04/ 4105 Assessing the accuracy of interval arithmetic estimates in space flight mechanics.

Budget 25 KEUR
Duration 4 months

Background:

Interval arithmetic, started from the pioneering works by Moore and Sunaga, has recently been applied to space related issues. In particular, the so-called verified integration technique has been shown to be quite relevant to a number of important issues such as solar system dynamics and asteroid orbits. In the works by Berz and his group, the Taylor model approach is applied to decrease the overestimation during the intermediate computations and to control the wrapping effect affecting the interval integration technique. This Taylor model technique is quite debated within the interval community and needs further comparative studies with existing verified techniques. The claims contained in the work [1] are quite impressive: a relative overestimation of about 10^{-5} is found when computing an asteroid orbit over three and a half years in time, starting from a large initial domain region of width 10^{-6}. The sole relativistic effects are accounted for in the paper, raising some doubts on the effective use of the results. A more useful approach to this problem could be that of considering the disturbances as unmodelled forces belonging to some predefined interval vector, or to include in the model effects due to solar radiation pressure or J_2 effects for Earth Orbit applications. The result of this last approach should be that of determining, within a given disturbance uncertainty, a bounding box for the considered object. Applications could range from collision avoidance of NEOs to limiting the search space in a lost-in-space situation to many other interesting cases.

Study Objectives

In the framework of interval integration techniques, this study proposes to estimate the resulting interval overestimation in the position of a spacecraft over time when different techniques are applied to different problems. The initial conditions of the spacecraft have to be considered as intervals (orbit determination errors) together with the disturbance forces (modeling uncertainties). The simple keplerian case should be assessed first by means of basic interval arithmetic (using the simpletic transition matrix given by Lagrange coefficients), a verified integration technique should then be used to introduce perturbances both as intervals and as modeled effects.

In summary, the study objectives are as follows:

- Assess the possibility of using interval arithmetic in common spaceflight mechanics problem such as the Kepler’s problem and the long term evolution of the perturbed Kepler’s problem.

- Find the best suitable interval integration technique that might tackle these problems (Taylor model results might be compared to the results obtained via alternative techniques).

- Perform long term integrations on the simple Kepler’s model and find the overestimation of the bounding box introduced by the differential technique (with time) by comparison with the results obtained by applying interval arithmetic to the analytical solution available in terms of Lagrange coefficients.

- Search for a practical case in which the proposed interval technique makes the difference with respect to other, more classical space-flight mechanics methods.

The study outcomes have to be at the level suitable for publication in a peer-reviewed journal. A publication of the results is encouraged.

References

[1] Berz, M., Makino, K. and Hoefkens, J.: “Verified Integration of Dynamics in the Solar System”. Nonlinear Analysis: Theory, Methods & Applications, 47, 179-190 (2001).

[2] Lohner, R.J.: AWA – software for the computation of guaranteed bounds for solutions of ordinary initial value problems. ftp://ftp.iam.uni-karlsruhe.de/pub/awa

[3] Lohner, R.J.: "Einschließung der Lösung gewöhnlicher Anfangs- und Randwertaufgaben und Anwendungen". Dissertation, Universität Karlsruhe (1988).

[4] Makino, K and Berz, M.: “Suppression of the wrapping effect by Taylor Model Based Validated Integrator”. http://bt.pa.msu.edu/pub/papers/VIRC03/VIRC03.pdf

[5] Nedialkov, N.S., Jackson, K.R., and Corliss, G.F.: “Validated Solutions of Initial Value Problems for Ordinary Differential Equations”. Appl. Math. & Comp., 105 (1), 21-68 (1999).

[6] Neumaier, A.: “Taylor forms – use and limits.” Reliable Computing 9, 43-79 (2002).