1.2)$$

encompasses most 2D dilaton gravity models of interest (cf e.g. [9]) including not only spherically reduced Einstein gravity (i.e., the Schwarzschild BH) [10–12], but also the Jackiw–Teitelboim model [13], string inspired BH models (including the Witten BH) [14] and others. The functions U, V define the model. The covariant derivative ∇ is associated with the Levi-Civita connection related to the metric $g_{\mu\nu}$, the determinant of which is denoted by g . In the absence of matter there are no propagating physical degrees of freedom, but the theory nevertheless is not empty: as opposed to (1.1) taken at $D = 2$ (which is just the Euler characteristic in the presence of boundaries and deficit angles, cf e.g. [15]) there are non-trivial EOM and interesting global physical properties, some of which will be recalled in the present work.

Spherical reduction of the action (1.1) leads to a 2D dilaton gravity action (1.2) with the potentials (λ is an irrelevant scale factor and may be set to 1)

$$V(X) = -\frac{\lambda^2}{2}(D-2)(D-3)X^{(D-4)/(D-2)}, \quad U(X) = -\frac{(D-3)}{(D-2)X}, \quad (1.3)$$

supplemented by the boundary term (ds is the arc-length on the boundary)

$$S_b^{(2)} = \int_{\partial\mathcal{M}} ds XK, \quad (1.4)$$

where we assume that corner contributions are absent. The presence of (1.4) makes the total action functionally differentiable with respect to the induced metric on the boundary (i.e., no normal derivative of the variations of the induced metric appears). This property does not depend on the potentials U and V , and, therefore, we also adopt (1.4) for all other dilaton theories (cf e.g. [12, 16, 17]). Note that (1.4) is not just a spherical reduction of the boundary term in (1.1), as one has to take into account a contribution from partial integration of the volume term in (1.1) as well.

In some applications one would like to consider null surfaces as boundary. This is problematic because the extrinsic curvature becomes undefined and thus it is not straightforward to implement, e.g., horizon constraints. In the context of the second-order formulation of 4D EH gravity null boundaries were discussed in [18], but neither the analysis of the constraint algebra nor the construction of the reduced phase space was performed. As will be shown below this problem may be circumvented in a formalism different from (1.1) and (1.2). Indeed, there exists a classically equivalent [19] reformulation of (1.2) as a first-order action [20], which renders constraint analysis and quantization surprisingly easy [9]. It is one of the prime goals of the present work to translate (1.4) into the first-order formulation and to study its impact on the constraint algebra⁵. This is not only of interest by itself, but a crucial

⁵ An existing formulation [21] does not address the main issues of our present work.

prerequisite to address questions regarding quantization in the presence of a horizon, and one would also expect relevant implications for the BH entropy.

To make this connection more concrete, let us briefly review some recent key results by Carlip [22]: he performed a Hamiltonian analysis of 2D dilaton gravity, restricting the initial/boundary data such that a ‘stretched horizon’ is present⁶. Two conditions are imposed: one sets the Killing norm ‘almost’ to zero, the other one makes the expansion of the hypersurface nearly zero. Although they are not constraints in the ordinary sense of constrained Hamiltonian dynamics, one may calculate the Poisson brackets between them and it is found that they convert the first class constraints generating gauge transformations into second class constraints. Introducing the Dirac bracket then establishes the Virasoro algebra with a classical central charge (which vanishes in the limit of a true horizon). Then, after fixing a relevant ambiguity, the Cardy formula [23] is exploited to recover the Bekenstein–Hawking relation [24]. Besides technical issues, this interesting analysis implies an important conceptual question: since the result seems to be valid for small, but otherwise arbitrary, ‘stretching’ of the horizon there appears to be no essential difference between a horizon as a boundary and some generic spacelike or timelike boundary. However, if entropy originates from some microstates attached to the BH horizon, one would expect the latter to play a special role in the analysis. To even address this question, it is therefore necessary to be able to implement ‘sharp’ horizon conditions. It is one of the main tasks of this work to do precisely that. Horizon boundary conditions will turn out to be quite special regarding the constraint algebra, the gauge symmetries and the physical phase space. Our result turns out to be concurrent with a recent idea by ‘t Hooft [25].

This paper is organized as follows: in section 2 the first-order formulation of 2D dilaton gravity is recapitulated briefly. Boundary terms are introduced in section 3. Then the constraint algebra (section 4) as well as the gauge symmetries (section 5) is analysed in detail. The classical physical phase space is studied in section 6, while the final section 7 concludes with an extensive discussion of our results and an outlook to generalizations and applications.

2. First-order formulation

Already classically, but especially at the quantum level, it is very convenient in 2D to employ the first-order formalism in terms of the ‘Cartan variables’ zweibein e^a and spin connection ω .⁷ The first-order gravity action [20]

$$S^{(1)} = - \int_{\mathcal{M}} [X_a T^a + X \mathcal{R} + \epsilon \mathcal{V}(X^a X_a, X)] \quad (2.1)$$

encompasses essentially all known dilaton theories. The fields X and X_a are Lagrange multipliers for curvature \mathcal{R} and torsion T^a , respectively. The former coincides with the dilaton field in the second-order formulation. Actually, for most practical purposes the special case

$$\mathcal{V}(X^a X_a, X) = X^+ X^- U(X) + V(X) \quad (2.2)$$

⁶ Actually, it is not a stretched horizon in the technical sense of the word, but some spacelike hypersurface which is ‘almost null’, i.e. a partial Cauchy hypersurface extending from the bifurcation point to lightlike infinity.

⁷ In our notation $e^a = e^a_\mu dx^\mu$ is the dyad 1-form. Latin indices refer to an anholonomic frame, Greek indices to a holonomic one. The 1-form ω represents the spin connection $\omega^a_b = \epsilon^a_b \omega$ with the totally antisymmetric Levi-Civita symbol ϵ_{ab} ($\epsilon_{01} = +1$). With the flat metric η_{ab} in light-cone coordinates ($\eta_{+-} = 1 = \eta_{-+}$, $\eta_{++} = 0 = \eta_{--}$) it reads $\epsilon^\pm_\pm = \pm 1$. The Levi-Civita symbol with anholonomic indices is fixed as $\tilde{\epsilon}_{01} = -\tilde{\epsilon}^{01} = 1$. The metric is determined by $g_{\mu\nu} = e^+_\mu e^-_\nu + e^-_\mu e^+_\nu$. The determinant of the zweibein is denoted by $e = \det e^a_\mu = e^-_0 e^+_1 - e^+_0 e^-_1$. The torsion 2-form is given by $T^\pm = (d \pm \omega) \wedge e^\pm$. The curvature 2-form R^a_b can be represented by the 2-form \mathcal{R} defined by $R^a_b = \epsilon^a_b \mathcal{R}$, $\mathcal{R} = d\omega$. The volume 2-form is denoted by $\epsilon = e^+ \wedge e^-$. The overall minus sign in (2.1) is the only difference to the notation used in [9].

is sufficient because (2.1) with (2.2) is equivalent to the second-order action (1.2) with the same potentials U, V . For $U \neq 0$, the action (2.1) implies non-vanishing torsion. Therefore, also \mathcal{R} should not be confused with the Hodge dual of R in (1.2) where the connection is torsionless (and metric compatible). It is useful to introduce the notation

$$Q(X) = \int^X U(y) dy \quad (2.3)$$

and

$$w(X) = \int^X V(y) \exp Q(y) dy. \quad (2.4)$$

The latter combination of the potentials U, V remains invariant under local Weyl rescalings $g_{\mu\nu} \rightarrow \Omega^2(X)g_{\mu\nu}$. Both definitions contain an ambiguity from the integration constant which may be fixed conveniently.

It has been pointed out in [20] that (2.1) is a particular Poisson- σ model (PSM),

$$S^{\text{PSM}} = - \int_{\mathcal{M}} \left[X^I dA_I - \frac{1}{2} P^{IJ} A_J \wedge A_I \right] \quad (2.5)$$

with a three-dimensional target space, the coordinates of which are $X^I = \{X, X^+, X^-\}$. The gauge fields comprise the Cartan variables, $A_I = \{\omega, e^-, e^+\}$. Because the dimension of the Poisson manifold is odd, the Poisson tensor

$$P^{X\pm} = \pm X^\pm, \quad P^{+-} = \mathcal{V}, \quad P^{IJ} = -P^{JI}, \quad (2.6)$$

cannot have full rank. Therefore, always a Casimir function,

$$\mathcal{C} = X^+ X^- \exp Q(X) + w(X), \quad (2.7)$$

exists whose absolute conservation,

$$d\mathcal{C} = 0, \quad (2.8)$$

has been investigated extensively [26, 27]. For spherically reduced gravity, \mathcal{C} is related directly to the ADM mass.

In the context of 2D dilaton gravity, equation (2.8) is sometimes called the ‘generalized Birkhoff theorem’ because from the exact solution for the line element (cf e.g. [9, 28]) following from (2.1) with (2.2), it is evident that there is always a Killing vector $k = k^\mu \partial_\mu$ with the norm

$$k^\mu k_\mu = 2X^+ X^- \exp Q(X). \quad (2.9)$$

This implies that the condition for a Killing horizon may not only be imposed on the worldsheet⁸,

$$\text{Killing horizon (world sheet)} : e_0^- e_0^+ = 0, \quad (2.10)$$

but also alternatively in the target space,

$$\text{Killing horizon (target space)} : X^+ X^- = 0. \quad (2.11)$$

Note that on-shell (2.7) allows us to express the Killing norm as a function of X and the value of the Casimir function. In axial gauge ($e_1^- = 1, e_1^+ = 0$), the metric simplifies to

$$g_{\mu\nu}^{(\text{EF})} = e_0^+ \begin{pmatrix} 2e_0^- & 1 \\ 1 & 0 \end{pmatrix}_{\mu\nu} \quad (2.12)$$

⁸ In (2.10), it has been assumed that the Killing horizon is a surface of constant coordinate x^1 and therefore the Killing norm is proportional to $e_0^- e_0^+$. This can always be achieved by a suitable choice of coordinates.

and the (worldsheet) condition for a Killing horizon reads $e_0^- = 0$. The first EOM in equation (A.1) then implies $X^- = 0$ since the dilaton is constant on the horizon. If additionally the dilaton is assumed to depend on x^1 only (which is always possible in the absence of matter and within a certain patch) then the gauge implied by (2.12) is of Eddington–Finkelstein type. We do not go further into details explaining 2D dilaton gravity without boundaries, but refer to the extensive review [9], upon which this work is based.

Subsequently we will often drop the attribute ‘Killing’ as it is understood that all horizons studied in the present work are Killing ones on-shell. However, it is emphasized that our results below should generalize to trapping horizons [29], which are defined by condition (2.11) but not (2.10). This point will be addressed in more detail in the conclusions where systems with matter are outlined.

3. Boundary terms in 2D dilaton gravity

As pointed out in the previous section, the first-order formulation of 2D dilaton gravity has many advantages over the second-order formulation. In this section we rewrite the boundary term (1.4) in a first-order form and discuss the boundary conditions which follow from varying the total action. First we have to find an alternative form of the extrinsic curvature which involves the connection ω .

To clarify the relation between ω , the Levi-Civita connection $\hat{\omega}$ and the extrinsic curvature, consider the Euclidean case for simplicity. Recalling the main definitions (see e.g. [30]), let \mathcal{M} be a manifold of dimension D , let \bar{e}_\perp be an inward pointing unit vector on the boundary, and let \bar{e}_i ($i = 1, \dots, D - 1$) be a local orthonormal frame for the tangent bundle of $\partial\mathcal{M}$. If one identifies \bar{e} with our dynamical vielbein e , this would mean partially fixing the gauge. Then the extrinsic curvature is given by $K_{ij} = \bar{\omega}^\perp_{ij}$ and its trace is denoted by K . Let us denote by a semicolon (;) the multiple covariant derivatives with respect to the Levi-Civita connection $\bar{\omega}$ (associated with \bar{e}), and by a colon (:) the multiple covariant derivatives with respect to the Levi-Civita connection on the boundary. The extrinsic curvature tensor measures the difference between these two connections:

$$v_{i;j} = v_{i;j} - K_{ij}v_\perp, \tag{3.1}$$

where v is a co-vector field on \mathcal{M} . We denote i, j, \dots by \parallel . For a two-dimensional manifold $\partial\mathcal{M}$ is one dimensional. Since $so(1)$ is trivial,

$$v_{\parallel; \parallel} = \partial_{\parallel} v_{\parallel}. \tag{3.2}$$

On the other hand, the expression

$$v_{\parallel; \parallel} = \partial_{\parallel} v_{\parallel} - \epsilon^\perp_{\parallel} \bar{\omega}_{\parallel} v_\perp \tag{3.3}$$

by virtue of (3.1) yields

$$K_{\parallel\parallel} = \epsilon^\perp_{\parallel} \bar{\omega}_{\parallel}. \tag{3.4}$$

It is recalled that $\epsilon_{\parallel\perp} = \tilde{\epsilon}_{01} = 1$ if the boundary is assumed to be at $x^1 = \text{const}$.

The connections $\bar{\omega}$ and $\hat{\omega}$ are related by a Lorentz transformation. Performing it after returning to Minkowskian signature obtains

$$K = \hat{\omega}_{\parallel} + \frac{1}{2} \partial_{\parallel} \ln \left| \frac{e_{\parallel}^+}{e_{\parallel}^-} \right|, \tag{3.5}$$

where e_{\parallel}^\pm means again the component of the zweibein parallel to the boundary. In terms of the full spin connection ω , the first-order version of the boundary term (1.4) reads

$$\int_{\partial\mathcal{M}} \left(X\omega + \frac{1}{2} X \, \text{d} \ln \left| \frac{e_{\parallel}^+}{e_{\parallel}^-} \right| \right). \tag{3.6}$$

It has also been obtained in [21], although from quite a different argument. A local Lorentz transformation (with infinitesimal or finite Lorentz angle γ),

$$\omega' = \omega - d\gamma, \quad (e^\pm)' = e^\pm \exp[\pm\gamma], \quad (3.7)$$

indeed leaves (3.6) invariant. For the sake of definiteness from now on, it will be supposed that $\partial\mathcal{M}$ is the *lower* boundary in the x^1 direction and that there is no boundary in x^0 direction. Then the full action reads

$$S = - \int_{\mathcal{M}} d^2x [X_a (D_\mu e_\nu^a) \tilde{\epsilon}^{\mu\nu} + X \partial_\mu \omega_\nu \tilde{\epsilon}^{\mu\nu} + e(U X^+ X^- + V)] \\ - \int_{\partial\mathcal{M}} dx^0 \left[X \omega_0 + \frac{1}{2} X \partial_0 \ln \left(\frac{e_0^+}{e_0^-} \right) \right] \equiv \int dx^0 \mathcal{L}. \quad (3.8)$$

We have dropped the absolute value in the last boundary term because we will exclusively discuss patches where the sign of the ratio e_0^+/e_0^- is (semi-)positive.

To prove that this action is indeed equivalent to the second-order action (1.2) with (1.4), one has to use twice the algebraic EOM for X^a [9, 31]. It is crucial that the variation of (3.8) with respect to X^a does not produce any boundary terms. Therefore, the proof of [9, 31] does not require any essential modifications due to the presence of boundaries.

The variation of (3.8) produces the EOM in the bulk (A.1) and the boundary terms

$$\int_{\partial\mathcal{M}} dx^0 \left[(\delta e_0^-) \left(X^+ - \frac{\partial_0 X}{2e_0^-} \right) + (\delta e_0^+) \left(X^- + \frac{\partial_0 X}{2e_0^+} \right) - (\delta X) \left(\omega_0 + \frac{1}{2} \partial_0 \ln \left(\frac{e_0^+}{e_0^-} \right) \right) \right]. \quad (3.9)$$

We shall exclusively consider boundaries on which the value of the dilaton is fixed to a constant,

$$X|_{\partial\mathcal{M}} = \text{const.} \quad \Rightarrow \quad \partial_0 X|_{\partial\mathcal{M}} = \delta X|_{\partial\mathcal{M}} = 0. \quad (3.10)$$

Then to cancel (3.9) it is sufficient to impose

$$X^+ \delta e_0^-|_{\partial\mathcal{M}} = 0, \quad X^- \delta e_0^+|_{\partial\mathcal{M}} = 0. \quad (3.11)$$

All boundary conditions which we consider in this paper do satisfy (3.10) and (3.11). Three cases are of particular interest. Fixing $e_0^\pm|_{\partial\mathcal{M}}$ corresponds to a *generic boundary*. The requirement $X^\pm|_{\partial\mathcal{M}} = 0$ will be called the *bifurcation point boundary condition* because on-shell it holds at the bifurcation point on the Killing horizon. And, finally, $X^-|_{\partial\mathcal{M}} = 0, e_0^-|_{\partial\mathcal{M}} = 0$ defines a *horizon boundary condition* (cf (2.11)). We stress that the condition $e_0^-|_{\partial\mathcal{M}} = 0$ is possible only if we also fix the dilaton to a constant on $\partial\mathcal{M}$, because otherwise the action (3.8) would become singular.

So far we have disregarded the possibility of corners. In their presence (3.8) has to be replaced by a similar action, but with the last boundary term partially integrated (cf section 5 of [21], (5.1)–(5.6)),

$$S^{\text{tot}} = S^{(1)} + \int_{\partial\mathcal{M}_s} ds X K + \sum X \alpha, \quad (3.12)$$

where K is the extrinsic curvature with respect to the full spin connection. Here $\partial\mathcal{M}_s$ extends over the smooth parts of the full boundary and the sum is over the corner points; α is again the local rapidity at each corner. The boundary terms in (3.12) contain a multiplicative factor X in comparison with those in (1.1). If $\delta X = 0$ at the corners, then no additional terms are produced by variation of (3.12) as compared to (3.9). Subsequently smoothness of $\partial\mathcal{M}$ will be assumed and hence corner terms will play no role.

Within the first-order formulation there are other choices of boundary terms which may look natural. For example, instead of the second line in (3.8) one can use

$$-\int_{\partial\mathcal{M}} dx^0 [X\omega_0 + X^+e_0^- + X^-e_0^+]. \quad (3.13)$$

This prescription has been studied by Gegenberg, Kunstatter and Strobl [32] (cf also [33]). However, in this approach the action is not invariant under the (unrestricted) Lorentz transformations, and, therefore, the corresponding first-order model cannot be equivalent to a second-order action which is formulated in terms of Lorentz invariant fields (the metric and the dilaton). On a more technical side, the variation of the action (3.13) with respect to X^\pm produces some boundary terms which violate the standard proof of the equivalence between the two formulations of dilaton gravity [31]. As we would like to make contact with the second-order formulation and the standard YGH result we shall concentrate on the action (3.8).

4. Constraint algebra in the presence of boundaries

In the Hamiltonian approach, the boundary conditions become constraints. Therefore, one has to analyse the constraint algebra which includes these new constraints, separate second class ones, and define the Dirac bracket. Some general aspects of this procedure were developed in [34] where one can also find further references. More recently, this approach was applied to Dirichlet branes [35].

Typically, in the presence of boundaries all gauge symmetries are broken, i.e. the action and the boundary conditions for the fields are gauge invariant only if the parameters of gauge transformations are restricted at $\partial\mathcal{M}$. Sometimes even such partial gauge invariance encounters serious obstacles (cf [36], where supersymmetric boundary conditions for supergravity were analysed). In this respect the situation with the gauge symmetries we observe below is quite specific. If the boundary corresponds to the BH horizon, one does not need to impose conditions on some of the gauge parameters to achieve full gauge invariance of the action and of the boundary conditions. This may be interpreted as a manifestation of gauge degrees of freedom localized on the horizon whose existence was proposed recently by 't Hooft [25].

The full set of the boundary conditions for the fields and for the gauge parameters requires several consistency conditions. Boundary terms in the Euler–Lagrange and in the symmetry variation of the action should vanish, and the set of boundary conditions on the fields should be closed under gauge transformation. All these requirements are satisfied in our scheme, so that we actually construct an *orbit of boundary conditions* in the sense of [37]⁹.

4.1. No boundaries: a brief review

Because appropriate generalizations are needed in the presence of boundaries, and also to fix our notation, we briefly review the constraint analysis in the absence of boundaries in the first-order formulation (2.1). By $\{, \}$, we always mean the Poisson bracket. Expressions of the

⁹ This notion was originally introduced to study supersymmetry in two dimensions, and its practical use included quantum corrections to the mass of supersymmetric solitons [38]. Gauge invariant boundary conditions for the four-dimensional perturbative quantum gravity were constructed in [39]. In Euclidean 2D $R^2 + T^2$ gravity, the corresponding analysis of boundary conditions and one-loop divergences was performed in [40]. In that paper it was demonstrated that the volume divergences are cancelled (in accordance with the absence of bulk degrees of freedom), but the boundary divergences are not. This result indicates that some boundary degrees of freedom are present in the model.

type $\{q, p'\}$ imply that q is taken at point x^1 and p at point $x^{1'}$. The shorthand notation δ for $\delta(x^1 - x^{1'})$ is used. No distinction need be made between upper and lower canonical indices, i.e. we will employ exclusively

$$\{q_i, p'_j\} = \delta_{ij}\delta, \quad \{q_i, q'_j\} = 0 = \{p_i, p'_j\}. \quad (4.1)$$

It is supposed that x^0 is our ‘time’ variable in the Hamiltonian formulation, although it is emphasized that x^0 might as well be a radial or lightlike coordinate. Whatever the physical interpretation of the coordinate x^0 , it is the quantity with respect to which the Hamiltonian generates translations. From the first line in the action (3.8), one can see immediately that the target space coordinates X, X^\pm are canonically conjugate to the 1-components of the gauge fields,

$$\text{coordinates: } (q_1, q_2, q_3) = (\omega_1, e_1^-, e_1^+), \quad (\bar{q}_1, \bar{q}_2, \bar{q}_3) = (\omega_0, e_0^-, e_0^+), \quad (4.2)$$

$$\text{momenta: } (p_1, p_2, p_3) = (X, X^+, X^-), \quad (\bar{p}_1, \bar{p}_2, \bar{p}_3), \quad (4.3)$$

whereas the 0-components of the gauge fields encounter no canonically conjugate partner. Consequently, primary first class constraints

$$\bar{P}_i = \bar{p}_i \approx 0 \quad (4.4)$$

are produced (≈ 0 means ‘weakly vanishing’). They generate secondary first class constraints $G_i \approx 0$, where

$$G_1 = \partial_1 p_1 + p_3 q_3 - p_2 q_2, \quad (4.5)$$

$$G_2 = \partial_1 p_2 + q_1 p_2 - q_3 \mathcal{V}(p_2 p_3, p_1), \quad (4.6)$$

$$G_3 = \partial_1 p_3 - q_1 p_3 + q_2 \mathcal{V}(p_2 p_3, p_1). \quad (4.7)$$

As expected for a reparametrization invariant theory, the Hamiltonian density is a sum over constraints, $\mathcal{H} = -\bar{q}_i G_i$. While all Poisson brackets between any of \bar{P}_i with any other constraint vanish trivially, the brackets between the secondary first class constraints yield the algebra

$$\{G_i, G'_j\} = C_{ij}{}^k G_k \delta. \quad (4.8)$$

It is one of the remarkable features of the first-order formulation (2.1) that the constraint algebra closes with delta functions and not with derivatives thereof. Thus it resembles a Lie algebra, albeit it is nonlinear, i.e. there are structure functions $C_{ij}{}^k(p_l) = -C_{ji}{}^k(p_l)$ whose non-vanishing components read

$$C_{12}{}^2 = -1, \quad C_{13}{}^3 = +1, \quad C_{23}{}^i = -\frac{\partial \mathcal{V}}{\partial p_i}, \quad (4.9)$$

and whose centre is generated by the Casimir function \mathcal{C} as defined in (2.7) and $\partial_1 \mathcal{C}$ [27]. Since the structure functions only depend on p_i and because the Poisson brackets of p_i with G_j also yield some functions of p_i alone, one obtains a finite W-algebra if p_i are included in the set of generators of the algebra. For details on such algebras, cf e.g. [41]. Physically, the constraint G_1 may be identified as the generator of local Lorentz transformations, while (certain combinations of) G_2 and G_3 constitute the two diffeomorphism constraints. In the presence of minimally coupled matter¹⁰ the constraints are modified, but their algebra is not. For non-minimally coupled matter, the structure function $C_{23}{}^1$ acquires an additional, matter-dependent, contribution [42].

¹⁰ ‘Minimally coupled’ means no coupling to the dilaton field—for instance, a minimally coupled massless scalar field ϕ has the Lagrange density $\sqrt{-g}(\nabla\phi)^2$.

Because classically (2.1) is equivalent to (1.2), it is possible to recover the Virasoro algebra by appropriate recombinations of the constraints G_i . Indeed, by taking the linear combinations $\mathcal{E} = q_1 G_1 - q_2 G_2 + q_3 G_3$, $\mathcal{P} = q_i G_i$ the new constraints \mathcal{E}, \mathcal{P} fulfil [43]

$$\{\mathcal{E}, \mathcal{E}'\} = (\mathcal{P} + \mathcal{P}') \partial_1 \delta, \tag{4.10}$$

$$\{\mathcal{P}, \mathcal{P}'\} = (\mathcal{P} + \mathcal{P}') \partial_1 \delta, \tag{4.11}$$

$$\{\mathcal{E}, \mathcal{P}'\} = (\mathcal{E} + \mathcal{E}') \partial_1 \delta. \tag{4.12}$$

In the presence of a central charge c , the last Poisson bracket acquires an additional term $ic/(12\pi)\partial_1^3\delta$ on the right-hand side. We conclude this brief review by remarking that another linear combination of G_i is possible which makes the constraint algebra Abelian (not only locally, which is always possible [44] of course, but in a certain patch). This is discussed in detail in appendix B.

4.2. Modifications due to boundaries

It is recalled that exclusively the prescription (3.10), (3.11) is employed. We start the analysis by rewriting (3.8) in terms of canonical variables (4.2), (4.3),¹¹

$$S = \int_{\mathcal{M}} d^2x [p_i \partial_0 q_i + \bar{q}_i G_i^{\text{bulk}}] + \int_{\partial\mathcal{M}} dx^0 \left[p_2 \bar{q}_2 + p_3 \bar{q}_3 - \frac{1}{2} p_1 \partial_0 \ln \left(\frac{\bar{q}_3}{\bar{q}_2} \right) \right], \tag{4.13}$$

with G_i^{bulk} equal to G_i as defined in (4.5)–(4.7). It is useful to smear the constraints with test functions $\eta(x^1), \xi(x^1), \dots$. For the primary ones this yields (cf (4.4))

$$\bar{P}_1[\eta] = \int dx^1 \bar{p}_1 \eta, \tag{4.14}$$

$$\bar{P}_2[\eta] = \int dx^1 \bar{p}_2 \eta - \frac{1}{2} \frac{p_1}{\bar{q}_2} \eta \Big|_{\partial\mathcal{M}}, \tag{4.15}$$

$$\bar{P}_3[\eta] = \int dx^1 \bar{p}_3 \eta + \frac{1}{2} \frac{p_1}{\bar{q}_3} \eta \Big|_{\partial\mathcal{M}}. \tag{4.16}$$

The total Hamiltonian reads

$$H^{\text{tot}} = \int dx^1 (p_i \partial_0 q_i + \bar{p}_i \partial_0 \bar{q}_i) - \mathcal{L} + \sum_i \bar{P}_i[\lambda_i], \tag{4.17}$$

and the canonical one is defined as

$$H = - \int dx^1 \bar{q}_i G_i^{\text{bulk}} - (p_2 \bar{q}_2 + p_3 \bar{q}_3) \Big|_{\partial\mathcal{M}}. \tag{4.18}$$

As usual $H^{\text{tot}} = H + \sum_i \bar{P}_i[\tilde{\lambda}_i]$ with $\tilde{\lambda}_i = \lambda_i + \partial_0 \bar{q}_i$. The Poisson brackets between H and the primary constraints generate the secondary ones,

$$\{\bar{P}_i[\eta], H\} = G_i[\eta], \tag{4.19}$$

where

$$G_1[\eta] = \int dx^1 G_1^{\text{bulk}} \eta, \tag{4.20}$$

¹¹ As before $\partial\mathcal{M}$ refers to the lower boundary in the x^1 direction. Note that this implies some unusual minus signs in partial integration, e.g. $\int y \partial x = - \int x \partial y - xy \Big|_{\partial\mathcal{M}}$.

$$G_2[\eta] = \int dx^1 G_2^{\text{bulk}} \eta + \frac{\eta}{2\bar{q}_2} F \Big|_{\partial\mathcal{M}}, \quad (4.21)$$

$$G_3[\eta] = \int dx^1 G_3^{\text{bulk}} \eta + \frac{\eta}{2\bar{q}_3} F \Big|_{\partial\mathcal{M}}, \quad (4.22)$$

and

$$F \equiv \bar{q}_2 p_2 + \bar{q}_3 p_3. \quad (4.23)$$

The nice relation

$$H = - \sum_i G_i[\bar{q}_i] \quad (4.24)$$

shows that H is weakly zero, because in our formulation there are boundary terms in the Hamiltonian *and* in the constraints.

Among the Poisson brackets between the constraints only non-zero ones are given explicitly:

$$\{\bar{P}_2[\eta], G_2[\xi]\} = \frac{\eta\xi}{2\bar{q}_2^2} F \Big|_{\partial\mathcal{M}}, \quad (4.25)$$

$$\{\bar{P}_2[\eta], G_3[\xi]\} = -\frac{\eta\xi}{2\bar{q}_2\bar{q}_3} F \Big|_{\partial\mathcal{M}}, \quad (4.26)$$

$$\{\bar{P}_3[\eta], G_2[\xi]\} = -\frac{\eta\xi}{2\bar{q}_2\bar{q}_3} F \Big|_{\partial\mathcal{M}}, \quad (4.27)$$

$$\{\bar{P}_3[\eta], G_3[\xi]\} = \frac{\eta\xi}{2\bar{q}_3^2} F \Big|_{\partial\mathcal{M}}, \quad (4.28)$$

$$\{G_1[\eta], G_2[\xi]\} = -G_2[\eta\xi] + \frac{\eta\xi}{\bar{q}_2} F \Big|_{\partial\mathcal{M}}, \quad (4.29)$$

$$\{G_1[\eta], G_3[\xi]\} = G_3[\eta\xi] - \frac{\eta\xi}{\bar{q}_3} F \Big|_{\partial\mathcal{M}}, \quad (4.30)$$

$$\{G_2[\eta], G_3[\xi]\} = - \sum_i G_i \left[\eta\xi \frac{\partial\mathcal{V}}{\partial p_i} \right] + \frac{1}{2} \eta\xi F U \left(\frac{p_3}{\bar{q}_2} + \frac{p_2}{\bar{q}_3} \right) \Big|_{\partial\mathcal{M}}. \quad (4.31)$$

So far no specific boundary conditions have been imposed. Subsequently several different cases are studied, all of them being consistent with the variational principle (3.10), (3.11).

4.2.1. Generic boundary. At the boundary, the dilaton is given by a constant, p_1^b , by assumption, while in the generic case $X^\pm \neq 0$ there. Thus we impose the boundary constraints \hat{B}_i (E_0^\pm are given functions)

$$\hat{B}_1[\eta] = (p_1 - p_1^b)\eta \Big|_{\partial\mathcal{M}}, \quad (4.32)$$

$$\hat{B}_2[\eta] = (\bar{q}_2 - E_0^-(x^0))\eta \Big|_{\partial\mathcal{M}}, \quad (4.33)$$

$$\hat{B}_3[\eta] = (\bar{q}_3 - E_0^+(x^0))\eta \Big|_{\partial\mathcal{M}}. \quad (4.34)$$

They are obviously consistent with (3.10), (3.11). Clearly $\{\hat{B}_i, \hat{B}_j\} = 0$. The non-vanishing brackets with the other constraints are

$$\{\hat{B}_2[\eta], \bar{P}_2[\xi]\} = \eta\xi|_{\partial\mathcal{M}}, \quad (4.35)$$

$$\{\hat{B}_3[\eta], \bar{P}_3[\xi]\} = \eta\xi|_{\partial\mathcal{M}}, \quad (4.36)$$

$$\{\hat{B}_1[\eta], G_2[\xi]\} = -\eta\xi p_2|_{\partial\mathcal{M}}, \quad (4.37)$$

$$\{\hat{B}_1[\eta], G_3[\xi]\} = \eta\xi p_3|_{\partial\mathcal{M}}. \quad (4.38)$$

Thus, the only constraint which obviously remains first class is \bar{P}_1 . The other eight constraints $\phi_i, i = 1, \dots, 8$, yield a matrix $M_{ij}\eta\xi|_{\partial\mathcal{M}} = \{\phi_i[\eta], \phi_j[\xi]\}$ which has support only at the boundary. Its determinant

$$\det M_{ij} = \frac{F^2}{(\bar{q}_2\bar{q}_3)^2} (p_2\bar{q}_2 - p_3\bar{q}_3)^2 \quad (4.39)$$

vanishes on-shell, but has maximal rank-8 off-shell. The gauge-fixing procedure and the treatment of the delicate issue of bulk first class constraints which become second class at the boundary will be postponed until section 6. In the previous sentence and the discussions below the phrase ‘second class at the boundary’ is always understood in the sense that M_{ij} has support only at the boundary.

4.2.2. Horizon boundary conditions. From (2.10), (2.11), (3.10) and (3.11), an obvious set of boundary constraints that corresponds to a horizon is

$$B_1[\eta] = (p_1 - p_1^h)\eta|_{\partial\mathcal{M}}, \quad (4.40)$$

$$B_2[\eta] = \bar{q}_2\eta|_{\partial\mathcal{M}}, \quad (4.41)$$

$$B_3[\eta] = p_3\eta|_{\partial\mathcal{M}}. \quad (4.42)$$

For this choice $F|_{\partial\mathcal{M}} \approx 0$. It should be noted that on-shell B_3 is a consequence of B_1 and B_2 , so our boundary conditions are perfectly consistent with the classical EOM (A.1).¹² Again $\{B_i, B_j\} = 0$. Non-zero brackets with the other constraints read

$$\{B_2[\eta], \bar{P}_2[\xi]\} = \eta\xi|_{\partial\mathcal{M}}, \quad (4.43)$$

$$\{B_1[\eta], G_2[\xi]\} = -\eta\xi p_2|_{\partial\mathcal{M}}, \quad (4.44)$$

$$\{B_1[\eta], G_3[\xi]\} = B_3[\eta\xi], \quad (4.45)$$

$$\{B_3[\eta], G_1[\xi]\} = -B_3[\eta\xi], \quad (4.46)$$

$$\{B_3[\eta], G_2[\xi]\} = \mathcal{V}\eta\xi|_{\partial\mathcal{M}}. \quad (4.47)$$

Now an obstruction is encountered: the constraint B_2 appears in the denominator in many places in (4.25)–(4.31). This reflects the well-known difficulty in constructing a canonical formulation of gravity theories when the boundary coincides with a horizon. To be able to proceed further, one has to define the way one treats fractions of the constraints. For non-extremal horizons, we propose to assign the same order to F , B_2 and to the smearing function ξ corresponding to G_2 . Then $F\xi/\bar{q}_2|_{\partial\mathcal{M}} \approx 0$, and all fractions such as F/\bar{q}_2 should be considered as finite but undetermined. These simple rules are sufficient to make a separation between first and second class constraints. We stress that these rules cannot be derived from the canonical formalism, but they can be justified by considering the behaviour of corresponding quantities near the boundary when x^1 plays the role of a small parameter ($x^1 = 0$ is the

¹² This is no longer true if the horizon is ‘stretched’ on the worldsheet, i.e. if (4.41) is replaced by (4.33) with some ‘small’ $E_0^- > 0$.

boundary). Nevertheless, they are not just *ad hoc* assumptions: \bar{q}_2 is essentially the smearing function for G_2 because it appears in the Hamiltonian as a multiplier of this constraint. Thus, for consistency one ought to require the same boundary condition for the smearing function as for \bar{q}_2 . But this is not sufficient yet; we still need some insight into the scaling behaviour of p_3 and \bar{q}_2 near a horizon in order to be able to judge whether the ratio p_3/\bar{q}_2 is zero, finite or infinite. A straightforward analysis shows that the ratio is always finite on-shell; it may become zero for extremal horizons. Thus we have argued that for non-extremal horizons indeed F , B_2 and the smearing function ξ corresponding to G_2 are of the same order. If one accepts this rule, only one pair of second class constraints survives, namely B_2 and \bar{P}_2 . All other constraints are first class¹³.

As an alternative to (4.40)–(4.42), another boundary representing a horizon may be implemented in strict analogy to (4.32)–(4.34) as

$$\hat{B}_1 = B_1[\eta] = (p_1 - p_1^h)\eta|_{\partial\mathcal{M}}, \quad (4.48)$$

$$\hat{B}_2 = B_2[\eta] = \bar{q}_2\eta|_{\partial\mathcal{M}}, \quad (4.49)$$

$$\hat{B}_3[\eta] = (\bar{q}_3 - E_0^+(x^0))\eta|_{\partial\mathcal{M}}. \quad (4.50)$$

It is important to realize the physical difference between the two sets (4.40)–(4.42) and (4.48)–(4.50). In the former case the boundary is defined to be a horizon in the worldsheet ($\bar{q}_2 \approx 0$) as well as in the target space ($p_3 \approx 0$). In the latter case, the horizon is defined in the worldsheet only; it still may fluctuate in the target space. However, an off-shell treatment of (4.48)–(4.50) turns out to be problematic. As is obvious from (4.23), F no longer vanishes at the boundary and therefore divergent results in the brackets (4.25) and (4.26) remain unless one demands that the smearing function of \bar{P}_2 vanishes at the horizon. In contrast to G_2 , however, such a restriction cannot be motivated easily. Therefore, we conclude that a quantum treatment of the horizon should start from the constraints (4.40)–(4.42), which also agrees with the known technical difficulties to define the extrinsic curvature at a horizon in the second-order formalism (1.2) for the analogue of (4.32)–(4.34). Henceforth exclusively (4.40)–(4.42) will be employed to characterize the boundary as a horizon.

4.2.3. Bifurcation point boundary conditions. The bifurcation point boundary conditions $X^\pm = 0$ can be obtained from the horizon boundary conditions (4.40)–(4.42), but replacing B_2 by

$$b_2[\eta] = p_2\eta|_{\partial\mathcal{M}}. \quad (4.51)$$

Now only the brackets

$$\{b_2[\eta], G_1[\xi]\} = \eta\xi p_2|_{\partial\mathcal{M}} = b_2[\eta\xi], \quad (4.52)$$

$$\{b_2[\eta], G_3[\xi]\} = -\eta\xi\mathcal{V}|_{\partial\mathcal{M}} \approx -\eta\xi V|_{\partial\mathcal{M}}. \quad (4.53)$$

are non-vanishing. For the bifurcation point boundary conditions again $F|_{\partial\mathcal{M}} \approx 0$, and there are two pairs of second class constraints on the boundary, b_2 , G_3 and B_3 , G_2 . Since $\bar{q}_2 \neq 0$ in general, the canonical analysis does not require any additional assumptions. For the special case of an extremal horizon, $V|_{\partial\mathcal{M}} = 0$ is valid and thus all constraints become first class.

It should be noted that for this set of boundary conditions our assumption of a smooth boundary may not be accessible, i.e. corner contributions should be taken into account. This

¹³ Actually, the brackets (4.25), (4.26) and (4.43) are weakly non-vanishing. However, with $\phi_i = \{G_2, G_3, B_2, \bar{P}_2\}$ the corresponding matrix $M_{ij} = \{\phi_i[\eta], \phi_j[\xi]\}$ does not have full rank, so there are only two second class constraints instead of four.

can be achieved most conveniently by starting from the action (4.13), but with the partially integrated version of the last boundary term. Consequently, the canonical analysis will be modified. In the second-order formulation, bifurcation point boundary conditions have been investigated thoroughly in [45].

5. Symmetries

A natural interpretation of the canonical analysis in the previous section is that each pair of second class constraints breaks a gauge symmetry at the boundary, i.e. a boundary condition on the gauge parameter is required. One can check this statement without any use of the canonical methods by simply considering the known symmetries of the bulk action. One has to check that the action is invariant (i.e., that the symmetry variation of the action is zero) as well as the boundary conditions (i.e., that if some field is fixed on the boundary its symmetry variation is zero). As the formulae are more transparent, we convert within the Lagrangian formulation to the original notation of Cartan variables. The symmetries comprise local Lorentz transformations (with transformation parameter γ) and diffeomorphisms (with transformation parameter ξ^μ). Their infinitesimal action on the fields reads

$$\delta e_\mu^\pm = \pm\gamma e_\mu^\pm + \xi^\nu \partial_\nu e_\mu^\pm + (\partial_\mu \xi^\nu) e_\nu^\pm, \quad (5.1)$$

$$\delta \omega_\mu = -\partial_\mu \gamma + \xi^\nu \partial_\nu \omega_\mu + (\partial_\mu \xi^\nu) \omega_\nu, \quad (5.2)$$

$$\delta X = \xi^\nu \partial_\nu X, \quad (5.3)$$

$$\delta X^\pm = \pm\gamma X^\pm + \xi^\nu \partial_\nu X^\pm. \quad (5.4)$$

Within our choice of the boundary at fixed x^1 for any set of the boundary conditions, ξ^1 must obey

$$\xi^1|_{\partial\mathcal{M}} = 0. \quad (5.5)$$

This may be established either from $\delta X|_{\partial\mathcal{M}} = 0$ or from the condition that the gauge transformation of the action (3.8) must not produce a surface term. The latter does not yield additional constraints: Lorentz invariance is guaranteed by construction, the remaining diffeomorphisms produce total derivatives along the boundary only,

$$\delta_\xi (X\omega_0|_{\partial\mathcal{M}}) = \partial_0(\xi^0 X\omega_0), \quad (5.6)$$

$$\delta_\xi \left((\partial_0 X) \ln \frac{e_0^+}{e_0^-} \right) = \partial_0 \left(\xi^0 (\partial_0 X) \ln \frac{e_0^+}{e_0^-} \right). \quad (5.7)$$

Consequently, the action we consider is invariant under the diffeomorphism transformations iff ξ^1 vanishes on the boundary. This property is shared by the EH action with the YGH term in four dimensions. Naturally, it is preserved by the dimensional reduction.

Now we turn to our specific choices of boundary conditions. For the generic ones, (4.32)–(4.34), inspection of the remaining transformations immediately yields Dirichlet boundary conditions for all symmetry parameters. The situation becomes more interesting in the case of a horizon. For (4.40)–(4.42), it is obvious that local Lorentz transformations and diffeomorphisms along the boundary (ξ^0) are unconstrained, as they are multiplied in all relevant transformations by quantities that have been fixed to zero. These two results agree with the constraint analysis of the previous section; for the generic case, none of the G_i remains first class at the boundary (in the sense explained in section 4), and for the horizon we found that the Lorentz constraints G_1 and G_3 , which may be interpreted as diffeomorphisms

along the boundary, remain strictly first class. In the bifurcation point scenario, local Lorentz transformations are again unconstrained. However, both diffeomorphisms must obey Dirichlet boundary conditions although $X^\pm = 0$ at the bifurcation point, as this restriction does not hold along an extended (one-dimensional) boundary.

It should be emphasized that the symmetry transformations can have a non-trivial action at the boundary even if the corresponding parameter obeys Dirichlet boundary conditions, due to the derivative terms normal to the boundary acting on the symmetry parameters in (5.1) and (5.2). For instance, $\partial_1 \xi^1$ does not necessarily vanish at $\partial\mathcal{M}$. Similarly, the appearance of derivatives in the constraints (4.5)–(4.7) leads to the generation of residual gauge transformations at $\partial\mathcal{M}$ even if they do not have any support at the boundary or if they are second class there.

We would like to elaborate a bit on the connection between Hamiltonian symmetries and Lagrangian symmetries for the horizon scenario, since in this case we have encountered the peculiar property that the boundary constraints B_1, B_3 are first class and thus they generate gauge transformations, the meaning of which shall be clarified. Consider first the Hamiltonian side of the picture. The gauge transformations generated by the B_i read (cf e.g. [44])

$$\delta_\varepsilon f(q_i, p_i) = \varepsilon^j \{f(q_i, p_i), B_j\}, \quad j = 1, 3, \quad (5.8)$$

where f is a (differentiable) function on the phase space. The only non-trivial transformations generated by the B_i are

$$\delta_\varepsilon q_1 = \varepsilon^1, \quad \delta_\varepsilon q_3 = \varepsilon^3. \quad (5.9)$$

In order to understand the underlying symmetries better, we consider now the Lagrangian picture, cf (5.1)–(5.4). We attempt to construct the local parameters ξ^μ, γ such that (5.9) is recovered and no other quantities are being transformed. The first restriction comes from (5.5). Consistency with B_2 requires

$$\delta e_0^- = (\partial_0 \xi^1) e_1^- \stackrel{!}{=} 0. \quad (5.10)$$

Since ∂_0 is the derivative parallel to the boundary, (5.10) is fulfilled automatically without imposing any further restriction on ξ^1 . The constraint B_3 does not provide anything new either. Consistency with (5.9) requires the vanishing of all variations besides $\delta\omega_1$ and δe_1^+ . This establishes Dirichlet boundary conditions for all symmetry variation parameters, $\gamma|_{\partial\mathcal{M}} = \xi^\mu|_{\partial\mathcal{M}} = 0$, as well as for their ∂_0 derivatives, $\partial_0 \gamma|_{\partial\mathcal{M}} = \partial_0 \xi^\mu|_{\partial\mathcal{M}} = 0$. The remaining conditions,

$$\delta\omega_1 = -\partial_1 \gamma + (\partial_1 \xi^1) \omega_1 \stackrel{!}{=} \varepsilon^1 \quad (5.11)$$

and

$$\delta e_1^+ = (\partial_1 \xi^0) e_0^+ + (\partial_1 \xi^1) e_1^+ \stackrel{!}{=} \varepsilon^3, \quad (5.12)$$

provide further restrictions which must be valid in any gauge (no gauge conditions have been imposed so far). Thus, they have to hold in particular in the gauge $\omega_1 = 0 = e_1^+$, which is always accessible. Moreover, we may assume that the horizon is located at $x^1 = 0$. In such a gauge, one obtains

$$\gamma = -\varepsilon^1 x^1 \quad (5.13)$$

and

$$\partial_1 \xi^0 = \frac{\varepsilon^3}{e_0^+}. \quad (5.14)$$

The last equation is well defined because $e_0^+|_{\partial\mathcal{M}} \neq 0$ and the integration constant is fixed by the Dirichlet condition on ξ^0 .

To summarize, we have established that the gauge transformations in the phase space generated by B_1, B_3 may be interpreted as specific local Lorentz transformations and diffeomorphisms, respectively. Thus, the Hamiltonian picture is consistent with the Lagrangian one, as may have been anticipated on general grounds. This provides a further justification for our treatment of the constraint algebra in section 4.2.2 (cf the text below (4.47)).

6. The reduced phase space

In order to count the number of physical degrees of freedom, it is useful to take the following route: first, one may pretend that no boundaries are present, i.e. one constructs the reduced phase space for the domain $\mathcal{M} - \partial\mathcal{M}$. Then one applies a standard machinery of gauge fixing and solving constraints to construct the reduced phase space. One may then extend the results to the whole of \mathcal{M} , including $\partial\mathcal{M}$, provided the gauge-fixing functions do not contradict the boundary constraints. This circumvents the challenge to deal with constraints which are first class in the bulk and second class at the boundary (in the sense explained in section 4): after fixing the gauge, all constraints are second class and therefore may be treated on equal footing. Obviously, if one chooses a gauge which is not compatible with the boundary data inconsistencies may emerge. So one has to tread gingerly and check the consistency of all gauge-fixing functions with the boundary constraints.

For the purpose of treating all constraints on equal footing, we introduce the extended Hamiltonian $H^{\text{ex}} = H^{\text{tot}} + \sum_i G_i[\mu_i]$, where μ_i are Lagrange multipliers for the secondary constraints G_i , cf (4.20)–(4.22), and H^{tot} is defined in (4.17). The canonical variables \bar{q}_i now coincide with the zero components of the Cartan variables only for $\mu_i = 0$. To convert the primary first class constraints \bar{P}_i into second class constraints in the bulk, we impose the gauge-fixing conditions¹⁴

$$\chi_1 = \bar{q}_1 - \Omega_0(x^0, x^1, q_i, p_i), \quad (6.1)$$

$$\chi_2 = \bar{q}_2 - E_0^-(x^0, x^1, q_i, p_i), \quad (6.2)$$

$$\chi_3 = \bar{q}_3 - E_0^+(x^0, x^1, q_i, p_i). \quad (6.3)$$

Continuity requires that the limit of approaching $\partial\mathcal{M}$ coincides with the boundary values at $\partial\mathcal{M}$. Thus, the gauge-fixing conditions (6.1)–(6.3) are also valid on $\partial\mathcal{M}$. The functions Ω_0, E_0^-, E_0^+ are arbitrary in principle. However, it is very convenient—from a physical point of view perhaps even mandatory—to impose $\mu_i = 0$ at least at the boundary, so that boundary conditions on $\bar{q}_{2,3}$ coincide with those on e_0^\pm . This can be achieved easily by fixing the functions E_0^\pm in (6.2), (6.3) such that at $\partial\mathcal{M}$ they coincide with the corresponding boundary constraints discussed in section 4. We will return to this issue in more detail below, where we will always assume that the functions Ω_0, E_0^-, E_0^+ have been chosen such that $\mu_i|_{\partial\mathcal{M}} = 0$.

Since \bar{q}_i, \bar{p}_i are nondynamical now, the reduced phase space in the bulk may be constructed by solving $G_i = 0$ together with the corresponding gauge fixings which convert them into second class constraints. A convenient set of such conditions,

$$\chi_4 = q_1, \quad (6.4)$$

¹⁴ In this context we note that the constraints \bar{P}_i generate symmetries—namely shifts of \bar{q}_i —of the extended action with the Hamiltonian H^{ex} , but not of the original Lagrangian action (3.8). The conditions (6.1)–(6.3) and the constraints \bar{P}_i reveal that the variables \bar{q}_i, \bar{p}_i are nondynamical, as expected. These features are precisely the same as for quantum electrodynamics: \bar{P}_i, G_i and \bar{q}_i correspond to primary constraint, Gauss constraint and the zero component of the gauge potential, respectively.

$$\chi_5 = q_2 - 1, \quad (6.5)$$

$$\chi_6 = q_3, \quad (6.6)$$

will be imposed on $\mathcal{M} - \partial\mathcal{M}$. This gauge is simple, always accessible and implies (2.12) for the metric. Without loss of generality, we will suppose that the boundary is placed at $x^1 = 0$. At each point in the bulk one may now solve the constraints $G_i = 0$ (cf (4.5)–(4.7)):

$$G_2 : \quad p_2 = T(x^0), \quad (6.7)$$

$$G_1 : \quad p_1 = T(x^0)x^1 + p_1^b, \quad (6.8)$$

$$G_3 : \quad p_3 = \frac{M(x^0) - w(p_1)}{T(x^0)} \exp(-Q(p_1)). \quad (6.9)$$

By continuity the solution may be extended to $\partial\mathcal{M}$. As we require the dilaton to be constant at the boundary in all prescriptions, p_1^b has to be constant. Now all q_i , p_i , \bar{q}_i , \bar{p}_i are fixed and the reduced phase space has dimension zero as far as bulk degrees of freedom are concerned. There are, however, two arbitrary free functions, $M(x^0)$ and $T(x^0)$. Note that $M(x^0)$ coincides with the Casimir function (2.7). Thus, in the reduced phase space there appear to remain physical degrees of freedom related to the mass $M(x^0)$ and some conjugate quantity $T(x^0)$. We will study now in detail different cases to see whether these degrees of freedom are compatible with the boundary conditions.

6.1. Generic boundary conditions

The boundary constraint (4.32) is fulfilled identically because in our solution of the constraints we have assumed that $p_1^b = \text{const}$, cf (6.8). This property holds in all cases below. The functions in (6.2), (6.3) have to fulfil boundary conditions consistent with the boundary constraints (4.33), (4.34). Thus, B_2, B_3 are actually redundant since they follow from continuity of the gauge-fixing functions χ_2, χ_3 and consequently do not have to be counted as independent constraints. Therefore, these boundary constraints are nothing but gauge fixing constraints (6.2), (6.3) evaluated at the boundary, provided the latter are chosen to be consistent with the former. If these gauge fixing functions are chosen to be inconsistent with the boundary data then a contradiction is encountered and the gauge should be discarded as inaccessible.

A formal counting establishes $12 - 6 - 6 = 0$ bulk degrees of freedom, apart from possibly a finite number of global ones, which may be interpreted as being located at the boundary. Indeed, as the above analysis has shown there are two free functions $M(x^0)$ and $T(x^0)$ as boundary degrees of freedom. The boundary phase space is two dimensional.

6.2. Horizon boundary conditions

If we choose the gauge (6.1)–(6.6), it turns out that in this case there remains a residual gauge freedom. The most convenient way to fix it is by replacing (6.4) with

$$\chi_4^h = p_1 - x^1 - p_1^h. \quad (6.10)$$

As a technical sidenote, we remark that (6.10) is inaccessible in the generic case, unless the boundary constraints (4.33), (4.34) are fine tuned in a specific way. In the present case, however, the gauge is accessible because only one boundary condition on the zweibein, (4.41),

is imposed. Solving the constraints yields

$$G_1 : p_2 = 1, \quad (6.11)$$

$$G_2 : q_1 = 0, \quad (6.12)$$

$$G_3 : p_3 = (M(x^0) - w(p_1)) \exp(-Q(p_1)). \quad (6.13)$$

Note that on the surface of constraints condition (6.10) is more restrictive than (6.4) because it does not allow for a free function $T(x^0)$. The geometric reason behind this simplification is the residual Lorentz symmetry discussed in section 5.

The function in (6.2) has to fulfil Dirichlet boundary conditions consistent with (4.41). For (4.40), the analysis of the generic case applies. However, a crucial difference emerges due to B_3 as given in (4.42): the function $M(x^0)$ is no longer free but rather fixed by the requirement

$$p_3|_{\partial\mathcal{M}} = 0 \quad \Rightarrow \quad M(x^0) = w(p_1^h) = \text{const}. \quad (6.14)$$

With this condition imposed, the boundary constraints are again merely a consequence of the gauge fixing functions and continuity. In contrast to the generic case, however, the boundary data fix the value of the Casimir function as well. Therefore, as anticipated by the constraint analysis in section 4, the dimension of the reduced phase space is zero and there are indeed fewer physical degrees of freedom if the boundary is a horizon.

6.3. Bifurcation point boundary conditions

In the gauge (6.4)–(6.6) again there remains some residual gauge freedom. The alternative gauge fixing (6.10) breaks down at the bifurcation point. Instead, by analogy to [46], one may choose

$$\chi_4^b = p_1 - x^0 x^1 - p_1^h. \quad (6.15)$$

The bifurcation point is located at $x^0 = x^1 = 0$, the bifurcate horizon at $x^0 x^1 = 0$. Proceeding as above one may solve the constraints and obtain

$$G_1 : p_2 = x^0, \quad (6.16)$$

$$G_2 : q_1 = 0, \quad (6.17)$$

$$G_3 : p_3 = \frac{M(x^0) - w(p_1)}{x^0} \exp(-Q(p_1)). \quad (6.18)$$

The only crucial differences to (6.11)–(6.13) are the possibility for p_2 to change sign and the appearance of a denominator x^0 in (6.18). The boundary constraints (4.40) and (4.51) are fulfilled automatically. The remaining one, (4.42), imposes the restriction (6.14). Therefore, close to $x^0 = 0$ the expansion $p_3 = -V(p_1^h)x^1 + \dots$ is regular, and also for the bifurcation point boundary conditions the dimension of the reduced phase space is zero.

6.4. Comparison with classical EOM

The above results may be compared with the analysis of the classical EOM in appendix A, where it is also found that the boundary constraints impose certain restrictions on the solutions. It is emphasized that they arise already from the study of a single boundary. The only arbitrariness that remains is the freedom to choose a constant c in (A.17) which corresponds to the on-shell value of the Casimir function $M(x^0)$. For the boundary conditions corresponding

to the horizon constraints (4.40)–(4.42), there are two residual gauge symmetries, whence the solution of the EOM is found to be unique without any free parameter to adjust.

Summarizing our main results, we can state that for the horizon scenario there are fewer physical degrees of freedom in the reduced phase space as compared to generic boundary conditions, concurring with the constraint analysis in section 4 where more gauge symmetries have been found for this case, and also in agreement with the analysis of EOM in appendix A.

7. Conclusions and outlook

We have analysed how the presence of a horizon, interpreted as a boundary, changes the constraint algebra and the physical phase space compared to the presence of a general boundary. In order to reduce clutter, we have restricted the EH action to its spherically symmetric sector. Actually, we were able to describe generic dilaton gravity in 2D on the same footing. Furthermore, as it is advantageous to reformulate the action in a first-order form, a careful treatment of boundaries started from the derivation of the first-order form of the YGH boundary term in section 3. The constraint algebra in the presence of boundaries, studied extensively in section 4, led to three cases: generic, horizon and bifurcation point boundary conditions. We focused on the former two, because our main interest lies in an analysis which can be generalized to the case where matter degrees of freedom are present—in that case no bifurcation point is expected to exist because the horizon does not bifurcate for a physical BH (as opposed to an eternal BH).

Two different sets of boundary conditions characterizing a horizon have been found. Apart from fixing the dilaton field, one can either impose ‘mixed’ horizon constraints (on the worldsheet *and* on the target space) or implement the horizon solely on the worldsheet. The latter is a limiting case of a general boundary and consequently it can be ‘stretched’ while the former cannot be deformed in this way. Where applicable, the results for both possibilities agree. The mixed one turned out to be the preferred set in the Hamiltonian analysis, as for the other alternative the limiting case of a generic boundary led to singularities we could not handle.

Somewhat surprisingly, the horizon constraint algebra revealed more first class constraints, i.e. more generators of symmetries, as compared to the generic case. Studying the symmetries in section 5 showed consistency between the Lagrangian and Hamiltonian formalism and also provided the appropriate boundary conditions for the transformation parameters. Generically, Dirichlet boundary conditions are required for all transformation parameters, except at horizons or bifurcation points, where some of the symmetries (in particular local Lorentz transformations) survive. For our preferred set of horizon conditions additionally diffeomorphisms along the boundary are possible.

Collecting the evidence obtained so far—in particular the enhancement of symmetries at the horizon—we were led to study the reduced phase space in order to decide whether or not horizon boundary conditions are special from a physical point of view (section 6). A pivotal observation has been the consistency between the boundary constraints and the gauge fixing functions. This allowed us to regard the former as a continuation of the latter to the boundary. Due to a convenient (Eddington–Finkelstein) choice of the gauge-fixing functions, it was possible to solve all constraints exactly. In the generic case, there remain two degrees of freedom in the physical phase space. This is consistent with the results of Kuchař [11], cf appendix B. However, for the horizon scenario no free function is found because the boundary condition defining the horizon in the target space implies the fixing of the remaining unknown function (the Casimir function). As an independent check, the solutions of the EOM in the presence of a boundary are presented in appendix A. Full agreement with the

Hamiltonian analysis was found. Thus, we conclude that physical degrees of freedom are converted into gauge degrees of freedom on a horizon.

A similar statement can be found in the 2003 Erice lectures by 't Hooft [25]. Within the ‘brick wall’ model for BHs, he argued that the local gauge degrees of freedom on a horizon could represent lost information. This suggests studying the symmetry of the horizons and relations to the black hole entropy with our methods. There exist various ways to count the microstates by appealing to the Cardy formula and to recover the Bekenstein–Hawking entropy. However, the true nature of these microstates remains unknown in this approach, which is a challenging open problem. Many different proposals have been made [6, 47], some of which are mutually contradictory.

From a technical point of view, the boundary Hamiltonian in the presence of null surfaces has been constructed in the second-order formulation of 4D EH gravity in [18]. Also there some differences between generic variational problems and those which involve null surfaces were found (cf (26)–(27) and the surrounding paragraphs in that work). However, no constraint analysis has been performed and no construction of the physical phase space has been attempted. We also mention similarities to isolated horizons [48]. In the first-order formulation of 4D EH gravity, boundary conditions have recently been studied in the Lagrangian [49] as well as in the Hamiltonian picture [50]. However, sharp horizon constraints are not implemented and a study of the constraint algebra is not performed.

At this point we would like to compare with [22] (cf the penultimate paragraph of section 1). An important difference appears to be the role of diffeomorphisms along the horizon. While in [22], the vector field that generates horizon diffeomorphisms blows up, these transformations in our case are regular and belong to the residual gauge symmetries discussed in section 5. As pointed out above, the horizon boundary conditions described in section 4.2.2 cannot be achieved through some ‘stretching procedure’, which may account for the qualitative differences between our results and [22].

Minimal [51] and non-minimal [52] supergravity extensions of the model considered are known and it would be interesting to extend our work to these cases. Recently, it was found [36] that local supersymmetry of the EH action *and* of the boundary conditions requires vanishing extrinsic curvature K of the boundary. If one adds the surface tension of the boundary to the action, the extrinsic curvature must be related to this surface tension [53]. It is important to find out how these schemes work in the case of horizon boundary conditions where the extrinsic curvature is not defined, strictly speaking. The condition $K|_{\partial\mathcal{M}} = 0$ may be of interest by itself because in this case we do not have to fix the dilaton on the boundary to cancel the corresponding part of (3.9).

From our experience with the constraints in the bulk alone [9], the analysis will not differ very much after matter has been included. There are, however, some relevant changes regarding the boundary constraints. A crucial observation in this context is that the horizon condition in the target space (2.11) still describes a trapping horizon! One can derive this statement by analysing the EOM (A.1) suitably extended to include matter (for spherically reduced gravity, it is particularly transparent because the fields X^\pm correspond to the expansion spin coefficients). Thus, the corresponding horizon constraint (4.42) may still be imposed. Moreover, the coordinate x^1 can again be chosen constant along the boundary. However, the trapping horizon will not be a Killing horizon in general and thus the—in the matterless case equivalent—worldsheet condition (2.10) is no longer true. Accordingly, the (Killing) horizon condition (4.41) has to be replaced by (4.33). Moreover, the dilaton field need not be constant along the boundary anymore, which enforces a further generalization. Note that the boundary is no longer a null surface and therefore the presence of the logarithmic contribution to the YGH term in (3.8) is no longer dangerous. In the present derivation, the horizon condition on

the target space (4.42) plays a central role in the symmetry enhancement on the horizon. As it still holds after matter has been included, it may be expected that the physical phase space is again smaller as compared to the case of generic boundary conditions. It would be nice to verify this conjecture.

Any degrees of freedom which are observed in the reduced phase space formalism should also be seen in the path integral. Actually, we already have made some progress towards generalizing the path integral for 2D gravity [9, 54, 55] in the presence of boundaries. What is missing is the BRST charge, which is a technically demanding calculation, but no essential difficulties are expected. With a path integral at hand one may study, for example, the interaction between virtual black holes [55] and boundaries.

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Appendix A. Analysis of EOM

Variation of (2.1) in the bulk yields the EOM,

$$\begin{aligned} dX + X^- e^+ - X^+ e^- &= 0, \\ (d \pm \omega) X^\pm \mp \mathcal{V} e^\pm &= 0, \\ d\omega + \epsilon \frac{\partial \mathcal{V}}{\partial X} &= 0, \\ (d \pm \omega) e^\pm + \epsilon \frac{\partial \mathcal{V}}{\partial X^\mp} &= 0, \end{aligned} \tag{A.1}$$

in the covariant form. To simplify their analysis, let us impose the gauge conditions (6.4)–(6.6), i.e. $\omega_1 = 0 = e_1^+$ and $e_1^- = 1$. These gauge conditions leave unbroken diffeomorphisms and local Lorentz transformations provided the transformation parameters fulfil

$$\xi^0 = \xi^0(x^0), \quad \gamma + \partial_1 \xi^1 = 0, \quad \partial_1 \gamma = 0. \tag{A.2}$$

From the Hamiltonian point of view, there are three sets of EOM. The first one is obtained by varying the bulk part of the action (3.8) with respect to \bar{q}_i :

$$\partial_1 p_1 - p_2 = 0, \tag{A.3}$$

$$\partial_1 p_2 = 0, \tag{A.4}$$

$$\partial_1 p_3 + \mathcal{V}(p_2 p_3, p_1) = 0. \tag{A.5}$$

These equations coincide with the bulk parts of corresponding constraints G_i after the gauge conditions (6.4)–(6.6) have been taken into account. The second set is produced by the variations with respect to q_i ,

$$\partial_0 p_1 - p_2 \bar{q}_2 + p_3 \bar{q}_3 = 0, \quad (\text{A.6})$$

$$\partial_0 p_2 + p_2 \bar{q}_1 - \bar{q}_3 \mathcal{V} = 0, \quad (\text{A.7})$$

$$\partial_0 p_3 - p_3 \bar{q}_1 + \bar{q}_2 \mathcal{V} = 0, \quad (\text{A.8})$$

and the last set is generated by p_i ,

$$\partial_1 \bar{q}_1 - \bar{q}_3 \frac{\partial \mathcal{V}}{\partial p_1} = 0, \quad (\text{A.9})$$

$$\partial_1 \bar{q}_2 + \bar{q}_1 - \bar{q}_3 U p_3 = 0, \quad (\text{A.10})$$

$$\partial_1 \bar{q}_3 - \bar{q}_3 U p_2 = 0. \quad (\text{A.11})$$

Generic boundary. The boundary conditions are given by (4.32)–(4.34). There is no residual gauge freedom associated with these boundary conditions because ξ^1 fulfils Dirichlet boundary conditions and neither ξ^0 nor γ (nor any combination thereof) may be chosen freely without violating the boundary conditions. The solution of (A.3)–(A.5) is provided in (6.7)–(6.9). Note the emergence of two integration functions $M(x^0)$ and $T(x^0)$. We solve some of the remaining EOM in the order (A.11), (A.7), (A.6):

$$\bar{q}_3 = E_0^+(x^0) e^{\mathcal{Q}}, \quad (\text{A.12})$$

$$\bar{q}_1 = \frac{\bar{q}_3 \mathcal{V}}{T} - \frac{\dot{T}}{T}, \quad (\text{A.13})$$

$$\bar{q}_2 = T^{-1}(p_3 \bar{q}_3 + x^1 \dot{T}), \quad (\text{A.14})$$

where $\dot{T} = \partial_0 T$. The lower integration limits in the functions

$$Q(p_1) \equiv \int_{p_1^h}^{p_1} dy U(y), \quad (\text{A.15})$$

$$w(p_1) \equiv \int_{p_1^h}^{p_1} dy V(y) e^{\mathcal{Q}(y)}, \quad (\text{A.16})$$

have been chosen for convenience. All integration constants are captured by the two free functions T and M and the boundary value E_0^+ . Thus, at this stage all canonical variables are determined uniquely up to T , M and E_0^+ .

Equations (A.9) and (A.10) are satisfied automatically, but equation (A.8) yields the condition

$$\dot{M} = 0 \quad \Rightarrow \quad M(x^0) = c, \quad (\text{A.17})$$

with some constant c . The boundary condition (4.33) fixes the remaining integration function:

$$T(x^0) = \sqrt{c \frac{E_0^+(x^0)}{E_0^-(x^0)}}. \quad (\text{A.18})$$

The only arbitrariness that remains is the freedom to choose the constant c . This quantity corresponds to the on-shell value of the Casimir function (2.7). Its sign is fixed by the

requirement of reality of T ; in particular, if E_0^+ and E_0^- are (semi-)positive as assumed in the main text, then c is (semi-)positive as well.

Horizon boundary conditions. We impose the boundary conditions corresponding to the constraints (4.40)–(4.42). There are two residual gauge symmetries corresponding to $\xi^0(x^0)$ (diffeomorphisms) and $\gamma(x^0)$ (Lorentz). The general solution of (A.4) reads $p_2 = T(x^0)$. By an x^0 -dependent finite Lorentz transformation, one can always achieve $T(x^0) = 1$, so that

$$p_2 = 1. \quad (\text{A.19})$$

Then equations (A.3) and (A.5) have unique solutions satisfying the boundary conditions defined by (4.40) and (4.42):

$$p_1 = x^1 + p_1^h, \quad (\text{A.20})$$

$$p_3 = -e^{-\mathcal{Q}} w, \quad (\text{A.21})$$

where $x^1 = 0$ corresponds to the boundary and the definitions (A.15), (A.16) have been used. It is pivotal that no ambiguity is present in (A.21) because p_3 obeys Dirichlet boundary conditions.

Next we use the residual gauge freedom associated with $\xi^0(x^0)$ to make

$$(\bar{q}_3 = e_0^+) |_{\partial\mathcal{M}} = 1. \quad (\text{A.22})$$

Then (A.11) is solved uniquely,

$$\bar{q}_3 = e^{\mathcal{Q}}. \quad (\text{A.23})$$

The quantities \bar{q}_1 and \bar{q}_2 can be found from (A.7) and (A.6), respectively:

$$\bar{q}_1 = e^{\mathcal{Q}} \mathcal{V}, \quad (\text{A.24})$$

$$\bar{q}_2 = e^{\mathcal{Q}} p_3. \quad (\text{A.25})$$

The remaining equations (A.8)–(A.10) and the boundary condition (4.41) are satisfied automatically. The solution is fixed uniquely by the boundary conditions.

Bifurcation point boundary conditions. Solving the EOM in the same manner as before and adopting the choices (A.15) and (A.16) yields

$$p_1 = x^0 x^1 + p_1^h, \quad (\text{A.26})$$

$$p_2 = x^0, \quad (\text{A.27})$$

$$p_3 = -\frac{w}{x^0} e^{-\mathcal{Q}}, \quad (\text{A.28})$$

$$\bar{q}_1 = -\frac{E_0^+(x^0)(wU - w') + 1}{x^0}, \quad (\text{A.29})$$

$$\bar{q}_2 = \frac{x^0 x^1 - E_0^+(x^0) w}{(x^0)^2}, \quad (\text{A.30})$$

$$\bar{q}_3 = E_0^+(x^0) e^{\mathcal{Q}}. \quad (\text{A.31})$$

The residual local Lorentz transformations can be exploited to make $E_0^+(x^0) = E_0^+ = \text{const}$. Regularity of the coordinate system at $x^0 = x^1 = 0$ requires

$$E_0^+ = \frac{1}{w'(p_1^h)}. \quad (\text{A.32})$$

The quantity $w'(p_1^h)$ is essentially surface gravity, so for non-extremal horizons E_0^+ is well defined. Again the solution is fixed uniquely by the boundary conditions.

Alternative horizon boundary conditions. In contrast to the constraint algebra, Lagrangian symmetries and the EOM can be analysed for the alternative horizon prescription (4.48)–(4.50) as well. At first glance the situation appears to be different, as there remains only one residual gauge transformation $\xi^0(x^0)$ provided the other transformation parameters fulfil

$$\gamma E_0^+(x^0) + \partial_0 (\xi^0 E_0^+(x^0)) = 0, \quad \xi^1 = x^1 \gamma. \quad (\text{A.33})$$

However, this is sufficient to achieve (A.19) and thus the above analysis also applies to this case, with the sole replacement of 1 by E_0^+ on the right-hand side of (A.22). The reason for this equivalence is that on-shell the constraint $p_3 = 0$ is implied by the boundary constraint $\bar{q}_2 = 0$.

Appendix B. Abelianized constraints: relation to Kuchař

In [11], Kuchař constructed a reduced phase space for the Schwarzschild BH which comprises the BH mass and extrinsic time as canonical variables, in a way very different from the main text. Here we make contact with Kuchař's work. To this end it is very convenient to further simplify the constraint algebra (4.8).

In the absence of boundaries within the PSM formulation, it has been realized that the constraint algebra may be Abelianized not only locally, but also in a certain patch which covers, e.g., the whole exterior of a BH [33, 56]. Following appendix E of [42], we define a new set of constraints

$$G_1^c := G_1, \quad (\text{B.1})$$

$$G_2^c := e^{-Q(p_1)} \frac{p_3 G_2 - p_2 G_3}{2p_2 p_3}, \quad (\text{B.2})$$

$$G_3^c := e^{Q(p_1)} (\mathcal{V} G_1 + p_3 G_2 + p_2 G_3) = \partial_1 \mathcal{C}, \quad (\text{B.3})$$

where G_i are the bulk constraints defined in (4.5)–(4.7) and \mathcal{C} is defined in (2.7). This redefinition of constraints is well defined in a patch where $p_2 p_3 \neq 0$, which by virtue of (2.11) requires the absence of a Killing horizon in the whole patch. Thus, the Abelianized constraints (B.1)–(B.3) are not useful for horizon boundary conditions, but they may be employed for generic boundary conditions in the outside region of a BH. It is convenient to introduce a new set of canonical variables:

$$q_1^c := q_1 - \frac{q_2 p_2 + q_3 p_3}{2p_2 p_3} \mathcal{V}, \quad p_1^c := p_1, \quad (\text{B.4})$$

$$q_2^c := q_2 p_2 - q_3 p_3, \quad p_2^c := \frac{1}{2} \ln \frac{p_2}{p_3}, \quad (\text{B.5})$$

$$q_3^c := e^{-Q(p_1)} \frac{q_2 p_2 + q_3 p_3}{2p_2 p_3}, \quad p_3^c := p_2 p_3 e^{Q(p_1)} + w(p_1) = \mathcal{C}. \quad (\text{B.6})$$

Both the Jacobian in the transformation of the constraints and the Jacobian in the canonical coordinate transformation are equal to unity. One may easily check that

$$\{q_i^c, p_j^{c'}\} = \delta_{ij} \delta, \quad \{q_i^c, q_j^c\} = 0 = \{p_i^c, p_j^c\}. \quad (\text{B.7})$$

The constraints G_i^c in terms of these variables are very simple:

$$G_1^c = \partial_1 p_1^c - q_2^c, \quad (\text{B.8})$$

$$G_2^c = (\partial_1 p_2^c + q_1^c) e^{-Q(p_1^c)}, \quad (\text{B.9})$$

$$G_3^c = \partial_1 p_3^c. \quad (\text{B.10})$$

Their interpretation is as follows: G_1^c generates Lorentz transformations (being canonically conjugate to the Lorentz angle p_2^c), G_2^c generates radial translations (essentially being canonically conjugate to the dilaton p_1^c), and G_3^c generates time translations (being the derivative of the mass p_3^c which, roughly speaking, is canonically conjugate to ‘time’). The constraints are Abelian up to boundary terms¹⁵,

$$\{G_i^c, G_j^{c'}\} = 0, \quad \forall i, j. \quad (\text{B.11})$$

It is possible to make them Abelian even in the presence of a boundary by adding to the smeared constraint $G_2^c[\xi]$ a boundary term $\xi p_2^c \exp(-Q(p_1^c))|_{\partial\mathcal{M}}$.

The Hamiltonian action now reads

$$S^{(c)} = \int_{\mathcal{M}} d^2x (p_i^c \dot{q}_i^c + \bar{q}_i^c G_i^c) + S_B, \quad (\text{B.12})$$

where S_B denotes a boundary term. The new Lagrange multipliers may be expressed in terms of the old ones by requiring equivalence of the bulk Hamiltonians, $\bar{q}_i G_i = \bar{q}_i^c G_i^c$.

With the experience of the main text, it is convenient to make a shortcut rather than repeating the more elaborate analysis of sections 3–6: because the last term in (3.8) vanishes for a constant dilaton, we already know that arguments of functional differentiability (which may be invoked to establish the first boundary term in (3.8)) are sufficient if the dilaton is constant at the boundary, which again will be assumed to be a surface of $x^1 = \text{const}$. Thus, it suffices to declare which quantities are held fixed at the boundary in order to construct S_B . The first condition is constancy of the dilaton,

$$\delta p_1^c|_{\partial\mathcal{M}} = 0. \quad (\text{B.13})$$

For the second condition, we note that the Lagrange multiplier

$$\bar{q}_2^c = e^{Q(p_1)} (\bar{q}_2 p_2 - \bar{q}_3 p_3), \quad (\text{B.14})$$

vanishes on-shell. Thus, a choice consistent with the classical EOM is

$$\bar{q}_2^c|_{\partial\mathcal{M}} = 0. \quad (\text{B.15})$$

No contribution to S_B has been produced by conditions (B.13) and (B.15). If one would like to have $S_B = 0$, then the relation

$$\bar{q}_3^c \delta p_3^c = 0, \quad (\text{B.16})$$

has to be fulfilled. The quantity

$$\bar{q}_3^c = e^{-Q(p_1)} \frac{\bar{q}_2 p_2 + \bar{q}_3 p_3}{2p_2 p_3} \quad (\text{B.17})$$

may be interpreted as a lapse function and there is no geometric reason why it should vanish. Fixing p_3^c at the boundary means that the mass must not fluctuate there. This is a valid choice,

¹⁵ This seems to be the proper place to make a parenthetical remark regarding the stability of the constraint algebra against deformations due to addition of matter: the Abelian algebra (B.11) is very unstable in the sense that addition of minimally coupled matter already deforms it in a complicated way. Thus, for considerations of quantum effects or inclusion of matter it seems to be of little use. The ‘original’ constraint algebra with G_i is stable against addition of minimally coupled matter and even for non-minimally coupled matter only the structure function C_{23} ¹ is changed.

but it is something that Kuchař wants to avoid in his approach. Therefore, we now consider another possibility.

We may choose in (B.12)

$$S_B = - \int_{\partial\mathcal{M}} dx^0 \bar{q}_3^c p_3^c. \quad (\text{B.18})$$

This coincides with Kuchař's result if we call $p_3^c = M$ and $\bar{q}_3^c = N$. With this addition, the variation at the boundary now yields instead of (B.16)

$$p_3^c \delta \bar{q}_3^c = 0. \quad (\text{B.19})$$

This means that in order to get a non-vanishing mass ($p_3^c \neq 0$), we were forced to assume that the variation of the lapse vanishes at the boundary, $\delta \bar{q}_3^c = 0$. To avoid this requirement one can now proceed as Kuchař does, i.e. introduce a 'proper time function' τ instead of \bar{q}_3^c given by

$$\dot{\tau} = -\bar{q}_3^c. \quad (\text{B.20})$$

Consequently, one may keep variations of τ and \bar{q}_3^c arbitrary at the boundary and obtain the boundary action

$$S_B = \int_{\partial\mathcal{M}} dx^0 \dot{\tau} M. \quad (\text{B.21})$$

After going to the reduced phase space, these are the only remaining physical degrees of freedom, and therefore $\dot{p}_3^c = \dot{M} = 0$, i.e. mass conservation is implied on-shell (see section 3.5 in [11]). In principle, one could apply the 'trick' of introducing an additional derivative as in (B.20) several times, thereby producing an arbitrary number of 'time' derivatives in the boundary action. However, after the second time mass conservation is no longer implied on-shell, so in this sense the action (B.21) is preferred.

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