Automatic Extraction and Characterization of Structures in Area-preserving Maps

Type of activity: Standard Study

1. Background and study motivation

Many natural phenomena found in various areas, such as orbital mechanics, fluid dynamics or quantum mechanics, can be described in terms of Hamiltonian dynamics. Multi-dimensional Hamiltonian systems often exhibit chaotic behaviour, which makes their analysis difficult. The use of surfaces of sections, such as a Poincaré return map, is commonly employed to get insight into the phase space of dynamical systems, which present both, regular and chaotic behaviour.

1.1 Surfaces of section for the study of dynamical systems

In the last decade, maps have been used extensively in a wide range of scientific and engineering problems to understand the dynamical structure of complex systems. A Poincaré surface of section, as illustrated in Figure 1, can be interpreted as a discrete dynamical system with a state space that is one dimension smaller than the original continuous dynamical system. On the map, a periodic trajectory becomes a point, while a non-periodic trajectory is represented by a set of points.

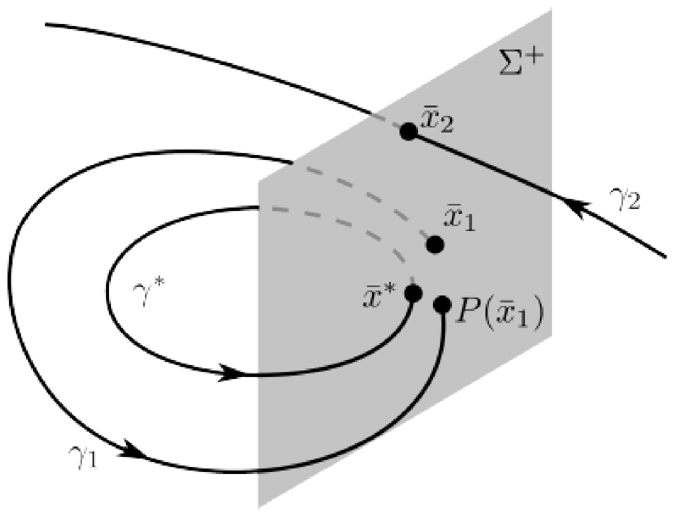


Figure 1: Schematic of a Poincaré surface of section (courtesy of Haapala [1])

One of the advantages of the Poincaré map lies in its power as a visualization tool. Such a map reduces the order of the problem, condensing quantities of information into a lower-dimensional image. Poincare maps reveal, at a glance, regions of well-ordered behaviour, despite the chaotic nature of the underlying problem. An example of a Poincaré map is represented in Figure 2. This map is generated in the Planar Circular Restricted Three-Body Problem (PCR3BP), so that the system is initially four-dimensional. To create the map, a grid of initial conditions is selected and integrated forward in time. The intersections of each trajectory with the surface of section create the Poincaré map. Three types of behaviours are easily identified on the map represented in Figure 2: periodic orbits, quasi-periodic motion, and chaotic trajectories.

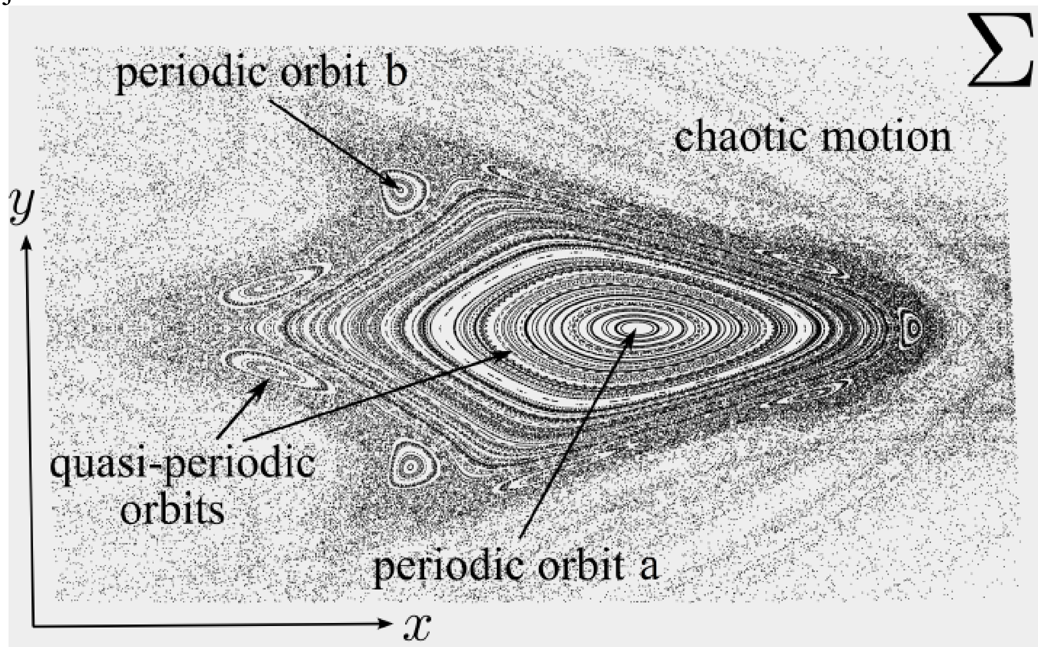


Figure 2: Example of Poincaré map (courtesy of Haapala [1])

Generating maps provides a global picture of the phase space for complex dynamical systems and offers features that might be difficult to identify otherwise. However, despite how valuable these maps are, their implementation is not practical. Two main limitations prevent their utilisation:

- The computation of the maps often takes time. To generate a Poincaré surface of section, the integration of the full nonlinear dynamics is usually required.
- There is no easy way to extract the information from the maps. Typically, the selection of initial conditions is obtained by hand, by zooming on some regions of interest on the maps.

Therefore, new efficient ways to employ these maps need to be evaluated. An automatic method to detect and extract structures quickly from the map is required.

1.2 Visualization of structures in maps

An effective analysis of maps remains a difficult task. The complexity of surfaces of sections often makes the identification of the topology challenging. In computer visualization, discrete dynamical systems and area-preserving maps are not typical research topics. Helman and Hesselink introduce some topological approaches to reproduce a topologic skeleton [2]. Peikert and Sadlo employ a Poincaré map approach to the visualization of vortex rings [3,4]. Recently, Tricoche, Garth and Sanderson presented a method to automatically extract and characterise structures on area-preserving maps [5]. Figure 3 illustrates the algorithm of Tricoche et al., which captures very subtle structures in the individual islands of the map.

The precision of the characterisation and extraction of area-preserving maps depends heavily on an accurate and efficient integration of the flow map. The more iterations are used to create the map, the more accurate the extraction can become, but the longer the whole process gets. Hence, trade-offs need to be made between the process that generates the map and the visual algorithm that detects and extracts the information from the map.

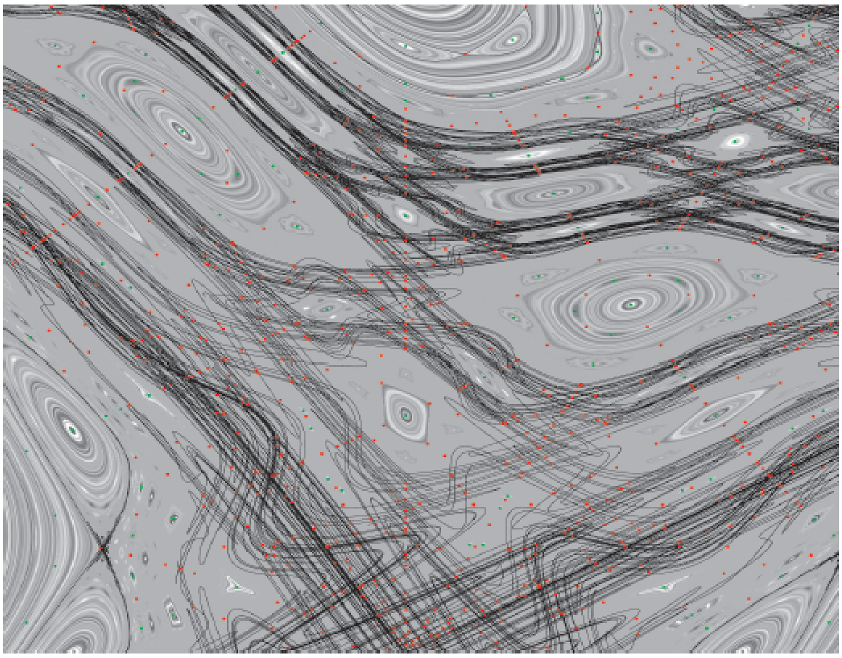


Figure 3: Structures of a Poincaré map captured via visualization algorithm (courtesy of Tricoche [5])

1.3 The Keplerian map for resonant transfers between Jovian's moons

Different maps can be generated for different applications. Besides the traditional Poincaré map, other types of maps are the Periapsis Poincaré maps or Keplerian maps, which display the semi-major axis of each trajectory as it evolves over time as a function of the initial periapsis angle ω [6]. A Keplerian map is represented in Figure 4, where the semi-major axis is denoted 'a' on the y-axis of the map.

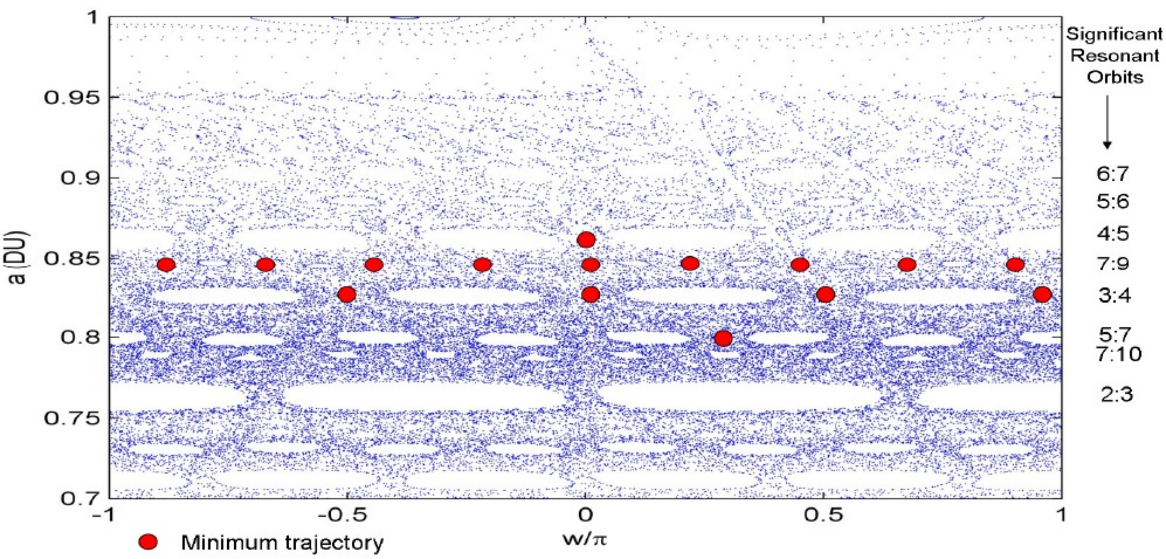


Figure 4: Phase space of the Keplerian map (courtesy Lantoine [7])

Keplerian maps offer some advantages compared to the traditional Poincaré maps:

- The geometry and structures of Keplerian maps are relatively straight-forward to detect. These maps are composed of resonant structures, which govern transport from one orbit to another. The random scattered points correspond to chaotic motion whereas blank 'holes' represent stable resonant islands. For every semi-major axis value, corresponding to a K:L resonance, there is a band of L islands. For instance, the large dots in Figure 4 give the successive resonant path to lower or increase the semi-major axis of a spacecraft's orbit.
- The Keplerian maps are typically generated via integration of the full dynamics in the circular restricted-three body model, using a Poincaré surface of section at periapsis. For orbits nearby Keplerian energies, the dynamics of the PCR3BP can be approximated via a symplectic twist maps, which capture well the dynamics of the full equations of motion. Using symplectic maps as an approximation of the full dynamics considerably reduce the computation and enable the design of quick algorithms for low-energy transfers and optimization procedures.

Some authors consider Keplerian maps to determine the long-time evolution of nearly parabolic comets [8,9]. In this investigation, Keplerian maps are employed to identify resonant transfer trajectories applicable to spacecraft in a planet-moon system [6,10]. A very challenging part in the design of a planetary moon tour, such as a multi-moon orbiter in the Jupiter system, is the orbital transfer from one planetary moon to another for low-energy transfers. Multiple gravity assists by moons could be used in conjunction with ballistic capture to drastically decrease fuel usage. In planetary systems, the strong dependence on the three-body regimes of motion precludes the use of a patched conic approach. Instead, some recent approaches employ patched three-body models to enable multiple "resonant-hopping" gravity assists. An example of a low-energy inter-moon transfer via resonant gravity assists is represented in Figure 5.

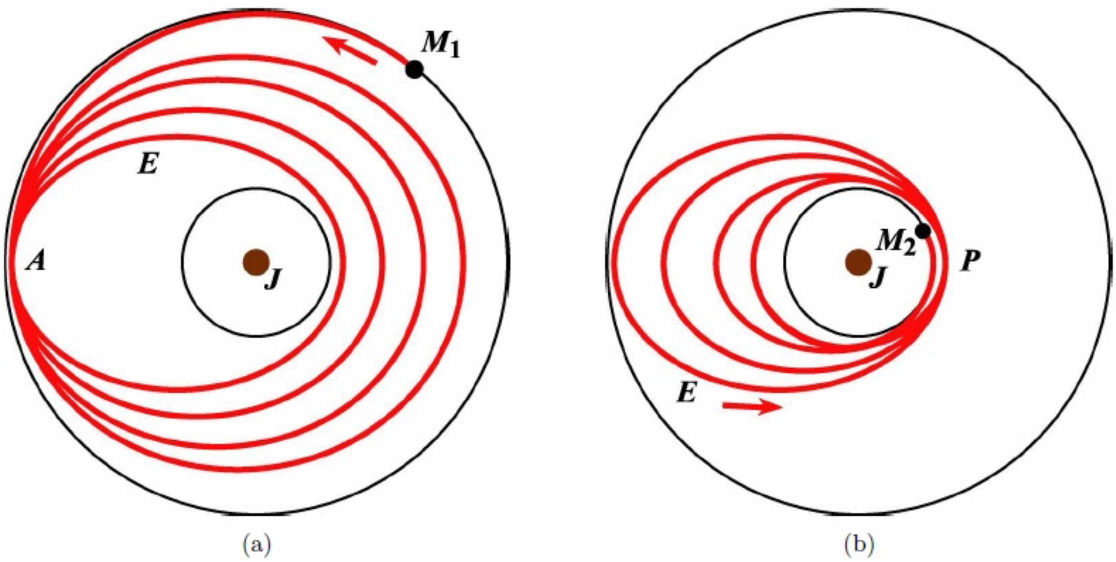


Figure 5: Inter-moon transfer via resonant gravity assists in the Jupiter system (courtesy Ross et al. [10])

In Figure 5(a), the spacecraft lowers its perijove by a sequence of successive resonant orbits with the outer moon M_1. Once the spacecraft's orbit comes close to grazing the orbit of the inner moon M_2, the inner moon takes "control." The spacecraft orbit where this occurs is denoted E. In Figure 5(b), the spacecraft now receives gravity assists from the inner moon at perijove and decreases its apojove by following a sequence of successive resonant orbits. Then, the spacecraft gets ballistically captured by the inner moon M_2.

2. Study objective

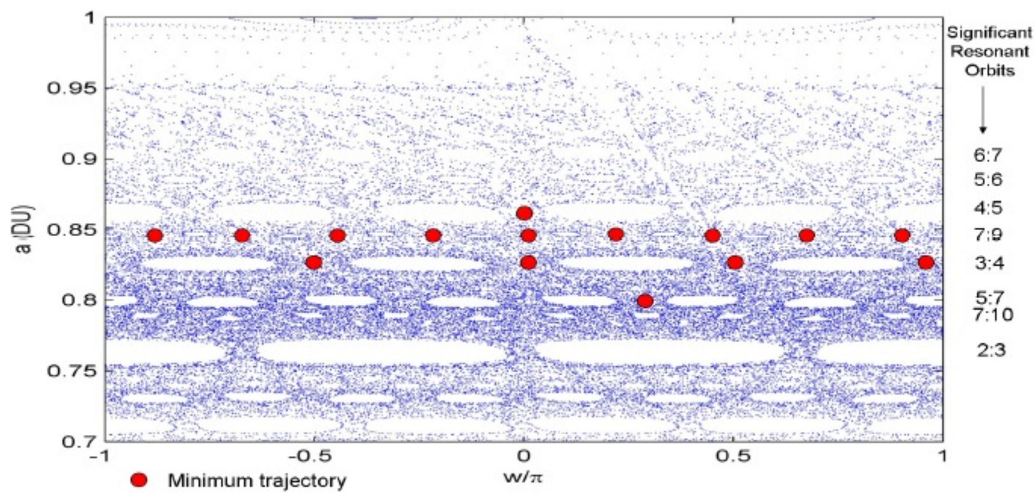
The goal of the Ariadna project is the implementation of a fast and accurate visualization algorithm to characterize and extract structures directly in area-preserving maps. In particular, the project is based on the features of Keplerian maps and intends to apply this algorithm to extract sequences of resonant orbits to generate low-energy inter-moon transfers.

3. Proposed Methodology

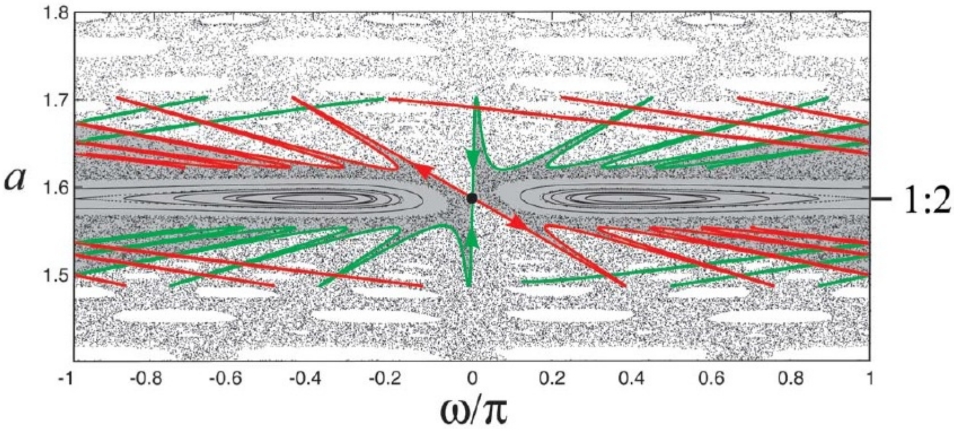
The following methodology is proposed for this study, and should be discussed in the proposal, though argued alternatives are welcome as long as they promise to achieve the project goals.

- Automatic characterisation of various features in Keplerian maps

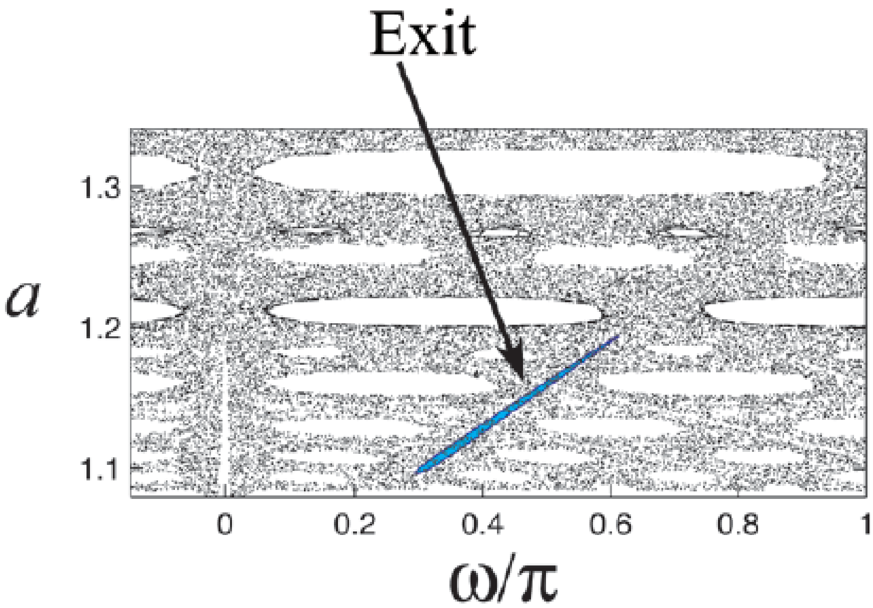
An efficient way to visualise the different structures on the maps should be proposed. To speed up the detection process, one feature at a time could be examined. For example, Figure 6 illustrates the same Keplerian map with different features added on top of the background. Unstable resonant orbits are represented by red dots in Figure 6(a). Then, stable/unstable manifolds associated with a 1:2 unstable resonant orbit are new features added to the map in Figure 6(b). Finally, an exit zone that corresponds to ballistic capture is also added on top of the map and is represented in Figure 6(c). The option to add and remove specific features of the map should be explored to aid in the detection process.



(a)



(b)



(c)

Figure 2: Various features on the same Keplerian map (courtesy Ross and Scheeres [6]); (a) Unstable resonant orbits; (b) Unstable/stable manifolds corresponding to a 1:2 resonant orbit; (c) Exit zone for ballistic capture.

- Automatic extraction of initial conditions directly from the map

An accurate way to extract data from the map should be introduced. In particular, the algorithm should extract accurate sequences of resonant orbits that can be employed to generate low-energy inter-moon transfers.

The efficiency of the detection and precision of the extraction are closely related to how well and how fast the Keplerian map is generated. The more iterations of the maps, the more accurate the extraction is but the slower it gets. Therefore, trade-offs need to be investigated between the process that generates the map and the visual algorithm that detects and extracts the information from the map.

This Ariadna project proposal is addressed at research groups with expertise in any of the following domains: computer science and vision, dynamical systems and chaotic motion, applied mathematics, orbital mechanics, astrodynamics and mission design.

4. ACT Contribution

The project will be conducted in close scientific collaboration with ACT-researchers. In particular, ACT-researchers will provide expertise in orbital mechanics and dynamical systems and will provide knowledge of Keplerian maps.

5. Bibliography

[1] Haapala, A., 'Trajectory Design using Periapse Maps and Invariant Manifolds,' MS. Thesis, Purdue University, West Lafayette, Indiana, 2010.
[2] Helman, J. L., Hesselink, L., 'Visualizing vector field topology in fluid Flows,' IEEE Computer Graphics and Applications, 11(3):36–46, May 1991.
[3] Peikert, R., Sadlo, F., 'Flow Topology Beyond Skeletons: Visualization of Features in Recirculating Flow,' H.-C. Hege, K. Polthier, and G. Scheuermann, editors, Topology-Based Methods in Visualization II, pp. 145–160. Springer, 2008.
[4] Peikert, R., Sadlo, F., 'Visualization Methods for Vortex Rings and Vortex Breakdown Bubbles,' A. Y. K. Museth, T. M¨oller, editor, Proceedings of the 9th Eurographics/IEEE VGTC Symposium on Visualization (EuroVis'07), pp. 211–218, May 2007.
[5] Tricoche, X., Garth, C., Sanderson, A., 'Visualization of Topological Structures in Area Preserving Maps,'IEEE Transactions on Visualization and Computer Graphics 17(12) (Proc. IEEE Visualization 2011), pp. 1765-1774, 2011.
[6] Lantoine, G., Russell, R., Campagnola, S., 'Optimization of low-energy resonant hopping transfers between planetary moons', IAC-09.C1.1.1.
[7] Malyshkin, L., Tremaine, S., 'The Keplerian Map for the Restricted Three-Body Problem as a Model of Comet Evolution,' Princeton University Observatory, Princeton, NJ 08544 http://arxiv.org/abs/astro-ph/9808172v2.
[8] Petrosky, T. Y., Broucke, R., 'Area-Preserving Mappings and Deterministic Chaos for Nearly Parabolic Motions,' Celestial Mechanics, Kluwer Academic Publishers 42 (1988), pp. 53-79.
[9] Ross, S., Scheeres, D., 'Multiple gravity assists, capture, and escape in the restricted three-body problem,' SIAM J. Applied Dynamical Systems, Vol. 6, No. 3, pp. 576-596.
[10] Ross, S. D., Koon, W. S., Lo, M. W., Marsden, J. E., 'Design of a Multi-Moon Orbiter,' 13th AAS/AIAA Spaceflight Mechanics Meeting, Ponce, Puerto Rico, February 2003. Paper No. AAS 03-143.

6. Additional reading

Tricoche, X., Garth, C., Sanderson, A., 'Visualizing Invariant Manifolds in Area-Preserving Maps,' Topology-Based Methods in Data Analysis and Visualization IV, Peikert, R., Hauser, H., Carr, H. (eds), Springer, to appear 2011.
Wiebel, A., Tricoche, X., Scheuerman, G.,'Extraction of Separation Manifolds Using Topological Structures in Flow Cross Sections,' Topology-based Methods in

Visualization II, Mathematics + Visualization, Springer, pp. 31-43, 2009.
Short, C., Howell, K., Tricoche, X., ‘Lagrangian Coherent Structures in the Restricted 3-Body Problem,’ 21st AAS/AIAA Space Flight Mechanics Meeting, New Orleans, Louisiana, February 2011, Paper No. AAS 11-250.
Barakat, S., Andrysco, N., Tricoche, X., ‘Fast Extraction of High-quality Crease Surfaces,’ Eurographics / IEEE Symposium on Visualization 2011 (EuroVis 2011), DOI: 10.1111/j.1467-8659.2011.01945.x, Volume 30 (2011), No 3.
Easton, R. W., Meiss, J. D., Carver, S., ‘Exit Times and Transport for Symplectic Twist Maps,’Program in Applied Mathematics, University of Colorado, Boulder, CO 80309-0526, November 1992.
Marsden, J. E., Ross, S. D., ‘New Methods in Celestial Mechanics and Mission Design,’ Bulletin (New Series) of the American Mathematical Society, V 43, No 1, pp. 43–73. S 0273-0979(05)01085-2, 2005.
Pan, M., Sari, R., ‘A Generalization of the Lagrangian Points: Studies of Resonance for Highly Eccentric Orbits,’ http://arxiv.org/abs/astro-ph/0302135v1.
Grover, P., Ross, S., ‘Designing Trajectories in a Planet-Moon Environment Using the Controlled Keplerian Map’, AAS 08-193.