Manipulation of Lightwave Through Coordinate Transformation

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Ariadna bidder code: ***21290***

Project plan

We are here!

Tasks / Time (mth)	0.5	**1**	1.5	**2**	2.5	**3**	3.5	**4**
Cylindrical cloak: Simplification								
Cylindrical cloak: Effect of order number								
Electromagnetic Concentrator								
Arbitrarily-shaped cloak								
Project report								

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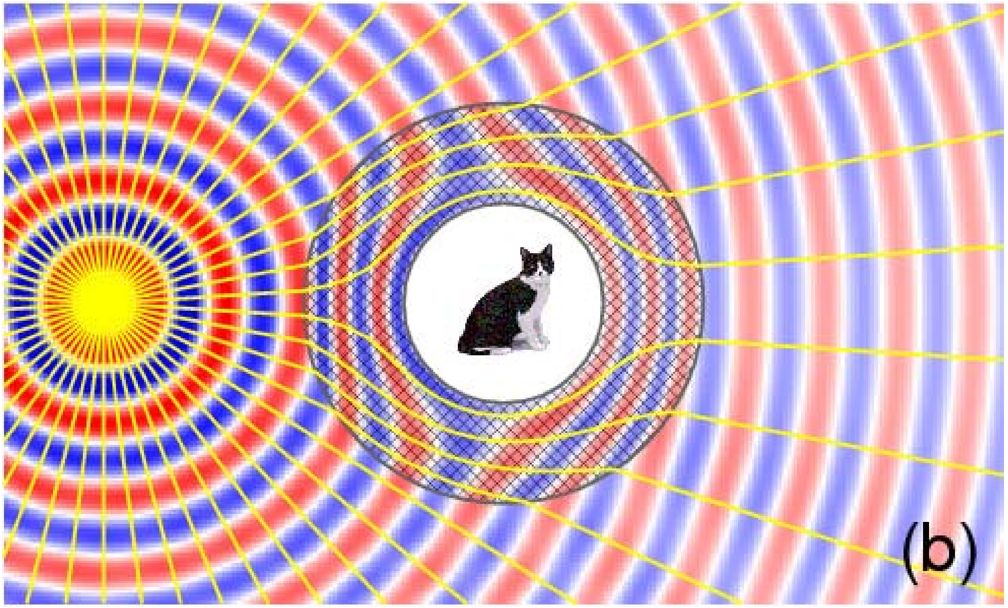
Introduction

Scattering $\rightarrow$ visibility

Flounder
Ostrich

? $\rightarrow$ invisibility

Coordinate transformation approach



1. J. B. Pendry, D. Schurig, and D. R. Smith, Science **312**, p.1780 (2006).
2. U. Leonhardt, Science **312,** p. 1777 (2006).

Coordinate transformation

Original Cartesian coordinate (*x*, *y*, *z*)

$$\nabla \times \mathbf{E} = -\frac{\partial \mathrm{B}}{\partial t}, \quad \nabla \times \mathbf{H} = \frac{\partial \mathrm{D}}{\partial t} + \mathrm{j}, \quad \nabla \cdot \mathrm{D} = \rho, \quad \nabla \cdot \mathrm{B} = 0.$$

$$\mathbf{D} = \epsilon_0 \overline{\overline{\varepsilon}} \cdot \mathbf{E}, \quad \mathbf{B} = \mu_0 \overline{\overline{\mu}} \cdot \mathbf{H},$$

Jacobian transformation matrix

$$\Lambda = \begin{bmatrix} \frac{\partial x}{\partial q_1} & \frac{\partial x}{\partial q_2} & \frac{\partial x}{\partial q_3} \\ \frac{\partial y}{\partial q_1} & \frac{\partial y}{\partial q_2} & \frac{\partial y}{\partial q_3} \\ \frac{\partial z}{\partial q_1} & \frac{\partial z}{\partial q_2} & \frac{\partial z}{\partial q_3} \end{bmatrix}.$$

New coordinate (q_1, q_2, q_3)

$$x = f_1(q_1, q_2, q_3), \quad y = f_2(q_1, q_2, q_3), \quad z = f_3(q_1, q_2, q_3).$$

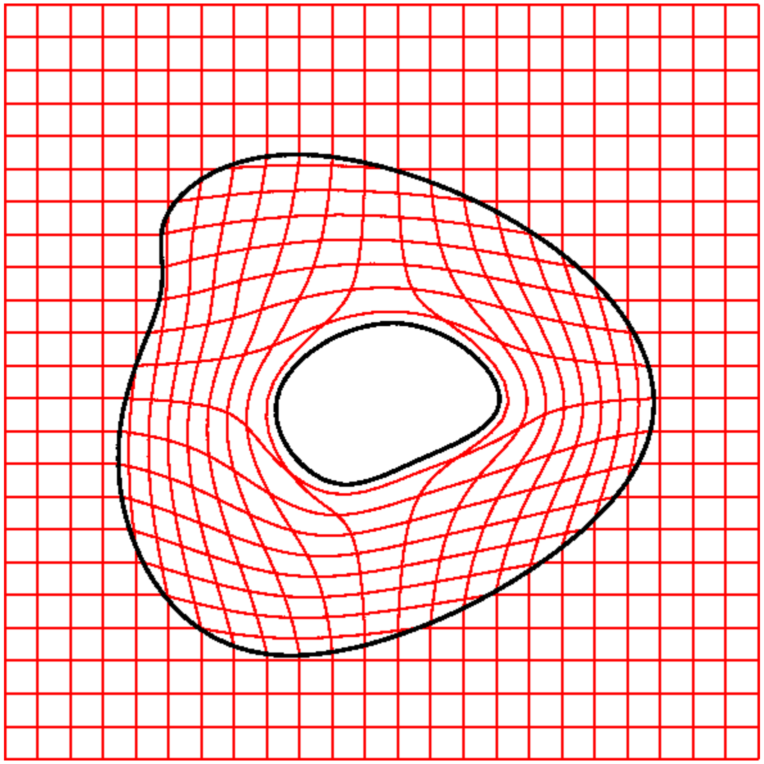
The Maxwell equations can take the invariant form as

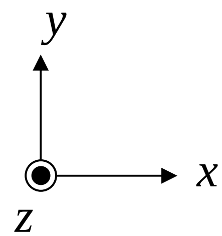
$$\nabla_q \times \widehat{\mathbf{E}} = -\frac{\partial \widehat{\mathrm{B}}}{\partial t}, \quad \nabla_q \times \widehat{\mathbf{H}} = \frac{\partial \widehat{\mathrm{D}}}{\partial t} + \widehat{\mathrm{j}}, \quad \nabla_q \cdot \widehat{\mathrm{D}} = \widehat{\rho}, \quad \nabla_q \cdot \widehat{\mathrm{B}} = 0$$

with

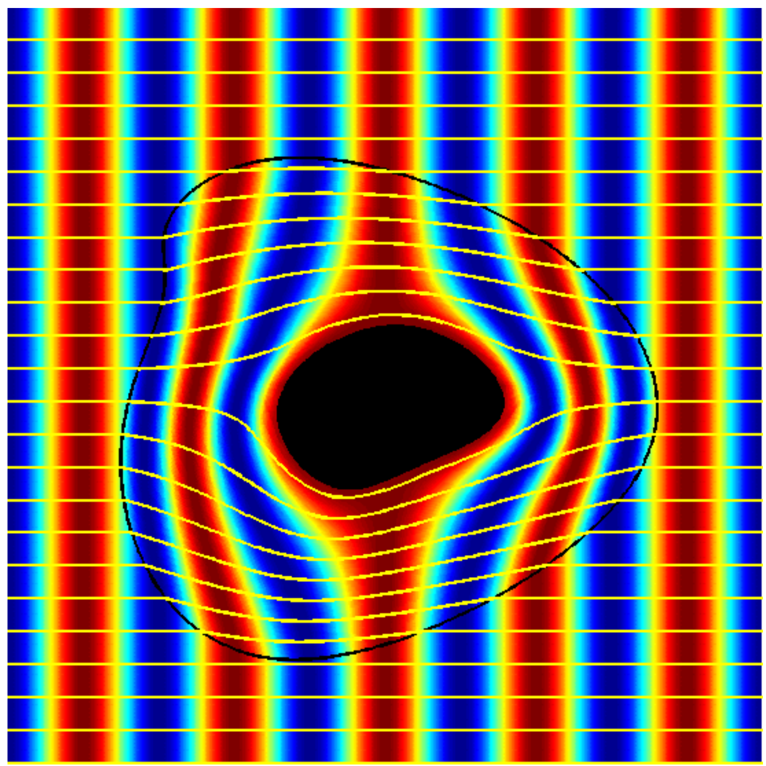
$$\widehat{\overline{\overline{\varepsilon}}} = \det(\Lambda)(\Lambda)^{-1} \overline{\overline{\varepsilon}} \Lambda^{-T}, \quad \widehat{\overline{\overline{\mu}}} = \det(\Lambda)(\Lambda)^{-1} \overline{\overline{\mu}} \Lambda^{-T},$$

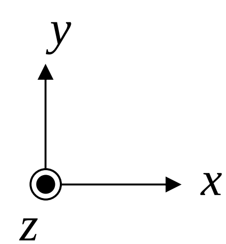
Coordinate transformation





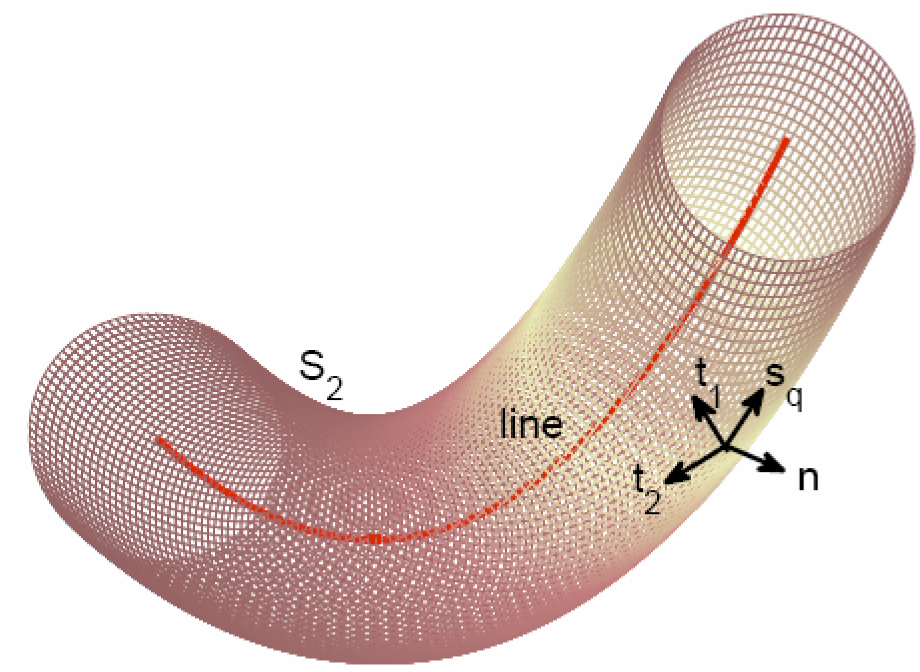
Coordinate transformation



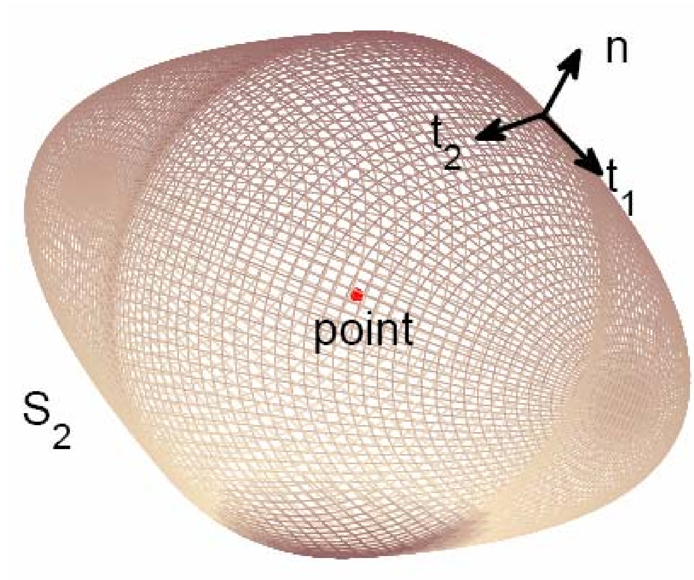


$$\widehat{\mathbf{E}} = \Lambda^T \mathbf{E}, \quad \widehat{\mathbf{H}} = \Lambda^T \mathbf{H},$$

Line-transformed cloak



Point-transformed cloak



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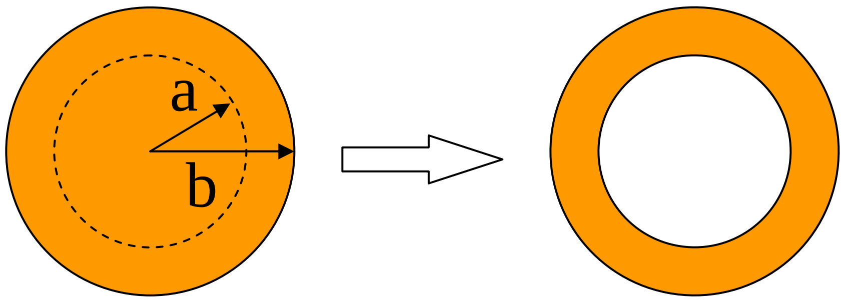
5. Conclusion	4'
Total:	45'

Ideal cylindrical cloak

Principle:

To compress EM fields within a cylindrical air region r' < b into the cylindrical annular region a < r < b.



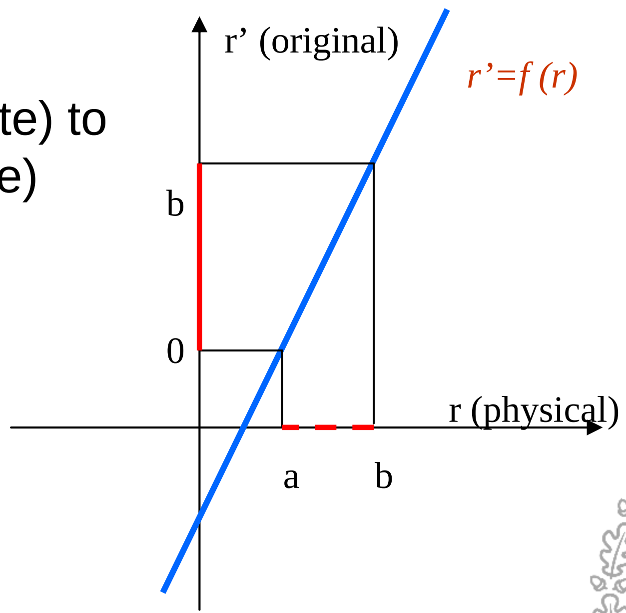
The simplest transformation:

from (r', θ', z') (original cylindrical coordinate) to (r, θ, z) (physical cylindrical coordinate)

$$r = \frac{b-a}{a} r' + a$$

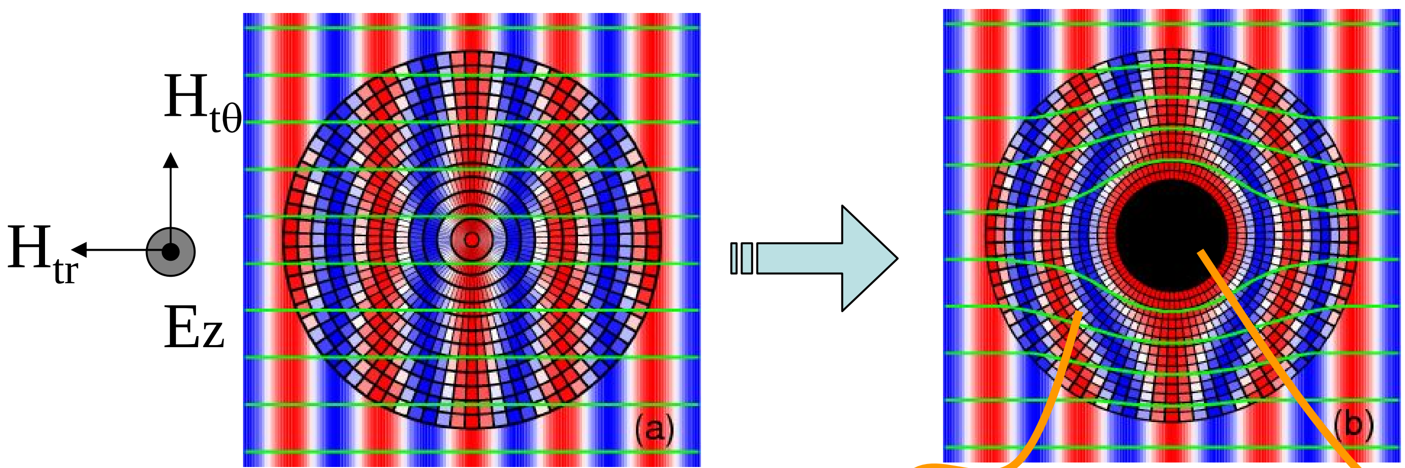
$$\theta = \theta'$$

$$z = z'$$



Ideal cylindrical cloak

Colormap shows E_z field



Field in original EM space

Field in physical space

$$\varepsilon_z = \left(\frac{b}{b-a}\right)^2 \frac{r-a}{r}$$

$$\mu_r = \frac{r-a}{r}$$

$$\mu_\theta = \frac{r}{r-a}$$

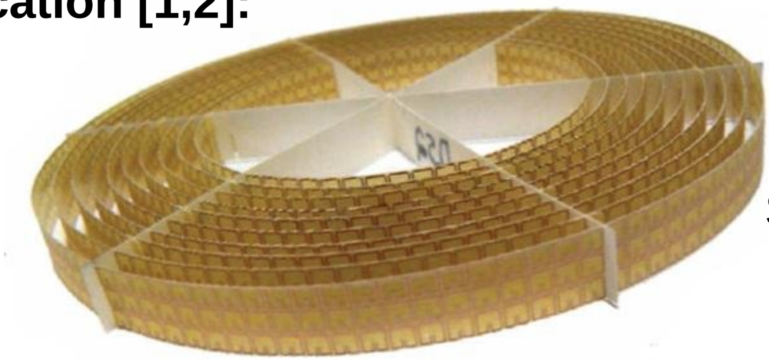
Electromagnetic null space!

Difficulty: ε_θ , μ_θ, ε_z , and μ_z diverge at r=a!

Simplified cylindrical cloaks

Reasons for material simplification [1,2]:

1. To avoid infinite parameters
2. To alleviate metamaterial engineering task



See Ref. 1

The material parameters are simplified as

$$\varepsilon_r = \mu_r = \frac{r-a}{r}, \quad \varepsilon_\theta = \mu_\theta = \frac{r}{r-a}, \quad \varepsilon_z = \mu_z = \left(\frac{b}{b-a}\right)^2 \frac{r-a}{r}.$$

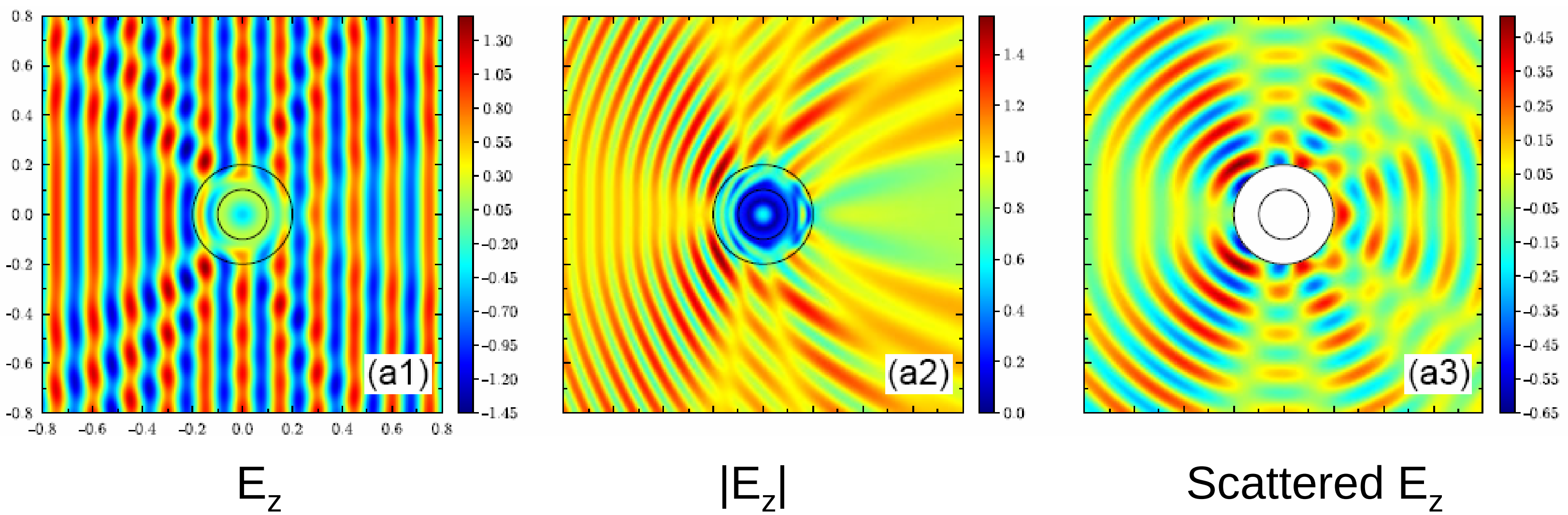
$$\Rightarrow$$

$$\varepsilon_r = \mu_r = \left(\frac{r-a}{r}\right)^2, \quad \varepsilon_\theta = \mu_\theta = 1, \quad \varepsilon_z = \mu_z = \left(\frac{b}{b-a}\right)^2.$$

$\mu_\theta \varepsilon_z$ and $\mu_r \varepsilon_z$ invariant

1. D. Schurig, et al., Science **314**, 977 (2006).
2. W. Cai et al., "Optical cloaking with metamaterials," Nat. Photonics **1**, 224–227 (2007).

Simplified cylindrical cloaks: not perfect



Origin of the problem:

Wave equation (*r*-dependent) in ideal cloak medium:

$$\frac{d}{dr}\left(\frac{r}{\mu_\theta}\frac{d\Psi}{dr}\right)+k_0^2 r\varepsilon_z\Psi-m^2\frac{1}{r\mu_r}\Psi=0.$$

Wave equation in simplified cloak medium:

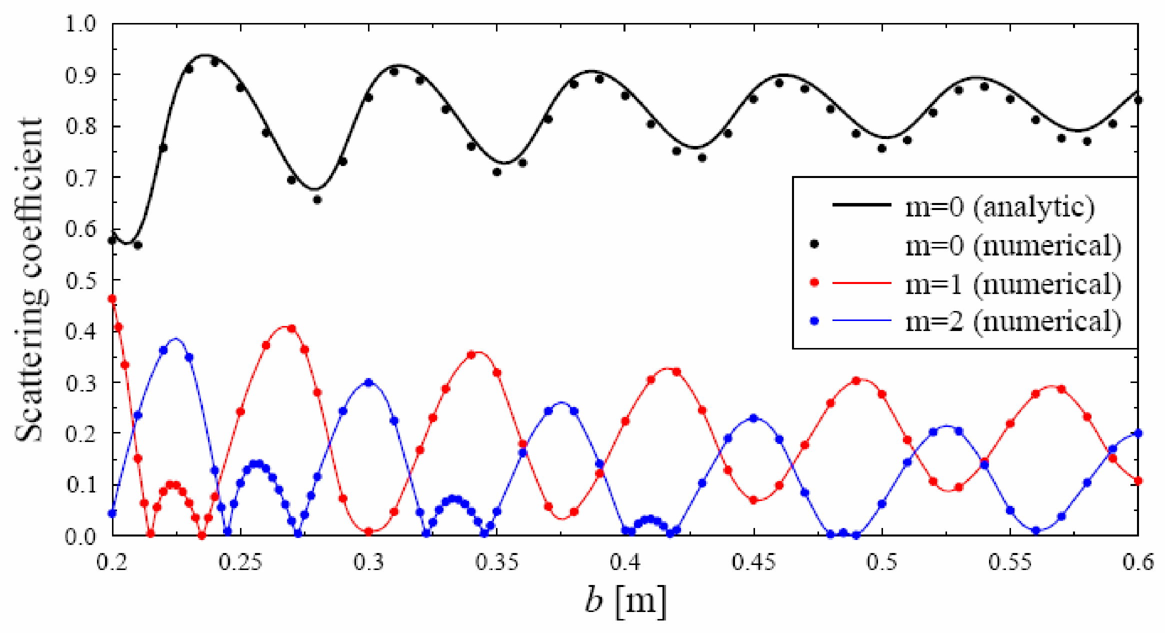
$$\frac{1}{\mu_\theta\varepsilon_z}\frac{d}{dr}\left(r\frac{d\Psi}{dr}\right)+k_0^2 r\Psi-m^2\frac{1}{r\mu_r\varepsilon_z}\Psi=0.$$

Wrong assumption: μ_θ is constant

where *m* is angular mode number

Scattering coefficients

Scattering coefficients in different cylindrical orders w.r.t. *Outer radius b* [1]*:*



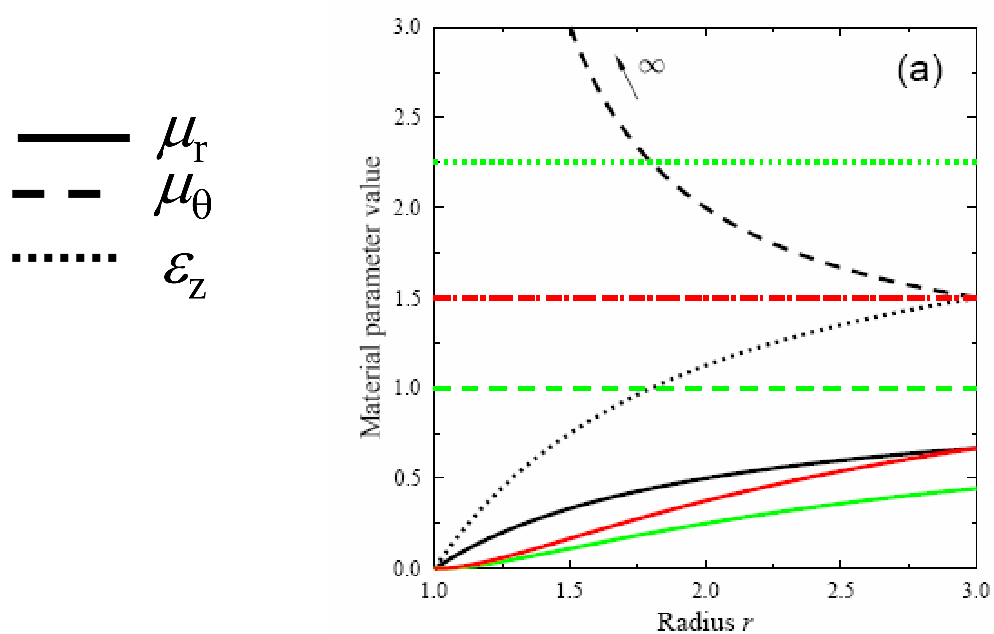
Parameters: f=2GHz; a=0.1m

1. M. Yan, Z. Ruan and M. Qiu, Phys. Rev. Lett. **99**, 233901 (2007).

Improved simplification

Ideal	Simplified [2]	Simplified (current work)
$\varepsilon_r = \mu_r = \frac{r-a}{r}$	$\varepsilon_r = \mu_r = \left(\frac{r-a}{r}\right)^2$	$\varepsilon_r = \mu_r = \left(\frac{r-a}{r}\right)^2 \frac{b}{b-a}$
$\varepsilon_\theta = \mu_\theta = \frac{r}{r-a}$	$\varepsilon_\theta = \mu_\theta = 1$	$\varepsilon_\theta = \mu_\theta = \frac{b}{b-a}$
$\varepsilon_z = \mu_z = \left(\frac{b}{b-a}\right)^2 \frac{r-a}{r}$	$\varepsilon_z = \mu_z = \left(\frac{b}{b-a}\right)^2$	$\varepsilon_z = \mu_z = \frac{b}{b-a}$

Same condition: $\mu_\theta \varepsilon_z$ and $\mu_r \varepsilon_z$ invariant



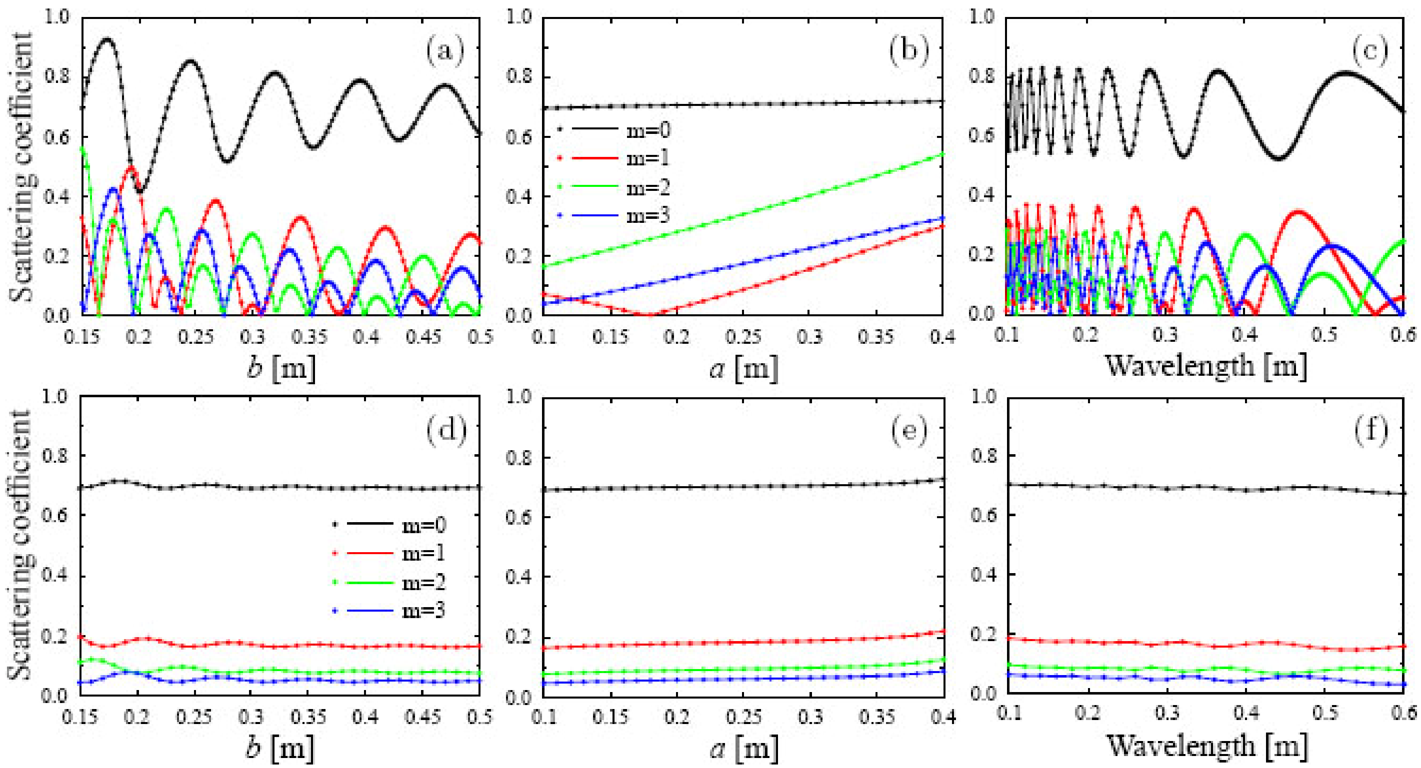
Advantage:
The outer interface is perfectly matched to exterior!

1. M. Yan, Z. Ruan, and M. Qiu, Opt. Express **15**, p. 17772 (2007).
2. D. Schurig, et al., Science **314**, 977 (2006).

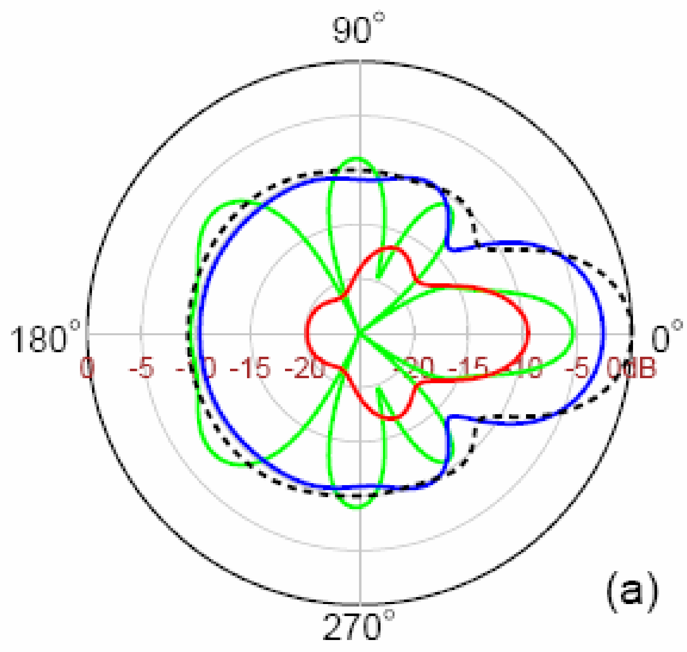
Improved simplification

Previous simplified

Improved

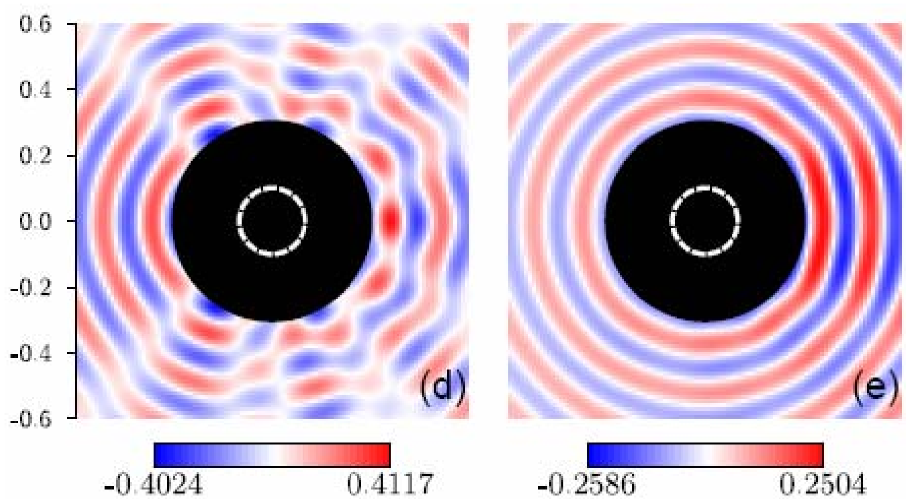


Improved simplification



- ······ Bare cylinder
- Simplified linear cloak [2]
- Simplified quadratic cloak [3]
- Simplified linear cloak (improved)

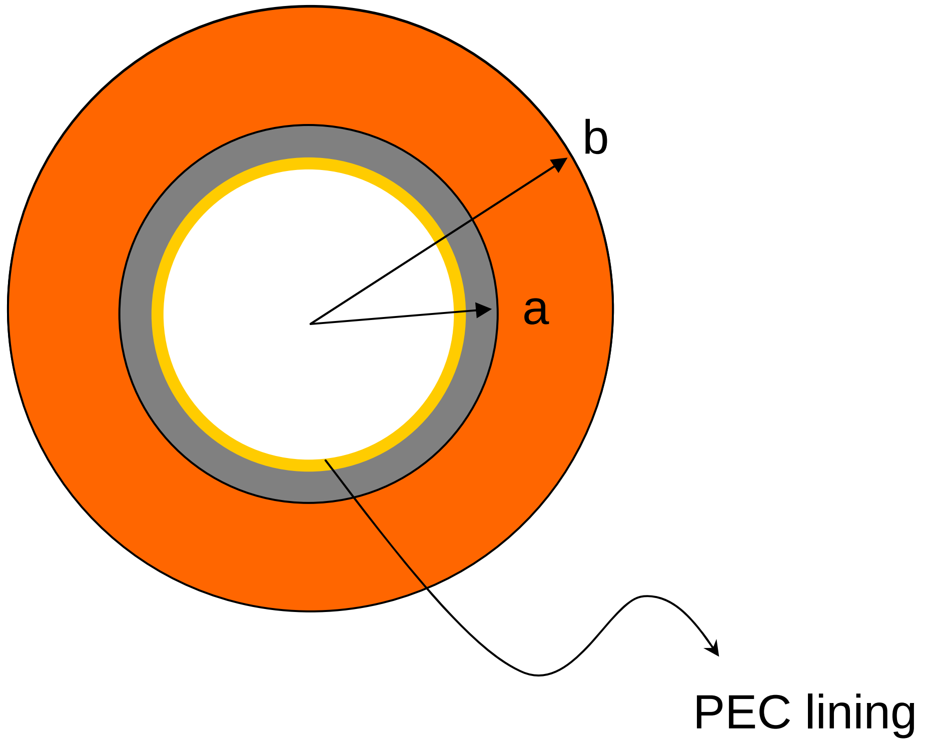
Linear simplified cloak | Current work



Scattered E_z field

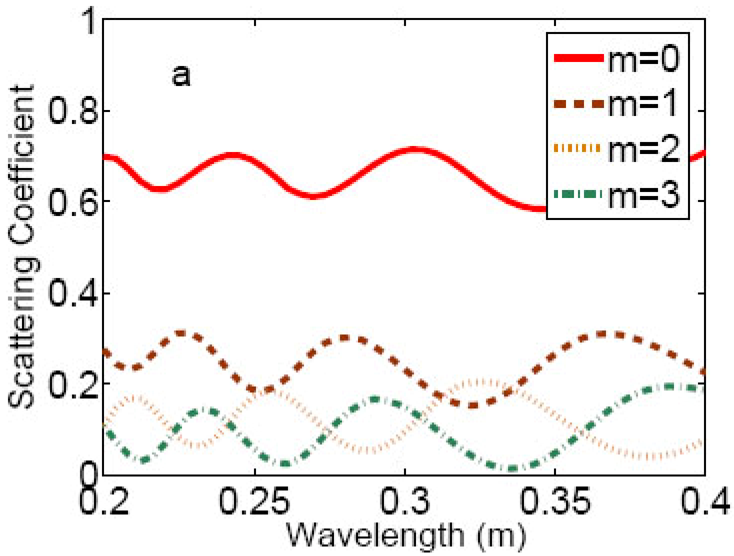
1. M. Yan, Z. Ruan, and M. Qiu, Opt. Express **15**, p. 17772 (2007).
2. D. Schurig, et al., Science **314**, 977 (2006).
3. W. Cai et al., Appl. Phys. Lett. **91**, 111,105 (2007).

Can we cancel the zeroth order scattering?

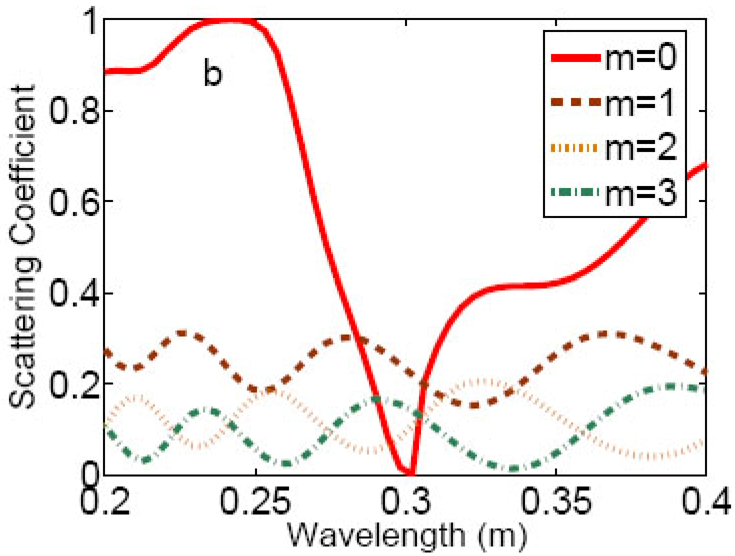


Cancellation of zeroth-order scattering

without monopole cancellation



with monopole cancellation

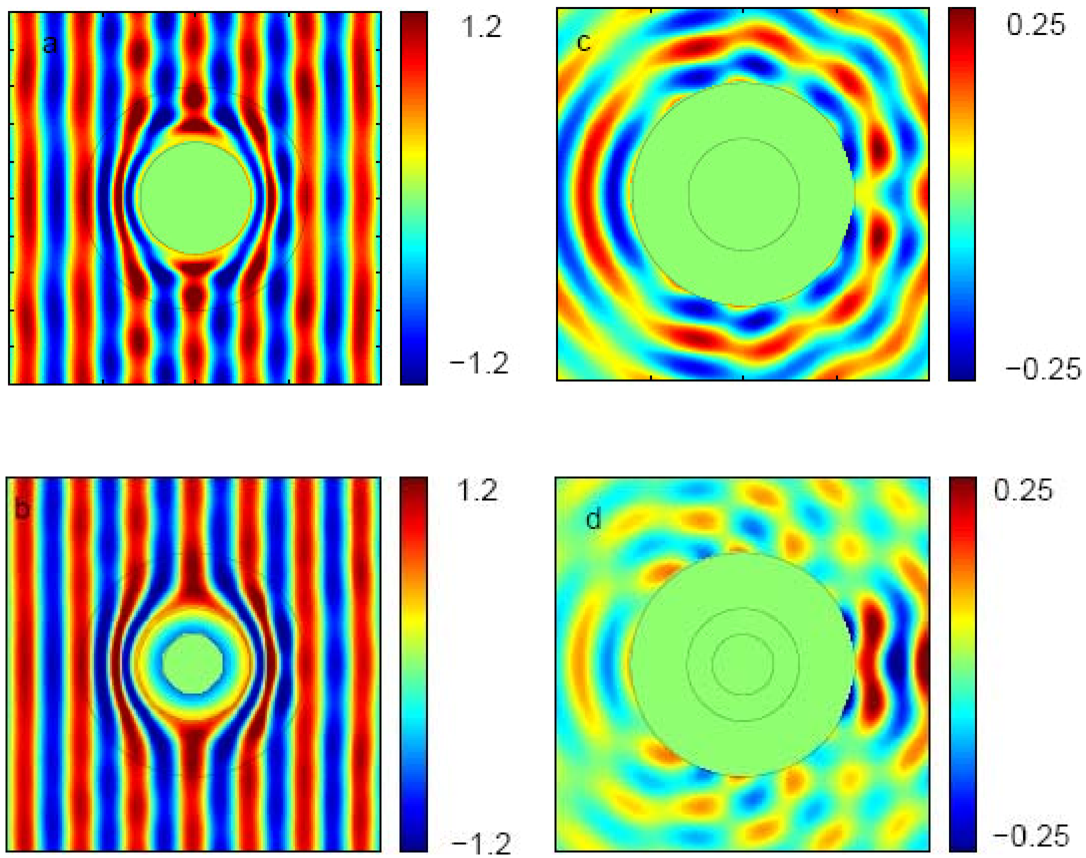


a = 0.3m, b = 0.6m

Cancellation of zeroth-order scattering

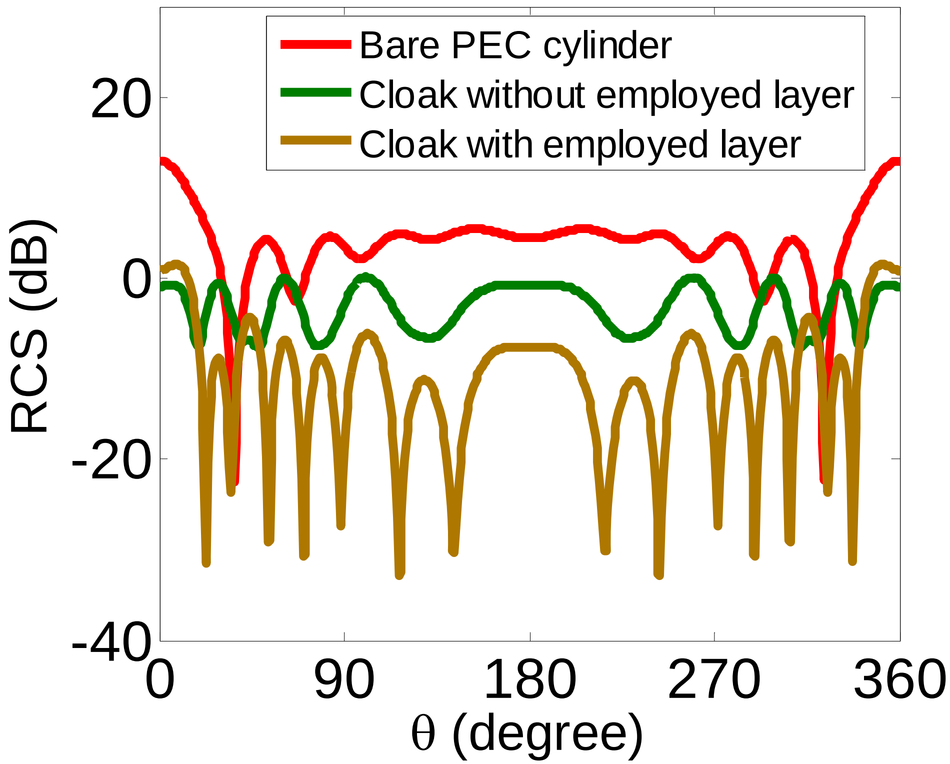
Without cancellation

With cancellation



λ=0.3m

Far field radiation



Remarks:

-Mostly high order scatterings remain

-Wavelength dependent

Ref:

W. Yan, M. Yan, and Min Qiu, "Non-magnetic simplified cylindrical cloak with near perfect invisibility by suppressing zeroth order scattering," Appl. Phys. Lett., in review.

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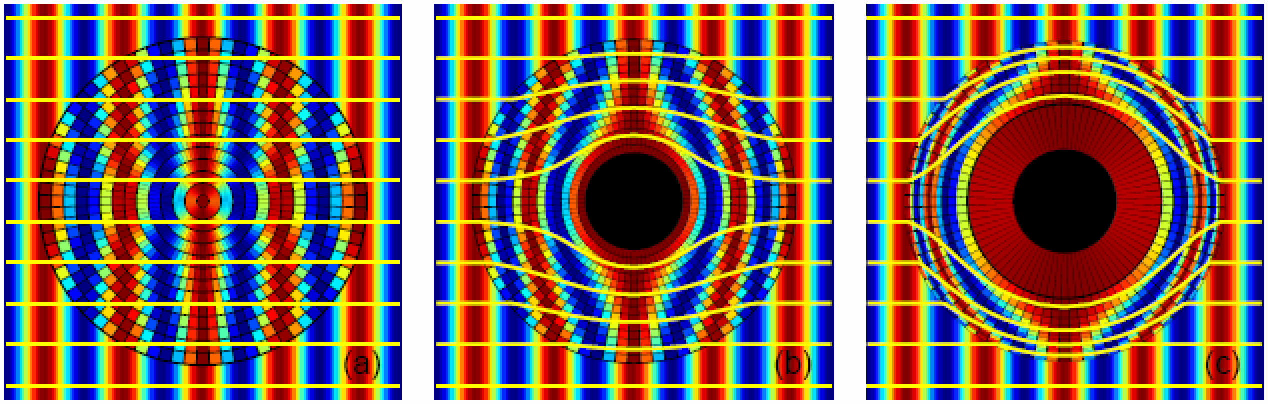
High-order cylindrical invisibility cloak

One possible class of transformation may take the form of

$$r' = \frac{b}{(b-a)^n}(r-a)^n,$$

n: transformation order.

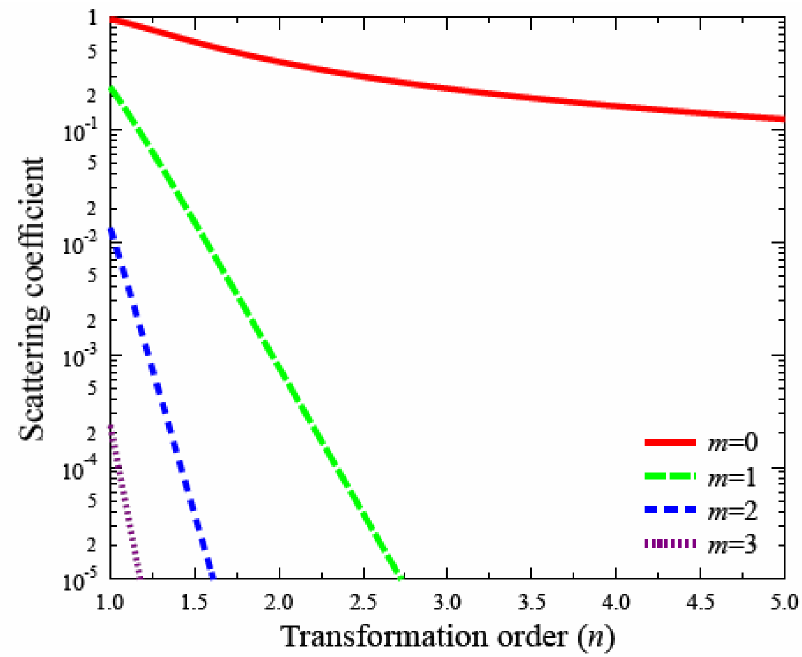


Virtual flat space　　n=1　　n=3

$$\varepsilon_r = \mu_r = \frac{r-a}{nr}, \quad \varepsilon_\theta = \mu_\theta = \frac{nr}{r-a}, \quad \varepsilon_z = \mu_z = \frac{nb^2(r-a)^{2n-1}}{(b-a)^{2n}r}.$$

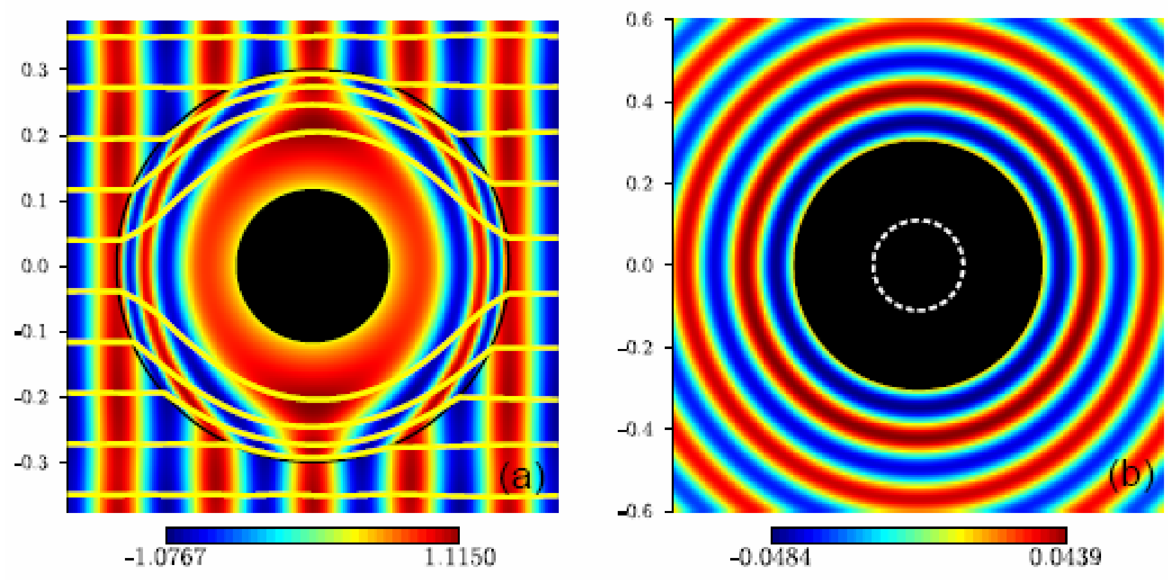
Infinite parameters at *r*=*a*!

Effect of transformation order on scattering coefficients



a = 0.1m
b = 0.3m
λ= 0.15m
Thickness of the layer peeled away: ***d*=0.01m**
PEC lining is present

Field distributions



Ez

Scattered Ez

Transformation order used: n=3

Ref

M. Yan, W. Yan, L. Zhang, M. Qiu, Cylindrical Invisibility Cloak: Properties and Strategies for Practical Realization, NATO Advanced Research Workshop, Metamaterials for Secure Information and Communication Technologies, 7-10 May, 2008, Marrakech – Morocco.

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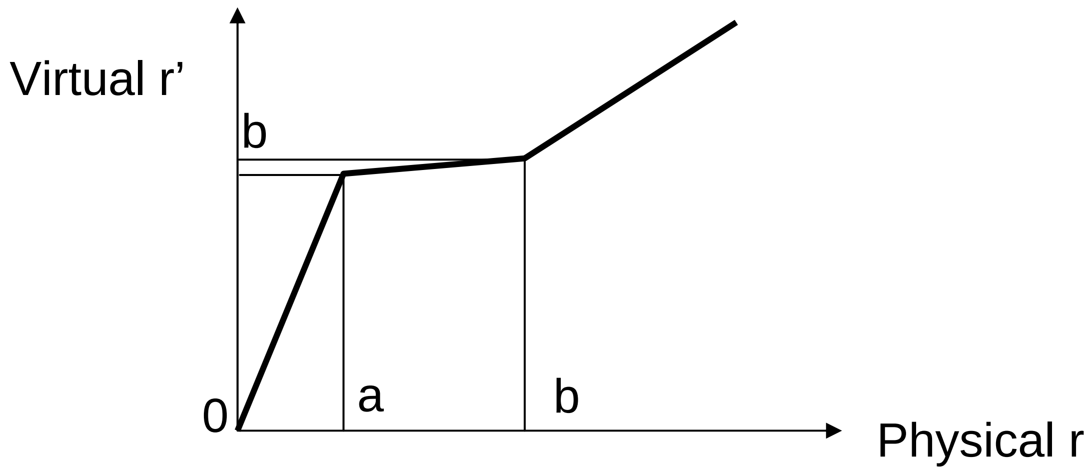
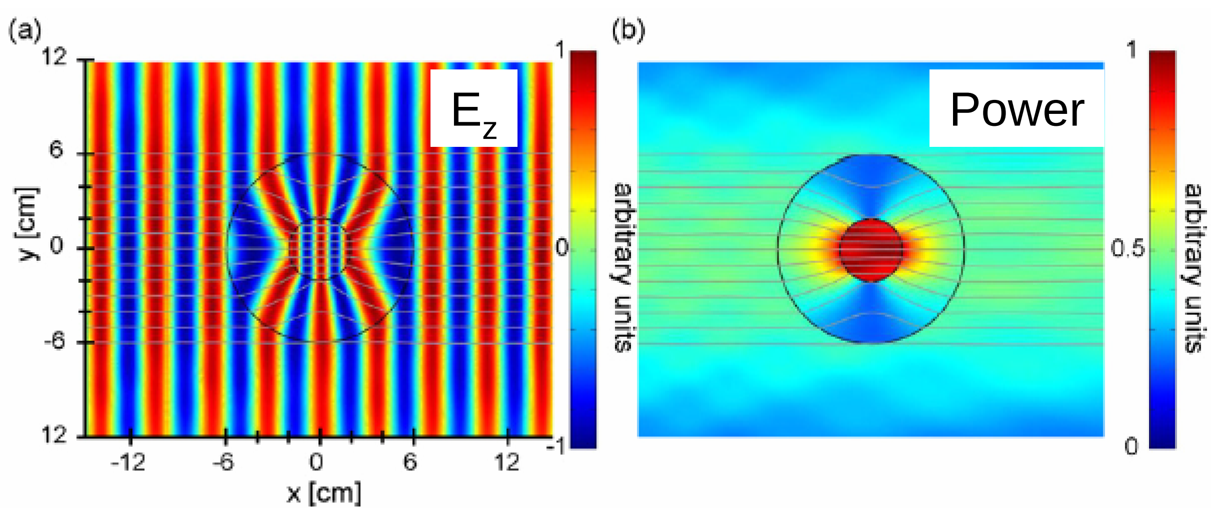
EM concentrator

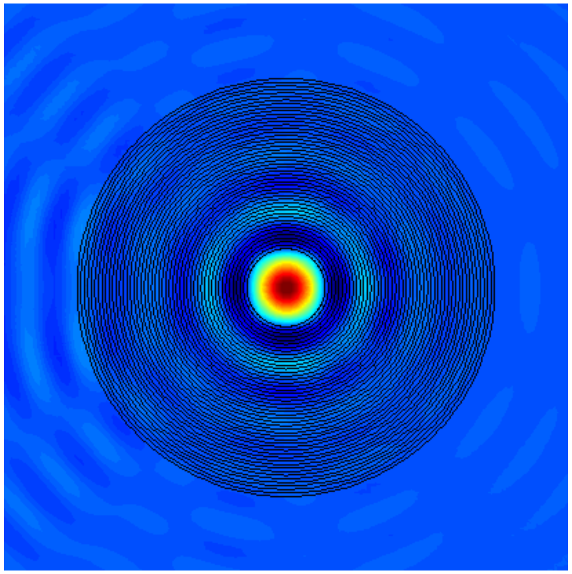


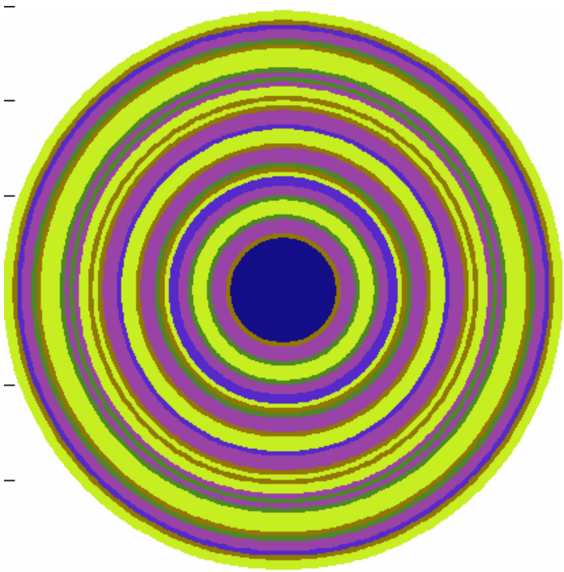


Application:
- Photovoltaic fiber
- Fluid heating

M. Rahm et al, Photonics and nanostructures - Fundamentals and Applications, vol 6, 87-95 (2008).

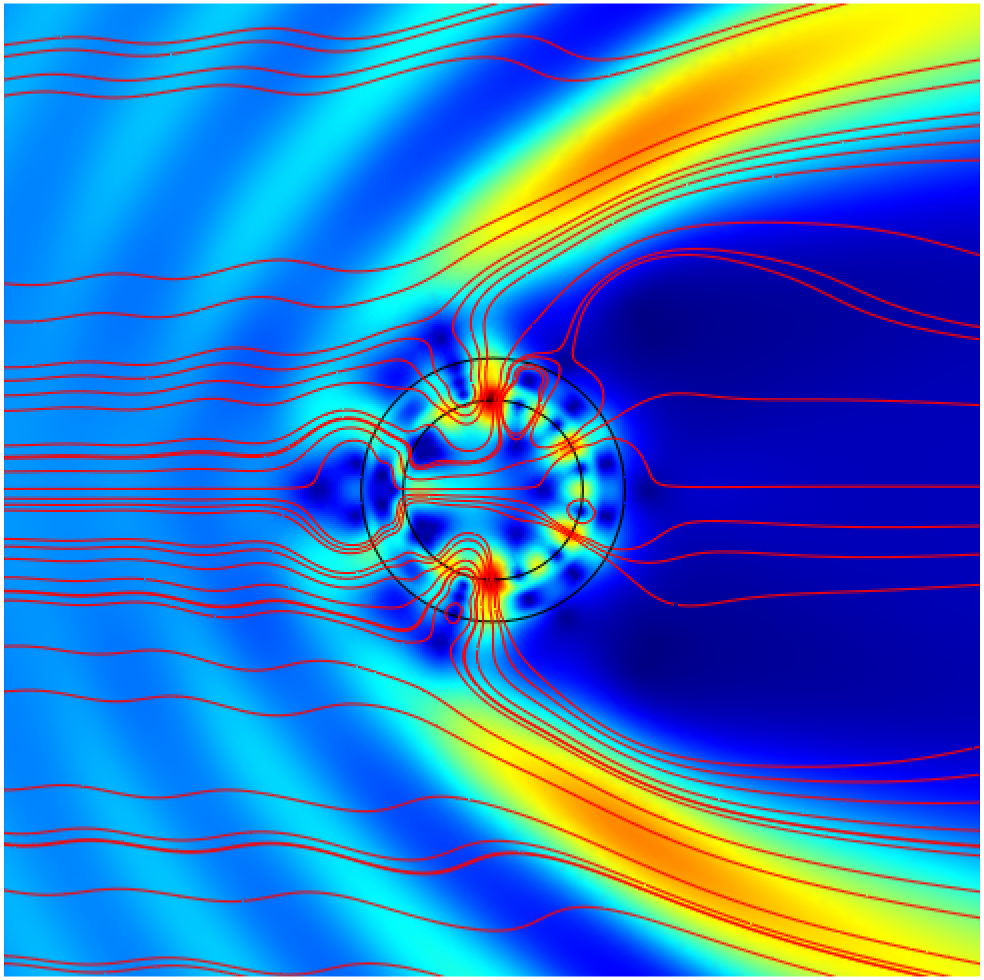
Resonance based EM concentrator





- Enhancement factor 230
- Wavelength dependant

Resonance based EM concentrator



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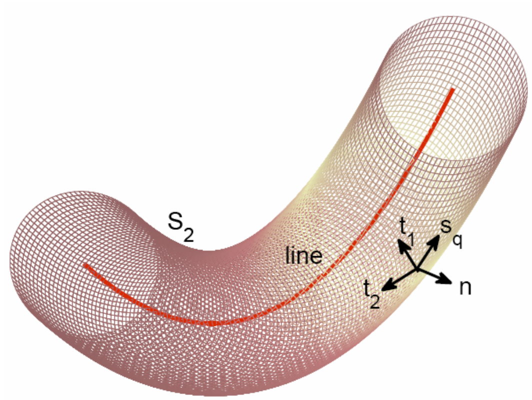
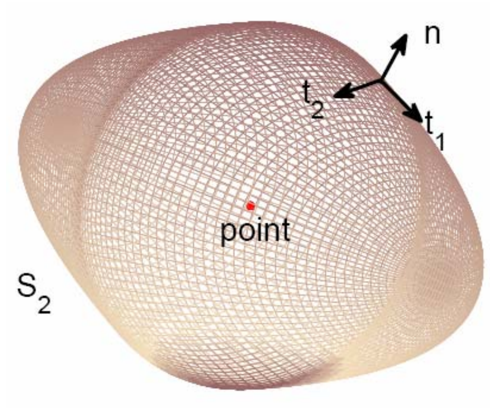
Arbitrarily-shaped cloaks

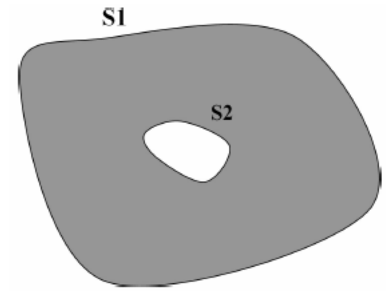


Blowing-up a line



Blowing-up a point

General cross-section:



Ref:
Wei Yan, Min Yan, Zhichao Ruan, Min Qiu, "Coordinate transformation makes perfect invisibility cloak with arbitrary shape," New Journal of Physics, vol. 10, 043040 (2008).

Arbitrarily-shaped cloak

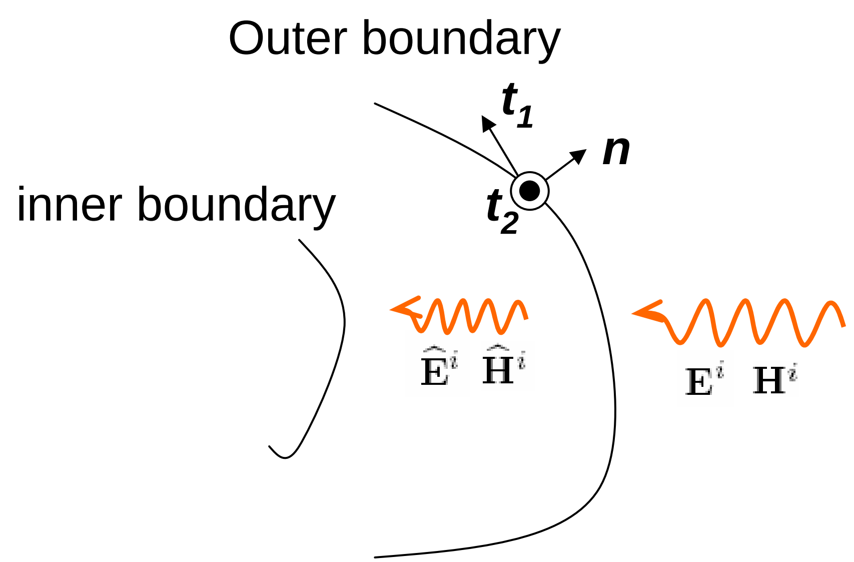
For both line-transformed or point-transformed cloaks,

$$\widehat{\mathbf{E}}^i = \Lambda^T \mathbf{E}^i, \quad \widehat{\mathbf{H}}^i = \Lambda^T \mathbf{H}^i$$

At outer boundary:

$$\widehat{E}^i_{t_1} = \mathbf{E}^i \cdot \widehat{t}_1, \ \widehat{H}^i_{t_1} = \mathbf{H}^i \cdot \widehat{t}_1,$$

$$\widehat{E}^i_{t_2} = \mathbf{E}^i \cdot \widehat{t}_2, \ \widehat{H}^i_{t_2} = \mathbf{H}^i \cdot \widehat{t}_2,$$



Arbitrarily-shaped cloak

For line-transformed cloak,

$$\widehat{E}^i_{t1} = \widehat{H}^i_{t1} = 0.$$

However, the other components of fields are not zero. In particular,

$$\widehat{E}^i_{t2} = (\widehat{s}\cdot\widehat{t}_2)[B_1,\ B_2,\ B_3]\mathbf{E}^i,$$

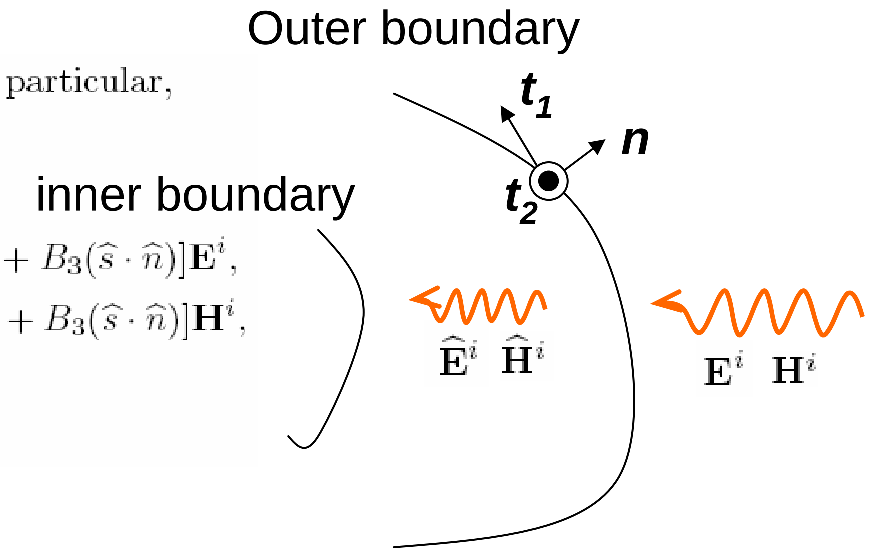
$$\widehat{H}^i_{t2} = (\widehat{s}\cdot\widehat{t}_2)[B_1,\ B_2,\ B_3]\mathbf{H}^i,$$

$$\widehat{E}^i_{n} = [F_1 + B_1(\widehat{s}\cdot\widehat{n}),\ F_2 + B_2(\widehat{s}\cdot\widehat{n}),\ F_3 + B_3(\widehat{s}\cdot\widehat{n})]\mathbf{E}^i,$$

$$\widehat{H}^i_{n} = [F_1 + B_1(\widehat{s}\cdot\widehat{n}),\ F_2 + B_2(\widehat{s}\cdot\widehat{n}),\ F_3 + B_3(\widehat{s}\cdot\widehat{n})]\mathbf{H}^i,$$

with

$$B_i = \sqrt{\partial b_i/\partial q_1^2 + \partial b_i/\partial q_2^2 + \partial b_i/\partial q_3^2}.$$



Remark: Surface current will be induced at the inner surface.

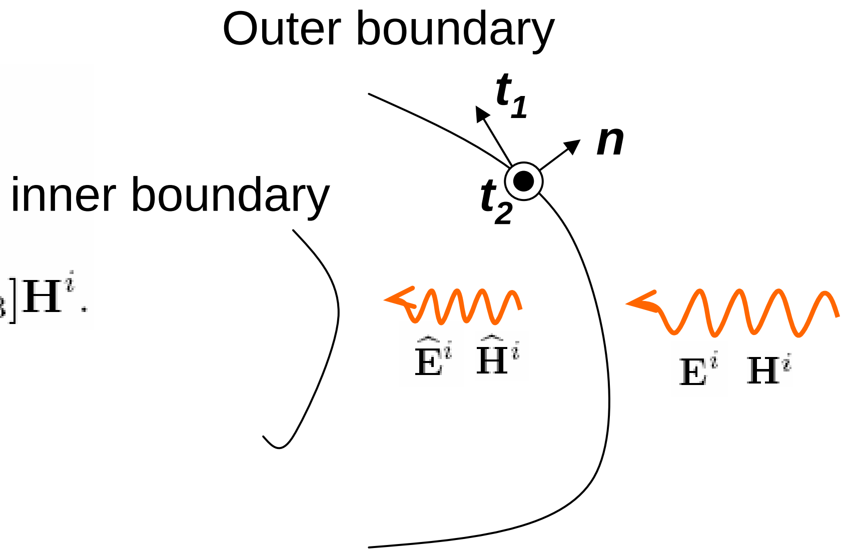
Arbitrarily-shaped cloak

For point-transformed cloak,

$$\widehat{E}_{t_1}^{i} = \widehat{H}_{t_1}^{i} = 0,$$

$$\widehat{E}_{t_2}^{i} = \widehat{H}_{t_2}^{i} = 0,$$

$$\widehat{E}_{n}^{i} = [F_1,\ F_2,\ F_3]\mathbf{E}^{i},\ \widehat{H}_{n}^{i} = [F_1,\ F_2,\ F_3]\mathbf{H}^{i}.$$



All tangential fields are zero.
Therefore no field discontinuity exists for perfect cloaking.

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Project plan

We are here!

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Cylindrical cloak: Simplification	■	■	■	■				
Cylindrical cloak: Effect of order number		■	■	■				
Electromagnetic Concentrator		■	■	■	■	□	□	□
Arbitrarily-shaped cloak			■	■	■	■	■	
Project report								■

Outcome

Papers:

1. Wei Yan, Min Yan, Zhichao Ruan, Min Qiu, "Coordinate transformation makes perfect invisibility cloak with arbitrary shape," New Journal of Physics, vol. 10, 043040 (2008).
2. M. Yan, W. Yan, L. Zhang, M. Qiu, Cylindrical Invisibility Cloak: Properties and Strategies for Practical Realization, NATO Advanced Research Workshop, Metamaterials for Secure Information and Communication Technologies, 7-10 May, 2008, Marrakech, Morocco.
3. W. Yan, M. Yan, and Min Qiu, "Non-magnetic simplified cylindrical cloak with near perfect invisibility by suppressing zeroth order scattering," Appl. Phys. Lett., in review.

Code:

Matlab code for scattering calculation of multilayered cylindrical structures.

Thank you!

Questions?