04/4104 **A search for an invariant relative satellite motion**
Budget 25 KEUR
Duration 4 months

Background

Relative satellite motion plays an important role in the design of a number of advanced space mission concepts. Rendezvous, docking, swarm constellation design are all issues closely related to the understanding of relative motion. The classical models used to model spacecraft relative dynamics, are the Clohessey-Wiltshire[1] (Hill's equations) and the Tshauner-Hempel[2] models. In the past years the attention on relative satellite motion has been revived due to a greater need for precise control capabilities in space operations. Analytical results have been made available[3-7] to better describe the problem dynamics, and many works on control issues have also been published (LQR control, impulsive control and other concepts have been investigated).

Mission design would greatly benefit from a better understanding of relative motion dynamic. This might be shown with a simple example. Say we chose to design a mission in which our desired relative motion is a solution to the Hill's equations. Then our control effort would be mainly used to cancel all the perturbations due to linearization effects, to small eccentricity effects and to differential drag and J2 effects. This could, depending on the orbits, result in a quite demanding control effort in terms of DV required per day. If, on the other hand, the whole mission was designed exploiting some other periodic relative motion, solution to a more complete set of dynamical equations, we could drastically reduce the control effort by exploiting, rather than canceling, the above quoted effects.

The DV needed to track periodic relative motions is, in-fact, quite relevant whenever the desired relative dynamic is a solution to an incomplete dynamical model (not taking into account, for instance, differential drag effects, differential J_2 effects or eccentricity effects) and may drop by properly selecting the relative motion[5,8] model. It is, then, of the greatest interest to understand whether there is a solution to the relative dynamic between two or more satellites that is naturally periodic when a complete dynamical model is accounted for, and to what extent space missions may benefit from these solutions. Some results have been found by Shaub[8], by taking into account the sole average differential J_2 effects. Invariants orbits, should, though, exist also in a more complete dynamical model such as that considered by Wiesel[3], in that case they should be found by looking to the Poincarè exponents of the relevant time-periodical system.

Study Objectives

The objective of this study is to search for periodic solutions in an as-complete-as-possible model of the relative motion dynamic and to investigate the impact that these solutions may have in real space missions. Mission concepts, based on drag controlled relative dynamics, should be investigated and eccentricity effects should be accounted for.

In summary, the study objectives are as follows:

- Review recent results on invariant orbits and on relative dynamics modeling.

- Propose a suitable model describing relative satellite motion in the complex case of perturbed dynamic. (If the identified model is linear, with time periodic coefficients, Floquet theory could be applied and the conditions for the Poincaré coefficients to be purely imaginary found).

- Search for analytical expressions for the initial conditions that make the identified model return periodic solutions (if Floquet analysis is possible this should be straight forward), or architect a numerical scheme able to find them.

- Simulate with a full non-linear dynamic scheme the real dynamical evolution.

- Assess the benefits that the relative dynamic control system has from an enhanced dynamic modeling.

The study outcomes have to be at the level suitable for publication in a peer-reviewed journal. A publication of the results is encouraged together with an as close as possible cooperation with the ACT.

References:

[1]Clohessy, W.H., and R.S. Wiltshire, “Terminal guidance for satellite rendezvous”. *Journal of the Aerospace Sciences*, Vol. 27, No. 9, 1960, pp. 653-658.

[2]Tschauner, J., and Hempel, P., “Elliptic Orbit Rendezvous”. *AIAA Journal*, Vol.5, No.6, 1967, pp.1110-1113.

[3]Wiesel, W.E., “Relative Satellite Motion about an Oblate Planet”. *Journal of Guidance, Control, and dynamics*, Vol. 25, pp. 776-785, 2002

[4]Schweighart, S.A., and Sedwick, R., “A high fidelity linearized model for satellite formation flying.” *Journal of Guidance Control and Dynamis*, Vol.25, No. 6, Nov-Dec. 2002, p.1073-80.

[5]Izzo D., Sabatini M., Valente C.: “A new linear model describing formation flying dynamics under J2 effect ”, *Proceedings of the XVII AIDAA congress*, pp.493-500, 2003.

[6]Melton, R.G., “Time-Expicit Representation of Relative Motion Between Elliptical Orbits.” *Journal of Guidance, Control, and dynamics*, Vol. 23, No. 4, 2000, pp. 604-610.

[7]Vadali, S.R., “An Analytical Solution for Relative Motion of Satellites.” *Proceedings of the 5th International Conference on Dynamics and Control of Structures and Systems in Space*, Cranfield University Press, 2002, pp.309-316.

[8]Schaub, H., and Alfriend, K.T., “J_2 Invariant Relative Orbits for Spacecraft Formation.” *Celestial Mechanics and Dynamical Astronomy*, Vol. 78, 2001, pp.77-95.