‘Analogue Transformational Acoustic’: An alternative theoretical framework for acoustic metamaterials (Ref: 11-1301)

Type of activity: Small study (15 k€)

1 Background & Motivation

1.1 Introduction.

The transformational paradigm has been very successful in the design of optical metamaterials. Since its first proposal [1] (and its extension [2]), this idea has been exploited in a variety of different ways [3-14]. The present study intends to fully extend this approach to the field of acoustics.

Contrary to electromagnetism, the theoretical framework of acoustics does not immediately generalize to a relativistic theory. It is not easy, therefore, to apply directly the techniques developed in transformational optics.

A first attempt to construct a design technique for acoustic metamaterials inspired by the approach of [1] was given in [15] and [16] in two and three dimensions respectively. Specifically, it has been shown (numerically) that it is possible to use transformational techniques build acoustic cloaks.

The aim of this proposal is to construct a new designing technique, which is more elegant than the one used in [15-16] and more powerful in terms of predicting power. This will be done using tools borrowed from General Relativity and specifically the so-called *Analogue Gravity* paradigm [17].

1.2 Analogue Gravity and acoustics.

The concept of Analogue Gravity is based on the development of analogies (typically based on condensed matter physics) to probe aspects of the physics of curved spacetime. The best-known of these analogies is the use of sound waves in a moving fluid as an analogue for light waves in a curved spacetime.

In particular the following theorem is proven in [17]:

Theorem: If a fluid is barotropic and inviscid, and the flow is irrotational (though possibly time dependent) then the equation of motion for the velocity potential describing an acoustic disturbance is identical to the d’Alembertian equation of motion for a minimally coupled massless scalar field propagating in a $(3+1)$–dimensional Lorentzian geometry

$$\Delta\phi \equiv \frac{1}{\sqrt{-g}}\partial_\mu \left(\sqrt{-g}\; g^{\mu\nu}\; \partial_\nu \phi\right) = 0. \tag{1}$$

Under these conditions, the propagation of sound is governed by an *acoustic metric* — $g_{\mu\nu}(t, \mathbf{x})$. This acoustic metric describes a $(3+1)$–dimensional Lorentzian (pseudo–Riemannian) geometry. The metric depends algebraically on the density, velocity of flow, and local speed of sound in the fluid.

Specifically

$$g_{\mu\nu}(t,\mathbf{x}) \equiv \frac{\rho}{c}\begin{bmatrix} -(c^2-v^2) & \vdots & -\mathbf{v}^T \\ \cdots\cdots\cdots\cdots & \cdot & \cdots\cdots \\ -\mathbf{v} & \vdots & \mathbf{I} \end{bmatrix}. \tag{1}$$

(Here $\mathbf{I}$ is the 3×3 identity matrix.) In general, when the fluid is non-homogeneous and flowing, the *acoustic Riemann tensor* associated with this Lorentzian metric will be nonzero. ♢

This theorem allows us to rewrite the sound equation as a relativistic equation for a scalar field. Traditionally it provides (at least in principle) a concrete laboratory model for curved space quantum field theory in a realm that is technologically accessible to experiment. In terms of transformational acoustic this theorem implies that we can apply in a straightforward way the line of reasoning of [2] for electromagnetism. In order to achieve this goal, however, one will need to redefine the acoustic metric (2) in such a way that (1) present explicitly the conductivity tensor and the unperturbed density, but this does not pose any real problem.
It is important to keep in mind that analogy is not identity and, as consequence, that the analogue model reflects only a certain number of important features of general or special relativity. This in our case will translate in some limit in the application of the analogue gravity to transformational acoustic. We will evaluate such limits during the first stage of this study.
The possibility of designing metamaterials with non-trivial acoustic properties constitutes the doorway to the development of interesting new technologies in the space sector. For example they could be useful to construct better shielding for a payload to be sent in orbit. Also, acoustic (perfect) lenses could have an important role in the geological analysis of asteroids and planetary surfaces for scientific and mining purposes.

1.3 Study objective.

In this project we plan to use analogue gravity as a tool to develop a transformational acoustic that is similar to the one proposed in [1-2] and, as consequence is more powerful and flexible than the one of [15-16]. In particular, we will use this new theoretical framework to explore the possibility to design materials able to generate, for example, a so-called “dumb hole” [17], the acoustic analogue of a “black hole” and other exotic General Relativistic object.

2 Study Description

2.1 Main objectives

This study is focused on the development and testing of a transformational acoustic based on Analogue Gravity.

The proposed structure of the study is:

1. The construction of the actual formalism. Using equations inspired from [17] a set of transformation as in [2] will be constructed that would allow the design of acoustic metamaterials. It will further also develop a “Geometric Acoustic” suitable for our purposes.

2. Using the new equations the study will design the acoustic cloaks described in [15] and [16]. This will serve as a checkpoint for the correctness of the calculations.

3. Use the new equation to design exotic acoustic metamaterials. In particular the study intends to investigate the existence of metamaterials able to generate dumb holes or the acoustic equivalent of trapped surfaces using also numerical simulations.

3. Collaboration with the Advanced Concepts Team of ESA

The project is mainly addressed at research groups working in the fields of General Relativity, Black hole theory and Analogue Gravity. Interest in material sciences and metamaterials (above all in the numerical simulation of their acoustic properties) would be of benefit.

The project will be conducted in close cooperation with ACT-researchers. They will be involved in the adaptation of the Analogue Gravity technique to transformational acoustics. They will also contribute to the application of the new method for the construction of acoustic cloaks and the other exotic metamaterials mentioned in section 4.

Main References

[1] J. Pendry, D. Schurig, and D. Smith, Science 312, 1780 (2006).
[2] Bergamin, L. , Phys. Rev. A 78, 043825 (2008)
[3] U. Leonhardt and T. G. Philbin, New J. Ph. 8, 247(2006).
[4] D. Schurig, J. Mock, B. Justice, S. Cummer, J. Pendry,
A. Starr, and D. Smith, Science 314, 977 (2006).
[5] D. Schurig, J. Pendry, and D. Smith, Opt. Express 15,14772 (2007).
[6] D. Smith and D. Schurig, Phys. Rev. Lett. 90, 077405 (2003).
[7] D. Smith, P. Kolilnko, and D. Schurig, J. Opt. Soc. Am. B 21, 1032 (2004).
[8] Z. Jacob, L. Alekseyev, and E. Narimanov, Opt. Express14, 8247 (2006).
[9] U. Leonhardt and T. Philbin, Prog. Opt. 53, 69-152 (2009), arXiv:0805.4778 [physics.optics].
[10] S. Tretyakov and I. Nefedov, in Metamaterials’, edited by F. Bilotti and L. Vegni (Rome, 2007), p. 474.
[11] S. Tretyakov, I. Nefedov, and P. Alitalo, New Journal of Physics, vol. 10, p. 115028, 2008 arXiv:0806.0489[physics.optics].
[12] J. B. Pendry, Phys. Rev. Lett. vol. 85, p. 3966, 2000.
[13] U. Leonhardt, Science 312, 1777 (2006).
[14] D. Smith, P. Kolilnko, and D. Schurig, J. Opt. Soc. Am. B vol. 21, p.1032, 2004.
[15] S. A. Cummer and D. Schurig *New J. Phys.* **9** 45 2007
[16] H. Chen and C. T. Chan, Appl. Phys. Lett. **91**, 183518 (2007); *ibid* 2010 J. Phys. D: Appl. Phys. 43 113001 (2008)
[17] C. Barceló, S. Liberati and M. Visser, “Analogue Gravity”, *Living Rev. Relativity*, 8, (2005), 12. [Online Article]: cited [*13/04/2011*], http://www.livingreviews.org/lrr-2005-12