Generic Effusion Cooling for Structures Exposed to Plasma

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Mass injection cooling is a technique for making structures exposed to hot gasses and plasmas resistant to the heat flux. For this aim, the outer parts of the structure are equipped with small boreholes or channels in the solid through which a coolant is conducted.

In former research activities at the IRS, mass injection cooling was used for the estimation of the enthalpy in plasma: By measuring the temperature of a hemispherical probe and knowing the coolant's mass flow, the plasma's enthalpy was concluded.

This contribution focuses on the inverse problem: A hemispherical body will be exposed to plasma of known enthalpy. The coolant's mass flow will be calculated so as not to exceed a given temperature.

Here, the model of this concept will be described with the respective equations. The transformation will be presented.

Key Words: Mass Injection Cooling, High Enthalpy Plasma

Nomenclature

B	:	mass addition momentum ratio
c_p	:	heat capacity at constant pressure
f	:	normalized tangential velocity
g	:	normalized enthalpy
h	:	mass specific enthalpy
j	:	exponent of body of revolution
$\dot{m}$	:	mass flow
p	:	pressure
Pr	:	Prandtl number
$\dot{Q}$	:	heat flux
$\dot{q}$	:	mass specific heat flux
r	:	radius
T	:	Temperature
$\vec{u}$	:	forced flow velocity vector
u	:	forced flow x-direction
v	:	forced flow y-direction
η	:	transformed tangential direction
μ	:	dynamic viscosity
ξ	:	transformed normal direction
ρ	:	volumetric density
Φ	:	dissipation function
∇	:	Nabla operator
Δ	:	Laplace operator

Subscripts

0	:	without coolant
∞	:	infinite bottom / upstream
cond	:	conductive
e	:	boundary layer limit or edge
ext	:	exterior
i	:	interface of structure to continuum
rad	:	radiative

1. Introduction

More and more applications in aerospace and in industry encompass the problem of extremely hot structures or structures exposed to excessively hot environment. Both the improvement of known technology and the implementation of advanced concepts may lead to such. For example, specific impulse of thermal propulsion might be better if it was possible to attain higher temperature in the combustion chamber. Unfortunately, known materials have melting temperatures below 4000 K[1)] constituting the dominant limitation. But there are various concepts to raise the enthalpy without melting or ablating the structure[2)].

The most prominent ones use an additional mass flow coming from the outside and cooling the structures. Because this mass flow is volitional, they can be qualified as *active* cooling in contrast to *passive* cooling like radiation or chemical processes[3)].

Different approaches in active cooling may be distinguished. The simplest one is dump cooling: The structure is given ducts through which coolant is flowing collecting heat transferred to the structure and dumping it outside of it. Another simple method is film cooling, which consists in adding the additional mass flow from the infinite upstream directly to the boundary layer around the structure. In the case of regenerative cooling, dump cooling is realized in a way combining it with film cooling. The coolant ducts lead to the point of the structure closest to the hot stream. There, one can find either a porous interface or bore holes through which the mass injection to the boundary layer is effected[3-5]. This approach can be generalized by making the mass addition occur all along the exposed surface of the structure. In the case of liquid coolants, this concept is called *transpirative cooling* and in the case of gaseous coolants *effusive cooling*.

Applications for such cooling methods other than the improvement of conventional combustion chambers and nozzles are among many others ceramic turbine blades, parts of plasma incinerators[6] or plasma space propulsion covering also fusion space propulsion[7-10] and diagnostics of high enthalpy plasma[4,5,11,12].

The latter has been studied at the Institute of Space Systems of Stuttgart University (IRS). In the mid 1990s, a probe has been developed by Stöckle et al.[4,11] to estimate the specific enthalpy of plasma currents in the facilities of IRS. The probe used the information of coolant mass flow and structure temperature.

The present contribution aims at presenting how to invert this method for the sake of estimating the necessary effusive cooling mass flow for a given applied stream's enthalpy and with respect to a structure's limiting temperature.

In the next section, the basic idea of the cooling concept is developed. In the following one, the boundary layer equations are given and explained before the transformation used by[5,12] is recapitulated and an extension for radiation proposed in section 4. The last section summarizes this modest contribution and offers an outlook.

2. Formulation of basic case for effusive cooling

First, an idealised porous semi infinite structure is considered for a field free two dimensional model. At its surface it is touching a hot, homogenous continuum with a forced steady-state laminar and compressible flow. At the structure's infinite bottom, a gaseous medium is injected at a mass specific enthalpy. Once the medium reaches the surface, it is at the mass specific enthalpy. The augmentation of enthalpy is delivered by the external heat flux $\dot{Q}_{ext}$. This system is depicted in figure 1.

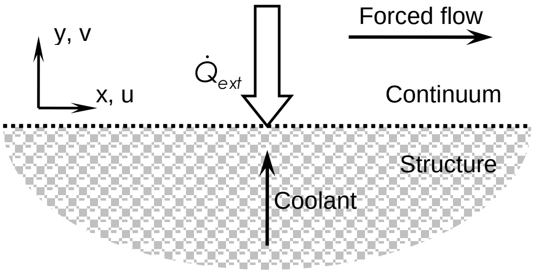

Fig. 1. Basic Case

Further, it is assumed that the structure with the contained medium is in steady state and that the transfer of heat to the medium is complete. Thus, the first law of thermodynamics is written

$$\dot{Q}_{ext} = \dot{m}\,(h_i - h_\infty). \qquad (1)$$

If the mass specific enthalpies – h_i at the interface, h_∞ at the infinite bottom – of the medium are known, this equation can be solved to conclude the mass flow necessary to fulfil this situation. If the structure is allowed for heat transfer and radiation, equation (1) modifies to

$$\dot{Q}_{ext} = \dot{m}\,(h_i - h_\infty) + \dot{Q}_{cond} + \dot{Q}_{rad}, \qquad (2)$$

in which a conductive part $\dot{Q}_{cond}$ of the heat flow on structure's side of the interface appears. The part $\dot{Q}_{rad}$ takes radiation from the structure surface in account. Note that the condition for this is a sufficiently transparent continuum on the other side, i.e. that the radiation's penetration depth is larger than 0.

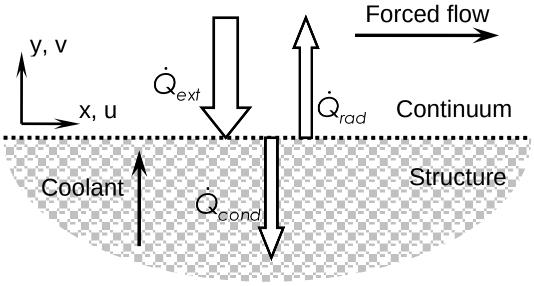

Fig. 2. Extended basic case

The part $\dot{Q}_{cond}$ will be dumped at the infinite bottom of the structure; the part $\dot{Q}_{rad}$ will be dumped in the continuum atop for finite penetration depths. Both parts will contribute to the cooling of the structure.

The part of heat stored into the gaseous medium for effusive cooling will be dumped in the infinite downstream of the continuum.

3. Formulation of a boundary layer problem

The equations of the boundary layer of the continuum described in the previous section are the equations of continuity[13-16], of momentum conservation[15] and of energy conservation[16] respectively in general form:

$$\nabla(\rho\vec{u}r^j) = 0\,, \qquad (3)$$

$$\rho\big((\vec{u}\nabla)\vec{u}\big) = -\nabla p + \mu\Delta\vec{u}\,, \qquad (4)$$

$$\rho\big(\vec{u}\nabla h_f\big) = \vec{u}\nabla p + k\Delta T + \mu\Phi\,. \qquad (5)$$

In these equations, ρ is the mass specific density of the continuum, $\vec{u}$ the flow velocity vector field, μ the fluid's dynamic viscosity, k the fluids thermal conductivity h_f and the fluids total enthalpy. The term $\nabla(\rho\vec{u}r^j)$ uses j = 0 in the case of two dimensional modelling and j = 1 in case of symmetries of rotation[5]. The last member of eq. (5) is the dissipation function Φ. The Nabla and Laplace operators for two dimensional Cartesian coordinates are used to fit the formulation of the problem.

The term ∇p designates the gradient of the pressure in the continuum. At the boundary limit, viscosity becomes negligible and hence the equation of momentum conservation can be approximated by an Euler equation[5,15]. Writing down eq. (4) at the limit of the boundary layer, one obtains

$$\frac{\partial p}{\partial x} = \frac{dp}{dx} = -\rho_e u_e \frac{du_e}{dx} \qquad (6)$$

and

$$\frac{\partial p}{\partial y} = 0 \,, \tag{7}$$

which is also the boundary layer equation of conservation of momentum in direction of y.

With

$$\frac{d^2 u}{dx^2} = 0 \tag{8}$$

eq. (4) becomes

$$\rho\left((\vec{u}\nabla)\vec{u}\right) = \rho_e u_e \frac{du_e}{dx} + \mu \frac{d^2 u}{dy^2} \tag{9}$$

in direction of x.

Respective boundary conditions for the problem are listed in table 1.

Table 1. Boundary conditions.

$\vec{u}(x,0) = \begin{pmatrix} 0 \\ v_i \end{pmatrix}$	Flow tangential component zero due to full viscosity on interface; normal component at interface crossing velocity of mass injection
$\vec{u}_e(x) = \vec{u}(x,\infty)$	Flow field at boundary layer's limit undisturbed
$h_f(x,0) = h_{f,i}$	Mass specific enthalpy at interface constant
$h_{f,e} = h_f(x,\infty)$	Mass specific enthalpy at boundary layer's limit as in undisturbed continuum flow

4. Transformation for hemispherical body

It is possible to apply solutions found for the problem given in the precedent section to problems with a point of stagnation. This is due to these problems' self-similarity of the boundary layer. The Lees-Dorodnitsyne transformation may be used to simplify the problem[5,12]. Its definition is

$$\xi = \int_0^x \rho_e \mu_e u_e r^{2j} dx \tag{10}$$

for the tangential component and

$$\eta = \frac{u_e}{\sqrt{2\xi}} \int_0^y \rho r^{2j} dy \tag{11}$$

for the normal one. Applying it to the equation of conservation of momentum, one obtains[12]

$$\left(f'' \frac{\rho\mu}{\rho_i \mu_i}\right)' + ff'' + \frac{1}{2}\left(\frac{\rho_e}{\rho} - f'^2\right) = 0 \tag{12}$$

and applying it to the equation of conservation of energy[12]

$$\left(g' \frac{\rho\mu}{\rho_i \mu_i \,\mathrm{Pr}}\right)' + fg' = 0 \quad , \tag{13}$$

in which Pr designates the Prandtl number.

In eq. (12) and eq. (13),

$$f'(\eta) = \frac{df}{d\eta} = \frac{u}{u_e} \quad , \tag{14}$$

the normalized velocity and

$$g(\eta) = \frac{h}{h_e} \quad , \tag{15}$$

the normalized mass specific enthalpy are used. As for the heat flux to the wall[5],

$$\dot{q} = \frac{k}{c_p} \frac{u_e \rho h_\infty}{\sqrt{2\xi}} \left.\frac{\partial g}{\partial \eta}\right|_i \quad , \tag{16}$$

it can be shown that it depends on the mass flow of gaseous coolant, as it can already be seen in eq. (1). Moreover, the following relation can be approximated using numeric approaches for the heat flux in the stagnation point[12]

$$\frac{\dot{q}}{\dot{q}_0} = 1 - 0.72B + 0.13B^2 \,, \tag{17}$$

with (heat flux without index) or without (heat flux with index) effusion with a mass addition momentum ratio

$$B = \frac{\rho_i v_i}{\rho_\infty v_\infty St_0} \quad . \tag{18}$$

Here, St_0 is the Stanton number.

It is possible, to find a minimum of eq. (17) by deriving it with respect to *B*. The first derivation yields

$$\frac{d}{dB}\left(\frac{\dot{q}}{\dot{q}_0}\right) = -0.72 + 0.26B \tag{19}$$

and the second one the positive constant 0.26, reassuring, that there is a minimum value for the ratio of heat flux with and without effusion. The mass addition momentum ratio is at about 2.8 and one obtains a ratio of about 0.003. A plot of the function is shown in fig. 3.

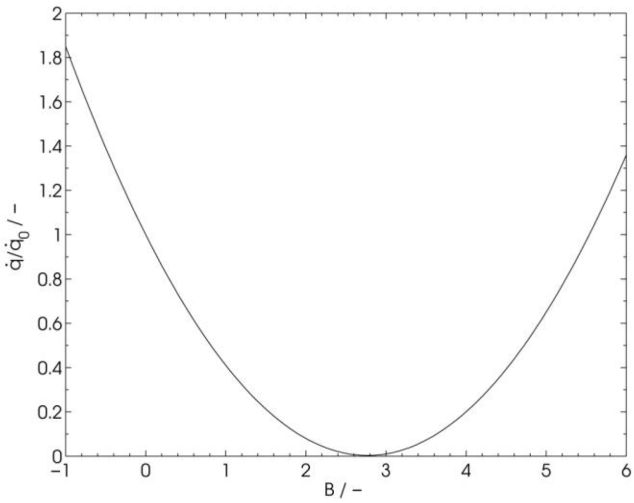

Fig. 3. Heatflux ratio as function of mass addition momentum ratio.

One thing, that can be seen in fig. 3 is that eq. (17) is giving heat flux ratios larger than 1 outside an interval of *B* ranging from 0 to about 5.6. On the left, there are negative *B* meaning that the effusion direction is backward, i.e. into the structure. Alternatively, one can state, that the interface velocity v_i is negative, i.e. fluid is entering the structure. However, the results on the right border of the interval need further investigation. They may state a limitation of the modelling and numerical solution. One has also to keep in mind, that it is actually possible to blow off the entire boundary layer, if the mass addition is too strong. In this case, the convective heat transfer ceases. Blow off has been reported[4].

For any further extension of the basic case which is connected to the condition depicted in fig. 1, one has to note, that there is a dependency of the heat flux ratio basing on the properties of the continuum. Hence, if extending to the case shown in fig. 2 a new relation of the type of eq. (17) will need to be detected.

This is due to the fact that the interfaces temperature will change in function of the applied coolant flow, and thus the power of the radiation will vary.

Also, considering radiation, properties like emissivity or absorptivity of the solid have to be considered. The refraction number of the boundary layer needs to be known[3)].

5. Summary and outlook

The present contribution aimed at summarizing knowledge about effusive cooling existing at the IRS. The working principle of a plasma probe based on effusion has been lined out. The proposal to extend the consideration by ration has been made. It has also been proposed to implement an inversion of the enthalpy probe principle for cooling applications. Related studies have started at the IRS. Among many others, parts of electric thrusters, plasma thrusters, advanced re-entry subsystems, experimental plasma generators and industrial or civil plasma incinerators (e.g. decomposition of bio hazards[6)], advanced fusion reactor and fusion propulsion concepts[7-10,17)], are applications aimed for. Currently, heavy activity began on the field of plasma and fusion propulsion[7-10)]. Results of the extended study of effusive cooling will play a decisive part in assessing certain fusion propulsion concepts' expected performance.

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