Chiral particles and transmission-line networks for cloaking applications

P. Alitalo, S. Tretyakov

13.6.2008
Outline

- Introduction
  - Project topics
- Chiral cloak
  - Design, simulations, feasibility
- Transmission-line cloak
  - Design, simulations, measurements, feasibility, applications
- Generalized field transformations
  - Principles, derivations, examples
Topics of the project

1. Coordinate-transformation cloak based on chiral inclusions
2. Cloak based on networks of transmission lines
3. Investigation of the potentials for realization of the cloaks studied in items 1 and 2 in the optical frequency range
4. Comparison of the two approaches in items 1 and 2 with other approaches for cloak design
5. Study of fundamental limitations of different types of cloaks (mainly the cloak types studied in items 1 and 2)
6. The ACT provided some examples of space applications, where the feasibility of the cloaks studied in items 1 and/or 2 were investigated
7. A generalized form of field-transforming metamaterials has been developed
Chiral cloak: Outline

- Basic cloak structure
- Simulations
- Design of chiral particles
- Cloak design
- Feasibility
- Fundamental limitations
- Future work
Basic cloak structure

- Coordinate-transformation cloak

\[ \varepsilon_\rho = \mu_\rho = \frac{\rho - a}{\rho}, \quad \varepsilon_\varphi = \mu_\varphi = \frac{\rho}{\rho - a}, \quad \varepsilon_z = \mu_z = \left(\frac{b}{b - a}\right)^2 \frac{\rho - a}{\rho} \]

Basic cloak structure

- Coordinate-transformation cloak composed of SRRs (TE-pol.)

\[ \varepsilon_z = \left( \frac{b}{b-a} \right)^2, \quad \mu_\rho = \left( \frac{\rho - a}{\rho} \right)^2, \quad \mu_\varphi = 1 \]

Basic cloak structure

- Coordinate-transformation cloak composed of needles (TM-pol.)

\[ \mu_z = 1, \quad \varepsilon_{\varphi} = \left( \frac{b - a}{b} \right)^2, \quad \varepsilon_{\rho} = \left( \frac{b}{b - a} \right)^2 \left( \frac{\rho - a}{\rho} \right)^2. \]

Basic cloak structure

- Coordinate-transformation cloak composed of chiral particles

\[(\text{BOTH TE & TM})\]

\[\varepsilon_\rho = \mu_\rho = \left(\frac{b}{b-a}\right)\left(\frac{\rho-a}{\rho}\right)^2, \quad \varepsilon_\varphi = \mu_\varphi = \varepsilon_z = \mu_z = \left(\frac{b}{b-a}\right)\]

FEM-simulations have been conducted to confirm the cloaking effect with the simplified material parameter distribution.

- Particle design

\[ \varepsilon_r = 1 + \frac{n\alpha_e}{\varepsilon_0 - \frac{1}{3}n\alpha_e}, \quad \mu_r = 1 + \frac{n\alpha_m}{\mu_0 - \frac{1}{3}n\alpha_m} \]  

(Clausius-Mossotti)

By demanding \( \varepsilon_r = \mu_r \) we find that the polarizabilities must satisfy

\[ \frac{\alpha_e}{\varepsilon_0} = \frac{\alpha_m}{\mu_0} \]

For the canonical spiral the condition reads

\[ l = k\pi R^2 \]
Next, we can provide the desired profile of the permittivity and permeability along $\rho$ by varying the density of inclusions $n$. This can be solved from

$$n = 3\varepsilon_0 \Re \left\{ \frac{1}{\alpha_e} \right\} \frac{\varepsilon_\rho - 1}{\varepsilon_\rho + 2}$$
Feasibility

- Microwave demonstration is currently being done (outside this project)
- Feasibility for high-frequency operation was studied (visible range)

Fundamental limitations (of coordinate-transformation cloaks)

- Energy propagation

Fundamental limitations (of coordinate-transformation cloaks)

- Dispersion and frequency bandwidth

Fundamental limitations (of coordinate-transforming cloaks)

- Losses $\implies$ ideal cloaking is impossible with passive materials (the total scattering cross section should be zero!)

  cylindrical case:

  \[
  \varepsilon_\rho = \frac{1}{\varepsilon_\varphi} \\
  \mu_\rho = \frac{1}{\mu_\varphi}
  \]

  spherical case:

  \[
  \varepsilon_\rho = \frac{1}{\varepsilon_\varphi} = \frac{1}{\varepsilon_\theta} \\
  \mu_\rho = \frac{1}{\mu_\varphi} = \frac{1}{\mu_\theta}
  \]

- In addition, losses destroy perfect matching of the cloak
Future work

- Experimental demonstration, first at microwaves…
Microwave transmission-line networks as electromagnetic cloaks

Pekka Alitalo, Sergei Tretyakov

13.6.2008
Outline

- Transmission-line networks
- Background
- Principle of cloaking
- Transmission-line cloak design
- Cloak examples and simulation results
- Experiment
- Fundamental limitations
- Examples of possible applications
Three-dimensional transmission-line networks

- A 3D isotropic host TL network:

- Note that here the isotropy refers to *isotropic propagation of waves of voltages and currents!*

Three-dimensional superlens

- Superlens structure:

  A backward-wave (\(\varepsilon<0,\mu<0\)) TL network sandwiched between two half-spaces composed of forward-wave (\(\varepsilon>0,\mu>0\)) TL networks

Dispersion and isotropy

- Dispersion equations and isotropy have been confirmed with full-wave simulations (Ansoft HFSS)

[Alitalo and Tretyakov, *Metamaterials*, 1, pp. 81-88, 2007]

![Graph](image)

\[Z_{\text{TL,FW}}=66 \text{ Ohm}, \quad Z_{\text{TL,BW}}=89 \text{ Ohm}, \quad d=13 \text{ mm}, \quad L=6.8 \text{ nH}, \quad C=3.3 \text{ pF}\]
Analytical equations for impedance were derived for matching the networks.
Superlens operation

- Transmission from source plane to image plane as a function of the transverse wavenumber (realistic losses)
- Resolution enhancement as a function of the distance between source and image

$$2l \approx 0.6\lambda$$
Superlens operation

- Focusing of propagating modes
- Enhancement of evanescent modes
Experiment

- Experimental demonstration of the proposed design was given in [Alitalo et al., *J. Appl. Phys.*, 99, 124910, 2006]
- Forward/backward-wave propagation in the networks was confirmed with measurements
- Amplification of evanescent modes was confirmed with measurements (essential to sub-wavelength resolution)
Matching to free space

- A transition layer can couple waves from free space to a TL network and vice versa

Matching to free space

- Negative refraction was confirmed with simulations in [P. Alitalo, O. Luukkonen, and S. Tretyakov, *Phys. Lett. A*, 372, 2720, 2008]
Matching to free space

- Dependence on the frequency and the incidence angle (in $H$-plane), obtained from the simulations
- Good transmission also for oblique incidence in $E$-plane

Normal incidence:

Oblique incidence ($f = 4$ GHz):
Transmission-line approach to cloaking

- Work started in our group in spring 2007
- Illustration of the operation principle:

Cloak design

- Design issues:
  1. Dispersion (2D)

Cloak design

- Design issues:
  - 2. Impedance (2D)

Unloaded or loaded?

- Unloaded
  + phase velocity equal to group velocity
  + large bandwidth
  + simple design, simulation, manufacturing
  - non-ideal phase velocity (how much will this affect?)

- Loaded
  + ideal phase velocity
  - ...
  - ...
  - ...
Scattering from an unloaded TL-cloak

- Effect of the non-ideal phase velocity on cloaking has been studied:

Scattering from an unloaded TL-cloak

- Effect of the non-ideal phase velocity on cloaking has been studied:

(a) 1 GHz
(b) 6.4 GHz
A rectangular cloak with a cylindrical incident wave

- An example cloak network was designed for operation around 5 GHz
- Transmission lines realized as parallel metal strips
- Transition from free space to the network (widened strips)
A cloak slab with a normally incident plane wave

- Plane wave excitation (normal incidence) to a transversally infinite cloak slab

(a) With cloak  
(b) Without cloak
Electrically small cylindrical cloak

- Transmission-line network designed for operation around the frequency 2 GHz.
- Cloak diameter equals $6 \times$ period = 48 mm $\approx \lambda/3$ @ 2 GHz.

Electrically small cylindrical cloak

- HFSS simulations at 2 GHz for cloaked object (a mesh of PEC rods) with and without the cloak

![Graph showing SCS results](image)

Electrically small cylindrical cloak

- HFSS simulations at 2 GHz for cloaked object (mesh of PEC rods) with and without the cloak

Within the project, we have modified the cloak to obtain more insight on the bandwidth of operation.

Redesigning the structure for higher frequencies (to enable future measurements, $f = 3 \text{ GHz}$ is chosen).

The period is made smaller to ensure isotropy.

$D \approx \lambda/3 @ 3 \text{ GHz}$. 
Modified electrically small cylindrical cloak

- Total scattering cross section, as compared to the reference case

![Graph showing the total scattering cross section for modified electrically small cylindrical cloak. The graph plots frequency (f) on the x-axis in GHz, and the ratio of total scattering cross section for the cloak to that of the reference case on the y-axis. The data points suggest a decrease in scattering cross section as frequency increases, reaching a minimum around 3 GHz before increasing again.]
Modified electrically small cylindrical cloak

- E-field at 3 GHz:
Electrically large cylindrical cloak

- Electrically large cloak was designed and simulated
- \( D \approx 4\lambda \) @ 3 GHz.

Electrically large cylindrical cloak

- The total scattering cross section, as compared to the reference case
- The optimal electrical size was approximated based on previous analysis

![Graph showing the total scattering cross section ratio versus frequency](image)

Electrically large cylindrical cloak

- HFSS simulations at 3 GHz for a cloaked object (a mesh of PEC rods) with and without the cloak

![Image of E Field in V/m with and without the cloak]
Modified electrically large cylindrical cloak

- Impedance of the network tuned for 3 GHz

P. Alitalo and S. Tretyakov, “Broadband microwave cloaking with periodic networks of transmission lines,” submitted to *Metamaterials’2008*
Measurement

- Cloak structure
- Produced by etching from copper
Measurement setup

- A big parallel-plate waveguide (1x1 m²) was fabricated
- A copper mesh placed into a hole in the upper plate
- Coax feed inside the waveguide
Measurement setup
Measurements

- Measurements conducted at 1 – 10 GHz (designed structure is expected to have best impedance matching characteristics at 5 – 6 GHz)

- By measuring $\text{RE}[S_{21}]$ and $\text{IM}[S_{21}]$, we obtain the complex distribution of electric field inside the waveguide → we can animate the time-harmonic electric field propagation inside the waveguide (and obtain also magnitude, phase, etc.)

- We have measured
  1) empty waveguide
  2) reference object (15x15 rods)
  3) reference object and cloak
Measurement results

- Best performance (corresponding to simulated field plots) was found around 5.85 GHz
- Snapshots of the time-harmonic electric field (empty waveguide)
Measurement results

- Snapshots of the time-harmonic electric field (reference object)
Measurement results

- Snapshots of the time-harmonic electric field (ref. object and cloak)
Measurement results

- Phase plots in front of the reference object / cloak
Measurement results

- $|E|$ distributions behind the ref. object/cloak (normalized to the field amplitude in front of the ref. object/cloak)
Fundamental limitations

- Cloaking of electrically large "bulky" objects
  - Only a volume limited by a three-dimensional mesh can be cloaked in the general (3D) case
  - A 2D network can cloak a larger volume than a 3D network
Fundamental limitations

- Operation at very high frequencies ($f > 300$ GHz)
  - no practical possibility to realize subwavelength transmission lines at the present stage of technology
Applications

- Antennas, lenses, support structures, ...
Applications

- A structure blocking a horn antenna (Gaussian beam excitation)
- Incident field (left) and metal object (right)
Applications

- A structure blocking a horn antenna (Gaussian beam excitation)
- Incident field (left) and cloaked metal object (right)
Applications

- A structure blocking a dipole antenna
Applications

- A structure blocking a dipole antenna
Applications

- "Invisibility shutter"

- The problem is that at high frequencies (wavelength 7-14 micrometers) it is not possible to fabricate the needed types of transmission lines.
With our transmission-line approach, the *refractive index* and the *impedance* of a "material" can be designed separately

→ possibility to obtain microwave lenses having refractive indices similar to dielectric lenses, *while* having impedance-matching with e.g. free space
Matched microwave transmission-line lenses

- Focusing with an example lens \( n=4.66 \)
- Refracts the wave similar to a dielectric lens, AND, power reflection is significantly lower!
Matched microwave transmission-line lenses

- Simulated phase patterns for normally incident plane wave excitation
Thank you!

References:


Generalized field transformations using metamaterials

S. Tretyakov, I. Nefedov, P. Alitalo

13.6.2008
Outline

- Introduction
- Constitutive parameters of generalized field-transforming metamaterials
- Example cases
Let us define the following transformation:

\[
E(r) = F(r, \omega)E_0(r) + \sqrt{\frac{\mu_0}{\varepsilon_0}} A(r, \omega)H_0(r)
\]

\[
H(r) = G(r, \omega)H_0(r) + \sqrt{\frac{\varepsilon_0}{\mu_0}} C(r, \omega)E_0(r)
\]

Here, scalar functions \( F(r, \omega), G(r, \omega), A(r, \omega) \) and \( C(r, \omega) \) are arbitrary differentiable functions.
Material parameters

- Inserting the previous equations into the Maxwell equations gives us $B$ and $D$ as functions of $E$ and $H$ (material relations):

$$B = \sqrt{\frac{\mu_0 j}{\varepsilon_0 \omega FG - AC}} (F \nabla A - A \nabla F) \times H + \mu_0 \frac{1}{FG - AC} \left( F^2 + A^2 \right) H$$

$$+ \frac{j}{\omega} \frac{1}{FG - AC} (G \nabla F - C \nabla A) \times E - \sqrt{\varepsilon_0 \mu_0 \frac{1}{FG - AC}} (AG + CF) E$$

$$D = \sqrt{\frac{\varepsilon_0 j}{\mu_0 \omega FG - AC}} (C \nabla G - G \nabla C) \times E + \varepsilon_0 \frac{1}{FG - AC} \left( G^2 + C^2 \right) E$$

$$+ \frac{j}{\omega} \frac{1}{FG - AC} (A \nabla C - F \nabla G) \times H - \sqrt{\varepsilon_0 \mu_0 \frac{1}{FG - AC}} (AG + CF) H$$
Material parameters

- The previous equations describe a bi-anisotropic medium, of the form

\[
B = \overline{\mu} \cdot H + \sqrt{\varepsilon_0 \mu_0 (\overline{\chi} + j\overline{\kappa})^T} \cdot E,
\]

\[
D = \overline{\varepsilon} \cdot E + \sqrt{\varepsilon_0 \mu_0 (\overline{\chi} - j\overline{\kappa})} \cdot H.
\]
Material parameters

- We can identify the material parameters as

\[
\overline{\varepsilon} = \varepsilon_0 \frac{1}{FG - AC} \left( G^2 + C^2 \right) \overline{I} + \sqrt{\varepsilon_0} \frac{j}{\mu_0 \omega} \frac{1}{FG - AC} (C \nabla G - G \nabla C) \times \overline{I}
\]

\[
\overline{\mu} = \mu_0 \frac{1}{FG - AC} \left( F^2 + A^2 \right) \overline{I} + \sqrt{\mu_0} \frac{j}{\varepsilon_0 \omega} \frac{1}{FG - AC} (F \nabla A - A \nabla F) \times \overline{I}
\]

\[
\overline{\chi} = -\frac{1}{FG - AC} (AG + CF) \overline{I} + \frac{1}{\sqrt{\varepsilon_0 \mu_0 \omega}} \frac{j}{2} \frac{1}{FG - AC} \left[ (A \nabla C - F \nabla G) - (G \nabla F - C \nabla A) \right] \times \overline{I}
\]

\[
\overline{K} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \frac{1}{2 \omega} \frac{1}{FG - AC} \left[ -(A \nabla C - F \nabla G) - (G \nabla F - C \nabla A) \right] \times \overline{I}
\]
Example cases

\[ B = \bar{\mu} \cdot H + \sqrt{\varepsilon_0 \mu_0} (\bar{\kappa} + j\bar{\kappa})^T \cdot E, \]
\[ D = \bar{\varepsilon} \cdot E + \sqrt{\varepsilon_0 \mu_0} (\bar{\kappa} - j\bar{\kappa}) \cdot H. \]

4.1. Backward-wave medium

Consider the case of the field transformation \( E_0 \rightarrow \sqrt{\frac{\mu_0}{\varepsilon_0}} H , \quad H_0 \rightarrow \sqrt{\frac{\varepsilon_0}{\mu_0}} E , \) which corresponds to \( F = G = 0 \) and \( A = C = 1 \) in (1),(2). Transforming electric field into magnetic field and vice versa, we reverse the propagation direction of plane waves in the transformation volume, so we expect that this would correspond to a backward-wave material filling the volume. And indeed we see that the material relations (3) and (4) reduce to

\[ B = -\mu_0 H, \quad (20) \]
\[ D = -\varepsilon_0 E, \quad (21) \]

which are the material relations of the Veselago medium.
4.2. Field rotation

Consider the case of the field transformation $E_0 \rightarrow \sqrt{\frac{\mu_0}{\varepsilon_0}} H$, $H_0 \rightarrow -\sqrt{\frac{\varepsilon_0}{\mu_0}} E$, which corresponds to $F = G = 0$ and $A = -1$, $C = 1$ in (1), (2). This kind of transformation relates to changing the polarization of the field by 90 degrees. The material relations (3) and (4) reduce to

$$B = \mu_0 H,$$

$$D = \varepsilon_0 E,$$

which are the material relations of free space. The reason why we do not see the change of polarization here is simply due to the fact we wanted to consider $F$, $G$, $A$ and $C$ as scalars. The media (original or transformed) do not know the polarization of the fields, so the rotation does not have any influence on the material relations.
4.3. *Isotropic Tellegen medium*

Consider next a more general case when all the transformation coefficients do not depend on the position vector inside the transformation domain. In this case, gradients of these functions vanish, and we see that the material relations are that of an isotropic Tellegen material:

\[
\mathbf{B} = \mathbf{\mu}_0 \left( \frac{1}{FG - AC} \left( F^2 + A^2 \right) \mathbf{H} - \sqrt{\varepsilon_0 \mu_0} \frac{AG + CF}{FG - AC} \mathbf{E} \right),
\]

\[
\mathbf{D} = \varepsilon_0 \left( \frac{1}{FG - AC} \left( G^2 + C^2 \right) \mathbf{E} - \sqrt{\varepsilon_0 \mu_0} \frac{AG + CF}{FG - AC} \mathbf{H} \right).
\]  \hspace{1cm} (24)

An interesting thing about the Tellegen medium is that the wavenumber of the medium inherently stays the same regardless of the applied transformation of the fields. This can be shown by using the equation for the wavenumber in a Tellegen medium [11]

\[
k^2 = k_0 \sqrt{\varepsilon_r \mu_r - \chi^2}, \quad \varepsilon_r \mu_r - \chi^2 = \frac{(G^2 + C^2)(F^2 + A^2)}{(FG - AC)^2} - \frac{(AG + CF)^2}{(FG - AC)^2} = 1
\]  \hspace{1cm} (26)
Example cases

\[\mathbf{B} = \bar{\mu} \cdot \mathbf{H} + \sqrt{\varepsilon_0 \mu_0} (\bar{\varepsilon} + j\bar{\kappa})^T \cdot \mathbf{E},\]
\[\mathbf{D} = \bar{\varepsilon} \cdot \mathbf{E} + \sqrt{\varepsilon_0 \mu_0} (\bar{\varepsilon} - j\bar{\kappa}) \cdot \mathbf{H}.\]

4.4. Nihility

Let us consider again the case where all the coefficients are position-independent, that is,

\[\nabla F = \nabla G = \nabla A = \nabla C = 0\]  \hspace{1cm} (26)

and also demand that the permittivity and permeability of the transforming medium become zero. This takes place when either

\[A = jF, \quad C = jG\]  \hspace{1cm} (27)

or

\[A = -jF, \quad C = -jG\]  \hspace{1cm} (28)

The material relations of the medium become

\[\mathbf{B} = -j\sqrt{\varepsilon_0 \mu_0} \mathbf{E}\]
\[\mathbf{D} = -j\sqrt{\varepsilon_0 \mu_0} \mathbf{H}\]  \hspace{1cm} (29)

for the case (27) or

\[\mathbf{B} = j\sqrt{\varepsilon_0 \mu_0} \mathbf{E}\]
\[\mathbf{D} = j\sqrt{\varepsilon_0 \mu_0} \mathbf{H}\]  \hspace{1cm} (30)

for the case (28).
Moving omega media

\[ \nabla \times \mathbf{E}(r) = -j\omega \mu_0 \frac{F(r)}{G(r)} \mathbf{H}(r) + \frac{1}{F(r)} \nabla F(r) \times \mathbf{E}(r), \]
\[ \nabla \times \mathbf{H}(r) = j\omega \varepsilon_0 \frac{G(r)}{F(r)} \mathbf{E}(r) + \frac{1}{G(r)} \nabla G(r) \times \mathbf{H}(r). \]

\[ \mathbf{D} = \frac{1}{\sqrt{1-v^2/c^2}} \left( \varepsilon_0 \mathbf{E} + \frac{1}{c^2} \mathbf{v} \times \mathbf{H} \right) \approx \varepsilon_0 \mathbf{E} + \frac{1}{c^2} \mathbf{v} \times \mathbf{H} \]
\[ \mathbf{B} = \frac{1}{\sqrt{1-v^2/c^2}} \left( \mu_0 \mathbf{H} - \frac{1}{c^2} \mathbf{v} \times \mathbf{E} \right) \approx \mu_0 \mathbf{H} - \frac{1}{c^2} \mathbf{v} \times \mathbf{E} \]
Moving omega media

\[ \mathbf{E}(\mathbf{r}) = F(\mathbf{r}, \omega) \mathbf{E}_0(\mathbf{r}) + \sqrt{\frac{\mu_0}{\varepsilon_0}} A(\mathbf{r}, \omega) \mathbf{H}_0(\mathbf{r}) \]

\[ \mathbf{H}(\mathbf{r}) = G(\mathbf{r}, \omega) \mathbf{H}_0(\mathbf{r}) + \sqrt{\frac{\varepsilon_0}{\mu_0}} C(\mathbf{r}, \omega) \mathbf{E}_0(\mathbf{r}) \]

- \( \nabla F / F \) is imaginary, e.g., \( F(z) = e^{-j \alpha z} \) with a real parameter \( \alpha \). In this case the effective “velocity” is real. This metamaterial simulates moving media. Note that the required material relations have the form of the relations for slowly moving media even if the equivalent velocity is not small.

- \( F(z) \) is a purely real function, e.g., \( F(z) = e^{-\alpha z} \), where \( \alpha \) is real. In this case the effective “velocity” of the medium is imaginary. Note that the material relations of a medium traveling faster than light formally have an imaginary vector coefficient in the second term of (40). However, one cannot say that this metamaterial simulates media moving faster than light, because in that case also the effective permittivity and permeability would be imaginary quantities.
Transmission through a slab of FT-metamaterial

\[ E(r, \omega) = F(r, \omega)E_0(r) + \sqrt{\frac{\mu_0}{\varepsilon_0}} A(r, \omega)H_0(r) \]

\[ H(r, \omega) = G(r, \omega)H_0(r) + \sqrt{\frac{\varepsilon_0}{\mu_0}} C(r, \omega)E_0(r) \]

\[ F = G \text{ and } A = C = 0 \]

Inside the slab (normal incidence):

\[ E(z) = F(z)(ae^{-jk_0z} + be^{jk_0z}) \]

Transmission coefficient depends only on the interface values of the transforming function:

\[ T = \frac{F(d)}{F(0)} e^{-jk_0d} \]
Thank you!

- We have only “scratched the surface” of the new possibilities…
Conclusions - The goals of the project were met

1. Coordinate-transformation cloak based on chiral inclusions
2. Cloak based on networks of transmission lines
3. Investigation of the potentials for realization of the cloaks studied in items 1 and 2 in the optical frequency range
4. Comparison of the two approaches in items 1 and 2 with other approaches for cloak design
5. Study of fundamental limitations of different types of cloaks (mainly the cloak types studied in items 1 and 2)
6. The ACT provided some examples of space applications, where the feasibility of the cloaks studied in items 1 and/or 2 were investigated
7. A generalized form of field-transforming metamaterials has been developed
Conclusions - Publications resulting from the project


