Game Theoretic Analysis of the Space Debris Dilemma

Final Report

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Date: June 9, 2016

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Available on the ACT website
http://www.esa.int/act

Ariadna ID: 15/8401
Ariadna study type: Standard
Contract Number: 4000113919/15/NL/LF/as

PROJECT REPORT

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June 9, 2016

Abstract

In this study we analyse active space debris removal efforts from a strategic, game-theoretical perspective. Space debris are non-manoeuvrable, human-made objects orbiting the Earth. They pose a threat to operational spacecraft, especially in low-Earth orbit. As a consequence, active debris removal missions have been considered and investigated by different actors (space agencies, private stakeholders, etc.) as part of a strategy to alleviate the problem, thus protecting valuable assets present in strategic orbital environments. An active debris removal mission is a costly endeavour that has a positive effect (or risk reduction) for all satellites in the same orbital band. This leads to a dilemma: each actor has an incentive to delay its actions and wait for others to respond. Specifically, each actor is faced with the choice between acting now or postponing to take action, i.e., either take an individually costly action of debris removal, which has a positive impact on all players; or wait until others jump in and do the 'dirty' work, as this will be at their own benefit and reduce their own costs. The risk of the latter action is that, if everyone waits the joint outcome will be catastrophic leading to what in game theory is referred to as the 'tragedy of the commons'. Such a scenario is unlikely if we look at the current debris population and the threat it poses, but it may become increasingly realistic as the debris population grows further and as more and more private stakeholders enter the business of exploiting the Earth orbital environment.

We introduce and thoroughly analyse this dilemma using empirical game theory in a two and three player settings and in terms of strategic properties and equilibria of the game, and show how the cost/benefit ratio of debris removal strongly affects the game dynamics. Our results are based on simulations of the long term evolution of the debris population accounting for new launches, new collision events, orbital decay and possible player actions such as debris removal.

Contents

1 INTRODUCTION

This work has been carried out in the context of the Ariadna Study "Game Theoretic Analysis of the Space Debris Removal Dilemma" – a cooperation between the European Space Agency, the University of Liverpool, and DFKI. Specifically, the study targeted to (1) introduce a realistic debris removal game that can be used to perform a theoretic analysis including the computation and approximation of equilibria, basins of attraction, and stability properties of the different equilibria or solutions of the game, e.g. in context of the replicator equations; (2) to gain insight in the evolutionary dynamics of, and determine potential optimal strategies in, the debris removal game by means of heuristic payoff tables; and (3) to design the structure of the debris removal game in such a way that the players' behaviour can be steered towards fair and truthful solutions for all involved parties. In this report we address these challenges and discuss the methods and experimental studies undertaken to tackle them.

Since the late 1950s a number of public and private actors have launched a multitude of objects into Earth orbits with low or no incentive to remove them after their life span. As a consequence, there are now many inactive objects orbiting Earth, which pose a considerable risk to active spacecraft. By far, the highest spatial density of such objects is in the Low Earth Orbit (LEO) environment, defined as the region of space around Earth within an altitude of 160 km to 2,000 km. According to most simulations and forecast, the density of objects in LEO is destined to increase due to the rate of new launches, on-orbit explosions, and object collisions being higher than the capability of the LEO environment to clean itself using the natural orbital decay mechanism.

NORAD tracks and catalogues objects in orbit, currently listing around 15,000 objects of 10cm^2 and larger.[1] However, it is estimated that the true number of objects is several orders of magnitude larger, with estimates of over 100,000 pieces of untracked debris of sizes 1-10cm^2 [3]. At orbital speeds of approximately 7.5 km/s such small pieces can cause considerable damage to active satellites.

In recent years there have been several incidents producing a high number of debris. Two of them have been especially severe: (i) a 2007 Chinese anti-satellite missile test producing more than 1,200 catalogued pieces of debris, and an estimated 35,000 pieces of size 1cm and larger, resulting in the most severe orbital debris cloud in history [34]; (ii) the collision of the Iridium-33 and Kosmos-2251 satellites in 2009, which was the first accidental hyper-velocity collision of two intact spacecraft [35]. More than 823 debris objects were catalogued forming two debris clouds in LEO. This incident introduced a high risk of potential collisions to many active objects in LEO. For example, the International Space Station (ISS) had to perform a manoeuvre in March 2011 to avoid a piece of debris from the 2009 Iridium-Kosmos collision [36].

These two incidents are only examples that show how space debris is a serious problem with potentially disastrous consequences. The problem is one that the scientific community is well aware of. Already in 1993 the Inter-Agency Space Debris Coordination Committee (IADC) was established with the task of a worldwide coordination of activities related to the issues of man-made and natural debris in space. To date IADC includes ASI (Agenzia Spaziale Italiana), CNES (Centre National d'Etudes Spatiales), CNSA (China National Space Administration), CSA (Canadian Space Agency), DLR (German Aerospace Center), ESA (European Space Agency), ISRO (Indian Space Research Organisation), JAXA (Japan Aerospace Exploration Agency), KARI (Korea Aerospace Research Institute), NASA (National Aeronautics and Space Administration), ROSCOSMOS (Russian Federal Space Agency), SSAU (State Space Agency of Ukraine) and the UK Space Agency. Their 2002 report [10] introduced some guidelines for mitigation including passive measures such as the end-of-life management of satellites via de-orbiting or graveyard orbits. Similar guidelines have since been released also by the United Nations Committee on the Peaceful Uses of Outer Space Mitigation, the International Organization for Standardization Space Debris Mitigation, ESA [18], and a multitude of national and international organizations. However, these measures, even if applied to all newly launched spacecraft, are deemed by many observers as not sufficient to prevent a potential exponential build-up of debris [24,29]. In this scenario, the use of active space debris removal missions, though very costly, may offer a solution [12,19]. An active

[1] `https://celestrak.com/NORAD/elements/`

debris removal mission, if successful, has a positive effect (or risk reduction) for all satellites in the same orbital band. This may lead to a dilemma: each stakeholder has an incentive to delay its actions and wait for others to respond.

The objective of this study is to model this effect and understand its consequences. We thus introduce a non-cooperative game between self-interested agents in which the agents are the owners of space assets. Using a high-fidelity simulator we estimate payoffs to the agents for different combinations of actions taken, and analyse the resulting game in terms of best-response dynamics and (Nash) equilibria. Contrary to the urgency of the space debris dilemma there has not been much attention to this problem in scientific circles. To the best of our knowledge we are the first to consider this dilemma in the context of multi-agent strategic decision making using empirical game theoretic techniques.

This report proceeds as follows. Firstly, we position our study in the context of related work. Next, we present our space debris simulator which includes a collision model, a break-up model, and an orbital propagator. We then outline our game theoretic methodology. Using our simulator we analyse the potential impact of several removal strategies on the orbital environment, and present a game theoretic analysis. Finally, we outline steps for (ongoing) further study, and conclude.

2 RELATED WORK

Our study can be placed in the context of two different areas of related work. Firstly, from a simulation modelling perspective various attempts have been made to accurately predict the evolution of space debris and the resulting risk of collisions for active spacecraft. Secondly, from a game theoretic perspective, researchers have utilised similar methods to study related problems of environmental pollution, and the shared exploitation of scarce resources [40].

One of the earliest analyses of the projected evolution of space debris was done by Donald J. Kessler in 1978 [15, 16]. This study led to the definition of the “Kessler Syndrome”, a particular scenario where the density of objects in LEO becomes high enough to cause a cascade of collisions, each producing new debris and eventually saturating the environment, rendering future space missions virtually impossible. In 2002, the Inter-Agency Space Debris Coordination Committee (IADC) outlined mitigation measures that should be implemented in newly launched spacecraft to limit the future growth of the debris population [10]. While effective [1], it is now widely believed that mitigation alone is not enough to prevent a further build-up of the debris population in LEO [24, 25, 30].

As a result, *active debris removal* (ADR) methods, in which spacecraft are deployed to capture and de-orbit larger pieces of debris and out-of-service satellites, are now considered by many as a necessary step to ensure sustainability of LEO [17, 28]. Several studies have been published recently in which the authors consider in detail the effect of active removal strategies to mitigate the space debris problem [26–28]. For example, Liou and Johnson [27] present a sensitivity analysis on object removal strategies. They propose removing 5, 10, or 20 objects per year, which can be seen as a single-agent approach. The authors compare these mitigation strategies with baselines “business as usual” or “no new launches” and show the effectiveness of object removals. The objects to be removed are chosen according to their mass and collision probability. We base our study on Liou and Johnson’s approach but, in contrast, consider a multi-agent scenario in which different space actors independently choose their removal strategy. In our model we implement individualised object removal criteria based on the potential risk to important assets of each of the actors.

The space debris removal dilemma is in many ways similar to other environmental clean-up efforts that have been studied using game theoretic tools in the past. For example, Tahvonen models carbon dioxide abatement as a differential game, taking into account both abatement costs and environmental damage [40]. More complex models have been studied as well, including for example the ability to negotiate emission contracts [7]. Another related model is the Great Fish War of Levhari and Mirman [23]. Although not the same as environmental clean-up, this scenario deals with shared use of a scarce common resource, which potentially leads to the same dilemma in game theoretic terms, known as the “tragedy of the commons” [6]. However, each of these studies has focused solely on a (simplified) mathematical model of the underlying system. In contrast, we use a complex simulator to obtain an approximate model using empirical game theoretic methods.

Analysis of complex strategic interactions using game theoretic tools is often hindered by the large action-spaces available to the agents in such scenarios. For example, in the space debris removal dilemma, each possible piece of debris to remove is potentially an action. Additionally, it is often impossible to define payoffs to all (combinations of) actions in advance. This has led recently to the advent of *empirical game theory* [42, 44]. The main idea is to limit the strategy space of each agent by introducing high level generic profiles, or meta-strategies, that capture the main aspects of the interaction. Then, the payoff function for this reduced strategy space can be estimated empirically, either by analysing data from a real system, or by simulating a model of the system. Standard methods and techniques from (evolutionary) game theory can then be applied to the estimated payoff function, e.g. to find approximate equilibria [14].

Such empirical game theoretic analysis has proven valuable in gaining insights into various complex real-world domains, such as automated trading [45], auction mechanism design [37], the game of Poker [38], collision avoidance in multi-robot systems [8], adaptive cyber-defence strategies [46], and large-scale bargaining [9]. In this work we follow a similar approach but focus on the domain of space debris removal.

3 SPACE DEBRIS SIMULATION MODEL

Our simulator[2] is built on top of the Python scientific library PyKEP [11], which provides basic tools for astrodynamics research including the SGP4 satellite orbit propagator we employed and various other utilities to interface to online databases. To simulate the future development of space debris in Low Earth Orbit (LEO) we developed several sub-modules, including a collision model and a break-up model, which we describe below.

The input data to our model comes from two satellite catalogues/databases: (i) the SATCAT[3] database containing descriptions of all objects on earth orbits that have ever been documented, and (ii) the TLE (two-line element set)[4] database providing up-to-date information on all active (not decayed) objects on earth orbits, including the orbital elements which uniquely identify an object's orbit, and which are used for orbit propagation.

Figure 1 shows how the orbital elements of all objects (less than 10 years old) in LEO are distributed. We compare the orbital elements of debris (including rocket bodies) and important assets (active satellites). One can observe that a large number of debris and active satellites share the same inclination (close to 100) and eccentricity (close to 0).

3.1 Collision model

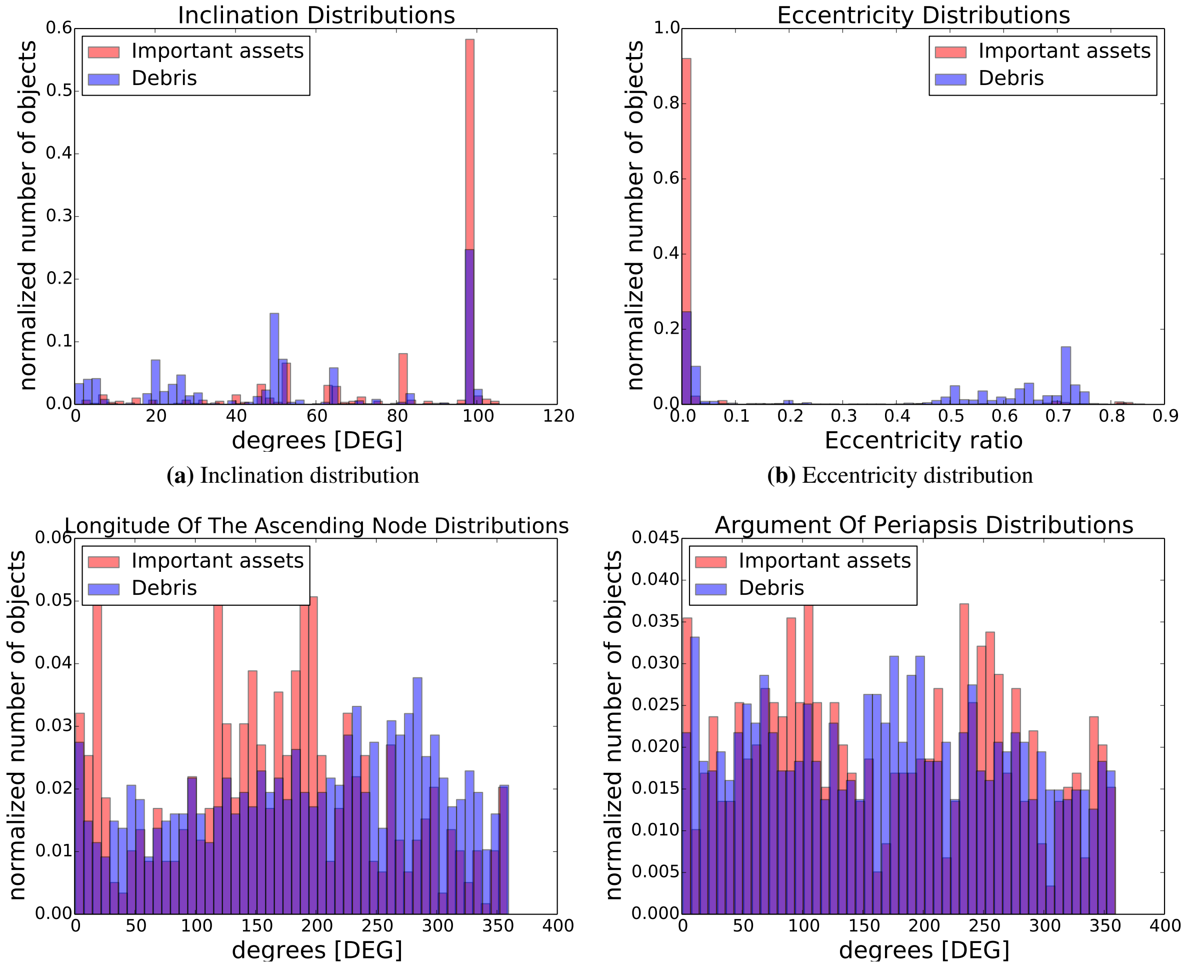
To evaluate probability of collision between objects we follow the *Cube* approach [31]. The Cube approach samples uniformly in time rather than space and is thus compatible with any orbital evolution simulation as it does not impose assumptions on the orbital geometry. This is particularly important in LEO, where orbital progression is significant in the considered time frame. We use the SGP4 [41] orbital propagator to calculate the evolution of the ephemeris (i.e., position and velocity) of an orbiting object given its TLE description. Ephemerides of all objects are calculated at regular time intervals. Space is then partitioned by a regular 3D-lattice and for any pair i, j of objects that fall into the same volume, the collision probability is evaluated as follows:

$$P_{i,j} = s_i s_j V \sigma \delta U ,$$

[2] https://github.com/richardklima/Space_debris_removal_model

[3] https://celestrak.com

[4] https://www.space-track.org/



(a) Inclination distribution

(b) Eccentricity distribution

(c) Longitude of the ascending node distribution

(d) Argument of periapsis distribution

Figure 1: Distribution of osculating elements of all important assets (active satellites, in red) and debris (in blue) in LEO which are currently less than 10 years old. The values on the y-axis are normalised to 1 for better comparison.

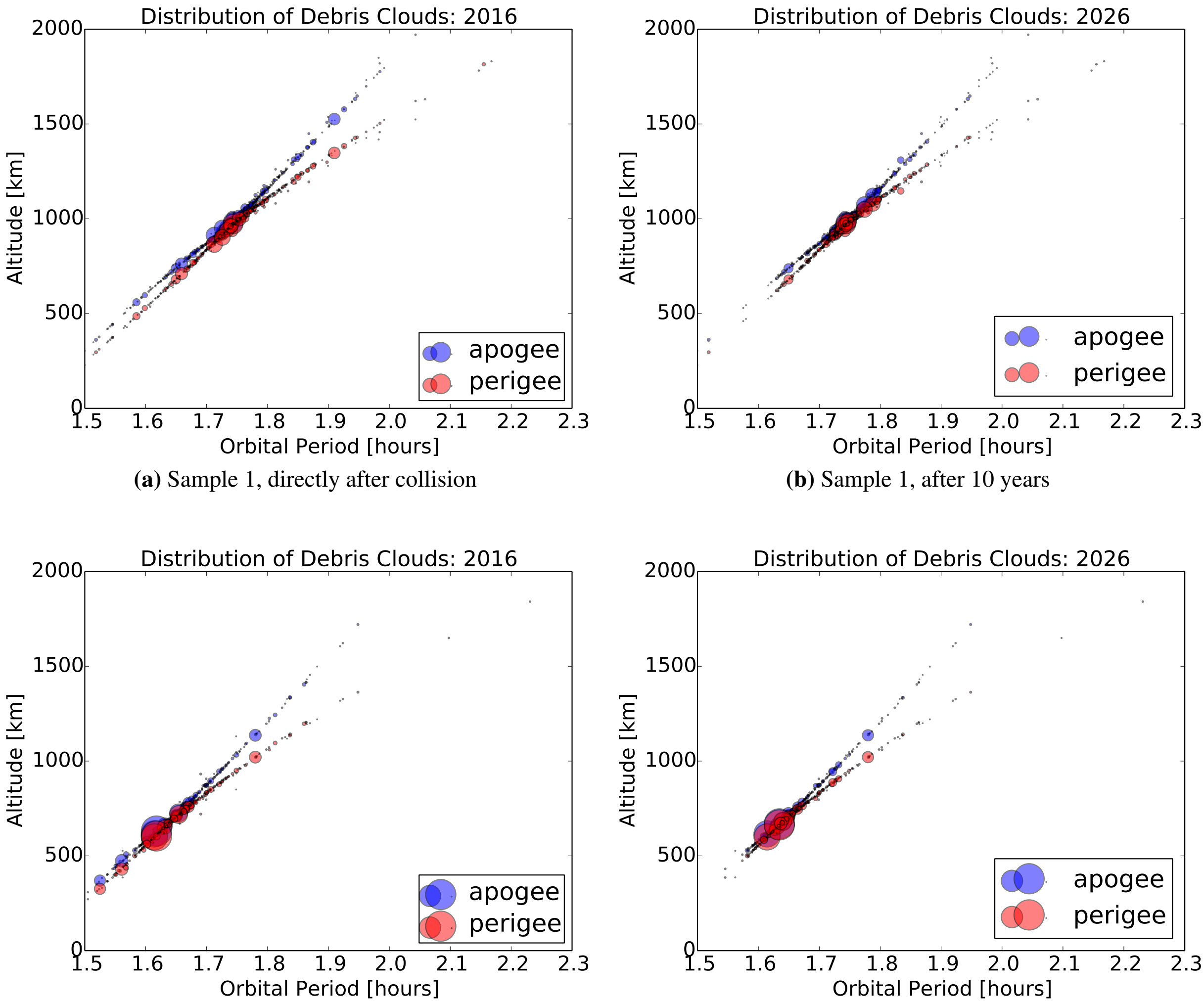
where $s_i = s_j = \frac{1}{\delta U}$ are the spatial densities of object i an j, $\sigma = 2\pi(r_i + r_j)$ is the cross-sectional collision area, V is the collision (relative) velocity of the two objects, and δU is the volume. For each pair, a pseudo-random number x is generated from a uniform distribution over the interval $[0, 1)$; if $P_{i,j} > x$, a collision event is triggered.

3.2 Breakup model

We use NASA's standard breakup model [13] to generate the population of fragments resulting from a collision event.[5] The NASA/JSC breakup model represents a widely accepted understanding of the fragmentation process of in-orbit collisions and explosions based on multiple ground-tests and radar observations of past events.

The model provides distributions for size, mass and ejection velocity of the fragment population parametrised by total mass and collision velocity of the parent objects. The number of fragments larger than a characteristic length-scale follows a power-law, the area-to-mass ratio follows a multivariate normal distributions, and the ejection velocity is sampled from a log-normal distribution. For details we

[5]This sub-module is implemented in *breakup.py* (`https://github.com/richardklima/Space_debris_removal_model`)



(a) Sample 1, directly after collision

(b) Sample 1, after 10 years

(c) Sample 2, directly after collision

(d) Sample 2, after 10 years

Figure 2: Debris clouds from two sample collisions in our simulation. Each dot represents one piece of debris; the size represents their mass. Left figures show the clouds directly after the collision and right figures after 10 years.

refer the reader to the original paper [13] as well as the description of the model in [17]. For each sampled fragment, we create a new TLE entry and add it to the population of objects being propagated by SGP4. Although the breakup model covers also explosions as well as non-catastrophic collisions, we only consider catastrophic collisions (i.e., leading to complete disintegration) in this work.

Figure 2 shows two examples (top and bottom row) of debris clouds resulting from collisions in our simulation. The debris cloud is plotted both directly after the collision (left) and after 10 years (right). Each dot represents one piece of debris; the size represents their mass. We can observe that a number of debris objects decay during these 10 years, in particular those with low altitudes. Atmospheric drag slows these object down, further reducing their altitude until they burn up in Earth's atmosphere. An example of debris decay is given in Figure 3. The object decays once it approaches the Earth's surface, which is around 6371 km (Earth radius).

3.3 Repeating launch sequence

To simulate future launches of new satellites we assume a "business as usual" scenario based on past data. One can assume that future launches will differ to past launches by many factors, e.g. the mission

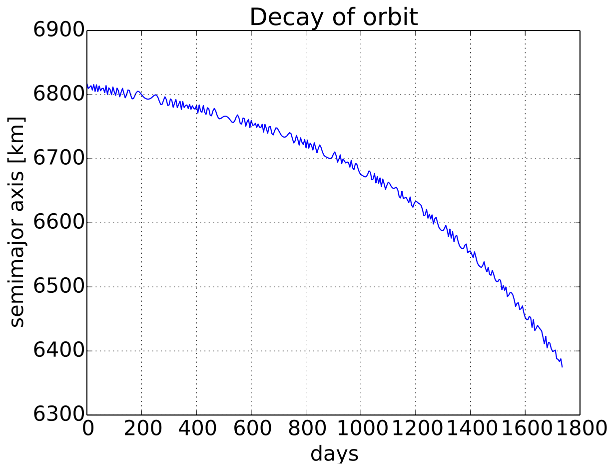


Figure 3: Orbital decay of a sample debris object in Earth's atmosphere around 6371 km (Earth radius) caused by atmospheric drag.

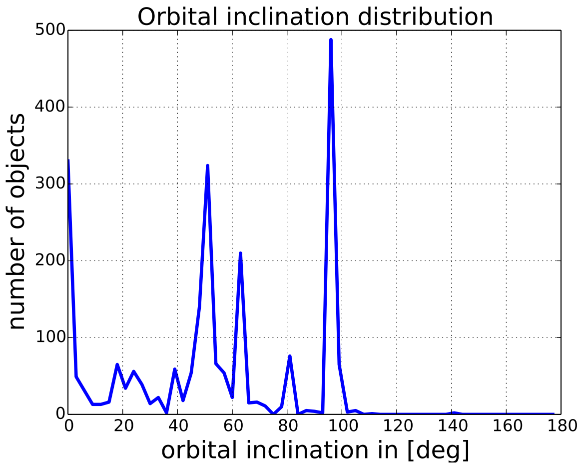
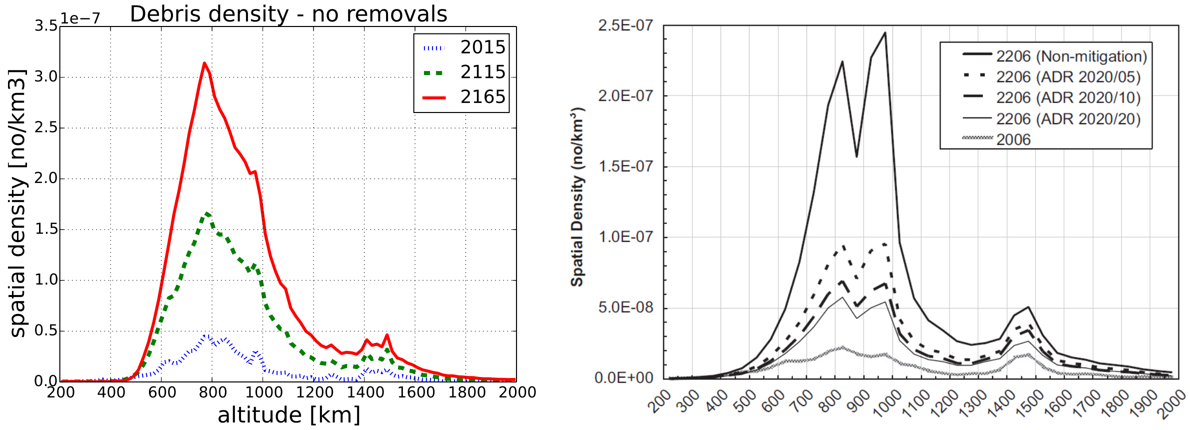


Figure 4: Orbital inclination distribution of objects being launched in last 10 years

purpose, the number of launches, their success rate and technology level, and the satellite's ability to decay in given time frame, etc. However, as a first step simplification we base our model on repeating a 10 year window from 2005 to 2015. From the SATCAT catalogue we filter all space objects introduced in this time window, excluding debris. For all these objects (both decayed and not decayed) we store the TLE data (for the decayed objects we store the last TLE recorded). We then repeat this 10 year launch sequence and introduce each month all the objects that were launched exactly (a multiple of) ten years ago. We keep all the orbital elements the same, except for the inclination, which we sample randomly from the distribution of inclinations of all objects in the repeated sequence. This way, newly launched satellites will have slightly different orbits, as can be expected. Figure 4 shows the distribution of orbital inclinations. We can see that the highest number of objects has an inclination of around 95°. One can see that Figure 4 differs from Figure 1 because it also includes already decayed objects.

We assume an increase over time in the number of launches due to technological development and changing needs. In addition to the 10 year repeating sequence, we increase the number of launches by 0.5% each year, by randomly sampling from the 10 year sequence. Note that each launch has a small probability of failing, due to the instability of some orbits resulting from the randomly sampled orbital inclination. Thus, some objects decay very soon after being launched, which can be thought of as e.g. unsuccessful launches, break-up during first stage, etc.



(a) Spatial density prediction of our model. The three curves depict prediction of spatial density in years 2015, 2115 and 2165.

(b) Spatial density prediction according to the model of Liou and Johnson [27]. The different curves represent spatial density predictions in year 2206 with no removal, with active debris removal (ADR) of 5, 10, 20 objects per year, and the reference density in the year 2006.

Figure 5: Spatial density prediction in LEO.

3.4 Validation

In order to validate our model we simulate the evolution of the total number of debris and compute the resulting spatial density in different altitude ranges for the next 150 years, and compare our findings to previously reported predictions. In Figure 5a we show our prediction of spatial density in LEO, assuming no mitigation strategies. The three curves in Figure 5a represent the current situation (the year 2015), and predictions for the years 2115 and 2165. One can observe that the highest spatial density is in the region around 800 kilometre altitude, caused by the Iridium-Kosmos collision (789 km) and the Chinese anti-satellite missile test causing the Fengyun-1C breakup (865 km). In our prediction, the spatial density increases significantly over time due to new collisions.

We compare our findings with those reported previously by Liou and Johnson [27], shown in Figure 5b for different possible removal strategies. The non-mitigation scenario in Figure 5b (for 2006 and 2106) shows a similar trend as the one observed in our model, with spatial density of debris increasing over time in particular in the altitude ranges around the Iridium-Kosmos collision and the Fengyun break-up. Small differences are likely due to different implementations, as the full details of the model of Liou and Johnson are not available.

4 GAME METHODOLOGY

Game theory models strategic interactions in the form of games. The most basic type are *normal-form games*, in which n players each have a set of actions to choose from. Without prior communication, each player selects an action, and the combination of actions by all player (the joint action) determines the payoff to each. Players are assumed to be rational, i.e. they will always want to play a *best response* (in terms of individual payoff) to the joint action of all remaining players. A joint action in which each action is a best response to all other actions is called a *Nash equilibrium* (NE).[6]

The space debris removal dilemma can be modelled as a game in which the players are space actors, their actions are debris removal strategies, and the payoffs are derived from removal costs as well as collision risks. The strategic interaction results from the fact that debris removal by one actor may affect the collision risks to others as well.

[6] For an introduction to game theory see [5].

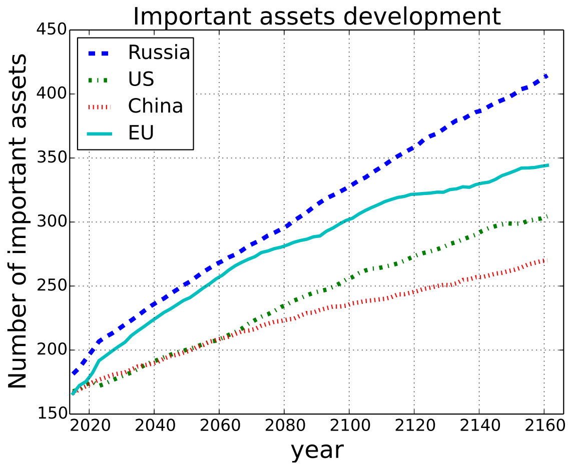


Figure 6: Prediction of the development of the number of important assets (active satellites) for each player based on the assumption of a 0.5% yearly growth in future launches.

Players — As a starting point we mainly focus our analysis on a two-player game, due to the amount of computation required to estimate the payoff function. Additionally, we show a smaller experiment with three players. As players we consider historically important actors, focusing in the two player game on (1) the United States (US) represented by The National Aeronautics and Space Administration (NASA), and (2) the European Union (EU) represented by European Space Agency (ESA) and all EU member states. In the three player scenario we add China (CN) as the third player; the fourth major space actor, Russia (Roscosmos), is not included in our game but does play a role in the simulator in terms of repeating past launch sequences.[7]

In further ongoing work we make use of a high performance computing (HPC) cluster which allows us to run a more extensive set of experiments with a larger number of players (see Section 7.3). Then, we can consider additional actors, such as those organisations mentioned in the introduction.

Important Assets — For each player we store a list of *important assets*. Important assets are all active objects owned by that player which are not debris, and which have been launched in the last 10 years (we assume a 10 year life span of satellites). The list of important assets is continuously updated during the simulation.

Figure 6 shows an example of the development of important assets for each of the actors. One can observe that a small difference in the number of important assets at the beginning causes a big difference at the end of the projection due to the repetition of launches from the same sequence, combined with the 0.5% yearly increase.

Actions — The players' actions are defined by the number of debris objects that will be removed each year. In our game, the players can remove 0, 1, or 2 risky objects every 2 years. We assume self-interested agents, meaning that each player first removes objects which directly threaten their important assets, and then removes objects which may potentially collide in general. The reasoning for the latter is that debris resulting from any collision may pose a potential future risk to a player's important assets. Therefore, removing any risky debris object (not only those that threaten important assets) may benefit all the players to some extent. Each player decides on their strategy at the beginning of the game, and does not change it later. Thus, we assume a one-shot normal form game.

Risks and Payoffs — During simulation we keep track of the risk of collision (see Section 3.1) to each player's important assets. The cumulative sum of these risks is taken as the overall risk to each player under the simulated scenario. Subsequently, we derive payoffs from the costs of losing important assets, and the costs of object removal. These payoffs are computed by multiplying the player's risk r by the associated cost of loosing an asset C_l, and adding the cost of removing one object each year

[7] This choice is arbitrary, we expect similar results if Russia would be included as active player instead.

Table 1: Payoff functions for the different strategies.

Strategy	Payoff function
Remove 2	$-(r \cdot C_l + T \cdot C_r)$
Remove 1	$-(r \cdot C_l + 0.5 \cdot T \cdot C_r)$
Remove 0	$-(r \cdot C_l)$

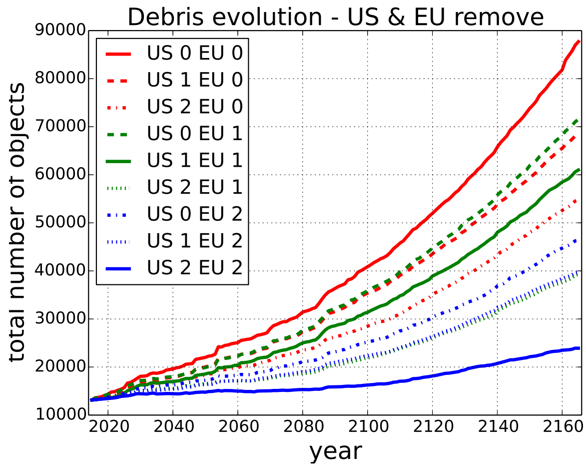


Figure 7: Debris evolution for next 150 years considering different strategies. The y-axis depicts the number of objects in low-earth orbit. Each curve represents a different combination of strategies (remove 0, 1 or 2 objects) taken by the two players (US and EU).

C_r multiplied by the number of removed objects and the time horizon T. Specifically, Table 1 lists the payoff functions that are used given the player's strategy. Since the term $r \cdot C_l$ is common to each strategy, we can assume without loss of generality that $C_l = 1$ (in arbitrary units) and focus only on C_r in the remainder.

5 SIMULATION RESULTS AND PROJECTIONS

We use our simulator to project the evolution of debris and collision risks with a time horizon of 150 years, i.e. the period 2016-2165, while repeating the launch history of 2006-2015 with a 0.5% yearly increase. We first focus on a 2-player 3-action game, with players US and EU, and the actions to remove 0, 1, or 2 objects every two years as described above. For each combination of actions we average over 160 Monte-Carlo runs to account for randomness in the collision and break-up modules. Error margins are omitted in the figures for readability, but are reported below in Table 2.

5.1 Debris evolution

Figure 7 shows the evolution of objects in LEO for different combinations of strategies taken by the US and the EU. We observe an exponential growth trend without mitigation, in line with findings previously reported by Liou and Johnson [27]. One can clearly see that removing risky objects has a positive effect as it leads to a much lower total number of objects in LEO. Note that when both players remove 2 objects every two years, this means that in total 300 objects are actively removed over the course of 150 years. In contrast, this leads to a reduction in total number of objects in LEO of over 60,000, due to a strong decrease in number of collisions and resulting debris. Also note that the total number of active satellites in each scenario is less than 1,500 (see 6), a small fraction of the total number of objects.

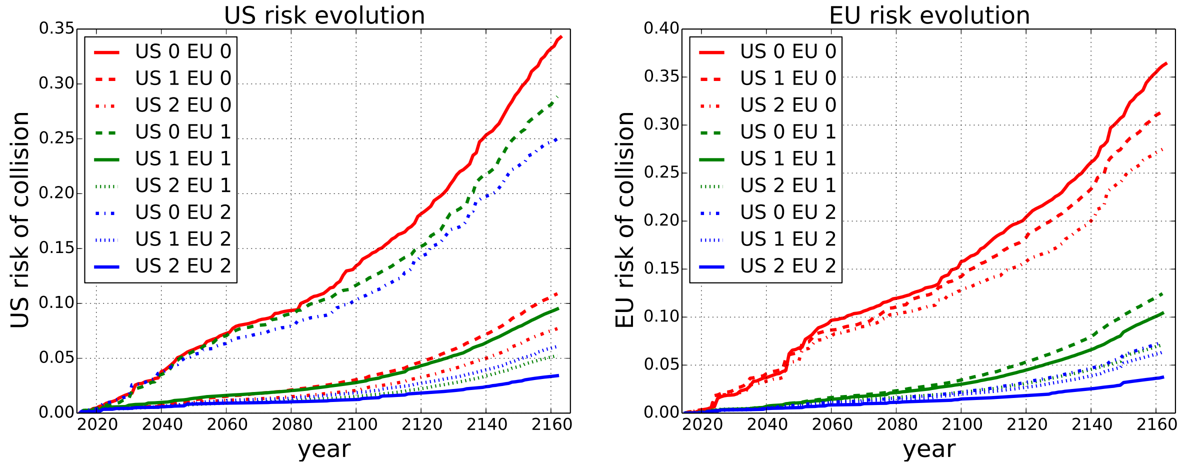


Figure 8: Evolution of the overall risk to important assets of each player, for different combinations of actions taken by both players.

5.2 Risk evolution

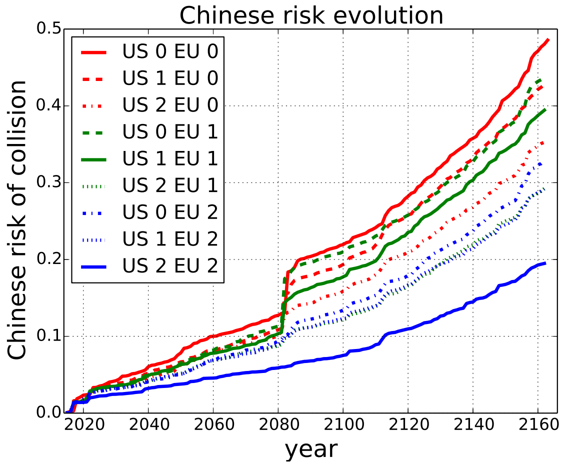
We now look at the potential risks to the players' important assets, as described in Section 4, that result from the debris evolution in LEO. Figure 8a shows the evolution of the expected overall risk to the US. One can observe that if the EU removes objects it helps US as well. However, objects removed by the US have greater impact on their overall risk, which is explained by the fact that each player removes firstly the objects that threaten their important assets directly, and only then they remove objects that pose a risk in general. Therefore, we can see that the joint action {US 1, EU 0} helps the US substantially more than {US 0, EU 2}, even though in the latter case more objects are removed in total.

In Figure 8b we can see the expected overall risk of losing important assets for the EU. We observe similar trends as in the previous figure: the EU is better off when they remove objects that directly threaten their assets. However, even when the EU removes nothing but the US does, the EU risks decrease. This means that, as expected, there is in fact a dilemma as each player benefits from mitigation efforts of others, without having to pay a cost (free-riding).

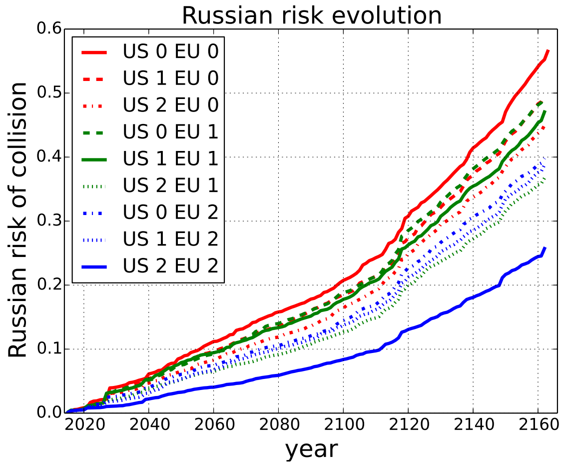
The free-riding effect can be observed as well when looking at the risk evolution for both China and Russia. Even though these actors did not take part in mitigation in our scenario (essentially playing the fixed action of *remove 0*), they still benefit from a reduced risk to their important assets. Figure 9a shows this for the case of China, and similar results are observed in Figure 9b for Russia. One can notice an abrupt increase in the Chinese risks around the year 2080, which is eliminated when more objects are removed in total. The joint efforts of the US and the EU in fact remove the one object which causes this high risk to the Chinese important assets. Note that this abrupt increase is persistent across simulation runs, caused by the deterministic nature of the orbital propagator.

6 GAME THEORETIC ANALYSIS

We now turn to the game theoretic analysis of the space debris removal dilemma. First, we use the results reported in Section 5 to derive a normal-form game representation of the two-player scenario. We then thoroughly analyse this game. Finally, we give an example of a three-player game.



(a) Risk to Chinese assets



(b) Risk to Russian assets

Figure 9: Free-riding effect in the overall risk to important assets for non-active players China and Russia (both removing 0), for combinations of actions taken by the US and EU. Non-active players benefit from other players removing objects. In the left figure we can observe an abrupt increase in the risks for China (around the year 2080) for several scenarios. This risk can be negated by actions of the active players (lower curves), further highlighting the free-riding effect.

6.1 Two player game

Using the simulation results of Section 5, we can now construct a normal-form game representation of the two-player space debris removal dilemma. First, we construct a risk matrix, showing the overall risks to the two players (US and EU) for each combination of actions. Then, we use this risk table together with the cost functions defined in Table 1 to derive payoff matrices for different removal costs C_r, and analyse all possible Nash equilibria outcomes. This game has some interesting properties due to the payoff structure and asymmetry of the studied game. The asymmetry comes from different levels of space programs of the agents resulting in different number of assets on the orbits. There are other factors contributing to the asymmetric property of the game model such as position of some assets on orbits with higher density of space debris and therefore higher potential risks to these assets. Table 2 shows the average cumulative risks accrued by both players, taken from the results in Figures 8a and 8b (time horizon 150 years, 160 runs for each scenario). A cumulative risk of 0.36385 for the EU in the no removal case can be interpreted as an expected loss of 0.36385 assets in total for the EU. The lower part of Table 2 shows the 95% confidence intervals for these averages. Clearly, when no removal costs are taken into account, it is in the best interest of each player to remove as many debris as possible. However, one should assume non-zero removal costs. Using the cost functions of Table 1 we can transform the risk matrix into a payoff matrix for any given cost C_r. Table 3 shows an example payoff matrix for cost $C_r = 0.003$ (in arbitrary units, see Section 4). The player's best responses as indicated in bold. One can see that there are two pure Nash equilibria in this scenario, {US 0, EU 1} and {US 1, EU 0}. Moreover there is one mixed equilibrium at where US and EU mix between removing 1 and 0 with probability $(0.488, 0.512)$ and $(0.218, 0.782)$, respectively.

We can identify two interesting regions in the range of costs C_r. For very low costs, removing 0 will never be a best response for either player. Similarly, for high costs, removing 2 will never be a best response. Therefore we can focus on two sub-games defined by the action-pairs $\{0, 1\}$ and $\{1, 2\}$. We compute Nash equilibria for a range of C_r, and visualise the results in Figures 10 and 11 for the sub-games $\{0, 1\}$ and $\{1, 2\}$, respectively. On the y-axis we have the probability of playing the first action in each sub-game (which equals 1 minus the probability of the second action) for US (top) and EU (bottom). The colours/line styles indicate the action pairs that make up the equilibria, e.g. the solid lines

Table 2: Risk matrix for both players for each combination of strategies. The risks are the average cumulative risk of losing an asset over the course of 150 years. We show 95 % confidence intervals in the lower table.

	EU 2	*EU 1*	*EU 0*
US 2	$0.03413, \mathit{0.03733}$	$0.05247, \mathit{0.07108}$	$0.07704, \mathit{0.27474}$
US 1	$0.06073, \mathit{0.06352}$	$0.09499, \mathit{0.10405}$	$0.10885, \mathit{0.31401}$
US 0	$0.25022, \mathit{0.07368}$	$0.28848, \mathit{0.12447}$	$0.34261, \mathit{0.36385}$

US 2	$\pm 0.00528, \pm \mathit{0.00563}$	$\pm 0.00712, \pm \mathit{0.00785}$	$\pm 0.00838, \pm \mathit{0.01874}$
US 1	$\pm 0.00689, \pm \mathit{0.00767}$	$\pm 0.00820, \pm \mathit{0.00938}$	$\pm 0.00994, \pm \mathit{0.01954}$
US 0	$\pm 0.01896, \pm \mathit{0.00786}$	$\pm 0.01685, \pm \mathit{0.01061}$	$\pm 0.01831, \pm \mathit{0.01859}$

Table 3: Payoff matrix for both players for each combination of strategies for $C_r = 0.003$. Players' best responses are in bold text.

	EU 2	*EU 1*	*EU 0*
US 2	$-0.48413, -\mathit{0.48733}$	$-0.50247, -\mathit{0.29608}$	$-0.52704, -\boldsymbol{\mathit{0.27474}}$
US 1	$-0.28573, -\mathit{0.51352}$	$-0.31999, -\mathit{0.32905}$	$-\mathbf{0.33385}, -\boldsymbol{\mathit{0.31401}}$
US 0	$-\mathbf{0.25022}, -\mathit{0.52368}$	$-\mathbf{0.28848}, -\boldsymbol{\mathit{0.34947}}$	$-0.34261, -\mathit{0.36385}$

in Figure 10 correspond to the pure Nash equilibria $(0, 0)$ (black) and $(1, 1)$ (red). The x-axis shows the ratio between the cost of removal C_r and the cost of losing an important asset C_l (assuming without loss of generality $C_l = 1$, as described in Section 4).

In both figures we see interesting transitions from the single Nash equilibrium at $(0, 0)$, to a situation where three equilibria exist at $(0, 1)$, $(1, 0)$ and one mixed, and finally back to a single pure equilibrium at $(1, 1)$. These transitions phases also include a stage in which only one of the asymmetric pure equilibria at $(1, 0)$ or $(0, 1)$ exists. The existence of these asymmetric equilibria is interesting, and results from the asymmetry that is inherent in the risk matrix due to actors having different numbers of assets and in different orbits.

6.2 Strategic substitutes and existence of pure equilibrium

In games we construct in this paper are finite strategic-form games. The celebrated result of Nash [32] shows that every finite game possesses at least one Nash equilibrium in mixed strategies. While mixing makes a lot of sense in some settings, e.g., zero-sum games like poker and sports matches, in other settings pure strategy equilibria are more compelling. In this section we discuss properties, relevant for the games we construct, that guarantees the existence of pure equilibria.

In general, active debris removal has a positive effect not only for the instigator of the removal but also for other players, and this is the cause of the dilemma that we are studying. In game-theoretic terminology, this suggests that we have games with a *weak strategic substitutes property*. The most well-known economic game with this property is Cournot oligopoly [33, 39]. First we formally define the property.

Our exposition is based on [4], but, for simplicity, is specialized to the setting of *finite* pure strategy sets. Denote the set of players by $N = \{1, 2, \dots, n\}$. Each player $i \in N$ has a finite pure strategy set S_i that is a subset of non-negative real numbers, i.e., $S_i \subset \mathbb{R}_{\geq 0}$. In our space debris removal games, S_i can be thought of as the set of choices of how much debris player i removes, so in Table 1, the three strategies *remove 0*, *remove 1*, *remove 2* would correspond to $S_i = \{0, 1, 2\}$. Let S denote the set of

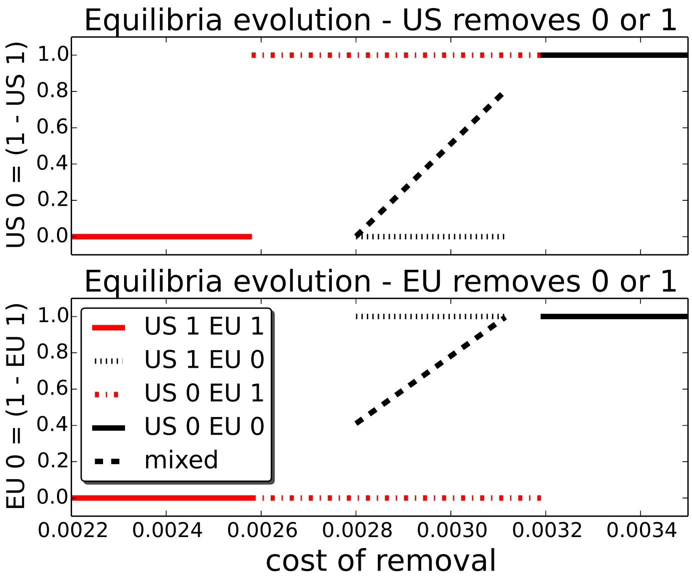


Figure 10: Equilibrium strategies for the sub-game {remove 0, remove 1} for a range of removal costs C_r. The y-axis shows the probability of each player (US and EU) removing 0 objects, which is equivalent to one minus the probability of removing 1. The x-axis shows the ratio between the cost of removal C_r and the cost of losing an important asset C_l (assuming $C_l = 1$).

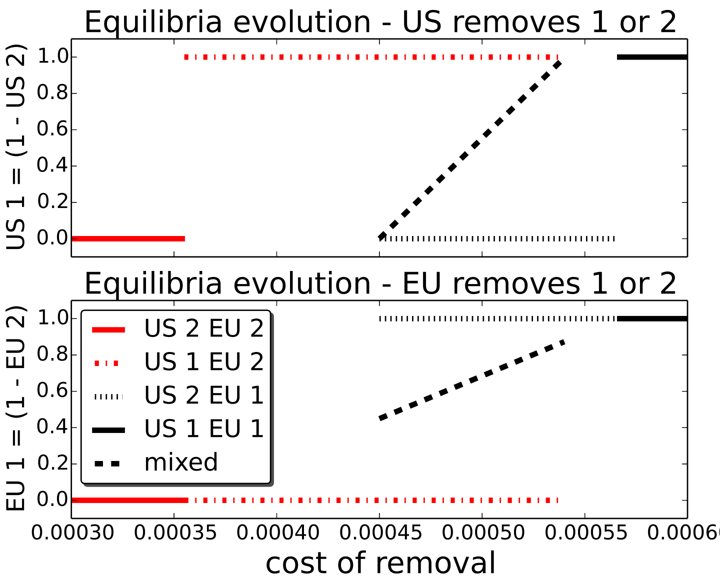


Figure 11: Equilibrium strategies for the sub-game {remove 1, remove 2} for a range of removal costs C_r. The y-axis shows the probability of each player (US and EU) removing 1 object, which is equivalent to one minus the probability of removing 2. The x-axis shows the ratio between the cost of removal C_r and the cost of losing an important asset C_l (assuming $C_l = 1$).

all pure strategy profiles, i.e., $S := S_1 \times S_2 \times \cdots \times S_n$. Denote the payoff function of player i by $\pi^i : S \to \mathbb{R}$.

For the purpose of stating known results on the existence of pure equilibria, we are going to assume that the payoff of player i depends only on his choice and the aggregate (i.e., sum) of the strategy choices of the other players. Formally, for any pure strategy profile $s = (s^1, s^2, \ldots, s^n) \in S$, we denote by $\bar{s}_{-i}$ the *additive aggregate* of other players strategies, i.e.,

$$\bar{s}_{-i} = \sum_{j \in N \setminus \{i\}} s^j \ .$$

Then we write our restricted payoff function as $\pi^i(s^i, \bar{s}_{-i})$. For any choice $s_{-i} \in \prod_{j \in N \setminus \{i\}} S^j$, the set $\beta_i(\bar{s}_{-i})$ of *best responses* of player i is given by

$$\beta_i(\bar{s}_{-i}) = \arg\max_{t \in S^i} \pi^i(s^i, \bar{s}_{-i}) \ .$$

Recall that $s = (s^1, s^2, \ldots, s^n) \in S$ is a (pure) Nash equilibrium if

$$s^i \in \beta_i(\bar{s}_{-i})$$

for all $i \in N$. For a given player $i \in N$, we denote by $\bar{S}_{-i}$ the set of all possible values of $\bar{s}_{-i}$, the additive aggregate of other players' strategies, i.e., $\bar{S}_{-i} = \{\bar{s}_{-i} \mid s \in S\}$. We say that a game like this has the *weak strategic substitutes property* if there exists a function $b^i : \bar{S}_{-i}$ for these games with restricted payoffs functions such that:

- $b^i(x) \in \beta^i(x)$ for all $x \in \bar{S}_{-i}$, [b^i selects a best response for i]
- $b^i(x) \leq b^i(y)$ whenever $x > y$. [b^i does not increase in $\bar{s}_{-i}$]

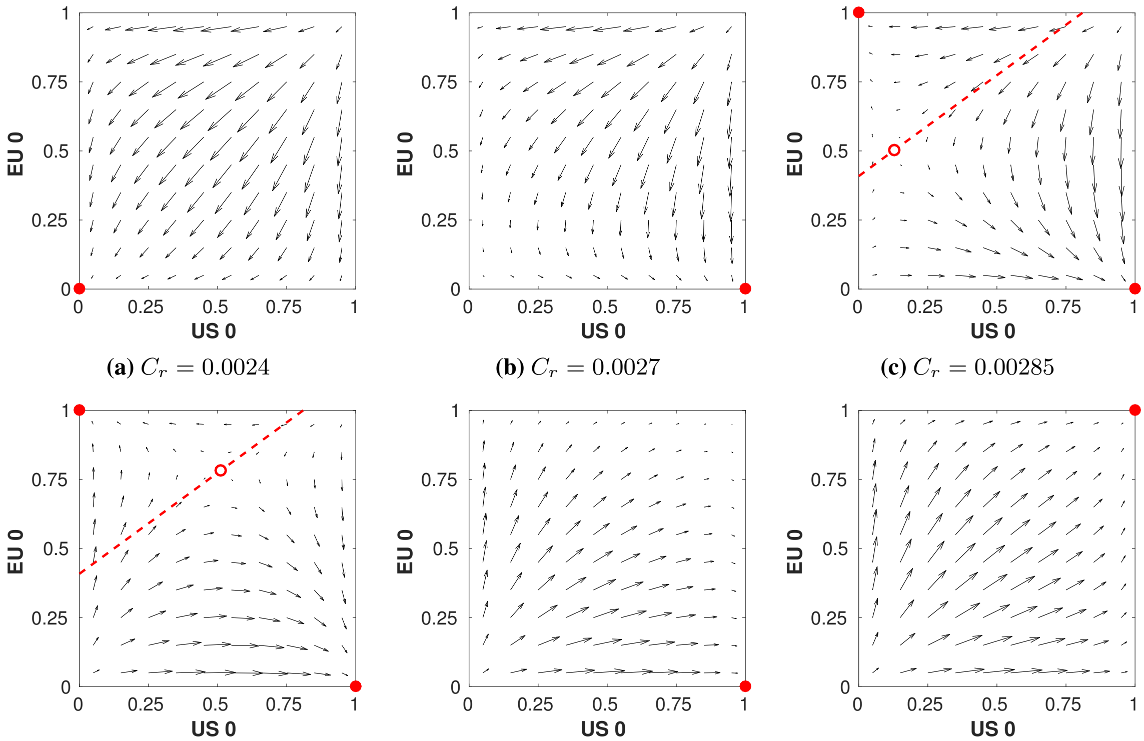
Such a game with the weak strategic substitutes property, and where payoffs depend only on one's own strategy and the sum of others' strategy, are known to always possess at least one pure strategy Nash equilibrium, which is shown via a potential-function type argument [4, 20, 21].

Notice that the weak strategic substitutes property can be defined as above even without the restriction that the payoffs are of the form $\pi^i(s^i, \bar{s}_{-i})$ for player i. However, in that case a pure equilibrium may not always exist. The games that we construct comprise payoffs that arise from (noisy) simulations and thus do not satisfy the restricted payoff form. However, the games we construct do either have the weak strategic substitutes property, or their violation of it is not statistically significant. Thus it is an interesting future direction to see if we can fit restricted payoff functions to closely approximate the empirical payoffs that arise from our simulations. We discuss this further below, where we also discuss slightly more general aggregation functions for defining restricted payoff functions that, along with the weak substitutes property, guarantee the existence of pure equilibria. First though we note that when considering only two players, the restriction of the payoff functions is without loss of generality, and so we have the following.

Observation 1. *Any two-player game that has the weak strategic substitutes property admits a pure equilibrium.*

For example, in Table 3, we can see that this game has the weak strategic substitutes property since, as the EU removes more (going from 0 to 1 to 2), the best responses of the US (as indicated by the boxes in Table 3 is to weakly remove less (going from 1 to 0 to 0, respectively), and similarly for the best responses of the EU as US changes pure strategy. This game has two pure equilibria (US 0, EU 1) and (US 1, EU 0) and one mixed equilibrium as one can also see in Figure 10.

As mentioned above, for games with the weak strategic substitutes property, the existence of pure equilibria is known for a wider class of games than just those where the payoff of i depends on his strategy and the sum of the others'. This aggregation of players' strategies done by $\bar{s}_{-i}$ can in fact be



(a) $C_r = 0.0024$ **(b)** $C_r = 0.0027$ **(c)** $C_r = 0.00285$

(d) $C_r = 0.003$ **(e)** $C_r = 0.00315$ **(f)** $C_r = 0.0033$

Figure 12: Evolutionary dynamics of the sub-game {remove 0, remove 1} for different values of C_r. Stable attractors are indicated with ● and unstable attractors with ○. The dotted line indicates the trajectory on which the mixed equilibrium ○ moves as C_r changes.

an arbitrary linear combination rather than just a sum, and further can include linear combinations of *products* of strategies too; for full details see [22]. Thus there is actually a lot of scope to fit payoff functions that meet these criteria for existence of a pure equilibrium due to strategic substitution and also are consistent with the payoff estimates that arise from our simulations. We leave this as an interesting direction for further work.

6.3 Evolutionary dynamics

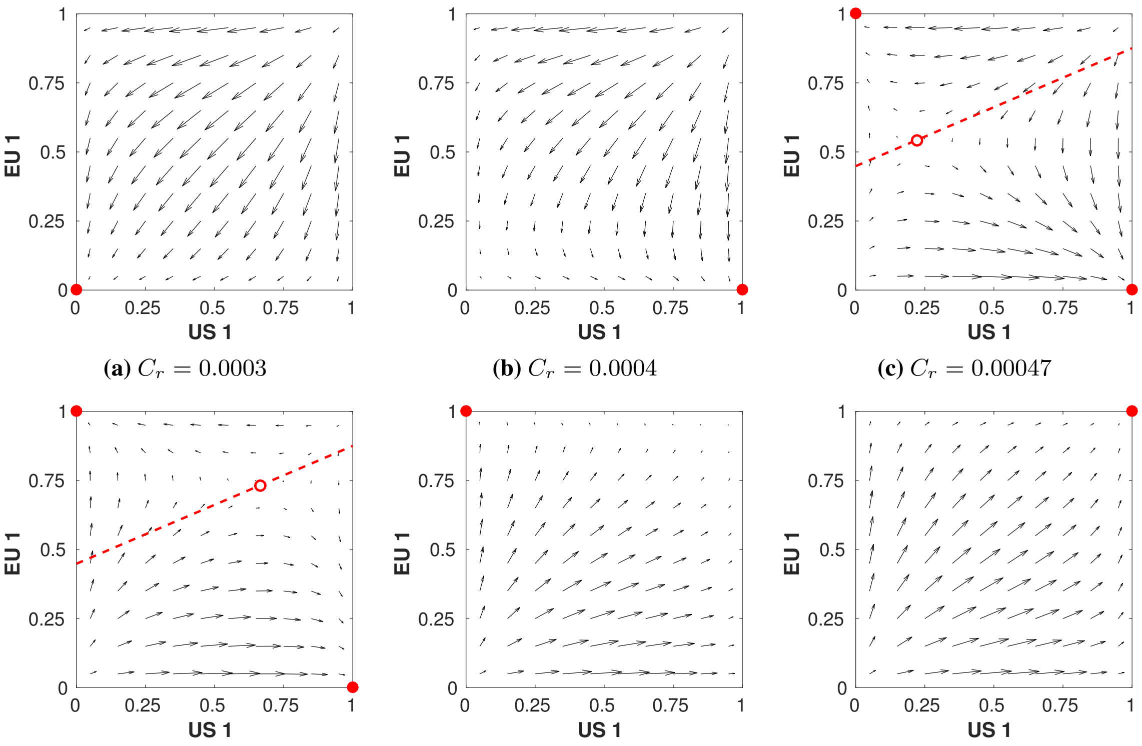
Another way to study the strategic properties of a game is by looking at the corresponding evolutionary dynamics. Evolutionary game theory[8] represents a player's strategy by a population of individuals, each of a certain type which corresponds to one of the player's possible actions. The fraction of the population belonging to each type indicates the probability with which the player will play the corresponding pure action. The *replicator dynamics* dictate how the fraction x_i of each type i in the population x changes over time due to evolutionary pressure:

$$\dot{x}_i = x_i[f_i(x) - \bar{f}(x)]$$

where $f_i(x)$ is the fitness (expected payoff) to type (action) i in the population, and $\bar{f}(x)$ is the weighted average fitness of the whole population. Under the replicator dynamics, types that do better than average will increase in abundance, whereas types that do worse will decline.

Figure 12 shows the directional field of the replicator dynamics for the sub-game {remove 0, remove 1} for different values of C_r corresponding to the different sets of equilibria observed in Figure 10. The axes show the probability with which both players play the action "remove 0" (US 0 and EU 0). The

[8]See [43] for an introduction.



(a) $C_r = 0.0003$ **(b)** $C_r = 0.0004$ **(c)** $C_r = 0.00047$

(d) $C_r = 0.00051$ **(e)** $C_r = 0.00055$ **(f)** $C_r = 0.0006$

Figure 13: Evolutionary dynamics of the subgame $\{\text{Remove } 1, \text{Remove } 2\}$ for different values of C_r. Stable attractors are indicated with ● and unstable attractors with ○. The dotted line indicates the trajectory on which the mixed equilibrium ○ moves as C_r changes.

arrows indicate the direction and magnitude of change. The replicator dynamics give insight into the stability of the different equilibria and their corresponding basins of attraction. We can conclude that the mixed Nash equilibria in panels (c) and (d) are unstable, as a small perturbation will cause the population to move towards one of the stable pure equilibria. Moreover we can see that the *basin of attraction* for the pure equilibrium {US 0, EU 1} (bottom right corner) is larger than for {US 1, EU 0} indicating that this equilibrium is more likely to arise when both players iteratively optimise their strategy. This is of particular interest when full knowledge of the game is not available and the players need to *learn by interacting*, e.g. when space actors mutually adapt their policy based on an estimate of other actors' policies. In fact, the replicator dynamics are descriptive of various multi-agent learning processes, and as such studying these dynamics provides valuable insights in the context of *adaptive agents* as well [2].

In Figure 13 we show the directional field of the replicator dynamics for the sub-game {remove 1, remove 2} for different values of C_r corresponding to the different sets of equilibria observed in Figure 11.

6.4 Three player game

So far we have only considered two active players. Here, we take a first step in analysing a larger game between three players (space actors): the US, the EU, and China (CN).[9] We focus on the two-action sub-game {remove 0, remove 1} only to facilitate analysis. Table 4 shows the cumulative risks for all three players, averaged over 180 Monte Carlo runs, as well as the corresponding confidence intervals. The risks for each player are distinguished by different font styles. We can see that the risks for China are considerably higher than for the US or the EU, even though their total number of important assets is

[9] This choice is arbitrary, we expect similar results if Russia would be included as active player instead.

Table 4: Risk matrix (top) and corresponding 95% confidence intervals (bottom) for a three-player two-action game. Players are US, *EU* and **China**, the font type shows risk values belonging to each player.

		EU 1	*EU 0*
CN 1	US 1	0.07013, *0.08621*, **0.10162**	0.09185, *0.31547*, **0.13320**
	US 0	0.27373, *0.10294*, **0.12226**	0.30759, *0.33980*, **0.15468**
CN 0	US 1	0.09229, *0.10067*, **0.38774**	0.11226, *0.32031*, **0.43121**
	US 0	0.28510, *0.12539*, **0.43225**	0.34400, *0.36335*, **0.49774**

		EU 1	*EU 0*
CN 1	US 1	±0.0061, ±*0.0074*, ±**0.0086**	±0.0071, ±*0.0174*, ±**0.0103**
	US 0	±0.0163, ±*0.0080*, ±**0.0098**	±0.0184, ±*0.0189*, ±**0.0110**
CN 0	US 1	±0.0077, ±*0.0087*, ±**0.0255**	±0.0093, ±*0.0183*, ±**0.0260**
	US 0	±0.0157, ±*0.0099*, ±**0.0275**	±0.0181, ±*0.0180*, ±**0.0279**

lower (see Figure 6). This interesting result may be due to the specific orbits used by each of the player, some being more dense in terms of debris than others, which requires further investigation.

We can again convert the risk matrix into a payoff matrix using the payoff functions defined in Table 1. In Figure 14 we visualise the Nash equilibria for varying costs of removal C_r. At the left part of the figure the cost of removal is low, and therefore it is in the best interest of all three players to remove debris. However, for increasing costs it becomes a best response for the US to stop removing, and there exists a pure equilibrium (US 0, EU 1, CN 1). The reason that the US opts out first is due to their lower overall risk compared to the two other players. In contrast, the higher risks to China mean it is in their interest to keep removing, even when both US and EU have opted out. When the cost rises even further (the right side of the figure), we see that for none of the players removing is viable.

Although for most removal costs C_r the strategic substitute property discussed previously holds, there is a range of costs for which the property is violated. However, the payoff differences leading to this violation are not statistically significant and may be resolved by increasing the number of Monte Carlo samples of our simulation, which is left for the future work.

6.5 Discussion

In our game-theoretical analysis we identified Nash equilibria for different levels of cost of removals; although the costs of active debris removal are still prohibitively high at the moment they are expected to decrease with future technological developments while the value of orbiting assets may increase. Additionally, we investigated the strategic substitute property that appears in this type of game scenario, and which guarantees existence of a pure equilibrium under certain conditions. Although a mixed equilibrium exists for some costs as well, it is often more desirable to focus on pure equilibria. Specifically, in our scenario, it cannot be expected that space actors will randomize over pure strategies to decide on their space debris removal policy. Another disadvantage of a mixed equilibrium in this game is its instability (as shown in Figure 12), which is undesirable in our scenario, where the choice of action taken has a huge impact on the earth orbit environment. The results of this study help space actors to better understand the debris removal problem and its short and long term consequences, in order to prepare for mitigation strategies. For instance, we show that removing just one high risk debris object every two years can already substantially decrease the risk of collision for active satellites. Additionally, removal of indirect

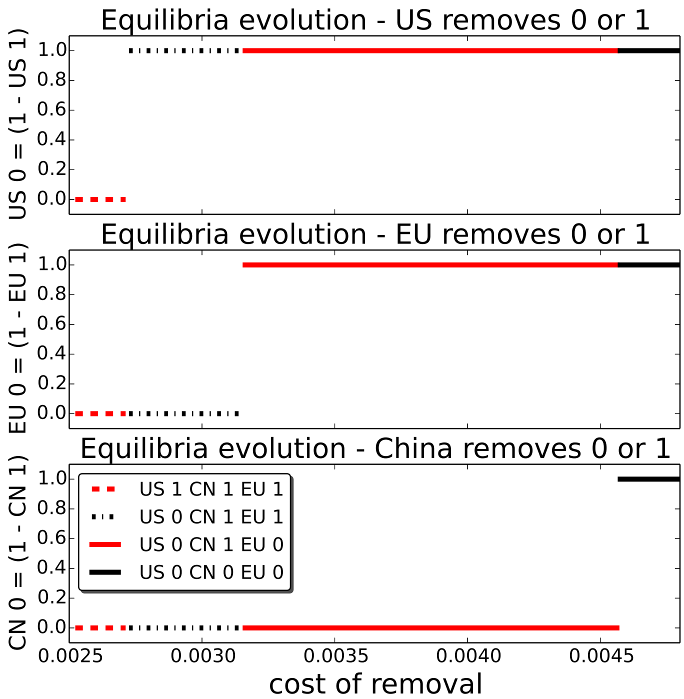


Figure 14: Equilibrium strategies for players US, EU and China (CN) of the game $\{$remove 0, remove 1$\}$ for a range of removal costs C_r. The y-axis shows the probability of each player (US, EU, and CN) removing 0 objects, which is equivalent to one minus the probability of removing 1. The x-axis shows the ratio between the cost of removal C_r and the cost of losing an important asset C_l (assuming $C_l = 1$).

collision risks is beneficial as well as it reduces the number of potential future on-orbit collisions.

7 NEXT STEPS

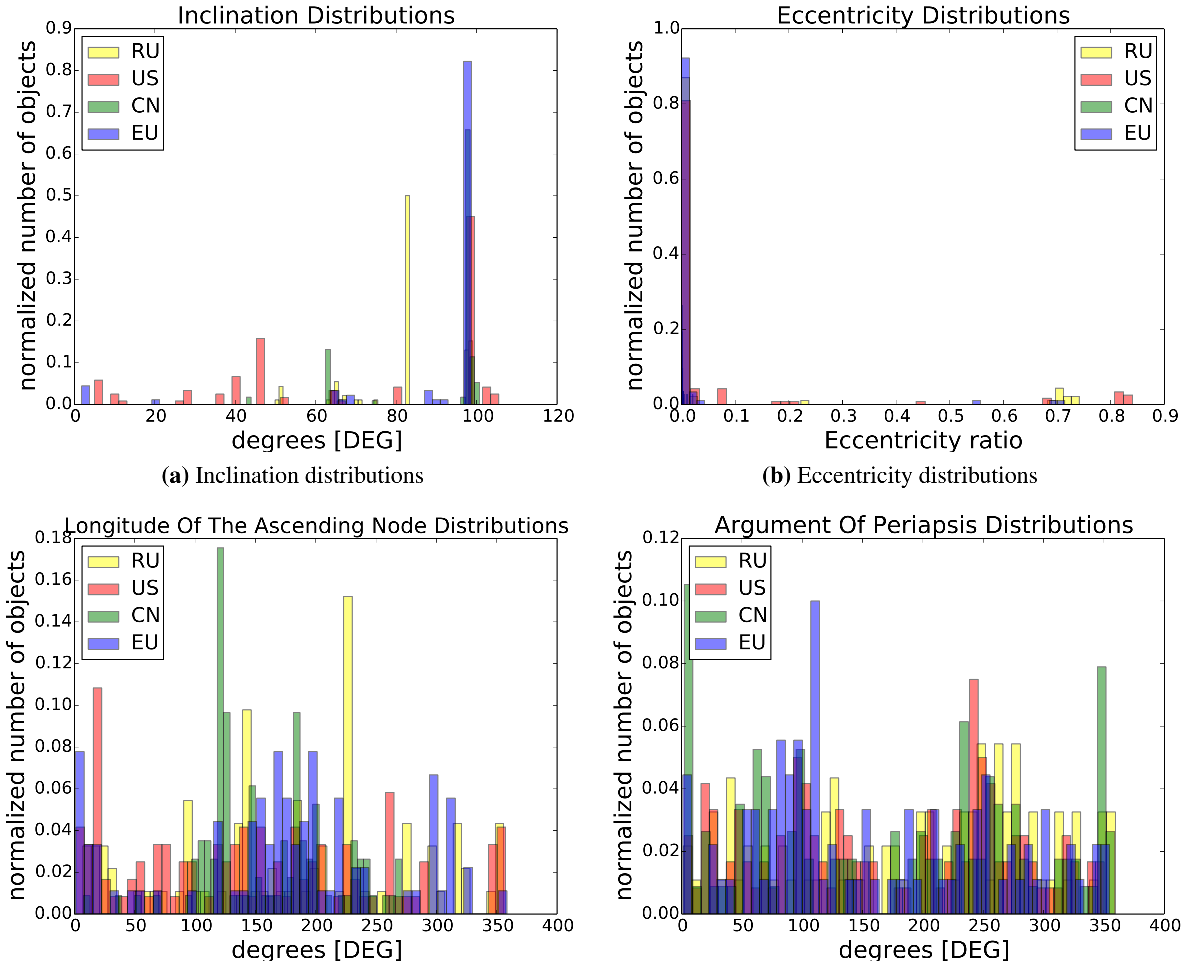
We have already initiated extensions of the study in different directions. In particular, we are working on a more realistic relaunching sequence, a more complex game formulation, and a *high performance computing* implementation to allow for more thorough simulations. We outline each of these below.

7.1 Realistic relaunching sequence

So far, we have assumed that space actors will launch new satellites that are highly similar to those launched in the past 10 years, only varying the inclination (see Section 3.3). More realistically, new satellites will be launched in slightly different orbits, due to changing needs, technological ability, and changes in the debris environment. In order to introduce more sophisticated future launching of space objects, we randomly sample features of past launch data.

In particular, we derive the distributions of the osculating Keplerian elements of past launches for each of the players:

- Eccentricity – e
- Inclination – i
- Longitude of the ascending node – Ω
- Argument of periapsis – ω



(a) Inclination distributions

(b) Eccentricity distributions

(c) Longitude of the ascending node distributions

(d) Argument of periapsis distributions

Figure 15: Distribution of the osculating elements of the important assets of the US, EU, China (CN) and Russia (RU), for assets less than 10 years old.

For each new launch, we randomly sample the osculating elements from these distributions. For now we sample the osculating elements independently, however we plan on investigating sampling from multivariate distributions as well. Finally, we sample the mean anomaly M uniformly from the range 0°to 360°.

Figure 15 shows the distributions of osculating elements for US, EU, China (CN) and Russia (RU) that are derived from past launch data. We can observe that the distributions differ, therefore it is desirable to sample new objects according to these distributions for each of the players individually.

7.2 More complex game formulations

From a game-theoretic point of view, our approach has been limited to a one-shot normal-form game, which assumes that the actors fix their removal policy for the entire time horizon up front. More realistically, these strategies may be adaptive and dependent on the state of the LEO environment as well as on current and past actions by others. Several game formulations are possible that would allows such interdependencies, however each requires careful thought and set-up of experiments in order to derive a meaningful heuristic payoff function.

Dynamic games: One option is to move from a one-shot game to a repeated stage game (or dynamic game), where the agents can decide on their strategy based on the history of past play. For example, one player may learn that the others are more prone to take clean-up actions, and based on that decide to opt

Table 5: Risk matrix and corresponding 95% confidence intervals for a four-player game. Players are US, EU, China and Russia, where US and EU removes 1 object per two years.

agent	total expected risk	confidence interval
US 1	0.06186	±0.00803
EU 1	0.07598	±0.00770
CN 0	0.32353	±0.02536
RU 0	0.43354	±0.02659

out – or vice versa. The main difficulty with this approach stems from the fact that actions influence the environment and hence the payoff matrix will change with each iteration of the game.

Stochastic games: When players take the state of the environment explicitly into account when deciding on their actions, we move to the domain of stochastic (or Markov) games. These games have multiple states, with probabilistic transitions between them that (may) depend on the actions taken by each of the players in the previous state. Since the space debris density and hence the resulting risks are essentially continuous variables, this will require to design a meaningful discretisation of the state space that preserves the dynamics of the continuous environment.

Extensive form games: Both normal-form and stochastic games assume that all players take their action simultaneously, and without communication. In reality, space organisations may publish their plans at separate points in time, and thus potentially be influenced by each others' decisions. Such sequential move games can be modelled as an extensive form game. Again, this more complex representation will require careful design in order to be meaningful, while still allowing to compute heuristic payoff functions in reasonable time.

7.3 HPC model extension

Projecting the future evolution of space debris itself is a very complex problem with many unknown variables and inputs, and therefore some necessary simplifications and assumptions have been made. Despite these simplifications the simulation is computationally demanding, which makes it difficult to obtain the necessary number of Monte Carlo runs, especially for larger games. We aim to use HPC clusters to obtain statistically significant results for more extensive scenarios, in which we can include more players (e.g. Russia, India) as well as more diverse debris removal strategies. In Table 5 there is an experiment example using HPC clusters. We modified our code using MPI package for Python to parallelize the process. We have successfully tested our model on HPC clusters running 110 sample runs of one game scenario (US and EU removes 1 object per two years). The initial setting of the clusters had restricted time window of 4 hours therefore some of the long lasting runs were unsuccessful and therefore we have not obtained the runs with high number of objects which means runs with high expected risks. One can observe by comparing Tables 4 and 5 that the risks in the latter table are lower due to not including some of the long lasting experiments. Nevertheless we can confirm successful modification of our model for HPC clusters use for future work, where we will be able to extend the clusters time window to obtain all the results. Using HPC clusters substantially reduced the time needed for each experiment, which will allow to run an extensive number of various experiments.

8 CONCLUSION

In this report we have shown that the active debris removal (ADR) dilemma can be modelled as a multi-player strategic game, which highlights how the rational behaviour of players varies depending on the cost of ADR versus the value of active satellites. When the cost is high, players are not inclined to engage in ADR, whereas a lower cost (or higher valued satellites) tip the scale in favour of removal actions.

We have introduced a multi-player non-cooperative game named the *Space Debris Removal Dilemma* based on prediction data from our space debris and satellite simulator. The simulator, built on top of

PyKEP, models the evolution of the object densities in the Earth orbital environment, taking into account new future launches, collisions of orbiting objects, and natural decay. Our model includes the possibility to account for different debris removal strategies.

In our game-theoretic analysis we identified which removal strategies for the different actors are in equilibrium with each other, i.e. which strategies purely rational actors are expected to decide on. We demonstrated the sensitivity of these equilibrium strategies to the ratio between cost of debris removal and the value of the active satellites. Although the costs of active debris removal are still prohibitively high at the moment they are expected to decrease with future technological developments while the value of orbiting assets may increase. The results of this study help to better understand the debris removal problem and its short and long term consequences.

In conclusion, coming back to the specific objectives that were set for this Ariadna study, we have (1) introduced a realistic debris removal game, and demonstrated how this game can be used for the computation and approximation of equilibria, basins of attraction, and stability properties of the different equilibria or solutions of the game; (2) gained insight in the evolutionary dynamics of, and determine potential optimal strategies in, the debris removal game by means of heuristic payoff tables; and (3) shown by means of the strategic substitutes property the types of equilibria we can expect in this game in general. The latter will allow to devise incentives (e.g. through taxation or subsidies) that would make sure each space actor plays their part in the clean-up effort. However, this is left for future work.

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