Manipulation of Lightwave Through Coordinate Transformation

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2008-06-13

Ariadna bidder code: 21290
## Project plan

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Introduction

Scattering $\rightarrow$ visibility

? $\rightarrow$ invisibility
Coordinate transformation approach

Coordinate transformation

Original Cartesian coordinate \((x, y, z)\)

\[
\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad \nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{j}, \quad \nabla \cdot \mathbf{D} = \rho, \quad \nabla \cdot \mathbf{B} = 0.
\]

\[
\mathbf{D} = \varepsilon_0 \varepsilon \cdot \mathbf{E}, \quad \mathbf{B} = \mu_0 \mu \cdot \mathbf{H},
\]

New coordinate \((q_1, q_2, q_3)\)

\[
x = f_1(q_1, q_2, q_3), \quad y = f_2(q_1, q_2, q_3), \quad z = f_3(q_1, q_2, q_3).
\]

The Maxwell equations can take the invariant form as

\[
\nabla_q \times \mathbf{\hat{E}} = -\frac{\partial \mathbf{\hat{B}}}{\partial t}, \quad \nabla_q \times \mathbf{\hat{H}} = \frac{\partial \mathbf{\hat{D}}}{\partial t} + \mathbf{j}, \quad \nabla_q \cdot \mathbf{\hat{D}} = \hat{\rho}, \quad \nabla_q \cdot \mathbf{\hat{B}} = 0
\]

with

\[
\hat{\varepsilon} = \det(\Lambda)(\Lambda)^{-1}\varepsilon \Lambda^{-T}, \quad \hat{\mu} = \det(\Lambda)(\Lambda)^{-1}\mu \Lambda^{-T},
\]

Jacobian transformation matrix

\[
\Lambda = \begin{bmatrix}
\frac{\partial x}{\partial q_1} & \frac{\partial x}{\partial q_2} & \frac{\partial x}{\partial q_3} \\
\frac{\partial y}{\partial q_1} & \frac{\partial y}{\partial q_2} & \frac{\partial y}{\partial q_3} \\
\frac{\partial z}{\partial q_1} & \frac{\partial z}{\partial q_2} & \frac{\partial z}{\partial q_3}
\end{bmatrix}.
\]
Coordinate transformation
Coordinate transformation

\[ \hat{E} = \Lambda^T E, \quad \hat{H} = \Lambda^T H, \]
Line-transformed cloak
Point-transformed cloak
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Ideal cylindrical cloak

**Principle:**
To compress EM fields within a cylindrical air region $r' < b$ into the cylindrical annular region $a < r < b$.

The simplest transformation:
from $(r', \theta', z')$ (original cylindrical coordinate) to $(r, \theta, z)$ (physical cylindrical coordinate)

\[
\begin{align*}
    r &= \frac{b - a}{a} r' + a \\
    \theta &= \theta' \\
    z &= z'
\end{align*}
\]
Ideal cylindrical cloak

Colormap shows $E_z$ field

Field in original EM space

Field in physical space

Electromagnetic null space!

**Difficulty:** $\varepsilon_\theta$, $\mu_\theta$, $\varepsilon_z$, and $\mu_z$ diverge at $r=a$!

\[
\varepsilon_z = \left(\frac{b}{b-a}\right)^2 \frac{r-a}{r} \\
\mu_r = \frac{r-a}{r} \\
\mu_\theta = \frac{r}{r-a}
\]
**Simplified cylindrical cloaks**

**Reasons for material simplification [1,2]:**
1. To avoid infinite parameters
2. To alleviate metamaterial engineering task

The material parameters are simplified as

\[
\varepsilon_r = \mu_r = \frac{r-a}{r}, \\
\varepsilon_\theta = \mu_\theta = \frac{r}{r-a}, \\
\varepsilon_z = \mu_z = \left(\frac{b}{b-a}\right)^2 \frac{r-a}{r}.
\]

\[
\varepsilon_r = \mu_r = \left(\frac{r-a}{r}\right)^2, \\
\varepsilon_\theta = \mu_\theta = 1, \\
\varepsilon_z = \mu_z = \left(\frac{b}{b-a}\right)^2.
\]

\[\varepsilon_\theta \varepsilon_z \text{ and } \mu_r \varepsilon_z \text{ invariant}\]

---

Simplified cylindrical cloaks: not perfect

Origin of the problem:

Wave equation (r-dependent) in ideal cloak medium:
\[
\frac{d}{dr} \left( \frac{r}{\mu_\theta} \frac{d\Psi}{dr} \right) + \frac{k_0^2}{r} \epsilon \Psi - m^2 \frac{1}{r \mu_r} \Psi = 0.
\]

Wave equation in simplified cloak medium:
\[
\frac{1}{\mu_\theta \epsilon} \frac{d}{dr} \left( \frac{r}{dr} \frac{d\Psi}{dr} \right) + \frac{k_0^2}{r} \Psi - m^2 \frac{1}{r \mu_r \epsilon} \Psi = 0.
\]

Wrong assumption: \( \mu_0 \) is constant, where \( m \) is angular mode number.
Scattering coefficients

Scattering coefficients in different cylindrical orders w.r.t. Outer radius $b$ [1]:

![Graph showing scattering coefficients]

**Parameters:** $f=2\,\text{GHz}; a=0.1\,\text{m}$

**Improved simplification**

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<tr>
<th>Ideal</th>
<th>Simplified [2]</th>
<th>Simplified (current work)</th>
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**Same condition:** \( \mu_\theta \varepsilon_z \) and \( \mu_\theta \varepsilon_z \) invariant

**Advantage:**
The outer interface is perfectly matched to exterior!

Improved simplification

Previous simplified

Improved
Improved simplification

Bare cylinder
Simplified linear cloak [2]
Simplified quadratic cloak [3]
Simplified linear cloak (improved)

Can we cancel the zeroth order scattering?
Cancellation of zeroth-order scattering

without monopole cancellation

with monopole cancellation

\[ a = 0.3 \text{m}, \quad b = 0.6 \text{m} \]
Cancellation of zeroth-order scattering

\[ \lambda = 0.3 \text{m} \]
**Far field radiation**

![Graph showing RCS (dB) vs θ (degree) for different configurations.](image)

**Remarks:**
- Mostly high order scatterings remain
- Wavelength dependent

**Ref:**
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High-order cylindrical invisibility cloak

One possible class of transformation may take the form of

\[ r' = \frac{b}{(b-a)^n(\rho - a)^n}, \]

\( n \): transformation order.

Virtual flat space \( n=1 \) \( n=3 \)

Infinite parameters at \( r=a \)!
Effect of transformation order on scattering coefficients

- $a = 0.1\text{m}$
- $b = 0.3\text{m}$
- $\lambda = 0.15\text{m}$
- Thickness of the layer peeled away: $d = 0.01\text{m}$
- PEC lining is present
Field distributions

Ez
Scattered Ez
Transformation order used: n=3

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EM concentrator

Virtual r' vs Physical r

Application:
- Photovoltaic fiber
- Fluid heating

Resonance based EM concentrator

- Enhancement factor 230
- Wavelength dependant
Resonance based EM concentrator
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Arbitrarily-shaped cloaks

Blowing-up a line

Blowing-up a point

General cross-section:

Ref:
Arbitrarily-shaped cloak

For both line-transformed or point-transformed cloaks,

\[ \hat{E}^i = \Lambda^T E^i, \quad \hat{H}^i = \Lambda^T H^i, \]

At outer boundary:

\[ \hat{E}^i_{t_1} = E^i \cdot \hat{t}_1, \quad \hat{H}^i_{t_1} = H^i \cdot \hat{t}_1, \]
\[ \hat{E}^i_{t_2} = E^i \cdot \hat{t}_2, \quad \hat{H}^i_{t_2} = H^i \cdot \hat{t}_2, \]
Arbitrarily-shaped cloak

For line-transformed cloak,

\[ \hat{E}^i_{t1} = \hat{H}^i_{t1} = 0. \]

However, the other components of fields are not zero. In particular,

\[ \hat{E}^i_{t2} = (\hat{s} \cdot \hat{t}_2)[B_1, B_2, B_3]E^i, \]

\[ \hat{H}^i_{t2} = (\hat{s} \cdot \hat{t}_2)[B_1, B_2, B_3]H^i, \]

\[ \hat{E}^i_n = [F_1 + B_1(\hat{s} \cdot \hat{n}), F_2 + B_2(\hat{s} \cdot \hat{n}), F_3 + B_3(\hat{s} \cdot \hat{n})]E^i, \]

\[ \hat{H}^i_n = [F_1 + B_1(\hat{s} \cdot \hat{n}), F_2 + B_2(\hat{s} \cdot \hat{n}), F_3 + B_3(\hat{s} \cdot \hat{n})]H^i, \]

with

\[ B_i = \sqrt{\partial b_i/\partial q_1^2 + \partial b_i/\partial q_2^2 + \partial b_i/\partial q_3^2}. \]

**Remark:** Surface current will be induced at the inner surface.
Arbitrarily-shaped cloak

For point-transformed cloak,

\[ \hat{E}^{i}_{t_1} = \hat{H}^{i}_{t_1} = 0, \]
\[ \hat{E}^{i}_{t_2} = \hat{H}^{i}_{t_2} = 0, \]
\[ \hat{E}^{i}_{n} = [F_1, F_2, F_3]E^{i}, \hat{H}^{i}_{n} = [F_1, F_2, F_3]H^{i}. \]

All tangential fields are zero. Therefore no field discontinuity exists for perfect cloaking.
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Outcome

Papers:

Code:
Matlab code for scattering calculation of multilayered cylindrical structures.
Thank you!

Questions?