Type of activity: Standard study (25 k€)

Space Trajectory Optimisation under Uncertainties

Background and Motivation

Introduction

The recent advances in low-thrust propulsion systems open a new class of space mission. Interplanetary low-thrust missions require less fuel than their chemical propulsion counterpart.
However, continuous-thrust propulsion usually leads to a significant increase of the total thrust duration (in the order of days or months). In addition, low-thrust propulsion systems provide very small thrust acceleration (in the order of 10^{-3} N). Consequently, along the trajectory, the accumulation of dynamical perturbations resulting in discrepancies between the flying trajectory and the designed one has an influence on the mission operations.
Using high-fidelity dynamics during the optimisation process of sensitive trajectories (e.g. asteroid rendezvous, halo orbit injection, …) would ensure that later, during operations, the spacecraft flies close to the designed trajectory. In this case, small correction manoeuvres can be performed efficiently. The problem is however that a model that would describe all the possible perturbations, a spacecraft may have to cope with, accurately is difficult to produce considering the different existing perturbations (e.g. owing to the dynamics and the systems in particular).
Note, even though low-thrust propulsion systems allow significant discrepancies between the flying trajectory and the designed trajectory with correction manoeuvres of small mass impact, the issue is that there may not be sufficient time to perform the manoeuvre. Allowing a mid-fidelity dynamical model, the problem is thus how far we are from the desired trajectory, and how much time we have to perform the corrections. We can then propose **mid-fidelity modelisation with margin errors** during the design phase.

For instance, we can use uncertain dynamics instead of high-fidelity dynamics to perform the trajectory design. The high order dynamical perturbations are modelled as bounded perturbations. **The optimization problem is thus posed using explicit uncertainties in the dynamics.**
Such an approach would ensure that even though the spacecraft flies far from the designed trajectory, we will always have fuel mass and time to perform necessary corrections as far as the actual physical perturbations lies within the domain of the generated disturbances.

Problem statement and Possible Approaches

Since accurate and high-fidelity dynamics are not always available, the proposed study addresses the problem of computing optimal control trajectories in low-thrust uncertain dynamics.

For this study, the dynamics are simplified to encompass only the two-body dynamical model. Perturbations however account for:

- **N-body effect**,
- **solar radiation pressure,**
- **non-spherical harmonics for orbital planetary mission.**

Other perturbations can be added if relevant for the case conducted. A mathematical formulation of the dynamics of an object into the uncertain environment could have the general form:

$$f(x,t)=f_0(x,t)+\sum_i \sigma_i(x,p_i,t)+\sum_j \omega_j(x,t)$$

Where f_0 is the dynamical model accounting for the gravitational force field and <u>well-defined dynamical perturbations</u>. The sum describes all the perturbations for which either little is known or it is difficult to characterise them accurately (lack of data). Essentially, $\boldsymbol{\sigma}_i$ are well-known dynamical perturbations depending on unknown parameters p_i, and $\boldsymbol{\omega}_i$ are continuous-time processes that can only be described by amplitudes of variation.
Along this problem, we are also interested in the problem of where the dynamics are well approximated, but the terminal constraints are uncertain.

Many approaches can be followed to solve an uncertain problem: min max approach, information-gap theory, stochastic optimisation, interval arithmetic, etc…
For instance, following a **stochastic problem formulation** [3][4], all unknown dynamics are modelled using Brownian noise and Wiener processes. The optimisation can be done using an optimal control approach with a preference for indirect methods because of their high fidelity integration and thus avoidance of numerical discrepancies. But any other methods that can provide an accurate trajectory should be considered as well.

Another approach is to consider **interval arithmetic**. Instead of formulating uncertainties as stochastic variables they are taken into account using intervals. These intervals should then be propagated along a trajectory, while optimising an objective function.

Practical Space Applications

The study can be validated on selected applications that would pose problem to operations in case of large discrepancies. For this study, we shall set up a fixed time problem with a maximum mass objective.

Considered applications are then:

- Mars aero-breaking manoeuvres.
- Asteroid rendezvous with unknown orbital parameters.
- Long duration missions.
- Transfer along manifolds.

We can define two classes of problem.

<u>Uncertain dynamics with fixed state constraints</u>

The first class of problem considered uncertain dynamics. The uncertainties can be owing to system perturbations [8] or dynamical perturbations.

A potential application is the case of a spacecraft performing an aero-breaking manoeuvre with an approximate atmospheric model. This manoeuvre usually requires a good model of the atmosphere, and good precision because of the low altitude. In practice, these manoeuvres are performed to change the orbital parameters of the spacecraft for planet orbit insertion. Aero-assist manoeuvres are not intended to rapid change of the parameters, and usually one might be interested by a long phase.
Another application is the case of transfer along manifolds, where a stability analysis is often required to ensure correct injection into a given orbit.
Lastly, it would be interesting to see the influence of the perturbation and mission margins for long duration missions.

Uncertain constraints with deterministic dynamical model
The second class of problem is about finding the optimal control with uncertain terminal constraints [7]. For instance, the second example deals with impacting an asteroid, where the orbital parameters are known only to a given precision. This example can be important in the case of deflection missions, when orbital parameter determination is not frequently possible, and thus the orbital parameters are usually known quite late. A tentative approach to solve this problem has been proposed in [1]. The idea would be to produce a trajectory that would allow late corrections for maximum impact efficiency.

Research and Study Objectives

The work proposes to use current state of the art methods to propose solutions to the spacecraft trajectory optimal control problem with uncertain dynamics . The aim of this study is to derive a solution method to at least one of the two, above described, problem classes: *Uncertain dynamics with fixed state constraints* and *Uncertain constraints with deterministic dynamical model*. A practical implementation should be proposed together with test cases, to be followed by a discussion on possible difficulties on the implementation.

Methodology

The following 3-step methodology is proposed:

1. selection of problem class and mathematical problem definition, including
   a) dynamics and constraints formulation.
   b) assumptions for modellisation of the perturbations.

2. Definition of the solution method including:
   a) description and argumentation of the choice of the method.
   b) definition of an algorithm based on the selected method.
   Possible methods include but are not limited to: indirect methods applied to stochastic control problems[2][3][5], interval arithmetic [6], heuristics or global optimisation techniques.

3. Application of the solution method for the selected problem class.
   a) Description of an application example, from the selected problem class, to be solved using the derived algorithm.
   b) Implementation of the derived algorithm from step 2 (e.g. Matlab, Fortran, etc …) for the solving of the example defined in step 3.a.
   c) Comparison of the solution for the problem with uncertainties to the solution of the problem without uncertainties. Minimum comparison parameters: stability and robustness (using for instance mission margins).

Proposals should cover points 1. and 2.a.

Contribution of the ACT

ACT researchers propose to contribute to phases 1 and 2, and to lead phase 3.c. In particular, the ACT will provide the solutions of the problem without perturbation.

References

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