Metamaterials for Space Applications

Manipulation of Lightwave Through Coordinate Transformation

Final Report

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Abstract

In this study, we apply the coordinate transformation method for designing optical devices that are able to control the propagation of electromagnetic waves in an unprecedented manner. The major part of our study is dedicated to invisibility cloaks, which includes an analytical analysis of the electromagnetic and material properties of arbitrarily-shaped cloaks, as well as detailed studies on approximate and simplified models of cylindrical invisibility cloaks. Electromagnetic concentrator is another type of coordinate transformed device covered in this project.

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1. INTRODUCTION

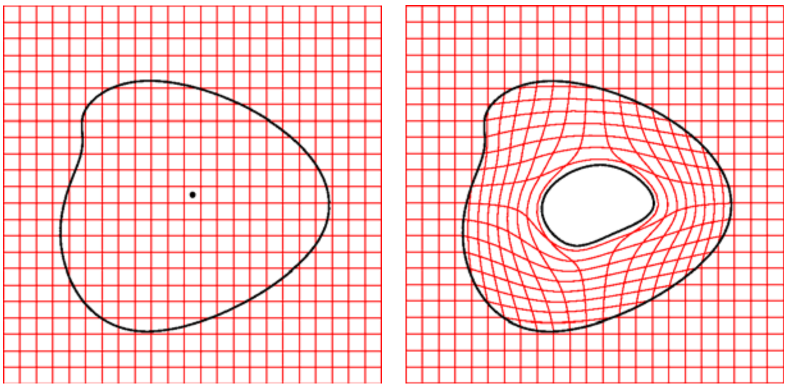
Recently the advent of artificial electromagnetic (EM) materials, referred to as metamaterials has opened up many new ways for us to interact with or control EM waves. Metamaterials so far have been realized in a broad spectral range, covering from radio frequency to near-optical frequencies (refer to the introduction of [1]). EM phenomena not existing in nature have been experimentally demonstrated. Negative refraction [2] and negative-index slab superlensing (focusing beyond diffraction limit) [3] are representative applications of this class of engineered materials. Metamaterials are not only able to tailor the permittivity and permeability values at our will, but also can provide a precise control over the material anisotropy and its spatial distribution. While it is not a problem for engineering each individual metamaterial unit, placing the units in an appropriate order for achieving a desirable macroscopic optical phenomenon remains a challenge. In simple language, with trees in various size and color at our disposal how should one form an enchanting forest? The recently proposed coordinate transformation method provides the very recipe for solving such design problems.

The impact of this unprecedented way of guiding light on space applications is huge. The performance of many optical devices can be improved thanks to the impedance matching nature of the devices designed via transformation optics. In addition, the family of invisible devices can provide an effective way to control the solar pressure. Many of the results obtained in this study are of great relevance when targeting the design of particular devices based on the principle of the coordinates transformation.

Figure 1(a) shows a Cartesian coordinate system. Consider a coordinate transformation from the Cartesian coordinate system (x, y, z) to a curved coordinate system (q_1, q_2, q_3). The two coordinate systems are related via

$$x = f_1(q_1, q_2, q_3),\ y = f_2(q_1, q_2, q_3),\ z = f_3(q_1, q_2, q_3)\,. \qquad (1)$$

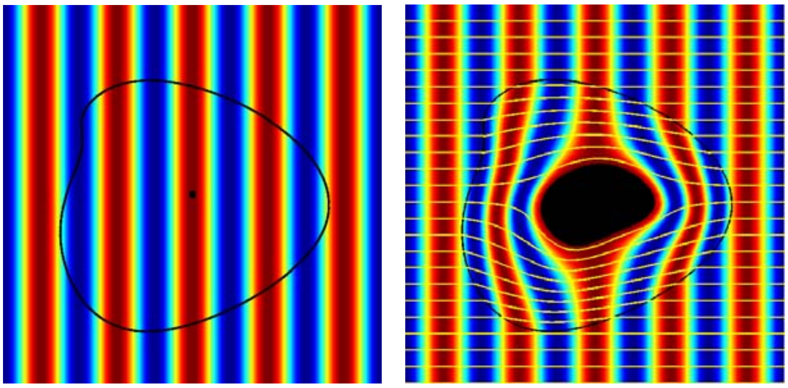
An example of the curved coordinate system, as interpreted in the original coordinate system, is shown in Fig. 1(b).



(a) (b)

Fig. 1. Coordinate transformation. (a) Cartesian coordinate; (b) Curved coordinate.

A change in coordinates does not change the behavior of EM waves as long as we change the distributions of material parameters accordingly, as results directly from the form invariance property of the Maxwell equations. However when we re-interpret the curved space (including the material parameters) back onto the physical space, we see light flowing in a distorted manner, in a way as suggested by how the curved coordinates are chosen. In other words, one mimics the effect of a curved space by introducing "artificial potentials", or filling space (where coordinates have been changed) with complex permittivity and permeability profiles. Thanks to the form invariance of the Maxwell equations this tool allows to derive straightforwardly the set of material parameters subject to a specific coordinate transformation, and hence a desired EM phenomenon. In Fig. 2(a) we show a right-propagating plane wave in the originally empty space with a Cartesian coordinate system. Assume that the system is invariant in paper-normal (z) direction, and that the plane wave is of single E_z polarization. In Fig. 2(b), we show how the field is distorted according to a coordinate transformation depicted in Fig. 1. The objective of the coordinate transformation, for the current example, is to obtain the ε and μ profiles in the annular region bounded by two black curves in Fig. 2(b), whose function is to exclude the field from the black region (while introducing no perturbation to the incident plane wave). More details about this coordinate transformation technique can be found in [4,5].



(a) (b)

Fig. 2. (a) Plane wave in flat Cartesian coordinate; (b) Distortion of plane wave caused by a metamaterial that mimics a curved coordinate.

What we have obtained in Fig. 2(b) is a rather general 2D *invisibility cloak*, one out of many types of *transformation media* as promised by the coordinate transformation method. First, the annular structure is invisible since the medium is *electromagnetically* equivalent to an air column. Second, due to the fact that the inner surface of the annular structure is obtained by blowing-up a line, no EM field can penetrate into the interior, and therefore cloaking of arbitrary objects is possible. Proposals of such invisibility cloaking devices first appeared in [6,7].

The potential of this technique for designing new optical devices is enormous. In terms of space applications, the control of the light flow can be decisive for improving the limitations of optical systems, thermal shielding systems and optically based telecommunications devices. Due to time limitation of the project, we focus mainly on invisibility cloaks. Invisibility cloaks can reduce the scattering cross section of an object (in theory by 100%). Therefore, it is not only promising for hiding space devices from foreign detection units, but also useful for (particularly in space applications) reducing solar pressure. Knowing the limitations of invisibility cloaking technology can also be helpful for counter-detecting foreign objects. EM concentrator is another subject that is covered in this study. A concentrator can somewhat be considered as an opposite to invisibility cloak: it is for collecting rather than avoiding radiations. Concentrators (or concentration optics) can be potentially deployed for photovoltaic fiber, solar cell, solar heating technologies.

2. RESEARCH SUMMARY

2.1. Invisibility cloaks in arbitrary shapes

In this work, we prove that an invisibility cloak in arbitrary shape, either based on blowing-up a line or based on blowing-up a point, is perfectly invisible. Moreover, no field can penetrate into the interior of the cloak. This work therefore, in a general sense, provides an independent validation for the invisibility cloaks designed according to the transformation approach.

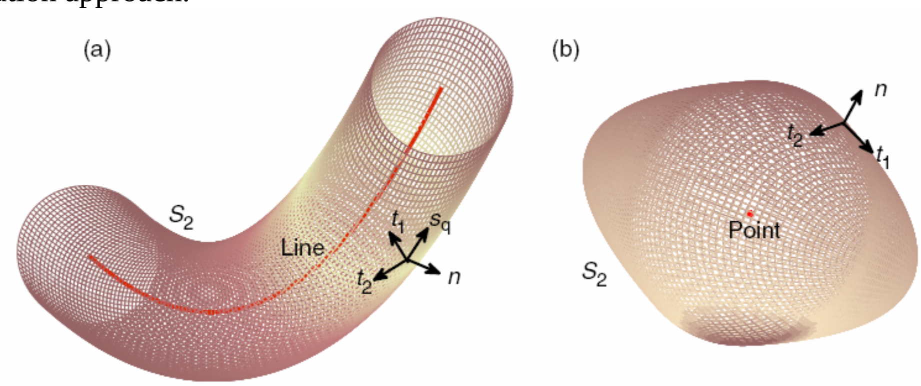


Fig. 3. Inner surfaces of two types of invisibility cloaks. (a) Inner surface based on blowing-up of a line. (b) Inner surface based on blowing-up of a point.

The inner surfaces of two types of invisibility cloaks are shown in Fig. 3. Instead of using numerical tools like the finite element method, we carry out the proof analytically by exploiting the fact that the eigenwave solution in a transformation medium is well related to that in the original space. In our proof, using the material parameters derived from the coordinate transformation, we show that, at the *outer interface* (not shown in Fig. 3) of a general cloak, all eigenwave components of the incident wave transmit across without causing reflection or excitation of new eigenwave component. This suggests that the outer surface of a cloak obtained through coordinate transformation is perfectly matched in impedance to outer space. At the *inner surface*, two types of cloaks are discussed separately. For the *line-transformed cloak*, it is found that there exists a tangential field discontinuity [t_2 component, see Fig. 3(a)] across the inner interface. The inner surface of the cloak has either infinite or zero material parameters, which is equivalent to a combination of both perfect electrical conductor (PEC) layer and perfect magnetic conductor (PMC) layer. Therefore the inner surface supports both electric and magnetic currents, which justifies the tangential field discontinuity. At the same time, the perfectly conducting inner surface prevents any field from going into the cloaked region. For the *point-transformed cloak*, all eigen-field components damp to zero at the inner surface of the cloak. Therefore no field can go into the enclosed region. At the same time no reflection occurs at the inner surface (owing to zero field amplitude there). All the details of this work can be found in Ref. [8].

This study provides better understanding of the EM behavior as well as awareness of metamaterial engineering challenges for invisibility cloaks in arbitrary shapes. Arbitrarily-shaped cloaks are important for devices whose overall shape is critical for certain functions and therefore should be preserved. For example, airplanes and some spacecraft have their shapes designed for certain aerodynamic purposes. Cylindrical or spherical cloaking structures certainly are not applicable to these devices.

2.2. Approximate models of cylindrical cloak

In this work, we investigate in particular the properties of cylindrical invisibility cloaks, which belongs to the line transformation class described above. Arguably, cylindrical cloaks are the simplest structures in terms of practical fabrication[1]. To realize a cylindrical cloak, we need only to take a coordinate transformation along the radial direction, i.e. from the electromagnetic space (r', θ', z') to the physical space (r, θ, z) with $\theta'=\theta$ and $z'=z$. A class of radial mapping functions can take the form of

$$r' = \frac{b}{(b-a)^n}(r-n)^n, \qquad (2)$$

where a and b are the inner and outer radii, respectively; n is called the transformation order number. The corresponding material property for the cylindrical shell is

$$\varepsilon_r = \mu_r = \frac{r-a}{nr}, \varepsilon_\theta = \mu_\theta = \frac{nr}{r-a}, \varepsilon_z = \mu_z = \frac{nb^2(r-a)^{2n-1}}{(b-a)^{2n} r}. \qquad (3)$$

The refraction of light rays by a cylindrical cloak designed with different n varies greatly. In Fig. 4, we show both the E_z field distribution and the refraction of light rays in (a) electromagnetic space, (b) a cloak designed with $n=1$, and (c) a cloak designed with $n=3$. It is observed from Fig. 4 that, with a cloak designed with a higher-order coordinate transformation, light rays are bent more heavily as they enter into the cloak medium. As a consequence, if we truncate away a thin layer of material from the inner surface of the cylindrical cloak, most rays tend to be guided around the shell still perfectly without noticing the structural perturbation at the inner surface. Since the inner surface of a cylindrical invisibility cloak inevitably has infinite material parameters, getting rid of the inner layer is a viable way for physical implementations of such a cloak.

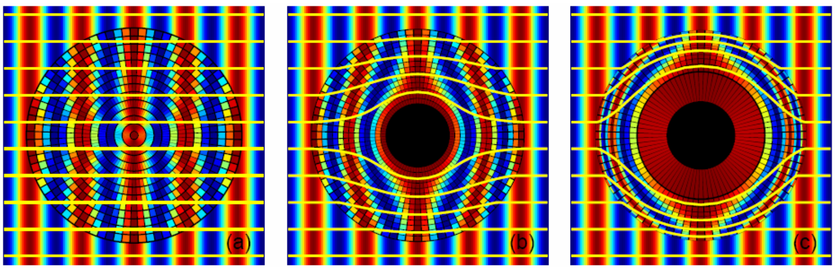


Fig. 4. (a) Virtual flat space; (b) Cloak with n = 1; (c) Cloak with n = 3. Colormap: Ez. Green lines: Poynting vectors. Invariant coordinate lines are imposed.

[1] Even so, an ideal cylindrical invisibility cloaks poses great fabrication challenges to metamaterial engineers at the current technology level. So far, the proof-of-concept demonstration of a cylindrical invisibility cloak [9] is only an approximated version targeted for normal wave incidence with single polarization.

We derived analytically the scattering coefficient in different cylindrical orders for a particular cloak whose inner surface is peeled away by a fixed width, and deduced the variation of the scattering with *n*. We focus on normal wave incidence and transverse-electric (TE) polarization (i.e. *E* is along *z* or paper normal direction). The relevant material parameters are therefore only μ_r, μ_θ, and ε_z. The result is shown in Fig. 5. For this particular analysis, we have used inner diameter *a* = 0.1m, outer diameter *b* = 0.3 m, and wavelength λ = 0.15 m. The thickness of the layer removed is *d*=0.01 m. PEC lining is present at the inner surface to prevent light penetration. Indeed, a high *n* value is noticed to be able to decrease the scattering coefficients (examined in different cylindrical orders) greatly, especially for the high order scattering coefficients.

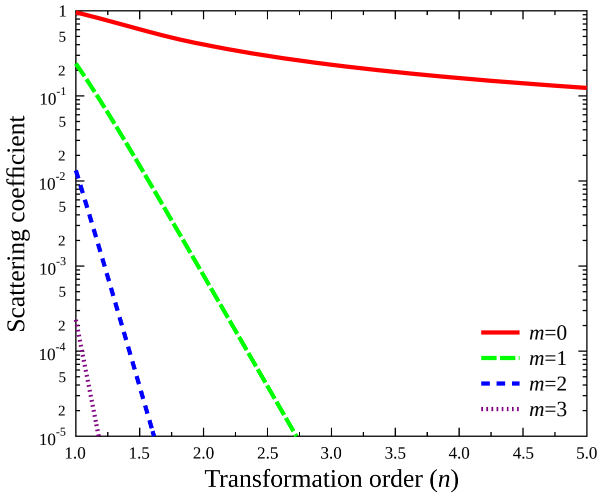


Fig. 5. Effect of transformation order on the cylindrical wave scattering coefficients.

The reason behind such an improvement can be explained by examining the material parameters at the cloak's inner surface *r* = *a*+*d*. That is,

$$\varepsilon_r = \mu_r = \frac{d}{(a+d)n},$$
$$\varepsilon_\theta = \mu_\theta = \frac{(a+d)n}{d}, \tag{4}$$
$$\varepsilon_z = \mu_z = \frac{nb^2}{d(a+d)}\left(\frac{d}{b-a}\right)^{2n}.$$

From Eq. (4), we see that, when *n* increases, ε_r (μ_r) decreases, ε_θ (μ_θ) increases, and ε_z (μ_z) decreases (valid for *d* << *a*). In other words, the parameters all approach to their ideal values (infinity or zero), hence the improvement for invisibility performance. The drawback is that, although we are removing the same thickness of layer from the inner surface, the required maximum and minimum material become more extreme as a larger *n* is deployed. A more convincing analysis is to see how the scattering coefficients vary with respect to *n*, but based on fixed values of maximum and minimum material parameters, rather than based on fixed thickness of peeled layer. Here, we fix the achievable extreme of the material parameters for μ_r and $\mu_\theta$as $\mu_{r,min}$=*0.08* and $\mu_{\theta,max}$=*12.5*, respectively. Under this restriction, the thickness of the layer to be peeled increases from *d* = 0.0087 m when *n*=1 to *d* = 0.0435 m when *n*=5. The variations of the cylindrical scattering coefficients with respect to n are shown in Fig. 6. Comparing Fig.6 to Fig. 5, we observe that a higher order number is still able to reduce the scattering coefficients efficiently, although the rates of decreasing in scattering for all cylindrical waves are not as fast.

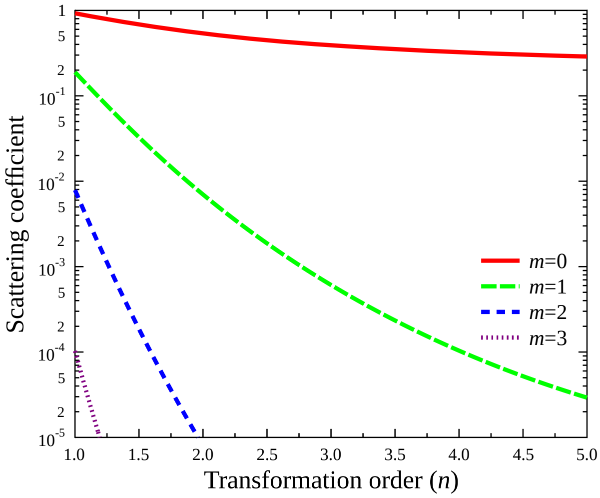


Fig. 5. Effect of transformation order on the cylindrical wave scattering coefficients, examined when the required extreme material parameters are fixed.

Part of the results shown in this sub-section can be found in our paper presented in [10]. A very similar work is also reported by us in [11]. This particular work enables us to construct "approximate" cylindrical cloaks with better invisibility performance with accessible material parameters.

2.3. Near perfect cylindrical cloak by suppressing zeroth-order scattering

A simplified model for cylindrical invisibility cloak was first proposed in [12], and later experimentally demonstrated in [9] at microwave frequency. The simplification is targeted for normal incidence and single-polarization scenario. Take TM polarization for example (only z magnetic field component exists). The relevant material parameters are ε_r, ε_θ, and μ_z. The simplification procedure argues that as long as we keep the products $\varepsilon_r\mu_z$ and $\varepsilon_\theta\mu_z$ identical, changing the individual material parameters ε_r, ε_θ, and μ_z will not alter the refraction of EM wave. Detailed theoretical descriptions can be found in [12] and in the Supporting Online Material for [9]. However, in our previous studies we have found that simplified cylindrical cloaks are inherently visible [13]. But nevertheless, such a simplification procedure does reduce the overall scattering cross section, especially when the outer surface of the cloak is maintained impedance-matched to the outer region subject to a proper choice of the simplified material parameters [14].

In this work, we extend our previous work on simplified cylindrical invisibility cloaks [14] and propose a method to realize a near-perfect cylindrical cloak with simplified material parameters at a certain wavelength. It was previously found that the overall scattering induced by a simplified cloak is always dominated by scattering of the zeroth-order cylindrical wave [13,14]. Here we place an extra layer of material (or even air gap) at the inner surface of a simplified cloak (see Fig. 6) to almost completely cancel out the zeroth-order scattering, hence to achieve near-perfect invisibility.

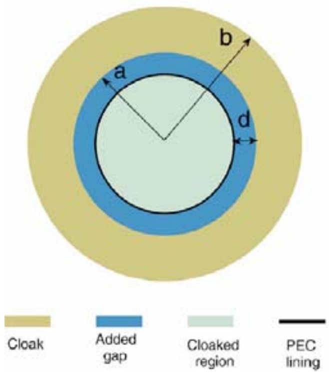


Fig. 6. Schematic picture of the proposed cloak structure. A gap of free space with width d is imposed between the cloak and the PEC lining.

First, we introduce the radial mapping function, in the form of *r'*= *f*(*r*), used in this work. Here *r'* denotes the radial coordinate in electromagnetic space, and *r* denotes the radial coordinate in physical space. The first simplified cylindrical model [9,12] is based on linear coordinate mapping, i.e. $f(r)=b(r-a)/(b-a)$. However, such a linearly coordinate transformed cloak can not lead to a simplified cloak that satisfies both of the following requirements: (1) impedance matching with outer space; (2) nonmagnetic metamaterial i.e. (μ_z = 1) for TM operation[2]. Motivated by the above two requirements, a quadratic radial mapping function was proposed in [15] to deduce a nonmagnetic simplified cylindrical cloak that is impedance matched to air. Unfortunately, the simplified cloak in [15] has a shortcoming: its outer radius *b* has to be at least twice as large as its inner radius *a*. This effectively disqualifies their approach for designing a thin-shelled simplified cloak. In the current work, we propose a different quadratic radial mapping function which circumvents the minimum thickness restriction and at the same time fulfills the two requirements stated above. The radial mapping takes the form of

$$f(r)=\frac{-ar^2+(b^2+a^2)r-ab^2}{(b-a)^2}\,. \tag{5}$$

The corresponding ideal material parameters for the cloak are

$$\varepsilon_r=\frac{(b^2-ra)(r-a)}{(a^2+b^2-2ar)r},\varepsilon_\theta=\frac{(a^2+b^2-2ar)r}{(b^2-ar)(r-a)},\mu_z=\frac{(a^2+b^2-2ar)(b^2-ar)(r-a)}{(b-a)^4 r}. \tag{6}$$

The deduced simplified material parameters are

$$\varepsilon_r=\frac{(b^2-ra)^2(r-a)^2}{(b-a)^4 r^2},\varepsilon_\theta=\frac{(a^2+b^2-2ra)^2}{(b-a)^4},\mu_z=1. \tag{7}$$

With this set of material parameters, not only that impedance is kept matched to free space at the cloak's outer surface, but also that there is no minimum thickness restriction. Therefore, reasonably good invisibility performance can be achieved at almost any cloak shell thickness.

In our case study, we choose a cylindrical cloak with *a*=0.3 m, *b*=0.6 m, and wavelength 0.3 m. An extra layer of air gap of width 0.1365 m (obtained with a careful calculation) is imposed at the cloak's inner surface, which is further terminated by a PEC. Figure 7 shows how the scattering coefficients in different cylindrical wave orders

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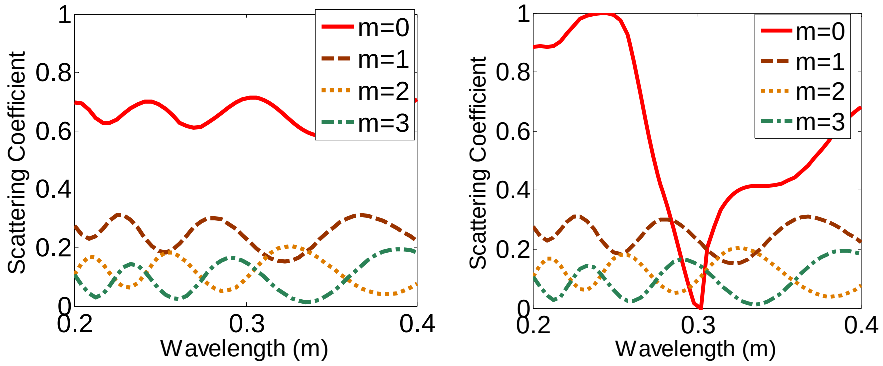
[2] In reality, metamaterials with magnetic response are hard to fabricate especially in the optical wavelength range.

vary with respect to wavelength, both with and without the extra air gap. It is clearly seen that at the targeted wavelength, 0.3 m, the zeroth-order scattering coefficient has been reduced to almost zero. Scattering coefficients for high-order cylindrical waves have not been modified, simply because the high-order waves can not penetrate into the cloak's inner surface, and therefore their scattering is not affected by the extra air layer introduced.



(a) (b)

Fig. 7. Scattering coefficients of cylindrical waves v.s. wavelength for: (a) simplified cloak without the gap, and (b) simplified cloak with the gap.

More information regarding this work can be obtained in Ref. [16]. This particular study positions us one step closer in achieving the goal of perfect invisibility cloaking. From a designing point of view the approach described above diminish the general character of the coordinate transformation technique. The extra inner lining is an interference element designed to work in the neighborhood of a given wavelength. However, although the perfect invisibility achieved in this manner happens at only one wavelength, it should be noticed that in reality all (meta)materials are inherently dispersive, and therefore perfect invisibility solely in a small range of wavelength is unavoidable. Our proposal should be considered then as an extra tool in the design of an invisibility device. Depending on the dispersion of the metamaterials eventually available and the application pursued it should be pondered the convenience of enhancing the performance in detriment of the bandwidth.

2.4. EM concentrator

In this part of the work we move to a different device, the EM concentrator (EMC), designed via the coordinates transformation method. The purpose of this device is to focus the incoming radiation in its interior. Although the transformation optics method permits to describe devices of many different shapes, for simplicity we will consider in what follows a cylindrical concentrator. Geometrically the concentrator *compresses* the electromagnetic field in its core by a gradual compression of the spatial coordinates. We show in Figure 8 an illustration of its working principle. More details about this proposal can be found in Ref. [19].

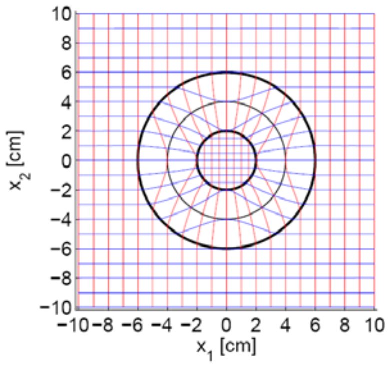


Fig. 8 Graphical representation of the spatial compression characteristic of the EMC's design. The inner bold line corresponds to R_1 and the outer one to R_3. The radius R_2 lies in between both lines. Figure extracted from Ref. [19].

The motivation behind studying the concentration of light is the large number of applications where it can play a role. Clear examples are heat concentration for energy conversion purposes or activation of actuators. One of the main advantages of this concept is the independence of the concentration factor on the azimuth angle. In most of the cases the design of a concentrator will consist of a homogeneous cylindrical fibre of radius R_1 with isotropic permittivity ε^0 and permeability μ^0. Rham et. al derived in Ref. [19] a compact set of equation establishing the required values that the device lining material must meet to achieve the concentration effect:

$$\varepsilon_{ij} = \mu_{ij} = \begin{cases} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \left(\frac{R_2}{R_1}\right)^2 \end{pmatrix} & 0 \le r \le R_1, \\ \begin{pmatrix} a & 0 & 0 \\ 0 & a^{-1} & 0 \\ 0 & 0 & \left(\frac{b}{c}\right)^2 a \end{pmatrix} & R_1 < r \le R_3, \end{cases} \tag{8}$$

where

$$a = \frac{e}{b}\frac{R_3}{r} + 1, b = R_3 - R_2, c = R_3 - R_1, \text{and } e = R_2 - R_1. \tag{9}$$

The transformation behind this set of equations is derived by shrinking the space occupied by a cylinder of radius R_2 into the space of a cylinder of radius R_1. Although the parameter R_3 can be freely chosen, its value will condition the properties of the materials constituting the lining[3]. From the equations it can be easily inferred that the core region must be anisotropic and magnetic, being only one component different from the value at vacuum. This condition restricts the available materials that can constitute the core of the fibre. In the most general case the concentration effect can not take place in a core made of a non-magnetic isotropic material, which constitutes the largest class of *natural* materials. It is also important to point out the absence of any reference

[3] Note that when R_3 tends to R_2 ε_{ij} and μ_{ij} tend to infinity or zero, i.e. become *singular*.

wavelength or polarization condition of the propagating fields. Indeed, the general character, elegance and compactness of the expression obtained by the transformation optics method are its main features.

In the particular case of radiation impinging normally to the cylinder surface with the electric field parallel to the cylinder axis[4] the only non-zero field components are E_z, B_r and B_ϕ . Therefore the relevant components of the electromagnetic parameter's tensor are ε_z, μ_r and μ_ϕ. Under these circumstances the concentrator lining can be applied to an isotropic non-magnetic fibre. The concentration factor computed as the power density averaged over the normal section of the core with and without lining is:

$$\eta = \frac{\int_{\text{core}} \langle S_{\text{conc.}}(r,\phi) \rangle \, dA}{\int_{\text{core}} \langle S_{\text{core}}(r,\phi) \rangle \, dA} = \frac{(R_2/R_1)^2}{\int_{\text{core}} \langle S_{\text{core}}(r,\phi) \rangle / \langle S_0 \rangle \, dA},$$

where $\langle S \rangle$ is the time-averaged Poynting vector and $\langle S_0 \rangle$ represents the power density of the incoming field in empty space.

To the best of our knowledge the performance of the EMC has not been reported yet. For this sake we need to compare the concentration factor achieved by the EMC with the one obtained by other technology. In this work we choose a multilayered cylinder as a reference technology. The calculation of the Poynting vector requires solving the problem of the scattering by a multilayer cylinder. Some approximations can be made in order to simplify the problem. When the wavelength is much bigger than the size of the scatterer (in this case the multilayered cylinder), the Rayleigh-Gans formulation results to be a good strategy. On the contrary, if the considered wavelength is much smaller than the scatterer size, then geometrical optics bring the solution very easily. In an intermediate case, i.e. the wavelength is of the same order as the size of the obstacle, the problem needs to be solved rigorously. In what rests of the section we will constrain ourselves to this last scenario. We calculate the solutions of the Maxwell's equation by means of the cylindrical wave expansion method, following a similar procedure as described by M. Yan in [20].

To obtain a benchmark able to qualify the performance of the EMC we adopt in this work an optimization strategy. In general, the multilayer structure is composed of N shells, each of them with a thickness d_i, a permittivity ε_i and permeability μ_i. The exploration of this space of parameters is a very demanding task, especially if many shells are involved. In this first approach to the problem we want to keep the complexity of the problem to a minimum, therefore we reduce the problem to a combinatorial one. We first impose a given thickness to all the layers, and then select a set of materials, which provide the available values of ε_i and μ_i. With the help of a genetic algorithm, we will find the arrangement of layers providing the best η.

We mentioned above that the EMC design is independent of the incoming radiation wavelength. However, this is not the case of a multilayer cylinder, which owing to interference phenomena responds in a different way to different wavelengths. Given

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[4] TE polarization taking as a reference the plane defined by a normal cylinder section or TM if instead the incidence plane is taken as a reference.

that in most space missions the main source of energy is the sun and that its emission is peaked at about 0.5 μm, we constraint our study to the range [0.4, 0.6] μm. We choose HfO_2[5] as the material for the fibre core. As possible materials for the coating we consider silver and a set of fictitious materials with permittivity 1.1, 2.2, 3.3, 4.4 and 5.5. The radius of the core is R_1=0.1 μm and the outer radius of the fibre is set to twice the inner radius, i.e. $R_2=2\times R_2$. Finally we choose a maximum number of layers equal to 25.

Figure 9 contains the results of the best multilayer concentrator found by the reverse searching algorithm. The green line represents the power density of the bare cylinder. The blue and the red lines show the enhancement in the power density due to the presence of the coating. The resonance appearing at ~550 nm in the bare cylinder is not recognisable anymore in the final device. It exhibits a rather flat behaviour over the spectrum ranger under study. It can also be inferred from the Figure that the silver layer does not play any crucial role for the configuration considered here. Perhaps when more layers are taken under consideration resonant plasmon modes could provide a way to increase the operational bandwidth.

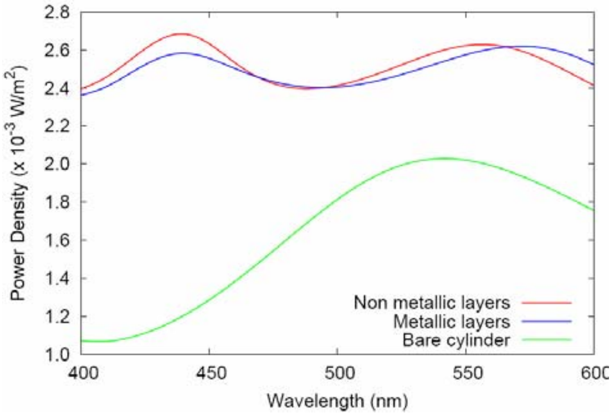

Fig. 9 Representation of the power density in a bare cylinder, and the same cylinder encapsulated in a refractive coating.

The comparison with the EMC is made in terms of the concentrator factor η defined above. Figure 10 shows the results corresponding to the optimized multilayer fibre and the EMC. For the latter we have considered that the radius R_1 and R_2 are the same as considered before, taking the parameter R_3 out of the discussion[6]. In general the EMC's η is twice the one of the multilayered cylinder.

[5] HfO_2 presents permittivity equal to 4.0 at 500 nm.

[6] In principle the value of R_3 does not affect the concentration factor, only the required materials for the shell.

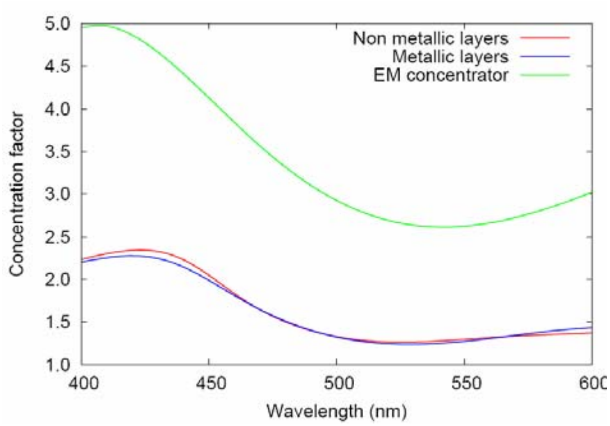

Fig. 10 Concentration factor for the multilayered cylinder and the EMC.

The general conclusion is that the EM concentration offers better performance than a simple multilayer structure. However, the enhancement factor is only two. Given the complexity in fabricating a metamaterial able to meet the requirements of Eq. 8, it could be preferable in some cases to employ a multilayer approach and sacrifice some performance. In addition, it is worth remembering that when the number of layers is small, high concentration of energy can take place via Mie resonances [21]. The main problem of this approach lies in the strong resonant character of the phenomena, meaning that the concentration takes place only in a very narrow bandwidth. Again the targeted application will determine which approach could be more convenient.

Further studies are to be carried out, which should fulfil the following two conditions: (1) impedance matching at the concentrator's inner and outer interface; (2) proper lensing effect of the multilayered cladding structure (which directs light into the core region). Recently a generalization of the coordinate transformation method has been reported [22]. There an isotropic and homogeneous EMC can be envisaged via a time transformation instead of a space transformation. However, a deeper study is required since the impedance matching boundary conditions are not met anymore, diminishing enormously the operability of the device.

3. OUTCOME

The outcome from this study comprises four publications: the papers are listed as Refs. [8,10,11,16].

4. CONCLUSION

In this Ariadna study, we have carried out theoretical investigations in aspects including: (1) EM and material properties of invisibility cloaks in arbitrary shapes; (2) approximate and improved simplified models of cylindrical invisibility cloaks; and (3) EM cylindrical concentrators. Our study provides better physical understanding of invisibility cloaks designed with the coordinate transformation method. Our proposal of improved simplified cylindrical cloak makes realization of near-perfect invisibility cloaking one step closer, especially at the optical wavelength range. During the project execution, we have maintained constant communications with the ESA Advanced Concepts Team (ACT). Our discussions and collaborative studies have even expanded from our original proposal. The work on "triple spacetime coordinate transformation" by Luzi Bergamin from ACT has motivated us to further look into transformation devices made of the so-called indefinite media. Indefinite media, when designed properly, can provide the same function as double-negative material (all principle material components for permittivity and permeability are negative); meanwhile their fabrication using the metamaterial technology are less demanding. Particularly, for slab superlens application, a lens made of indefinite media in bilayer configuration [17] has been found advantageous as compared to a simple negative index lens [18]. Currently, an immediate task is to extend the current coordinate transformation approach for design of indefinite media (so far the current coordinate transformation theory produces only definite media). The generalized coordinate transformation permits us to fully explore the potential of metamaterial technology. Finally we acknowledge ESA for giving us the opportunity to carry out the project.

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