

Triple Spacetime Metamaterials:

A generalization of transformation media.

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Maxwell's equations for linear media

Maxwell's equations in general coordinates

$$\begin{aligned}\nabla_i B^i &= 0, & \nabla_0 B^i + \epsilon^{ijk} \partial_j E_k &= 0, \\ \nabla_i \mathcal{D}^i &= \rho, & \epsilon^{ijk} \partial_j \mathcal{H}_k - \nabla_0 \mathcal{D}^i &= j^i,\end{aligned}$$

are considered with a linear, nondispersive constitutive relation

$$\begin{aligned}\mathcal{D}^i &= \epsilon^{ij} E_j + \kappa^{ij} \mathcal{H}_j, \\ B^i &= \mu^{ij} \mathcal{H}_j + \xi^{ij} E_j.\end{aligned}$$



Symmetries and invariant transformations

Maxwell's theory is invariant under diffeomorphisms, locally coordinate transformations

$$x^\mu \implies \bar{x}^\mu(x)$$

but

- two sets of equations

$$\epsilon^{\mu\nu\rho\sigma} \partial_\nu F_{\rho\sigma} = 0, \quad D_\nu H^{\mu\nu} = -J^\mu$$

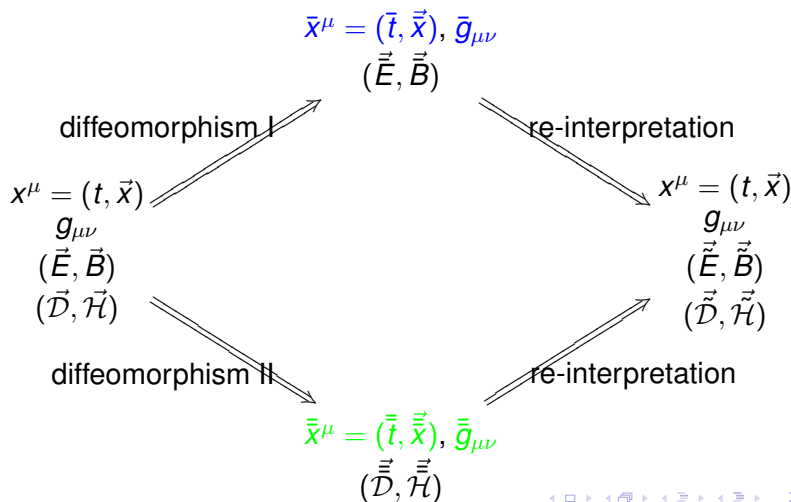
are invariant under **separate** diffeomorphisms;

- constitutive relation is not invariant under this manipulation.

Invariant transformation of the equations of motions, but no symmetry.



Generalization of transformation media



General constitutive relation

$$\begin{aligned}\tilde{\mathcal{D}}^i &= -\bar{s} \frac{\sqrt{-\bar{g}}}{\sqrt{\gamma} g_{\bar{0}\bar{0}}} g^{\bar{i}\bar{j}} \tilde{E}_j - \bar{s} \bar{s} \frac{\sqrt{-\bar{g}} \sqrt{-\bar{g}}}{\gamma g_{\bar{0}\bar{0}}} g^{\bar{i}\bar{k}} g^{\bar{0}\bar{l}} \epsilon_{klm} g^{\bar{m}\bar{j}} \tilde{\mathcal{H}}_j, \\ \tilde{B}^i &= -\bar{s} \frac{\sqrt{-\bar{g}}}{\sqrt{\gamma} g_{\bar{0}\bar{0}}} g^{\bar{i}\bar{j}} \tilde{\mathcal{H}}_j + \bar{s} \bar{s} \frac{\sqrt{-\bar{g}} \sqrt{-\bar{g}}}{\gamma g_{\bar{0}\bar{0}}} g^{\bar{i}\bar{k}} \epsilon_{klm} g^{\bar{j}\bar{0}} g^{\bar{m}\bar{l}} \tilde{E}_j,\end{aligned}$$

with

$$g^{\bar{\mu}\bar{\nu}} = \frac{\partial \bar{x}^{\mu}}{\partial x^{\rho}} \frac{\partial \bar{x}^{\nu}}{\partial x^{\sigma}} g^{\rho\sigma} = \bar{g}^{\mu\rho} \frac{\partial \bar{x}^{\nu}}{\partial \bar{x}^{\rho}} = \frac{\partial \bar{x}^{\mu}}{\partial \bar{x}^{\rho}} \bar{g}^{\rho\nu},$$

\bar{s} change of orientation in $x^{\mu} \rightarrow \bar{x}^{\mu}$,

\bar{s} change of orientation in $x^{\mu} \rightarrow \bar{\bar{x}}^{\mu}$.



Characteristics of generalized transformations

- $\bar{\bar{s}}\sqrt{-\bar{\bar{g}}}\mu^{ij} = \bar{s}\sqrt{-\bar{g}}\epsilon^{ij}$.
- ϵ^{ij} and μ^{ij} not symmetric if $\frac{\partial \bar{x}^\mu}{\partial \bar{\bar{x}}^\nu}$ not symmetric,
 \Rightarrow non-reciprocal materials, e.g. $\bar{x} = x - z$, $\bar{\bar{x}} = x + z$.
- Signs of eigenvalues of ϵ^{ij} can be chosen freely,
 \Rightarrow indefinite materials, e.g. $\bar{x} = -x$, $\bar{\bar{x}} = x$.
- Relative sign between eigenvalues of ϵ^{ij} and μ^{ij} not fixed.
 \Rightarrow evanescent waves for $\bar{t} = -t$ but $\bar{\bar{t}} = t$.
- Bi-anisotropic materials if time maps non-trivially,

$$g^{\bar{\bar{0}}\bar{i}} \neq 0, \quad g^{\bar{0}\bar{\bar{i}}} \neq 0.$$

κ^{ij} and ξ^{ij} need not be anti-symmetric.

