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The relativistic double-embedding problem for the Solar system

(3-month study, prepared by A. Rathke)

Already current interplanetary missions have exceedingly high requirements on the maximally allowed unmodeled disturbance forces. For example the NASA mission Galileo required an upper limit on unmodeled accelerations of the order *nano*meters per second squared (nm/s^2) [1]. The new science missions, currently under preparation, which will employ drag-free formations of spacecraft [2,3,4], will put even put stricter requirements on the knowledge of the disturbances exerted by the space environment. Currently the unmodeled disturbances are still dominated by spacecraft inherent effects such as fuel leaks and thermal radiation [1]. However, these effects are expected to be far better controlled in the new generation of missions mentioned above. Hence other effects will become the major source of spacecraft navigation uncertainties. These new challenges will on the one hand be approached by new navigational techniques und the other hand by better models of the space environment [5].

From the smallness of the relevant disturbances it becomes obvious that also general relativistic effects need reconsideration. For a long time relativistic corrections to the planetary ephemerides have been taken into account in trajectory design for planetary spacecraft. These relativistic corrections from the gravitational field of the Sun are implemented in post-Newtonian approximation and are well under control. At the order of magnitude of nm/s^2 for the accelerations under considerations also large scale relativistic effects may become relevant. This is easily seen by noting that the magnitude of the Hubble acceleration is of the order of 0.7 nm/s^2. Hence the cosmological state of the universe may actually have an influence on spacecraft trajectories, which may become relevant in view of the planned future space missions.

Unfortunately, it is far from trivial to determine the impact of the cosmic expansion on the local dynamics of the Solar system. Since the work of Einstein and Straus [6] it is clear that a Schwarzschild metric embedded into a cosmic solution at least partially resists the influence of the cosmic expansion.
The Einstein-Straus problem has been reconsidered by Gatreau [7] and has been applied to the Solar system in [7] and [8] with contradictory results. Whereas [8] states that the cosmic expansion might lead to spacecraft disturbances at the magnitude of the Hubble acceleration and relates this effect to the observed anomalous acceleration of the Pioneer 10 and 11 spacecraft [10] the conclusion in [9] is that the effect induced by the cosmic expansion is one order of magnitude smaller than the Hubble acceleration. The problem for the Solar system is however even more complicated than indicated by the recent discussion in the literature because in extension of the Einstein-Straus problem a proper treatment requires to embed both the Schwarzschild metric of the solar system into that of the Galaxy and to embed the Schwarzschild metric of the Galaxy into the Cosmic background solution.

The goal of this study is to obtain a solution to this double embedding problem. The solution shall be obtained for both a de Sitter universe and for closed, flat and open Friedman universes without cosmological constant as the cosmic background solution. The calculations may rely on either analytical or numerical methods. The results shall comprise both the effective metric for Solar system and a calculation of the accelerations induced by the cosmic background solution on interplanetary spacecraft. A discussion of the obtained results and a methodological comparison to the work of [8,9] should be given. The presentation has to be at the level suitable for publication in a peer reviewed physical journal. A publication of the results is encouraged.

Task summary

- Review the embedding formalisms for the Schwarzschild metric into a cosmic background solution.
- Derive the metric of the double embedding problem for the cosmic solutions listed above by the best suited of the methods reviewed or with a newly developed method.
- Justify the choice of method.
- Derive the forces and accelerations acting on bodies in the solar system in the derived metrics.
- Interpret and discuss the results in comparison with previous studies of the Einstein-Straus problem and explain possible discrepancies.

References

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