Analytical perturbative theories of motion in highly inhomogeneous gravitational fields (Ref: 11-5201)

Type of activity: Standard study (25 k€)

1. Background and Motivation

1.1 Introduction

The motion of bodies subject to non-keplerian gravitational fields is a classical subject of research in the context of celestial mechanics. Historically, the main focus has been on the study of the effect of the Earth's inhomogeneous gravitational field on the motion of natural and artificial satellites. Landmark results include the solution of the main problem of artificial satellite theory for small and moderate eccentricities [4] and precise analytical lunar ephemerides taking into account the effect of the Earth's figure effects on the motion of the Moon [9].

More recently, the focus of researchers is on the study of the effects on motion of the inhomogeneous gravitational field of other solar system bodies, including the Moon and other natural satellites [10]. The difficulty of dealing with smaller solar system bodies lies in the fact that typically such bodies feature shapes and density distributions more irregular than those of planets. Such irregularities break symmetries and require more complicated analytical expressions for their description, increasing noticeably the mathematical difficulties involved in such studies.

These obstacles are maximized in the case of bodies such as asteroids and comets, where shapes and density distributions are highly irregular and often unknown. On the other hand asteroids are at the centre of a number of recent advanced mission concepts. Asteroid deflection missions have been studied by ESA (i.e. "Don Quijote" concept [12]) and are being discussed for some time now in the scientific community, further asteroid sample return missions are considered and human missions to asteroids have entered the agenda of space agencies.

Numerical methods are today widely used to study the trajectories of objects orbiting irregular bodies [8]. Analytical methods, by contrast, are being used mainly in a limited and semi-numerical way (meaning that analytical expansions constitute the first step in such studies, which are then typically carried out from a numerical standpoint [11]). The main drawback of analytical methods is that their application in case of highly inhomogeneous bodies requires a prodigious amount of work in terms of algebraic manipulations. On the other hand, analytical methods can provide a full dynamical picture of the motion around irregular bodies that can be used to search and study particular classes of orbits useful, for instance, in the design of a space mission.

The subject of this study is to explore the potential of fully analytical perturbative methods carried out with the aid of computer-assisted algebra tools to study the orbital properties around bodies with arbitrary (known) shapes and density distributions.

1.2 Problem statement and Possible Approaches

The basic principle in analytical perturbative methods is to consider a complex physical system as the aggregate of a well-known system and some perturbation. The unperturbed part is exactly solvable and accounts for the most prominent and relevant features of the system; the perturbed part, which is typically not solvable exactly, induces a deviation from the unperturbed model and accounts for the finer details of the behaviour of the system.

In celestial mechanics, the exactly-solvable unperturbed model is often the two-body problem, consisting of two point masses moving under the reciprocal attraction resulting from Newton's theory of gravity. The trajectories followed by the two particles are conic sections, which can be described by the classical six orbital elements a (semi-major axis), e (eccentricity), i (inclination), w (argument of pericentre), W (longitude of the ascending node) and M (mean anomaly at epoch). In the two-body problem, these orbital elements are constants of motion. When this simple model is complicated by the addition of more realistic features, such as additional and/or non-spherical bodies, the resulting system is generally not solvable exactly.

Methods based on the variation of parameters have led to a number of important classical results in celestial mechanics, such as the existence of critical inclinations and of geosynchronous orbits, as well as the seminal works on analytical ephemerides by Charles-Eugene Delaunay and the discovery of planet Neptune [1]. The approaches used in these classical works are not suitable for problems of higher complexity because of the number and type of algebraic manipulations involved. This reason motivated the development of methods based on Hamiltonian formalism and canonical transformations through Lie series in the 60s, which can be considered as the basis of modern Celestial Mechanics [2,3,4]. The standpoint adopted in these seminal works was that of developing a technique that could be efficiently programmed into computer languages, thus delegating to a machine the task of performing the enormous amounts of calculations involved in perturbative methods. Such a standpoint has today become essential and the increasing power of computers allows tackling increasingly complex problems. Recent results on lunar orbital motion, on the long-term propagation and stability of the Solar System [5], on the peculiar motions of Jupiter and Saturn moons have all been enabled by these approaches, which require two fundamental elements: the method and a fast and efficient computer algebra system.

In this study we aim at determining whether modern computational performances allow the Lie series approach to tackle the problem of orbital motion around a highly irregularly shaped body such as that of an asteroid, possibly perturbed by a binary companion and by the Sun. In order to meet this objective the use of state of the art algebraic manipulators is a requirement.

2. Research and Study Objectives

The primary objective of this study is to determine whether modern computer performances are sufficiently developed to carry out the development of an analytical perturbative theory using the Lie series approach for an irregularly-shaped small body (e.g. asteroid). The study intends to determine in which orbital regimes such an approach is viable (e.g., distance from the body etc.). A secondary objective of the study is to use the developed theory to study orbital mechanics around asteroid systems.

It should be pointed out that, in many cases, for actual spacecraft trajectories other forces acting on the spacecraft than gravity and solar radiation pressure might play a substantial role (e.g. outgassing from cometary nuclei in cases of spacecraft orbiting comets). These are not subject to this research study, which focuses on the contribution of the gravitational field and includes only solar radiation pressure.

2.1 Proposed Research Plan

It is proposed to structure the study along the main elements of the research plan, which can be summarised as:

1. Identification of useful representations of irregular gravitational fields.
2. Automatic derivation (using a computer algebra system) of a perturbation theory assuming a known body shape, orbit and solar radiation pressure.
3. Use of the obtained analytical theory to study the existence of particular orbits.

As part of the proposal, the university team is encouraged to detail the intended approach each of the above points and to discuss some preliminary developments.

Ad. 1. Irregular gravitational fields

The first objective of the study is to identify the mathematical representations of an inhomogeneous gravitational field that are most promising for the purposes of this study. These should include spherical harmonics, multiple point masses and surface integral methods. This part of the study will be performed by the University with the collaboration of the ACT.

Ad. 2. Development of a perturbative theory

Once the most promising approach has been identified for the description of the gravitational field potential, the automated derivation of an analytical perturbative theory should be carried out using modern computer algebra tools able to deal with the highly dimensional series expansion likely to occur. This description needs to take into account also the solar radiation pressure. The University researchers will have the choice either to adopt own software tools to accomplish the task, or to use the Piranha computer algebra system [6,7], developed by one of the ACT researchers. In case the University decides to use in-house tools, the ACT will proceed to a parallel development of the perturbative theory using Piranha, both to provide a validation for the work performed by the University and possibly to investigate the effect of using another representation of the gravitational field.

Ad. 3. Search of orbits of interest for mission-design purposes

The third and last part of the study will be focussed on the use of the developed analytical theory to study, for some selected particular cases, or even in general when possible, particular class of orbits that are of interest for engineering studies.

3. Collaboration with the ACT

The Advanced Concepts Team is currently supporting the main developer of the open source software Piranha and will, if required, provide support to the correct use of the tool and/or extend its capabilities as necessary. The team will be providing the asteroid gravity model as well as the solar radiation pressure model for a few agreed study cases. The team's researchers will also work in parallel with the University team using the developed motion theory to identify interesting orbit classes.

References

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