Flower Constellations as Rigid Objects in Space

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Abstract

This paper summarizes the findings and the research status on Flower Constellations , a novel and revolutionary way to design satellite constellations that has been discovered and proposed at Texas A&M University. The theory of Flower Constellations is a natural consequence of the theory of compatible (or resonant) orbits. The most surprising aspect of the Flower Constellations is that the satellite distribution identifies the edges of rotating figures whose shapes are time invariant. The complex synchronized dynamics of the satellites preserves the shape of a space object. The whole Flower Constellation is an axial-symmetric rigid object in space that is spinning with prescribed angular velocity. The shape of this object can be deformed by playing with the Flower Constellation design parameters, and the object's axis of symmetry can be set to point to any inertial direction. In particular, when the axis of symmetry is aligned with the Earth's spin axis, the J_2 linear-dominant effect is identical for all the orbits. In this case, the J_2 effect deforms the object shape while preserving the axial-symmetry.

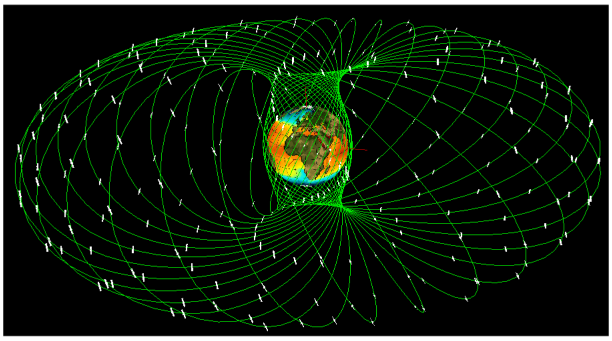
Introduction

The Flower Constellations constitute an infinite set of satellite constellations characterized by periodic dynamics. They have been discovered [1] on the way to the generalization of the concept of some existing satellite constellations. The dynamics of a Flower Constellation identify a set of implicit rotating reference frames on which the satellites follow the same closed-loop relative trajectory [2]. In particular, when one of these rotating reference frames is "Planet Centered, Planet Fixed", then all the orbits become compatible (resonant) with the Planet, and consequently, the projection of the relative trajectory on the planet becomes a repeating ground track.

As a particular case, the Flower Constellations can be designed as J_2 compliant [1, 3], that is with orbit compatibility that takes into account the linear effects of the J_2 perturbation. By considering the J_2 effect on these relative trajectories, it is possible to identify a set of critical inclinations associated with dynamically repeating relative trajectories, called repeating^2 ground track orbits [4], and to identify the two-way orbits [5], having identical and parallel perigee and apogee ground tracks, a property that allows us to design constellations observing the same geographical region from apogee and perigee, simultaneously. The recently proposed *Synodic* and *Relative* Flower Constellations [6, 7, 2] which use dual compatible orbits, as well as the results obtained in designing reconnaissance orbits for Earth sites [8] constitute key initial conditions for many potential research proposals as some of these designs would allow both long-term, stand-off surveillance, and episodic close-in inspection.

In the rotating reference frames the relative trajectories, which depend on five independent integer parameters, constitute a continuous, closed-loop, symmetric pattern of flower petals. Two integer parameters establish the orbit period and the other three distribute the satellites into an upper bounded number of admissible positions. One of the most important consequences of the Flower Constellation theory is that, for a particular set of the five integer parameters, the satellite distribution highlights the existence of *Secondary Paths* [9]. These Secondary Paths, which exhibit many beautiful and intricate dynamics and mysterious properties, are close to being fully understood, and the prediction of them appears to be linked to real algebraic geometry. Finally, the possibilities of re-orienting the Flower Constellation axis and playing with multiple Flower Constellations allow the design of a constellation of constellations, and constellations of formation flying schemes.

The Flower Constellation theory has been developed



at Texas A&M University. Along with the theory, the Flower Constellations Visualization and Analysis Tool (FCVAT) [10] has been developed and coded using Java and Java3D technologies. FCVAT software represents a truly fundamental breakthrough in satellite constellation design methodology, as it makes it easier to see and understand the complicated satellite dynamics, and to see the effects on the constellation of variations of the design parameters. This allows users to easily find different types of satellite formations which have been very difficult to construct using current methods. It is important to emphasize that, in order to design a Flower Constellation , a program like FCVAT must be first developed. Without such a specific program, or equivalent, the design (and the understanding) of Flower Constellation dynamics becomes very difficult or almost impossible.

The Flower Constellations are characterized by an axis of symmetry about which the constellation is rotating in the inertial space as a rigid body and with angular velocity of the rotating compatible reference frame. For Secondary Paths the angular velocity is related to four integer parameters, number of *peri-petals*, *peri-petal step*, number of *apo-petals*, and *apo-petal step* [7, 9].

The dynamics of a Flower Constellation can then be seen as consisting of two distinct parts: (1) an *internal part*, that describes the dynamics of the satellites within the "object-constellation", and (2) an *external part*, where the "rigid object" rotates in inertial space about a spin axis with an angular velocity that can be positive or negative. Some of the resulting shapes are shown in Figs 1 through 2, showing both the versatility and the infinite variety of possible shapes which we call "*choreographes*".

Flower Constellations thus open a new frontier in complex satellite constellations: in particular, these constellation-objects can be used as *building blocks* to construct configurations that can accomplish arbitrarily complex tasks. Indeed, just as the concepts of modularity and functionality gave important paradigm shifts in software design (allowing millions of similar tasks to be treated by one chunk of code), Flower Constellations provide building blocks to enable the creation of arbitrarily complicated ensembles of satellite orbits. Indeed, current approaches to satellites constellation are a simple by-product of the functionality they are designed for. By enabling the research community (and even the general population) to consider constellation as *rigid objects*, we enable new functionalities of satellites in urgently needed applications, and the study of even more intricate constellations for which functionalities have yet to be found.

Flower Constellations also allow us to profitably transform our intuition by thinking of trajectories in the solar system not just as shells like the LEO/MEO/GEO elliptical orbits, but rather as the union of several objects represented by different Flower Constellations . In interferometry, for example, a star-like Flower Constellation (see Fig. 17) can be thought as a unique radar-like antenna instead of as a collection of spacecraft.

Other interesting Flower Constellations are characterized by morphing capabilities, as for the morphing Flower Constellation shown in Figs. 3 and 4. This constellation has a dynamics that periodically changes from a five-loop aspect (Fig. 3) to an inscribed pentagon aspect (Fig. 4).

The particular dynamics of a Flower Constellation are obtained by introducing an automatic mechanism, ruled by a set of three integer parameters, to distribute the satellites into a limited set of "admissible locations". This is shown in Fig. 7, where 17 spacecraft are located on the same inertial orbit (green) and all of them belong to the same ECEF relative trajectory (red). These parameters rule the important phasing of the Flower Constellations . In this way, this new methodology to design satellite constellations has greatly simplified the constellation design problem and, thus, has provided the means to solve an extremely difficult family of problems.

Recently, two novel constellation design methodologies have been proposed [6, 2]. These are the Synodic and the Dual (or Relative) Flower Constellations , which constitute the important extension of Flower Constellations synchronized with the motion of two celestial objects (e.g., two planets) orbiting about the same gravitational mass. These two rotating reference frames can also be associated with natural or artificial satellites (e.g., moons, spacecraft) orbiting about a planet, and one of these frames can also be associated with the rotation of the central body itself. In particular, a Synodic Flower Constellation is made with orbits that are compatible with a reference frame rotating with a period suitably derived from the synodic period of the two objects, while a Dual Flower Constellation is made of orbits that are,

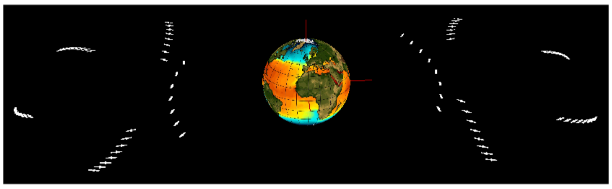


Figure 1: King Defense Flower Constellation

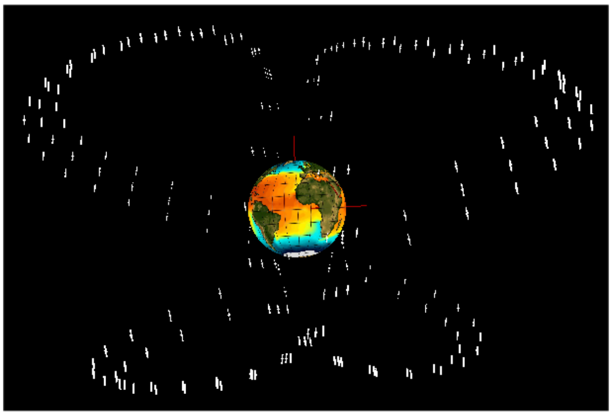


Figure 2: Two 3×3 formation flying loops

simultaneously, compatible with both the objects rotating reference frames. The latter, however, can be achieved under a very particular condition, that can be numerically approximated. The resulting constellation dynamics is synchronized with the dynamics of the geometrical rotation of the two objects.

The Flower Constellations , with its latest "*Synodic*" and "*Relative*" extensions, have already been partially investigated and some results for classical applications been obtained.

Compatible orbits

The Flower Constellation theory is built and derived from the theory of compatible (or resonant) orbits. The "compatibility" is a synchronization property between two rotating reference frames. Mathematically, the rotating reference frames, $\mathcal{F}_1$ and $\mathcal{F}_2$, are *compatible* or *resonant*, if their constant angular velocities, ω_1 and ω_2, satisfy the relationship

$$N_1\,\omega_2 = N_2\,\omega_1 \tag{1}$$

where N_1 and N_2 can be any integers. In the case the angular velocities are not constant, then $\mathcal{F}_1$ and $\mathcal{F}_2$ are compatible iff $\omega_1(t)$ and $\omega_2(t)$ are periodic functions. In this case, the compatibility is specified by the relationship

$$N_1 \int_0^{T_2} \omega_2 \, dt = N_2 \int_0^{T_1} \omega_1 \, dt \tag{2}$$

where T_1 and T_2 are the periods of the rotating frames.

The orbit mean motion n is a fictitious constant angular velocity associated with the periodic motion of the satellite along its orbit. Therefore, an orbit is compatible, with respect to a reference frame $\mathcal{F}$ rotating with angular velocity ω, if the orbit period T satisfies the relationship

$$N_p\,T = N_p\,\frac{2\pi}{n} = N_d\,\frac{2\pi}{\omega} \tag{3}$$

where N_p and N_d are two positive integers characterizing compatible orbits. Alternatively, the definition of a compatible orbit can be expressed by saying that an orbit is compatible when the ratio of its period with that of the rotating reference frame is rational.

Equation (3) simply states that after N_p orbital periods the rotating reference frame has performed N_d complete rotations and, consequently, the satellite and the rotating reference frame come back to their initial positions. This implies that the trajectory of the satellite in the rotating reference frame - the *relative trajectory* - is a continuous closed-loop trajectory that can be seen as a closed-loop 3-dimensional space track.[1] Two examples are provided in Figs. 5 and 6. In particular, $N_p\,T$ represents the time required by the satellite to repeat the entire relative trajectory.

In the Flower Constellation theory the two integers, N_p and N_d, are identified as the *Number of Petals* and the *Number of Days*, respectively. The reason for that arises because if the rotating reference frame is selected to be

[1]Even though compatible orbits are known as "repeating ground track" orbits, we want to highlight the distinction between these two definitions. A ground track is just the projection of the closed-loop trajectory on the Earth surface. This projection does not contain the full information of the 3-D trajectory. Moreover, the set of compatible orbits is just a subset of the repeating ground track orbits set (e.g., any equatorial orbit is repeating ground track!).

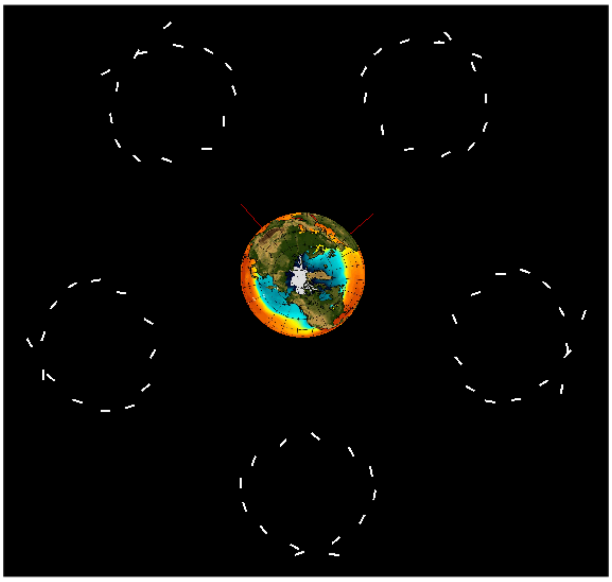


Figure 3: Morphing Flower Constellation : Five-loop

Earth-Centered Earth-Fixed, then N_d really represents the number of days to repeat the relative trajectory, while the *Number of Petals* N_p, that actually represent the number of orbit revolutions, finds its origin because of the petal-like shape of the relative trajectory.
It is important to understand that an orbit, that is compatible with respect to an assigned rotating reference frame, is also compatible with an infinite set of rotating reference frames. In fact, an orbit satisfying Eq. (3) is also compatible with all the reference frames $\mathcal{F}'$ rotating with angular velocity

$$\omega' = \omega \left(\frac{N'_d}{N'_p}\right) \left(\frac{N_p}{N_d}\right) \tag{4}$$

where N'_p, N'_d, and ω', satisfy the compatibility condition

$$N'_p \, T = N'_d \, \frac{2\pi}{\omega'} \tag{5}$$

Flower Constellation Phasing

Flower Constellations are built with the constraint that all the satellites belong to the same relative trajectory. In order to obtain the mathematical relationship stating this property, let us consider two identical compatible orbits having node lines displaced from each other by $\Delta\Omega$. Let us consider, as initial condition ($t = 0$), the satellite in the first orbit be located at pericenter ($M = 0$). This implies that the time interval Δt spent by the rotating reference frame to rotate of $\Delta\Omega$ (the relative trajectory is fixed in the rotating reference frame) must be identical to that associated with the increase of the mean anomaly of the spacecraft along its orbit. Therefore, we can write the relationship

$$\Delta t = -\frac{\Delta\Omega}{\omega} = \frac{\Delta M}{n} \tag{6}$$

where ω is the angular velocity of the rotating reference frame and n the orbit mean motion. The reason of the negative sign depends on the fact that for positive ω, the inertial orbits rotate clock-wise while Ω increases counter clock-wise. Figure 1 of Ref. [1] helps in understanding it.
Equation (6) shows a direct relationship between right ascension of the ascending node and mean anomaly. Substituting Eq. (3) in Eq. (6) we obtain

$$-N_p \, \Delta\Omega = N_d \, \Delta M \tag{7}$$

This relationship, which represents the fundamental equation of the Flower Constellation phasing, allows us to evaluate the "*admissible locations*" where to place the constellation satellites in order they all belong to the same relative trajectory. In other words, if a satellite is located at position M_1 of the orbit characterized by Ω_1 then, in order to belong to the same relative trajectory, a satellite on a different orbit characterized by Ω_2 must be placed at position M_2, where

$$M_2 = M_1 - (\Omega_2 - \Omega_1) \, \frac{N_p}{N_d} \tag{8}$$

In the case the second orbit coincides with the first orbit ($\Omega_2 - \Omega_1 = F_h \, 2\pi$, where F_h can be any integer), then Eq. (8) highlights all the admissible locations per orbit

$$M_{1,k+1} = M_{1,k} - F_h \, 2\pi \, \frac{N_p}{N_d} \tag{9}$$

where $F_h = 0, 1, \cdots, N_d - 1$. Equation (9) allows us to state the following:

1. Two satellites on the same orbit and displaced by $\Delta M = 2\pi \, F_h \, N_p/N_d$, where F_h can be any integer, belong to the same relative trajectory.

2. The number of admissible locations per orbit in Flower Constellation is N_d.

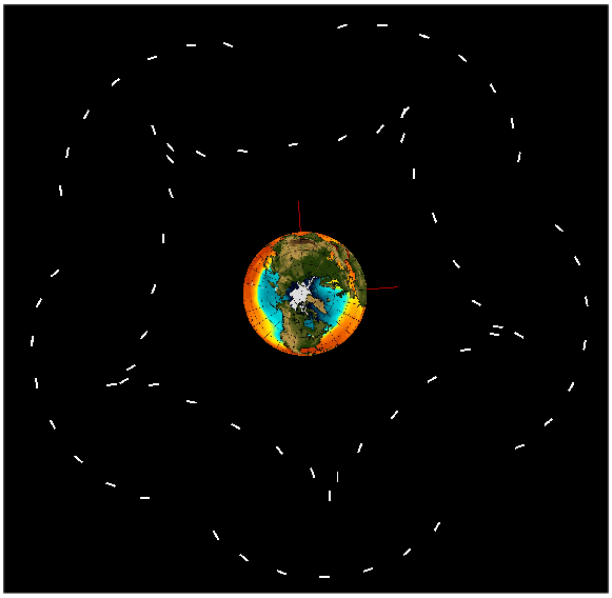


Figure 4: Morphing Flower Constellation : Inscribed pentagon

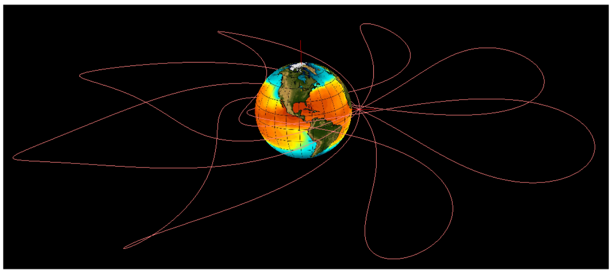


Figure 5: Relative trajectory example #1

Equation (8) provides us with the *natural* admissible location where to place the satellite. However, since there are N_d admissible locations per orbit, all the admissible locations in the orbit characterized by Ω_2 are provided by the relationship

$$M_2 = M_1 - (\Omega_2 - \Omega_1)\,\frac{N_p}{N_d} - F_h\,2\pi\,\frac{N_p}{N_d} \qquad (10)$$

Consequently: we have the complete free choice of where to place the first satellite (Ω_1, M_1), but when this is done then, for any assigned number N of orbits (not necessarily evenly distributed in 2π), the admissible locations are all defined. It is clear that evenly orbits distribution are preferred whenever the symmetry is desired. In the Flower Constellations this is obtained by selecting a rational value of 2π for the orbit node lines step

$$\Delta\Omega = \Omega_{k+1} - \Omega_k = 2\pi\,\frac{F_n}{F_d} \qquad (11)$$

where F_n and F_d can be any two integers. If the first orbit is selected having $\Omega_1 = M_1 = 0$, then Eq. (11) provides us with the orbit node lines sequence

$$\Omega_k = 2\pi\,\frac{F_n}{F_d}\,(k-1) \qquad (12)$$

while Eq. (8) the associated admissible locations

$$M_k = 2\pi\,\frac{F_n\,N_p + F_d\,F_h}{F_d\,N_d}\,(1-k) \qquad (13)$$

Equation (13) governs the sequence of the mean anomalies, which is dictated by the rational parameter $(F_n\,N_p + F_d\,F_h)/(F_d\,N_d)$. This ratio might be further simplified. To this end, let $C = \gcd(F_n\,N_p + F_d\,F_h, F_d\,N_d)$. This implies that Eq. (13) can be re-written in the following simplified way

$$M_k = 2\pi\frac{F_nN_p + F_dF_h}{F_dN_d}(1-k) = 2\pi\frac{R_n}{R_d}(1-k) \quad (14)$$

where $k = 1, 2, \ldots, N_s$, and where

$$R_n = \frac{F_n\,N_p + F_d\,F_h}{C} \quad \text{and} \quad R_d = \frac{F_d\,N_d}{C} \qquad (15)$$

Equation (14) implies that when we come back to the initial orbit with the sequence index $k = F_d + 1$, then the mean anomaly of the second satellite belonging to the first inertial orbit is

$$M_{F_d+1} = -2\pi\,\frac{R_n}{R_d}\,F_d \qquad (16)$$

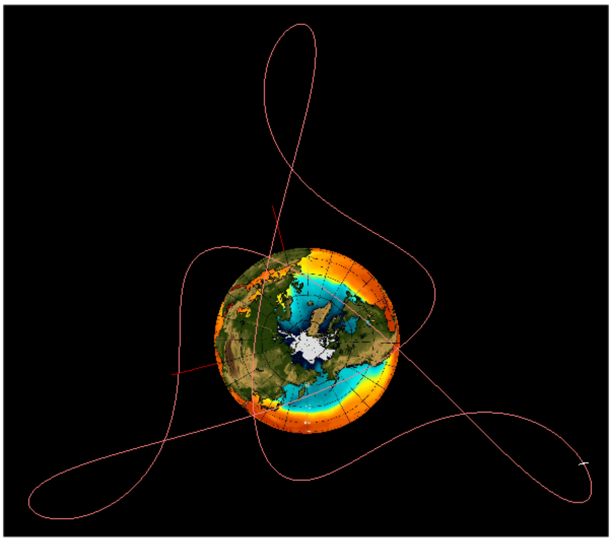


Figure 6: Relative trajectory example #2

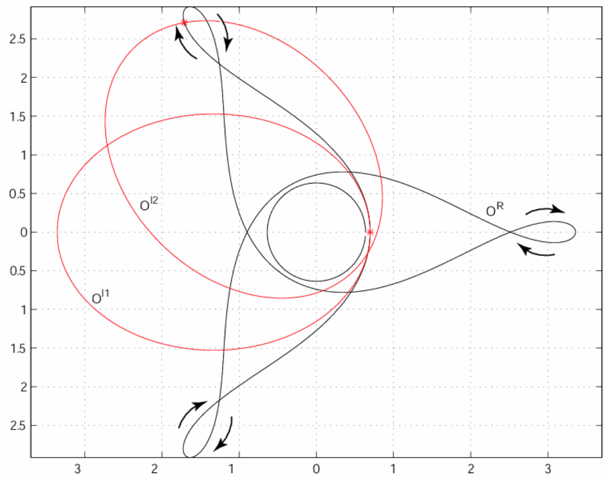


Figure 7: Phasing Geomery

Let $C_r = \gcd(F_d, R_d)$. Therefore, the integer

$$N_{so} = \frac{R_d}{C_r} \qquad \text{where} \qquad N_{so} \leq N_d \tag{17}$$

represents the number of satellites per orbit for the chosen distribution sequence and constellation. As a consequence, the total number of satellites will be

$$N_s = N_{so}\, F_d \leq N_d\, F_d \tag{18}$$

The parameter N_{so} also represents the number of loops around the Earth that are completed while placing satellites in a Flower Constellation . That is to say, if one places a single satellite in each of F_d orbits, it will take you N_{so} cycles to place all the satellites. Now, since N_d represents the overall number of admissible locations in one orbit, then N_{so} tells you how many of the N_d locations are filled in a given satellite distribution. Therefore, if $N_{so} = N_d$ then all the available admissible spots are filled, while if $N_{so} = 1$ then only one admissible spot (per orbit) is used. If $N_{so} < N_d$, then we define $N_s \equiv N_{so}\, F_d$ and describe the satellite distribution as forming a N_{so}/N_d *Secondary Path*.

In summary, associated with a given distribution sequence there is always an upper limit for the number of admissible locations where one can locate satellites. Therefore, a single Flower Constellation cannot be host to more than N_s satellites, where

$$N_s \leq N_d\, F_d \tag{19}$$

However, for an assigned sequence distribution, there exists the possibility that the sequence distribution does not fill all the $N_d\, F_d$ admissible locations. This happens when, during the satellite distribution, a satellite should be placed onto the initial location of the first satellite, that has already been occupied. When this happens the satellites are distributed along a *secondary path*, which is associated with a sequence distribution that creates a premature closing loop. Depending upon the number of satellites per orbit N_{so} constituting this particular distribution, a classification of the secondary paths is given. So, a secondary path having N_{so} satellites/orbit is called "secondary path of order N_{so}".

Secondary Paths

When all the admissible locations of a Flower Constellation are filled (especially when the number of these locations are many), the Flower Constellation dynamics reveals the shape of the relative trajectory by clearly showing the number of petals (apogees of the relative trajectory). In this case the whole constellation appears to be rotating, as a rigid body, with the angular velocity of the planet, if the orbits are compatible with the planet's spin rate. Sometimes, however, the phasing does not allow us to fill out all the admissible locations and it happens that the satellite distribution sequence comes back to the first position ($\Omega = 0$ and $M = 0$) before all the admissible locations are filled. When this happens, the Flower Constellation dynamics highlights the existence of *Secondary Paths* (SP) that have unexpected and beautiful shapes that are time invariant [3].

The immobility of the printed figure does not allow us to demonstrate the resulting complex shape-preserving dynamic. While complete Flower Constellations spin with a prescribed angular velocity (i.e. the same rate as that of the rotating reference frame), the spin rate of a secondary path should be quantified. Note that the angular velocity of a secondary path is *apparent* and not real. That is to say, the apparent angular rotation is not a motion that can be described by any particular dynamical relationship but rather is an artifact of the mathematics that generates a Flower Constellation . In other words, the appearing angular rotation IS NOT continuous but *appears* continuous. However, the continuity nor is discrete, as in the effect of the fast flow of photograms of motion pictures, because the satellites motion IS continuous. In effect, the angular motion pops up because of a particular combination of the continuity motion of a satellite along its orbit and the discrete separation of contiguous orbits.

Loops, petals, and jumping parameters

While in a complete Flower Constellation the satellites highlight the shape of the relative trajectory by moving along the single loop, in secondary paths the satellites can form single ($N_\ell = 1$) or multiple ($N_\ell > 1$) loops[2]. Figures (8) and (9) show a single- and a double-loop Secondary Paths, respectively. In the following, we introduce and explain the parameters characterizing the Secondary Paths and the relationships between them.

In addition to the number of loops N_ℓ, a Secondary Path is characterized by four integer parameters: the overall number of apogees (apo-petals), $N_{a\ell}$, the overall number of perigees (peri-petals), $N_{p\ell}$, and the two jumping-petal step parameters, $J_{a\ell}$ and $J_{p\ell}$, indicating the petal sequence visited by any satellite while moving from the petal k to the petal $(k + J_{a\ell})$ or $(k + J_{p\ell})$, where the apo/peri-petals are counted counter clockwise, and where $0 \leq J_{a\ell} < N_{a\ell}$ and $0 \leq J_{p\ell} < N_{p\ell}$.

Each loop is characterized by $N_{a\ell}/N_\ell$ apo-petals and by $N_{p\ell}/N_\ell$ peri-petals. Therefore, in a single loop the angles between any two consecutive apo-petals and between any two consecutive peri-petals are $2\pi\, N_\ell/N_{a\ell}$ and $2\pi\, N_\ell/N_{p\ell}$, respectively.

Most of the Secondary Paths are characterized by $N_{a\ell} = N_{p\ell}$. However, for some sets of design parameters, it is possible to have the number of apo-petals different from the number of peri-petals. As example, Fig. (10) shows a Secondary Path having $N_{a\ell} = 10$ and $N_{p\ell} = 5$.

In a Secondary Path the time required for a satellite to move from a petal to the next is, clearly, one orbit period. Therefore, in order to complete the loop, the value of the jumping parameter must be consistent with the number of petals. This consistency is mathematically defined by the following property: the greatest common divisor between the jumping parameter and the number of petals must be one for the apo-petals

$$\gcd(N_{a\ell}/N_\ell, J_{a\ell}) = 1 \tag{20}$$

and one for the peri-petals

$$\gcd(N_{p\ell}/N_\ell, J_{p\ell}) = 1 \tag{21}$$

This propriety assures the connection between all the petals of a loop, that is, it ensures that the satellites visit all the petals of the loop to which they belong.

After one orbit period, the satellite comes back to its initial position, but on a different petal of its loop[3]. After $N_{a\ell}$ orbit periods, the satellite has completed the visiting of all the apo-petals of his loop. In the case the Secondary Path has just an $N_\ell = 1$ single loop, $N_{a\ell} = 2$ apo-petals, and an $J_{a\ell} = 1$ jumping apo-petal step parameter, then after $N_{a\ell}$ orbit periods the Secondary Path is rotated by an angle 2π and, therefore, the Secondary Path angular velocity is $\omega = 2\pi/(N_{a\ell}\, T)$.

The expression for the general case, when the Secondary Path loop is characterized by any value of $N_{a\ell}$ and $J_{a\ell}$, can be easily derived. For the apo-petals we have two distinct solutions. One is associated with a clockwise loop rotation

$$\omega_{a\circlearrowright} = \frac{2\pi}{T} \cdot \frac{N_{a\ell} - J_{a\ell}}{N_{a\ell}} \tag{22}$$

while the second is associated with counter clockwise loop rotation

$$\omega_{a\circlearrowleft} = -\frac{2\pi}{T} \cdot \frac{J_{a\ell}}{N_{a\ell}} \tag{23}$$

[2]The shape of the loops in multi-loop Secondary Paths are all identical. They are just rotated one to another by an angle $2\pi/N_\ell$.

[3]If it comes back on the same petal, then the problem becomes trivial because it implies that the angular velocity of the Secondary Path is identical to that of the rotating reference frame. This cannot happen in Secondary Path but just in complete Flower Constellations .

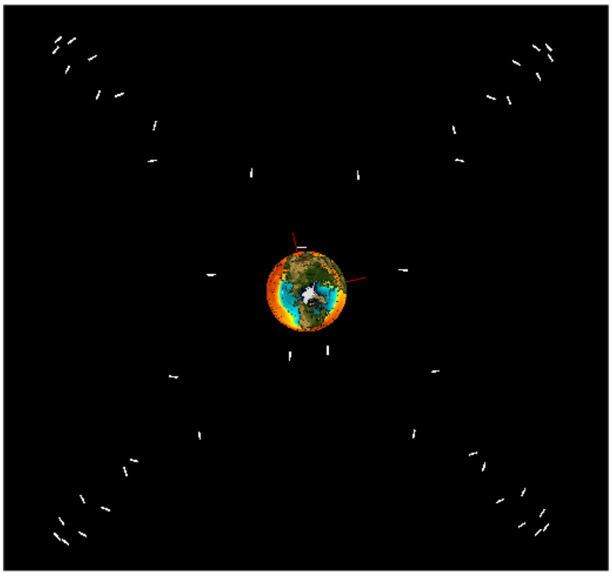


Figure 8: Single-loop Secondary Path

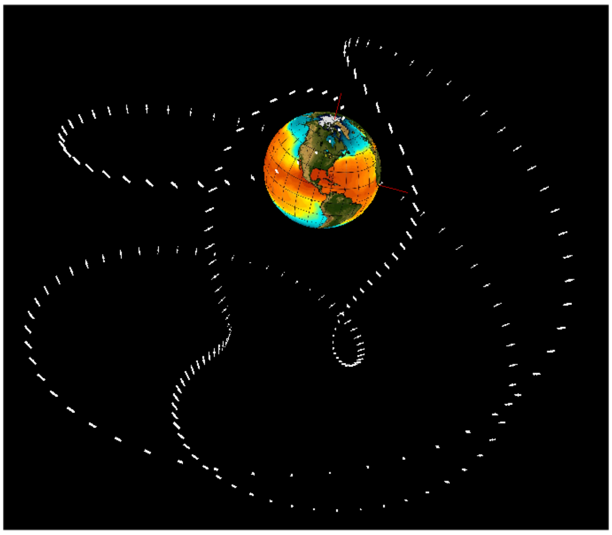


Figure 9: Double-loop Secondary Path

Analogously, for the peri-petals and in the general case when $N_{a\ell} \neq N_{p\ell}$ (and $J_{a\ell} \neq J_{p\ell}$), we have two distinct angular velocities: the clockwise

$$\omega_{p\circlearrowright} = \frac{2\pi}{T} \cdot \frac{N_{p\ell} - J_{p\ell}}{N_{p\ell}} \tag{24}$$

and the counter clockwise

$$\omega_{p\circlearrowleft} = -\frac{2\pi}{T} \cdot \frac{J_{p\ell}}{N_{p\ell}} \tag{25}$$

respectively. Equations (22) through (25) show that the angular velocity of a Secondary Path does not depend on N_ℓ,

Flower Constellation orientation

A non-oriented Flower Constellation has the characteristic property of having the axis of symmetry coincident with the planet's spin axis. The main reason is because two important orbital parameters - inclination and right ascension of the ascending node - are derived with respect to that axis. In the important case of choosing the constellation axis of symmetry as the planet's spin axis and the rotating reference as "planet-centered planet-fixed", then all the Flower Constellation satellites will travel along an identical repeating ground track. However, in general, the pointing of the axis of symmetry of a Flower Constellation is (as for the angular velocity of the reference rotating frame) a choice that is left completely free to users.

When choosing the constellation's axis of symmetry to

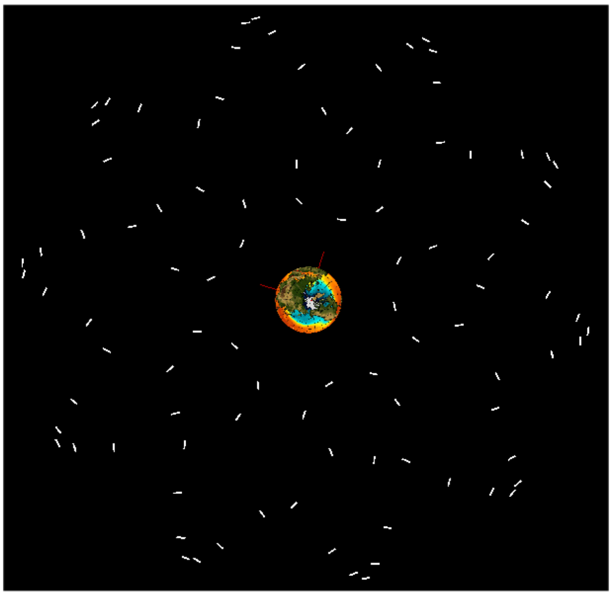


Figure 10: SP with $N_{a\ell} = 10$ and $N_{p\ell} = 5$

not be coincident with the planet's spin axis, then it is important to be aware that all the orbits of the constellation will have, in general, different inclination and right ascension of the ascending node. This implies that each orbit is subjected to different J_2 perturbations. Therefore, the deformation of the relative trajectory will be different for each orbit and, consequently, the beautiful symmetrical dynamics will be destroyed, unless using active control to compensate the relative perturbations and maintain the constellation dynamics.

To evaluate inclination and right ascension of ascending node of an oriented Flower Constellation we proceed as follows. Let $\mathbf{r}$ and $\mathbf{v}$ be the position and velocity inertial vectors (cartesian coordinates), respectively, of a generic satellite on a non-oriented Flower Constellation . In particular, $\mathbf{r}$ and $\mathbf{v}$ can be expressed in term of orbital parameters

$$\mathbf{r} = R_{OI}^{\mathrm{T}} \frac{p}{1 + e\cos\varphi} \begin{Bmatrix} \cos\varphi \\ \sin\varphi \\ 0 \end{Bmatrix} \tag{26}$$

and

$$\mathbf{v} = R_{OI}^{\mathrm{T}} \sqrt{\frac{\mu}{p}} \begin{Bmatrix} -\sin\varphi \\ e + \cos\varphi \\ 0 \end{Bmatrix} \tag{27}$$

where e is the orbit eccentricity, p the semilatus rectum, φ the true anomaly, and

$$R_{OI} = R_3(\omega)\, R_1(i)\, R_3(\Omega) \tag{28}$$

is the orthogonal transformation matrix moving from Inertial to Orbital reference frame. Matrices $R_1(\vartheta)$ and $R_3(\vartheta)$ are the matrices performing rigid rotation about the first and third coordinate axis

$$R_1(\vartheta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\vartheta & \sin\vartheta \\ 0 & -\sin\vartheta & \cos\vartheta \end{bmatrix} \tag{29}$$

and

$$R_3(\vartheta) = \begin{bmatrix} \cos\vartheta & \sin\vartheta & 0 \\ -\sin\vartheta & \cos\vartheta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{30}$$

Now, let

$$\hat{\mathbf{d}} = \begin{Bmatrix} \sin\alpha\,\cos\beta \\ \sin\alpha\,\sin\beta \\ \cos\alpha \end{Bmatrix} \tag{31}$$

be the direction of the desired Flower Constellation axis (where α and β are colatitude and longitude of the Flower Constellation axis in ECI). This implies that all the orbits of the Flower Constellation must be rotated by the angle α about the axis

$$\hat{\mathbf{a}} = \begin{Bmatrix} -\sin\beta \\ \cos\beta \\ 0 \end{Bmatrix} \tag{32}$$

The matrix performing such a rigid rotation is

$$R(\hat{\mathbf{a}}, \alpha) = I_3\, \cos\alpha + (1 - \cos\alpha)\, \hat{\mathbf{a}}\hat{\mathbf{a}}^{\mathrm{T}} + \tilde{A}\, \sin\alpha \tag{33}$$

where I_3 is the 3×3 identity matrix and

$$\tilde{A} = \begin{bmatrix} 0 & 0 & \cos\beta \\ 0 & 0 & \sin\beta \\ -\cos\beta & -\sin\beta & 0 \end{bmatrix} \tag{34}$$

is the skew-symmetric matrix performing the vector cross-product.

Now, the rotated orbit has, in general, new values for inclination, argument of perigee, and right ascension of ascending node that can be derived from the new rotated cartesian vectors

$$\mathbf{r}_n = R(\hat{\mathbf{a}}, \alpha)\,\mathbf{r} \qquad \text{and} \qquad \mathbf{v}_n = R(\hat{\mathbf{a}}, \alpha)\,\mathbf{v} \tag{35}$$

using well known transformations.

Dual-Compatible Flower Constellations

This section analyzes some particular Flower Constellations whose orbits are simultaneously compatible with two rotating reference frames. As it will be demonstrated later, for these Flower Constellations , we are no longer free to choose where to locate the orbit apsidal lines (i.e the values of Ω_k). Furthermore, the overall number of admissible locations strongly depend on the design parameters.

An orbit is Dual-Compatible (or dual-resonant) if, assigned the four integers N_{p1}, N_{d1}, N_{p2}, and N_{d2}, its orbital period T satisfies the two relationships

$$N_{p1}\,T = N_{d1}\,T_1 \qquad \text{and} \qquad N_{p2}\,T = N_{d2}\,T_2 \tag{36}$$

where

$$T_1 = \frac{2\pi}{\omega_1} \qquad \text{and} \qquad T_2 = \frac{2\pi}{\omega_2} \tag{37}$$

are the periods associated with two reference frames rotating with angular velocities ω_1 and ω_2, respectively.

Based on the above definition, any orbit characterized by orbit period T, is compatible with an infinity of rotating reference frames characterized by the set of angular velocities

$$\omega_k = \frac{2\pi}{T}\,\frac{N_{dk}}{N_{pk}} = n\,\frac{N_{dk}}{N_{pk}} \tag{38}$$

where n is the orbit mean motion.

In order to find out where to locate the satellite of a Dual-Flower Constellation , let us evaluate the RAAN variation between two consecutive satellites

$$\Delta\Omega = \Omega_{k+1} - \Omega_k \tag{39}$$

where to allocate one orbit $(k+1)$ with respect to the previous one (k). The satellite in the $(k+1)$-th orbit will have a variation of the mean anomaly with respect to the value of the previous satellite that can be evaluated using Eq. (10). The ΔM expression is

$$\Delta M = M_{k+1} - M_k = -\Delta\Omega\,\frac{N_p}{N_d} - 2\pi\,\frac{F_h}{N_d} \tag{40}$$

We can evaluate the variation ΔM using both sequences, $\mathfrak{F}_1 \triangleq \{N_{p1}, N_{d1}, F_n, F_d, F_h\}$ and $\mathfrak{F}_2 \triangleq \{N_{p2}, N_{d2}, F_n, F_d, F_h\}$. These two distributions, $\mathfrak{F}_1$ and $\mathfrak{F}_2$, must provide values for ΔM that can differ just of $2\pi\,\ell$, where ℓ can be any integer. Therefore, we can write that

$$\begin{aligned} \Delta M = & \; -\Delta\Omega\,\frac{N_{p1}}{N_{d1}} - 2\pi\,\frac{F_h}{N_{d1}} = \\ = & \; -\Delta\Omega\,\frac{N_{p2}}{N_{d2}} - 2\pi\,\frac{F_h}{N_{d2}} + 2\pi\,\ell \end{aligned} \tag{41}$$

This equation allows us to obtain an expression for $\Delta\Omega$ that is a function of the integer parameter ℓ

$$\begin{aligned} \Delta\Omega_\ell = & \; 2\pi\,\frac{F_h(N_{d2} - N_{d1}) + \ell\,N_{d1}\,N_{d2}}{N_{d1}\,N_{p2} - N_{d2}\,N_{p1}} = \\ = & \; 2\pi\,\frac{G_\ell}{G_d} \end{aligned} \tag{42}$$

which allows us to evaluate the values (therefore, the sequence) of the right ascension of the ascending nodes, Ω_k, where the two distributions locate satellites with the same values of the mean anomaly (same orbital position). In order to use Eq. (42), the condition $G_d \neq 0$, that is

$$N_{d1}\,N_{p2} \neq N_{d2}\,N_{p1} \tag{43}$$

must be satisfied. This condition implies that the case $T_1 = T_2$ should be avoided. In addition, the values of ℓ satisfying

$$\ell\,N_{d1}\,N_{d2} = F_h(N_{d1} - N_{d2}) \quad \leftrightarrow \quad G_\ell = 0 \tag{44}$$

which are associated with the condition $\Delta\Omega_\ell = 0$, allow us to obtain the sequence of all the solutions per orbit. This sequence can also be obtained for the values of ℓ giving $\Delta\Omega_\ell = 2\pi\,m$, which is satisfied when

$$\gcd\,(G_\ell, G_d) = G_d \tag{45}$$

For each value of $\Delta\Omega_\ell$ provided by Eq. (42), we have an associated value for ΔM_ℓ

$$\begin{aligned} \Delta M_\ell \;=\; & -\Delta\Omega_\ell\,\frac{N_{p1}}{N_{d1}} - 2\pi\,\frac{F_h}{N_{d1}} = \\ =\; & -\Delta\Omega_\ell\,\frac{N_{p2}}{N_{d2}} - 2\pi\,\frac{F_h}{N_{d2}} + 2\pi\,\ell \end{aligned} \tag{46}$$

Summarizing, a dual-compatible Flower Constellation is built using the phasing sequence

$$\left\{ \begin{array}{l} \Omega_{k+1} = \Omega_k + \Delta\Omega_\ell \\ M_{k+1} = M_k + \Delta M_\ell \end{array} \right. \tag{47}$$

where $k = 1, 2, \cdots$, and $\Delta\Omega_\ell$ and ΔM_ℓ are provided by Eqs. (42) and (46), respectively.

Examples and Potential Applications

The Flower Constellations and the recently introduced Dual Flower Constellations combine a number of new attractive features suitable for many potential classic applications (communications, Earth and deep space observation, coverage, navigation systems, etc.), as well as for new and advanced concepts. Some Flower Constellations schemes can be suitable for very futuristic applications while other can be of immediate use. Let us briefly describe soem of these potential applications:

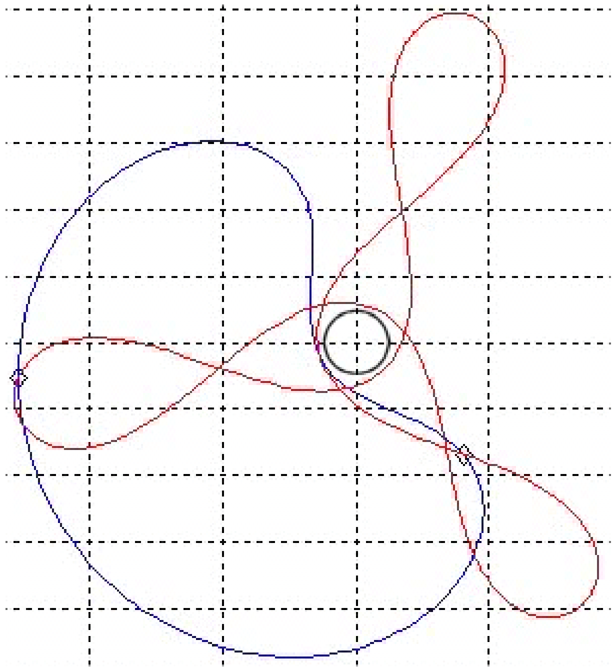


Figure 11: Dual Flower Constellation Example #1

1. *Space Network Architecture* (SNA) for planetary communications. Interplanetary communications are presently performed by means of single-hop links. In this simple architecture there is one node at the exploration planet (e.g. Mars) and one node at the Earth (specifically, the antennae of the NASA Deep Space Network). This simple architecture presents two severe constraints: it requires direct visibility (and hence limited duration operation) and it does not tolerate node failure. Using Dual Flower Constellations we can design a constellations that are synchronized with the motion of two rotating reference frames (e.g., Earth and planet orbital periods). Dual Flower Constellations [6, 7, 2] can provide solutions that avoid the mentioned critical constraints and would improve the communications necessary for human planetary missions. The design of a SNA using Dual Flower Constellation could potentially consist of multi-hop links, a constellation of spacecrafts connecting the Earth with a mission planet (or moon) and would drive to improve the connectivity of the deep space network. Reference [6] introduced this idea and proposed some approximated solutions to help communication for future missions to Mars and/or Jupiter.

   Reference [7] provided novel insights on the theory (specially on the phasing rules) while the complete mathematical theory (tractatus) on Flower Constellations will be presented in Refs. [2] and [9]. Figure 11 and 12 show two Dual Flower Constellation examples. In these figures the relative trajectories, associated with two distinct and independent reference frames are provided. The spacecrafts are located at some "admissible" intersections of the two relative trajectories that rotates with different constant angular velocities.

   The design of a *Space Network Architecture* for planetary communications must take into proper consideration the effects of the orbital geometry on the network topology, and the resulting effects of path delay and handover on network traffic (due to the great distances involved). In addition to these problems, a wide variety of requirements and constraints must also be satisfied. These are:

   (a) *Service continuity*: if any one of the nodes becomes inoperative (either, permanently or momentarily), then the communications are still guaranteed.

   (b) *Power efficiency*: minimize inter-node angle variations (to narrow the antennae FOV), minimize inter-node distances (to limit communication power), etc,

   (c) *Time efficiency*: minimize the overall distance (to limit communication times), and

   (d) *Fuel efficiency*: seek to minimize the orbit maintenance requirements by optimizing amongst feasible orbit configurations.

2. *Solar Global Navigation System*. This can be investigated using Flower Constellations synchronized with a reference frame rotating with the Earth orbit mean motion. The existing

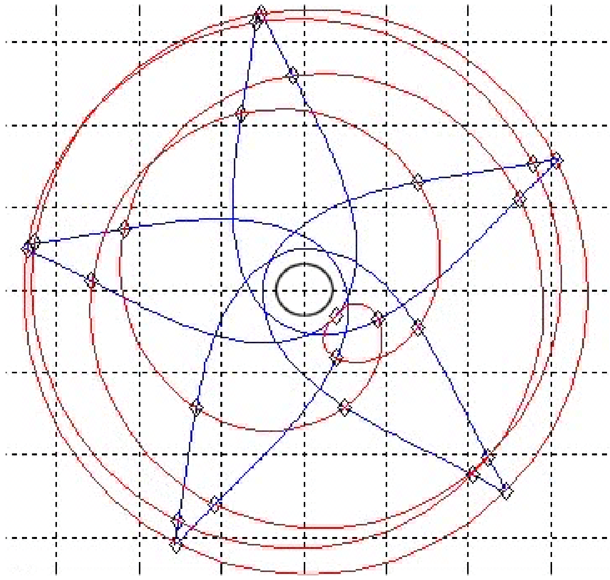


Figure 12: Dual Flower Constellation Example #2

Global Navigation systems (GPS, GLONASS, GalileoSat) are build using circular orbits (Walker constellations), only. European GalileoSat constellation is designed as in Fig. 13 with satellites lying on three orthogonal orbit planes.

This choice creates eight holes, one for each octant, which keep the satellites allocation far from being uniformly distributed in space. A first attempt to design a Global Navigation Flower Constellation (GNFC), has brought to the solution scheme shown in Fig. 14 [11, 12], where the optimality is defined as the most uniform satellite distribution along the relative trajectory. Using the same number of satellites as GalileoSat, GNFC provides better Geometric *and* Attitude Dilution of Precision parameters [11, 12] or the same level of accuracy with lesser satellites.[4]

3. *Space Dynamo*. The Faraday law of inductance states that a voltage is generated by a coil of wire when the magnetic flux enclosed by it changes. A space dynamo, for energy production in space, can be obtained using Flower Constellations with multiple Secondary Paths forming inclined circles. In this configuration, each Secondary Path can be considered a very long single wire where the circuit could be closed by electron cannons. By orbiting, each wire experiences the variation of planet's magnetic flux (planets are big magnets in space). In this way we pay for the energy induced on the wire by orbit decay. The orbit decay could be compensated by solar pressure if orbiting about the Sun.

Figures 15 and 16 show two Flower Constellations architectures for power production in space. To my knowledge no constellation architecture has been developed or proposed for power production in space. In the current thinking, Space Solar Power Satellites require the launching and assembling in space of a very large structure in order to be economically viable. I believe that some Flower Constellation concepts can remove the need for in-space assembly. This architecture would replace the bulky approach to producing large areas where energy is collected into smaller and cheaper components. The advantage would be to provide a means of assembling a large collecting area without making it a grand challenge. Figures 15 and 16 show two view of a potential configuration. We outline about the possibility to re-orient the whole constellation and the possibility that each circle can be differently oriented. The dynamics is double: the circles spin about their centers and all of them spin (as rigid body) about the Flower Constellation axis of symmetry.

4. *Pointing Architectures* for a hyper large directional instrument through the use of Flower Constellations . These architectures would have the following functional capabilities:

   (a) use the possibility to align satellites using elliptical orbits (see Fig. 17 where more alignments are combined to form a more complex "star"-object), to promote directional active and passive observation and transmission of particles and energy on length

[4]Reference [11, 12] have shown that a GalileoSat GDOP-accuracy can be achieved by a GNFC made of 27 satellites, only. Recent investigations on designing GNFC using genetic algorithms provide even better results, bringing down to 26 (or even 25) the total number of satellites.

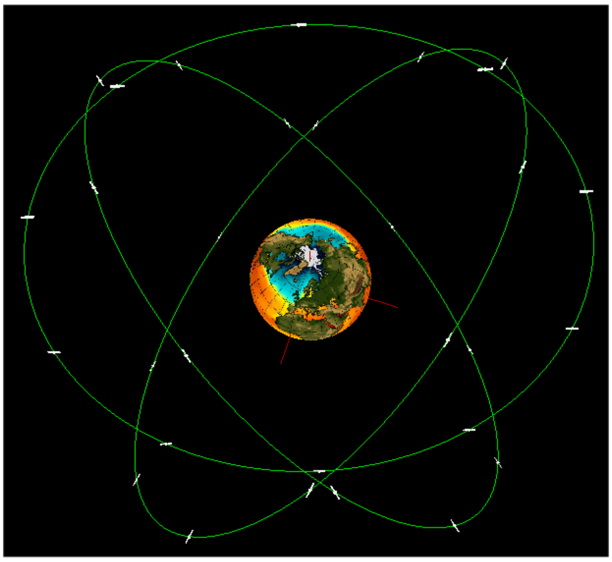


Figure 13: GalileoSat orbit planes

scale of the order of 1 AU or larger (passive observation can be performed through the pinpointing of a large lens assembly or the use of the interferometry configuration for the observation of planets of the solar system or other terrestrial planets outside the solar system)

(b) by collimating the Sun or other source of energy, one can provide active observation of materials of planets in our solar system by studying the properties of atmosphere and soil scattering of other planets with observatories on Earth. In particular, one could provide energy and particle transmission to future NASA mission probes beyond Jupiter by focusing energy or particles towards these probes, or provide a means of ablation to deflect asteroids.

(c) send signals beyond solar system by imitating common shapes on a very large scale.

(d) a small constellations fo surveillance and reconnaissance and well as for Space and Earth science.

5. *Laser Propulsion and Asteroid Deflection*. The idea is to design a Flower Constellation for laser propulsion of remote spacecraft using directional aligned lasers. The combination of lasers and satellites has captured the imagination of and interest of the scientific community. The potential applications, while futuristic at the current stage, are simply too important to discount: from laser beams that intercept and deflect earth-bound asteroids to laser propulsion in deep space, relaying and direction of the laser beams in the future will be accomplished by satellites. Despite the large advances in rocketry, the increases in payload capacity, and the effectiveness of orbit insertion, the possibility of putting massive lasers in orbit in the near future is minimal. If the laser beam pointing and steering can be accomplished by reflective and/or refractive elements on satellites, the potential exists for higher accuracy due to the lack of atmospheric beam steering, and reduction in the required laser power due to such accuracy. The Flower Constellation has one major advantage over other possible arrangements viz-a-viz laser beam manipulation: the existence of conserved paths with respect to the earth frame. One may think of ten satellites in a Flower Constellation as one large body in motion. Consequently, it is easier and more economical to aim one large-diameter beam towards the relaying Flower Constellation satellites than it is to aim several laser beams at every single satellite belonging in a general constellation. Severe beam decollimation occurs as laser beams propagate through the atmosphere, or even in vacuum. This is particularly true for the very powerful ultra-violet (UV) lasers that would have to be used for either laser propulsion or asteroid deflection. To some extent, the Flower Constellation turns this basic fact of laser propagation into an advantage: a circle of craft flying in a closed formation of a circle can be used for the relaying of a single laser beam of a given diameter. The precise choreography of the Flower Constellation craft makes it easier to generate these multiple laser beams in the first place.

The Flower Constellations represent a fundamental advance and a viable means to efficiently design new space objects, characterized by two, distinct, dynamics. The Flower Constellations represent a dramatic step forward with wide-ranging mission design impact, both

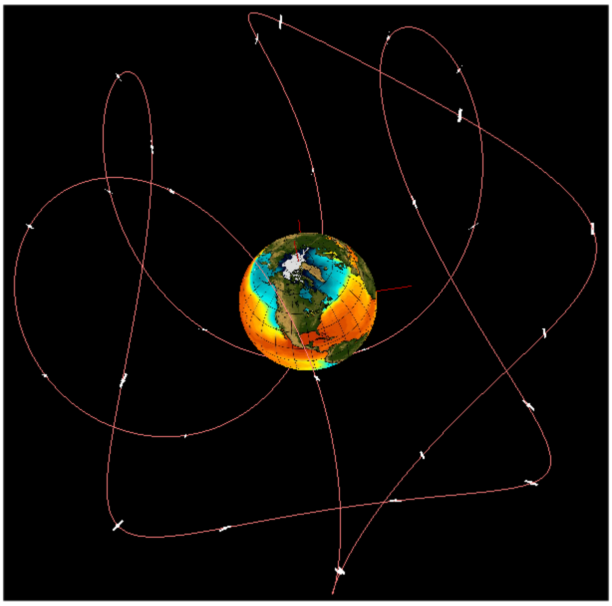


Figure 14: Flower Constellation Global Navigation

for future geocentric missions and the goals to move to the Moon, Mars, and beyond. In fact, new and more effective satellite constellations would strongly benefit many of the key strategic focus areas, already identified by NASA and ESA. In particular, we do expect to identify direct beneficiaries such as the Robotic and human lunar expeditions, the sustained, long-term robotic and human exploration of Mars, the robotic exploration across the solar system, the development of advanced telescopes searching for Earth-like planets and habitable environments, and the exploration of the Universe, of the dynamic Earth system, and of the Sun-Earth system.

It is obvious that in order to validate the proposed configurations, the analysis of the perturbations acting on each specific proposed constellations, must be done. Other validation criteria and performance metrics will be specific for each proposed solution. For instance, we may need to evaluate the type of particle/energy transmission capability that could benefit from these configurations (supercritical repeaters, collection of mirrors focalizing on one point) or evaluate the type of u-v-w plane capability for interferometry. In fact, in the interferometry systems currently evaluated by NASA, there is an expectation that a series of spacecrafts will be flying in constellation. This constellation flying constellation is likely to be restricted to a small baseline. We do expect to study how a line of sight between different spacecrafts can be attained using a Flower Constellation that enables a very large baseline. We do also expect to quantify the GNC system efforts needed to allow each spacecrafts to pinpoint in the right direction for interferometric observation.

Conclusion

Flower Constellations will have a large impact on future mission architectures and concepts. Flower Constellations are the 3-dimensional equivalents of “orbits with repeating ground tracks” which have been a staple for planet orbiting missions from Topex, EOS, to many of the planetary observation missions such as at Mars. Flower Constellations are ideal for studying 3-dimensional large scale structures and phenomena in space such as the detection of gravity waves, the study of magnetospheres, and radiation environment over vast regions around planets and moons. For example, such orbits could be used for the “Magnetospheric Constellation” that has been studied by NASA. Also, such orbits could provide the backbone for an “Inter-Planetary Network” for navigation and communication throughout the Earth-Moon system, at Mars and Jupiter, and eventually the entire Solar System. Such a “GPS” type network could enable the automation of many spacecraft functions including the adaptive on-board mission design and navigation in the future.

The 3-dimensional nature of these constellations enables the repeated visits of spatial locations to permit the study of the 3-dimensional structures of time varying phenomena in space around a planet. These constellations can be designed to enable complex, distributed instruments with virtual apertures extending 10s to 100s of km in diameter. As a bonus, if the axis of symmetry is aligned with the planet poles, these constellations actually have repeating ground tracks to boot.

The use of compatible orbits allows us to extend and enable the powerful techniques for studying planetary surfaces to 3-dimensional space around planets, as for

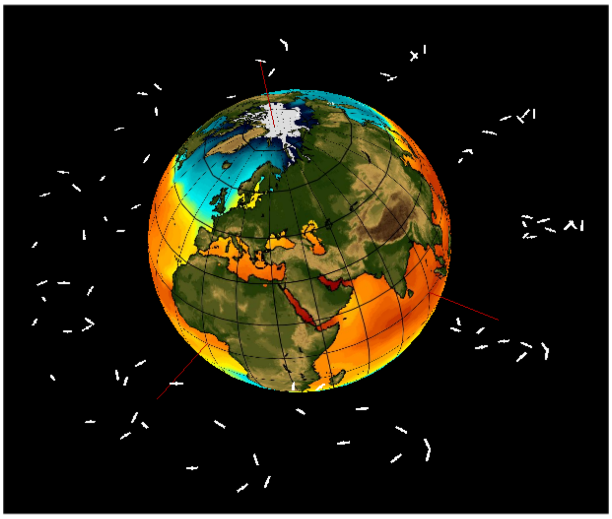


Figure 15: Flower Constellation Space Dynamo (view 1)

instance the planet's magnetosphere. By working with natural dynamics, Flower Constellations eliminate costly deterministic controls that limit current designs of more complex constellations around a planet.
Flower Constellations will have a dramatic impact on reducing cost and optimizing the functionality of satellite constellations for planetary exploration, as well as mapping the features of interest from orbit. The efficient use of natural dynamics reduces the number of control maneuvers required to maintain such constellations. This saves both propellant and operational costs. At the same time, the great variety of constellation patterns and 3 dimensionality of the constellation will enable new mission concepts and applications. Flower Constellations represent a fundamental advance in orbit design; the demonstration of a viable means to efficiently design Flower Constellations will represent a dramatic step forward with wide-ranging mission design impact, both for future geocentric missions and the goals to move to the Moon, Mars, and beyond.

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The author would like to thank Dr. Matthew Paul Wilkins and Christian Bruccoleri for being the two leading columns in the Flower Constellations discovery and research. In particular, Dr. Wilkins, one of my former Ph.D. student, has been instrumental in the Flower Constellations discovering and in the theory development. The results of his efforts are now published on his Ph.D. dissertation [3]. To help the developing and the understanding of the Flower Constellation theory, my Ph.D. student Bruccoleri developed FCVAT [10], an ad-hoc software program to visualize and to design Flower Constellations . Without FCVAT it would be practically impossible to take the Flower Constellations at today's stage of maturity.

I want also to express my gratitude to other two of my Ph.D. students: Dr. Keun Joo Park, and Dr. Ossama Abdelkhalik, presently still working with me as a post-doc. In particular, Dr. Park has been the first to design a Flower Constellation for Global Navigation System (GNFC), while Dr. Abdelkhalik has applied the theory to ground reconnaissance problem from space and has developed the theory of the two-way orbits.

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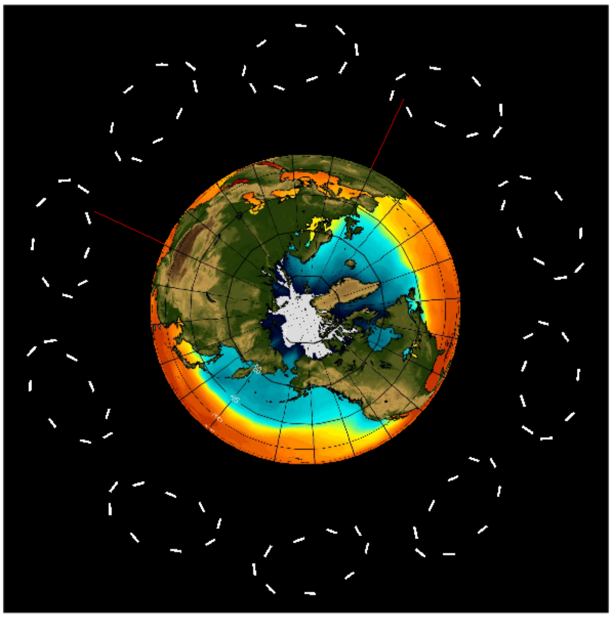

Figure 16: Flower Constellation Space Dynamo (view 2)

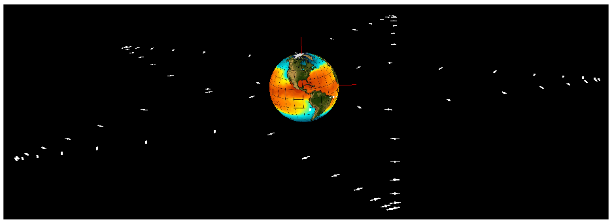

Figure 17: Lone Star Flower Constellation

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