Reverse engineering
in metamaterial based electromagnetic cloak

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The authors would like to thank the others members of DOME group :
N. Fabre, G. Houzet, X. Mélique, E. Lheurette, V. Sadaune, O. Vanbésien, Alejandro Lucas and F-L. Zhang.

Potential applications of metamaterial technologies)

Focusing

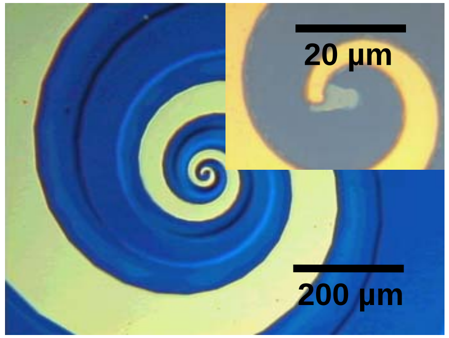
Superlens (near-field conditions)
Hyperlens (far-field condition)

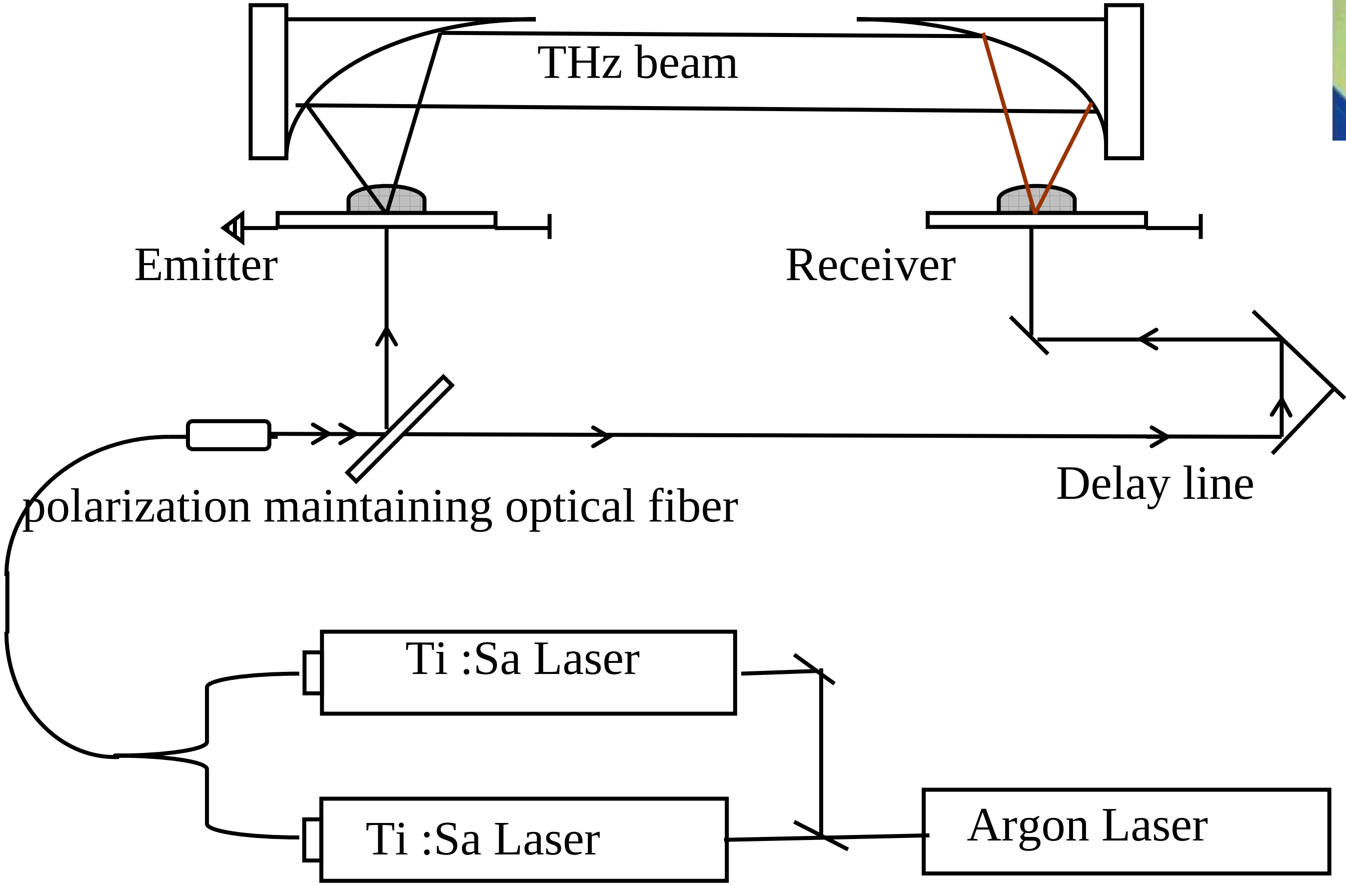
Antennas

AMC (Artificial magnetic conductors)
Partially reflected superstrates
Near-zero media

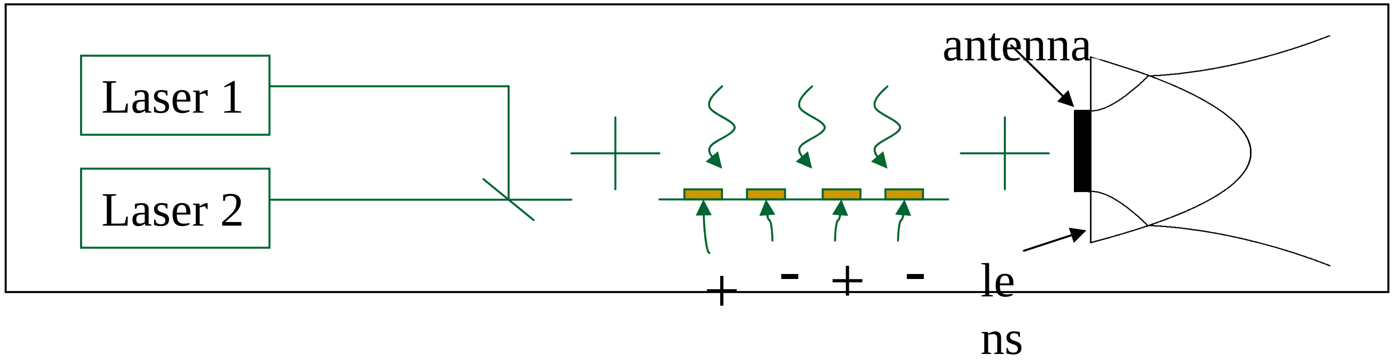
Non linear Terahertz Electronics

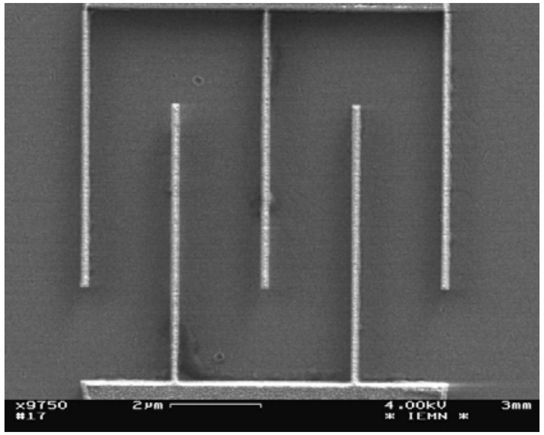
THz imaging and spectroscopy systems (homodyne system)

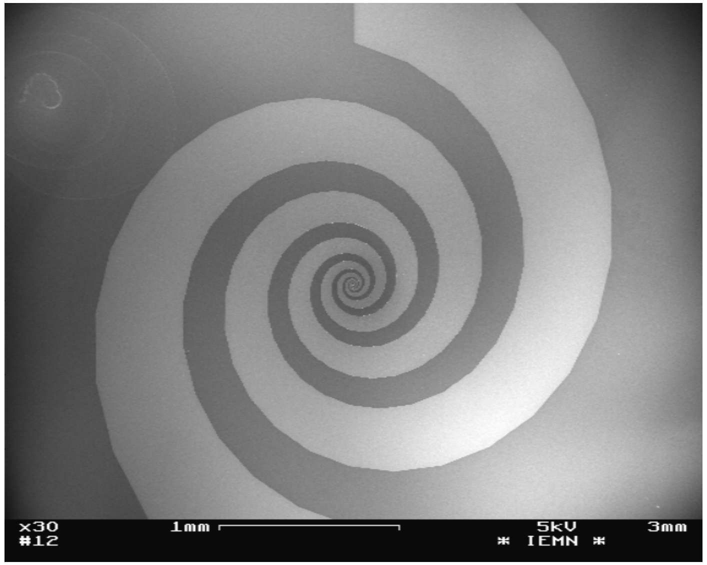




THz imaging and spectroscopy systems (homodyne system)







Transformations optics

Cloaking

Linear and non linear transformations

Reduced equations

Lensing

Diverging- converging systems

Channeling (near field)–Collimating (far field)

Outline

- Cloaking (Davy Gaillot - Jose Llorens Montolio)
  - Electric Cloak with metallic nanowires at optical wavelengths
    - Homogenization
      - Bruggeman and MG's approaches
    - Study of lossless and lossy cloaks
  - Magnetic Cloak with high-κ ceramics at THz frequencies
    - Basic principle of Magnetic Mie resonance
    - Cloaking performance and robustness
      - Wavelength-scaled cloak
      - Large cloak

- Transformation Optics (C. Croënne)
  - Channeling
  - High Resolution (HR) flat hyper lens
  - HR focusing devices

Studied Electromagnetic Cloaking Devices

GHz-THz Frequencies

Magnetic Cloak

$$0 < \mu_r < 1$$

IEMN

Electric Cloak

$$0 < \varepsilon_r < 1$$

IEMN + ACT

Optical Frequencies

High-κ Dielectrics

Zhao *et al.*, Appl. Phys. Lett., (2007)
O'Brien *et al.*, JPM (2002)

$$\varepsilon_z = \left(\frac{b}{b-a}\right)^2$$

$$\mu_r = \left(\frac{r-a}{r}\right)^2$$

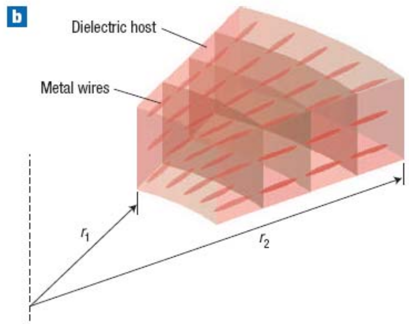
$$\mu_\theta = 1$$

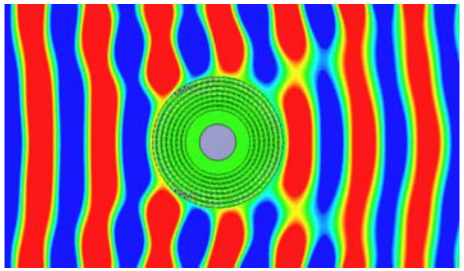
$$\mu_z = 1$$

$$\varepsilon_\theta = \left(\frac{b}{b-a}\right)^2$$

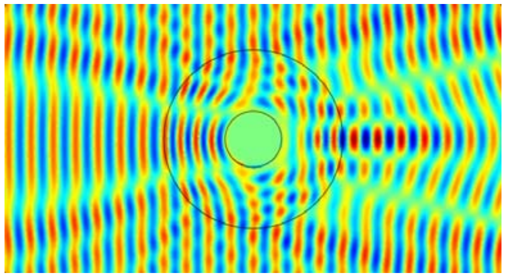
$$\varepsilon_r = \left(\frac{b}{b-a}\right)^2 \cdot \left(\frac{r-a}{r}\right)^2$$

Metallic Nanowires



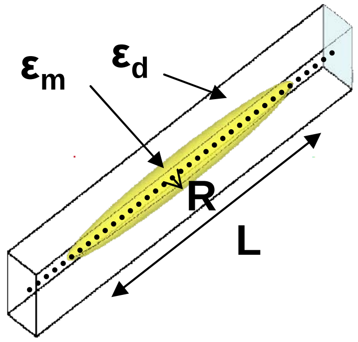


Gaillot *et al.,* Opt. Exp. (2008)



Cai *et al.,* Nat. Photon. (2007)

Electric Cloak : Homogenization of Particles



Aspect Ratio
α = L / R

Bruggeman's Formula

$$f\frac{\varepsilon_{\rm m}-\varepsilon_{\rm eff}}{\varepsilon_{\rm m}+\kappa\varepsilon_{\rm eff}}+(1-f)\frac{\varepsilon_{\rm d}-\varepsilon_{\rm eff}}{\varepsilon_{\rm d}+\kappa\varepsilon_{\rm eff}}=0$$

- Geometry imposed by the host dielectric medium properties
- Cai's solving approach is based on the fact that the filling fraction is a function of $f_a.(a/r)$
  - f_a is filling fraction of metal at the inner surface of the cloak

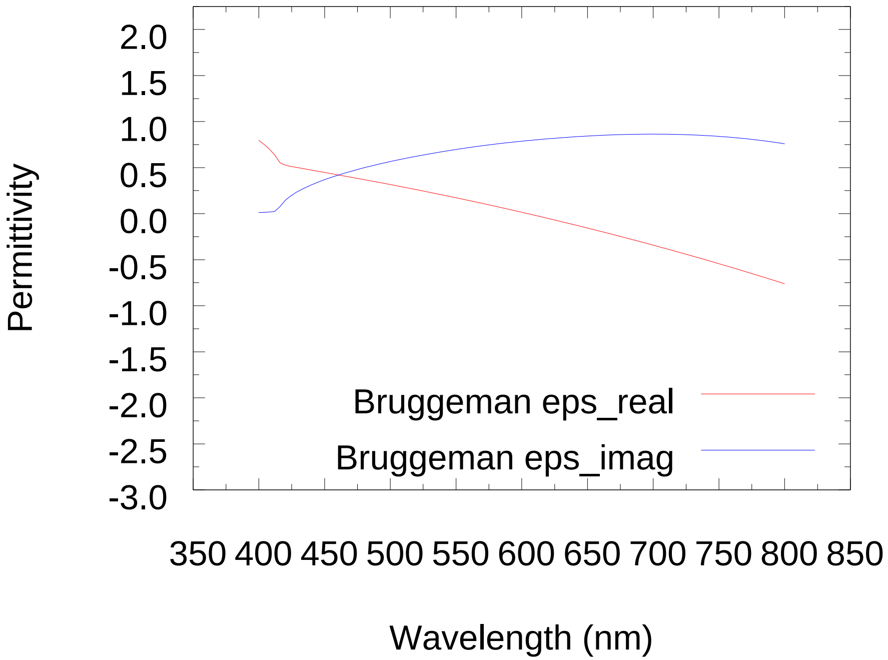
Maxwell-Garnett's Formula

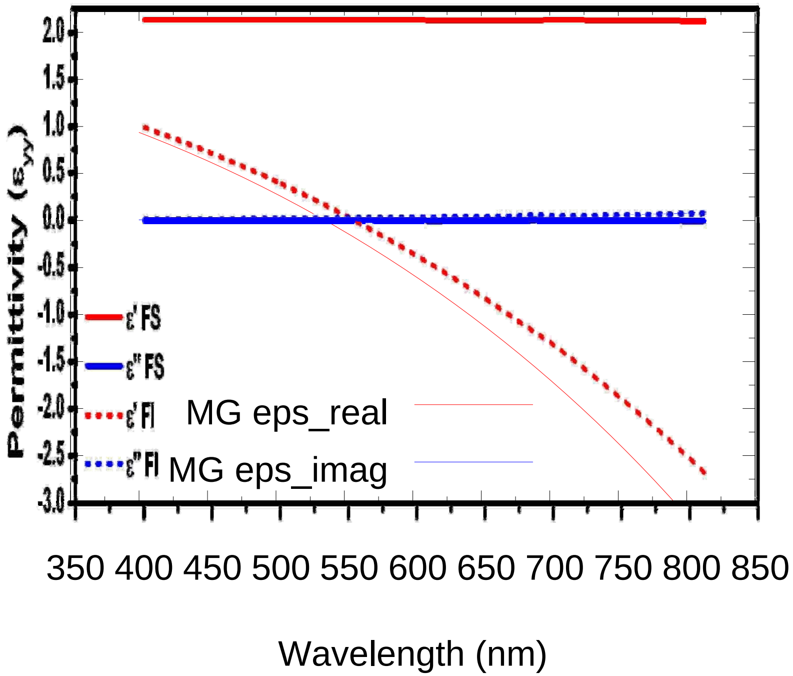
$$\frac{\varepsilon_{{\rm eff},r}-\varepsilon_{\rm d}}{\varepsilon_{{\rm eff},r}+\kappa\varepsilon_{\rm d}}=f\frac{\varepsilon_{\rm m}-\varepsilon_{\rm d}}{\varepsilon_{\rm m}+\kappa\varepsilon_{\rm d}}$$

- Optimization problem with parameters vector $x=\{\varepsilon_{\rm d},\varepsilon_{\rm m},a,b,\alpha,\lambda\}$
- One can only match one of the two parameters
  - ***Mismatch of $\varepsilon_{\Phi}(r)$***

Homogenization formula vs. numerical FS and FI

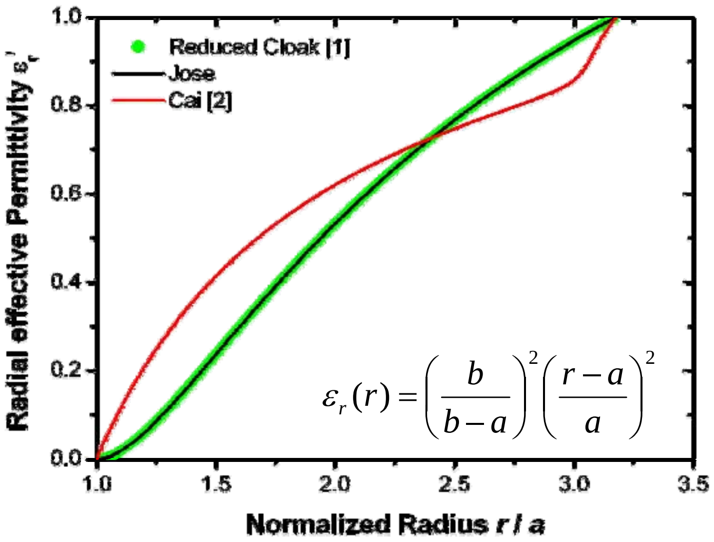
Computation of both analytical formulas for $\alpha = 11.83$ and $f = 0.1175$



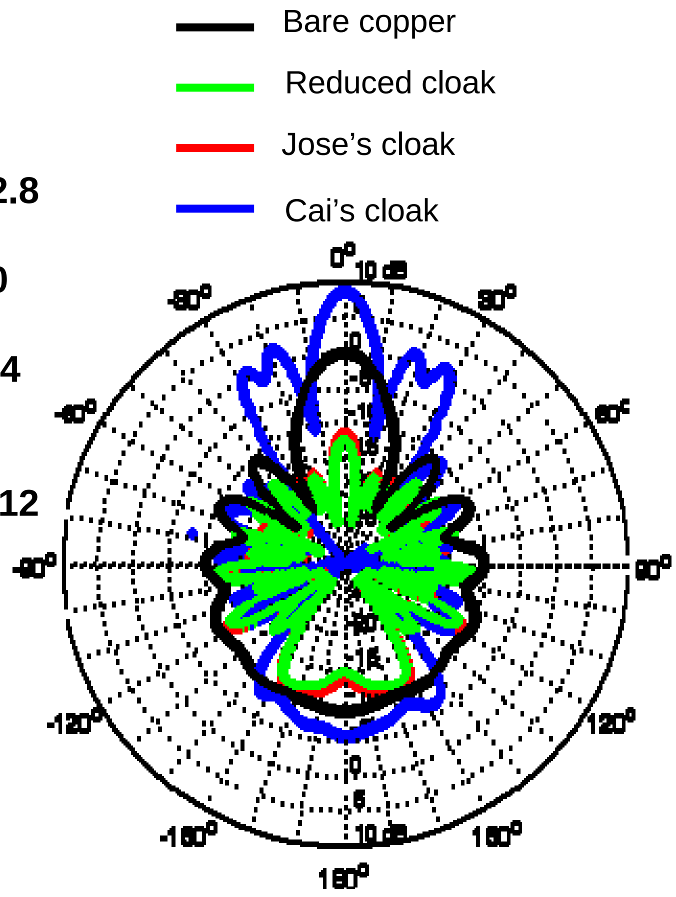


- Bruggeman formula shows a very poor agreement with FI results. Seems that the explanation of the high losses is due to the wrong use of Bruggeman formula.
- The Maxwell-Garnett formula exhibits an excellent agreement.

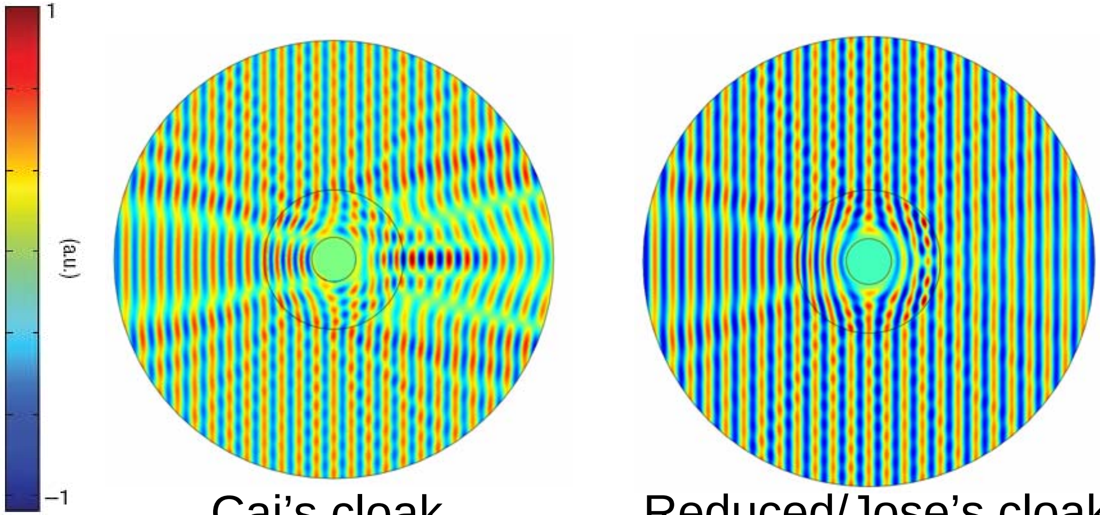
Bruggeman Homogeneous Lossless Model



λ = 632.8 nm
a = 800 nm
b = 2.54 µm
a / b =0.31512

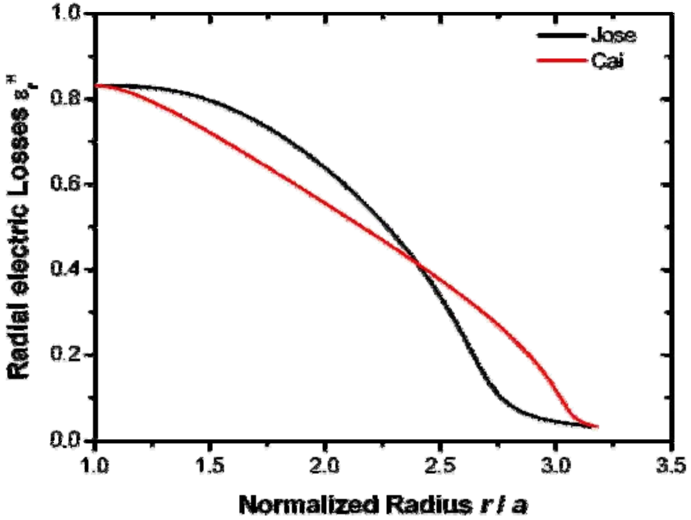


Scattering Pattern



Cai's cloak   Reduced/Jose's cloak

Bruggeman Homogeneous Lossy Model



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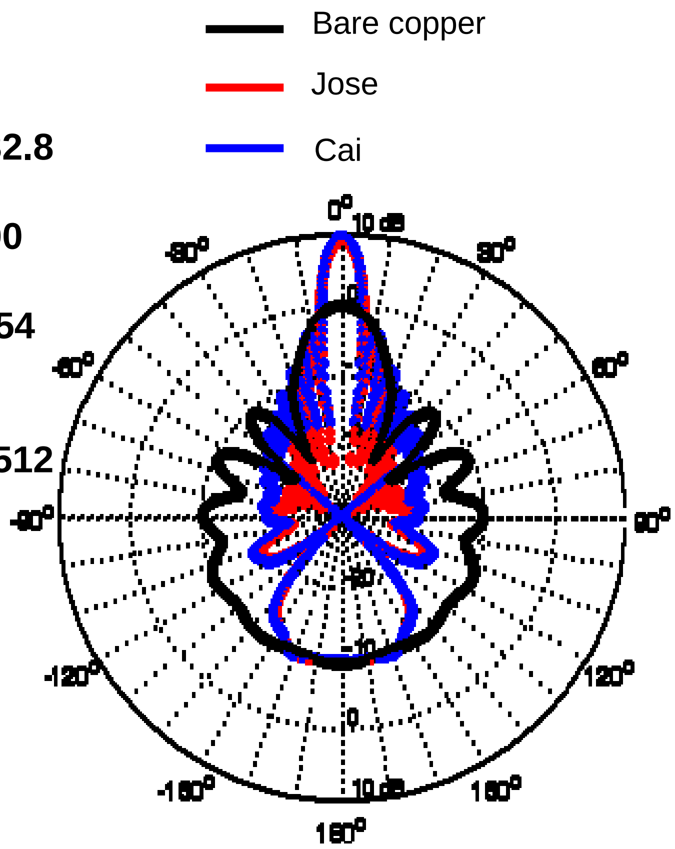

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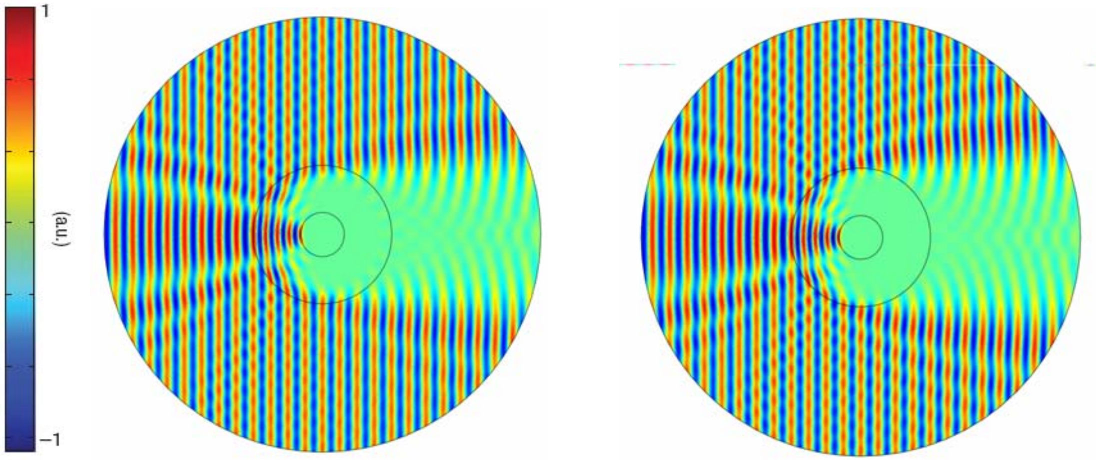
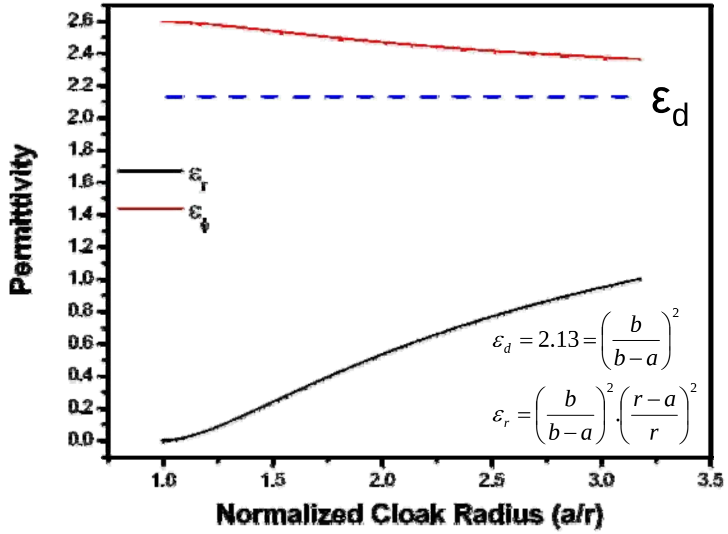
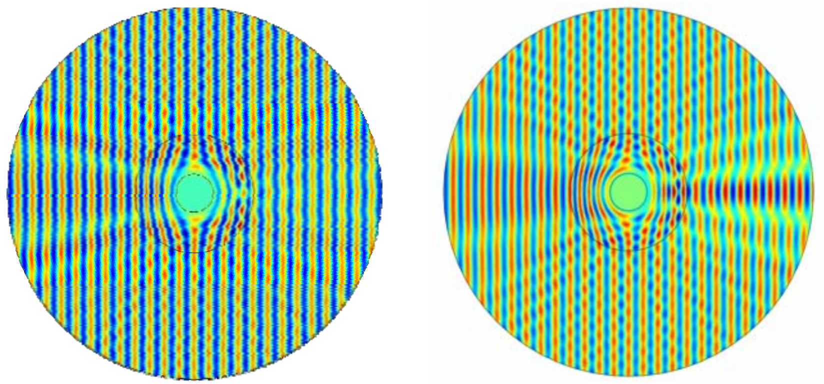


Reduced/Jose's cloak

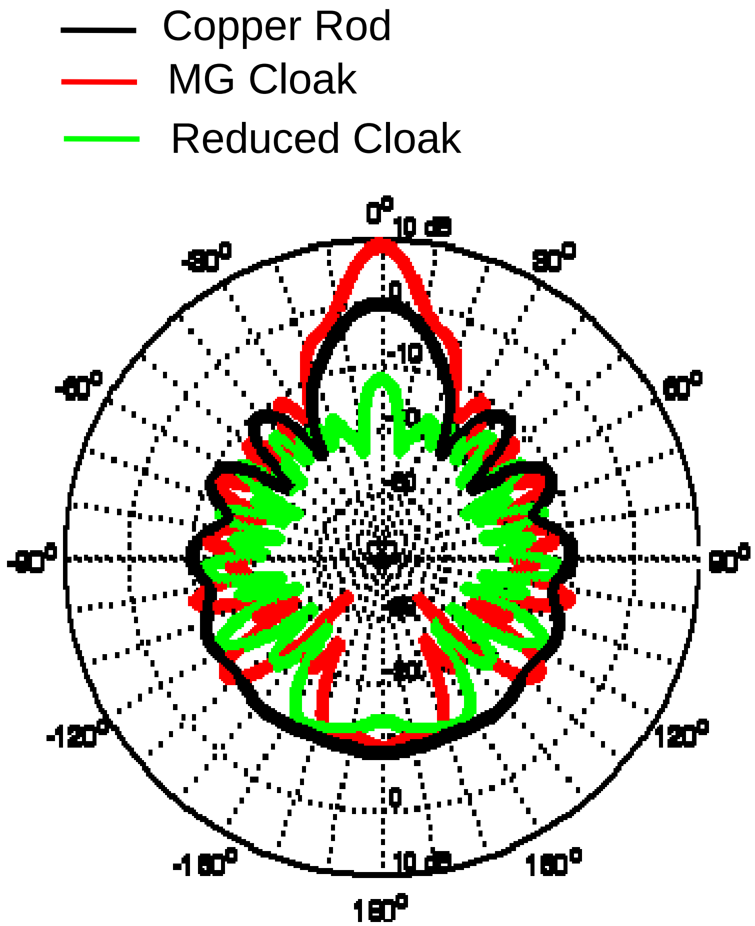
Cai's cloak

MG Homogeneous Lossless Cloak



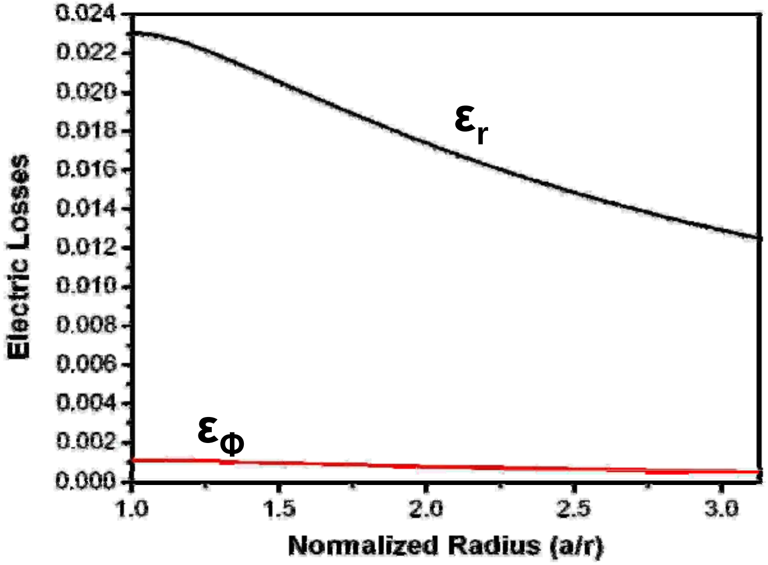


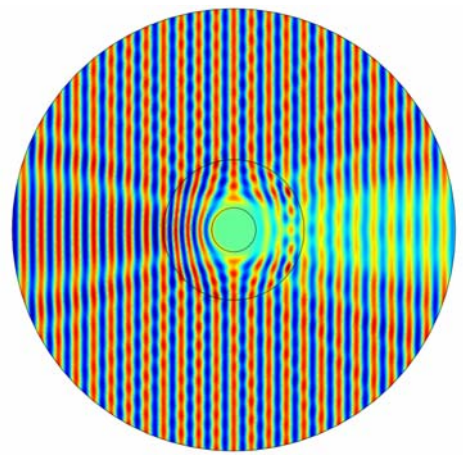
Reduced MG



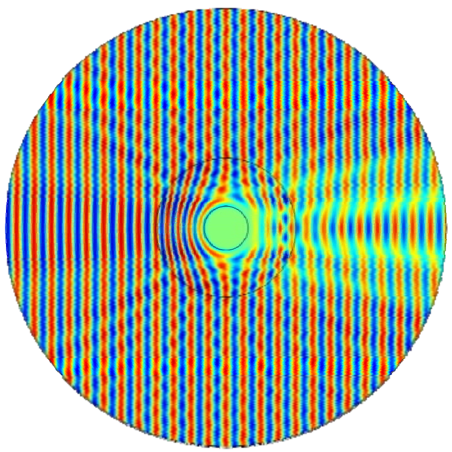
- Mismatched MG cloak does not perform well due to the mismatch of $\varepsilon_\Phi(r)$ with the reduced eqns.

MG Homogeneous Lossy Cloak

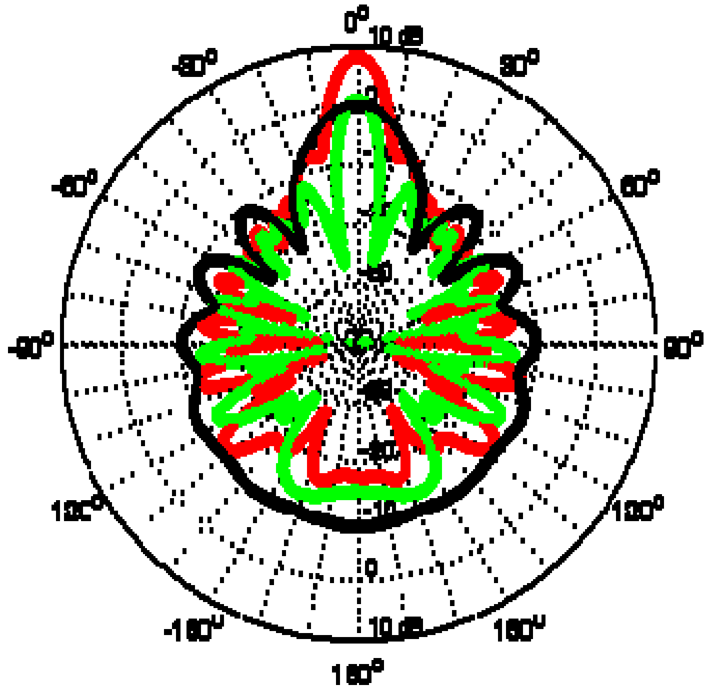




Reduced

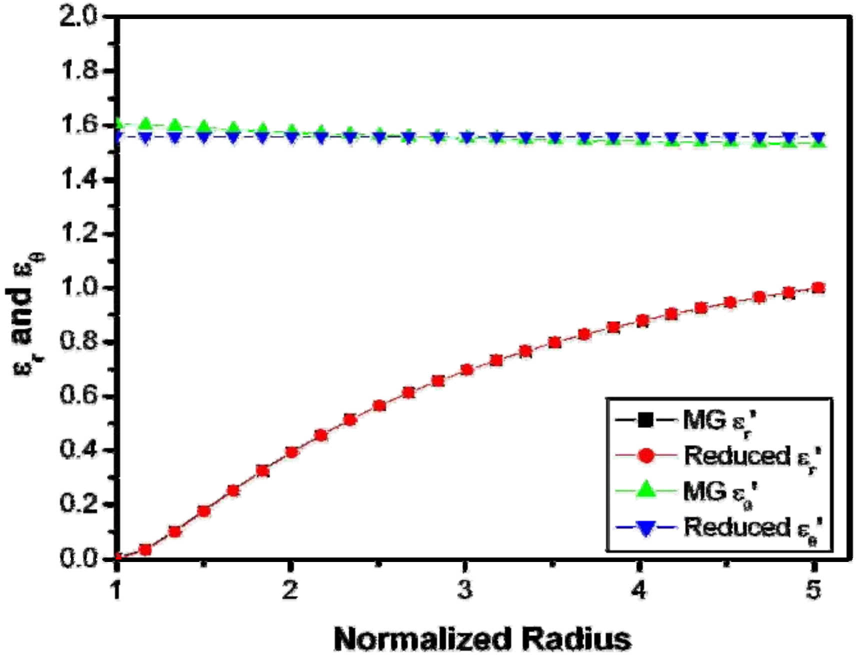


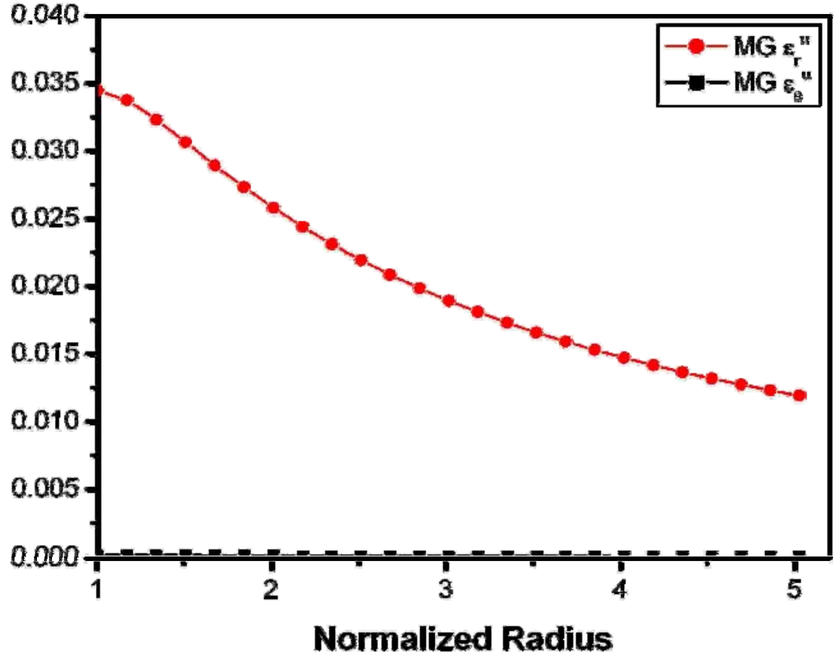
MG Cloak



Copper Rod
MG Cloak
Reduced Cloak

Optimized MG Cloak





- MG Data obtained from local optimization routine

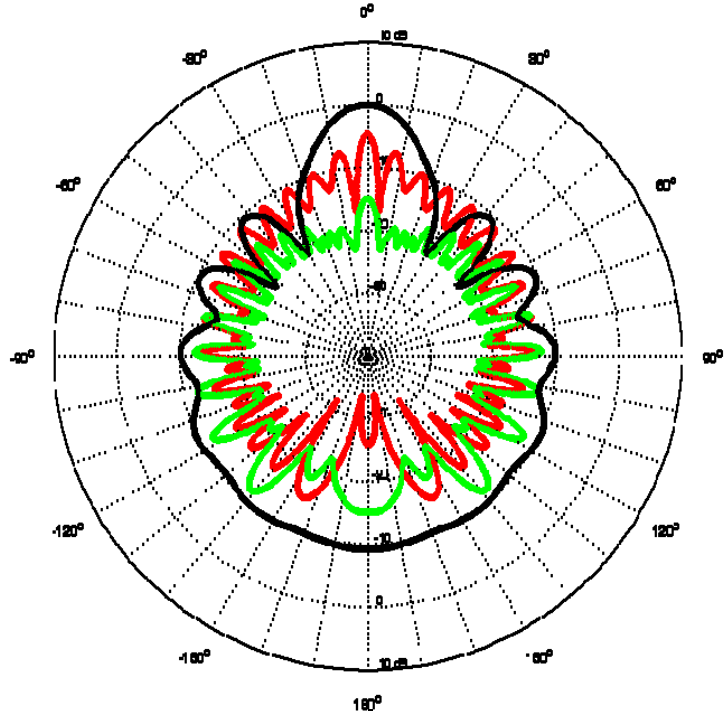
$$J = \left(\mathrm{Re}\left(\varepsilon_{\mathrm{eff},r}(r)\right) - \varepsilon_{\mathrm{theo},r}(r)\right)^2 + \left(\mathrm{Re}\left(\varepsilon_{\mathrm{eff},\theta}(r)\right) - \varepsilon_{\mathrm{theo},\theta}(r)\right)^2 + \mathrm{Im}\left(\varepsilon_{\mathrm{eff},r}(r)\right)^2 + \mathrm{Im}\left(\varepsilon_{\mathrm{eff},\theta}(r)\right)^2$$

- ε_d = 1.5 and α = 5.954009
- a = 1 and b = 5.022861 (a/b = 0.199)

Optimized Lossless MG Cloak

MG Cloak

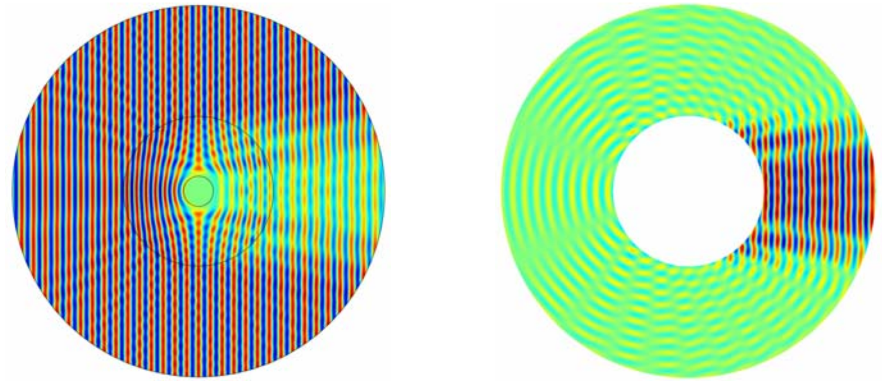
Reduced Cloak



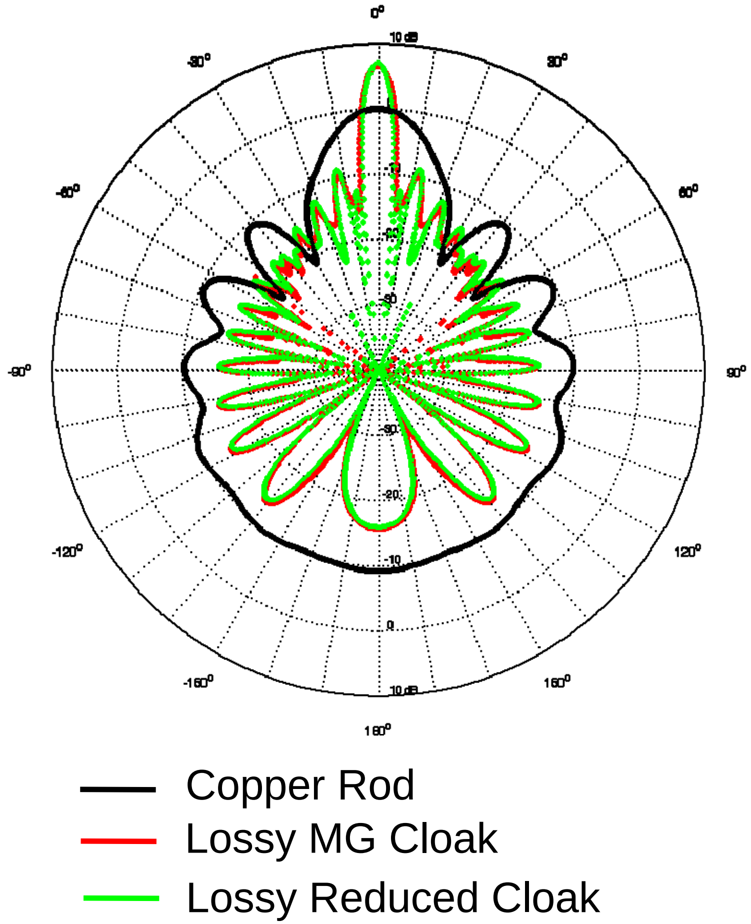
- Optimized cloak operates far better than original counterpart.
- Displays larger forward scattering but reduced backward scattering

Optimized Lossy MG Cloak

MG Cloak

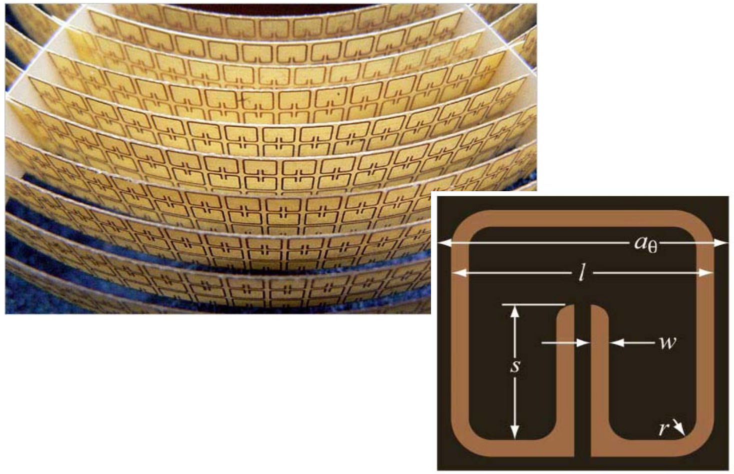


Reduced Cloak



- Reduced cloak losses calculated as the mean average from optimized data
- Field maps and radiation patterns are very similar indicating that losses dominate behavior

Magnetic Cloaking Devices Based on high-κ Dielectrics



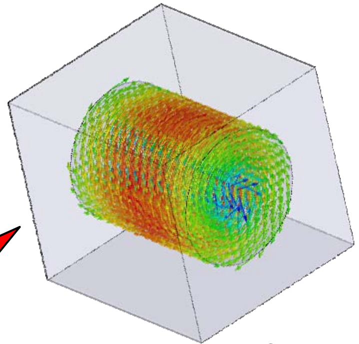
Schurig *et al.,* Science (2006)

- Need for a non-metallic metamaterial particle that operates up to THz frequencies
  - Strong magnetic response with adjustable magnetic plasma frequency
  - Manageable losses
  - Ease of assembly for cloaking or other purposes
  - Flexible fabrication for complex geometrical shape

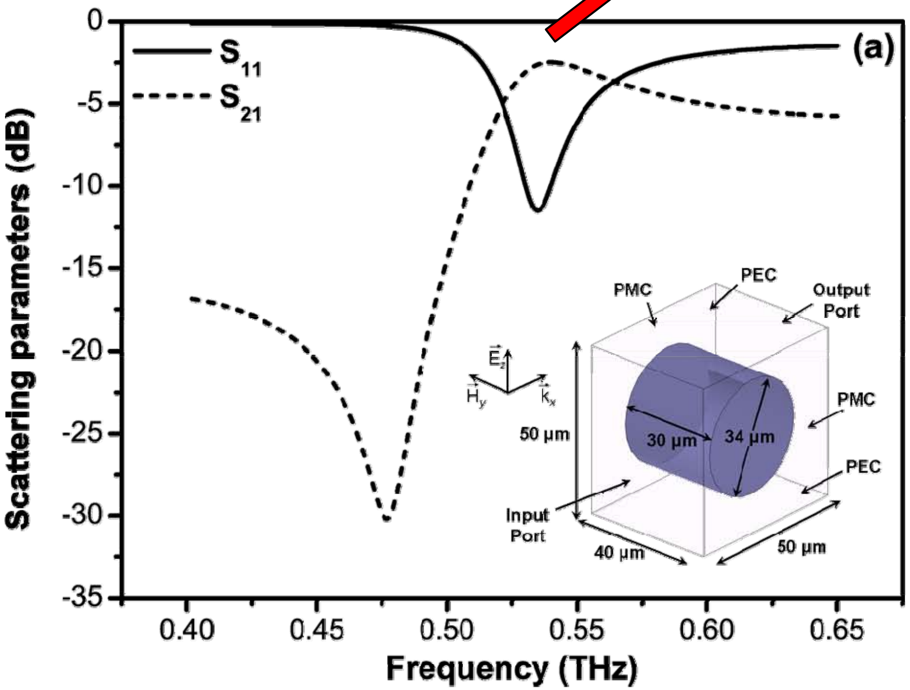
Magnetic Mie Resonances in Ferroelectrics at THz frequencies

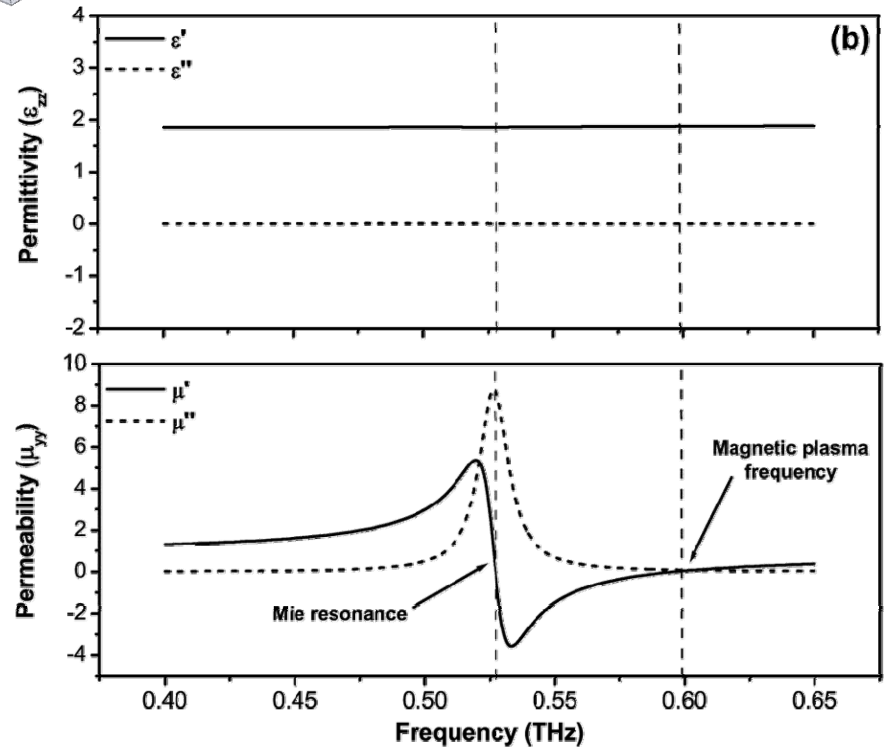
Example : $Ba_xSr_{1-x}TiO_3$ (BST)

ε = 200 + 5*j

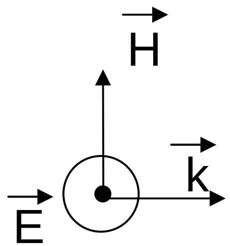


E-field map at Mie resonance





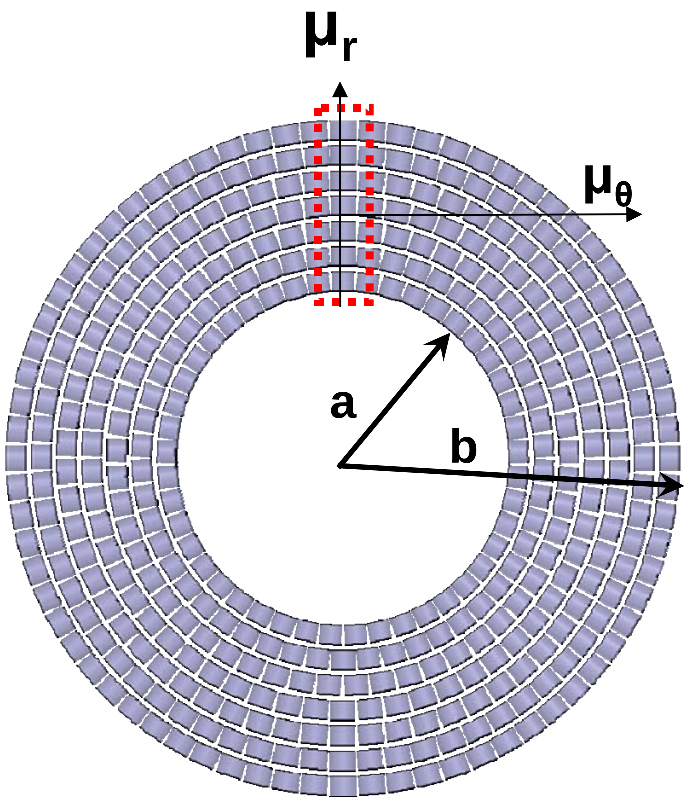
Wavelength-scaled Cloak Design using BST rods



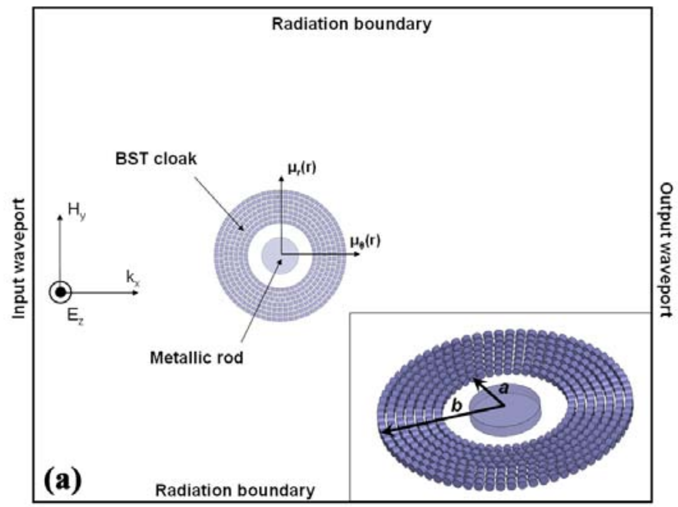
a = 280 µm

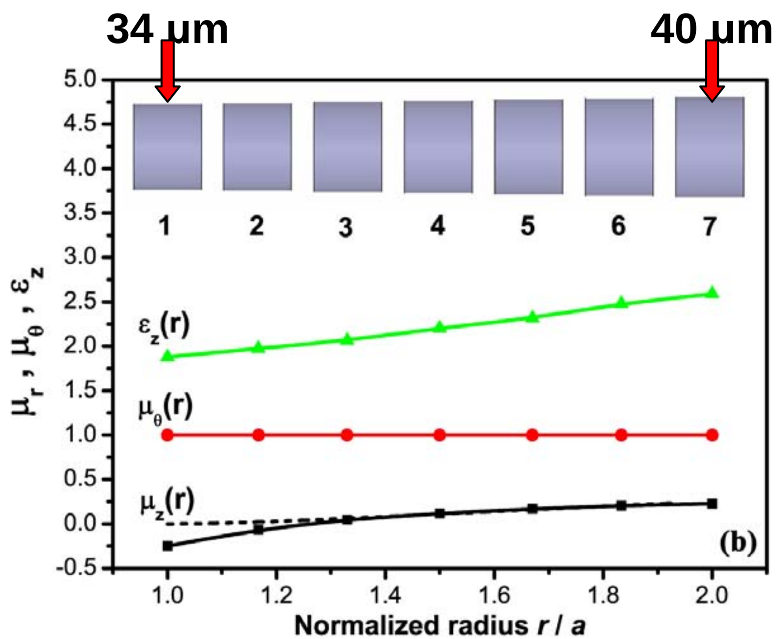
b = 560 µm

a / b = 0.5

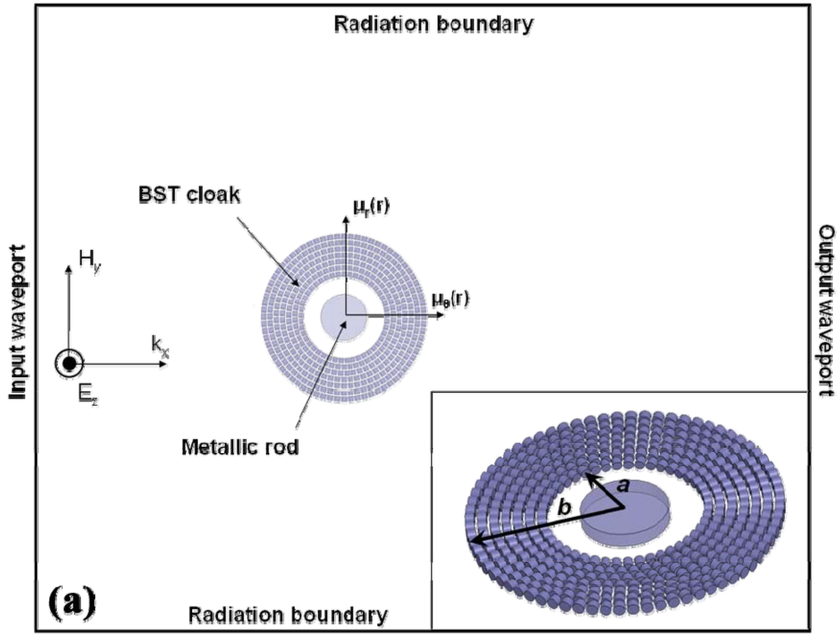


Top View of BST cloak





3D Full Wave Simulations of the BST cloak at 0.58 THz



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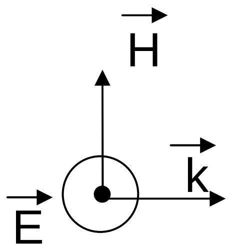
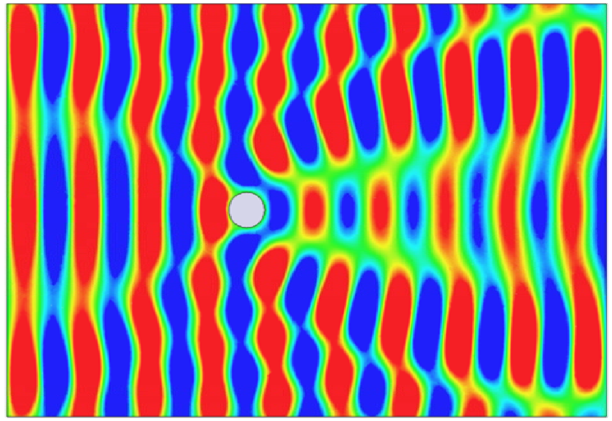
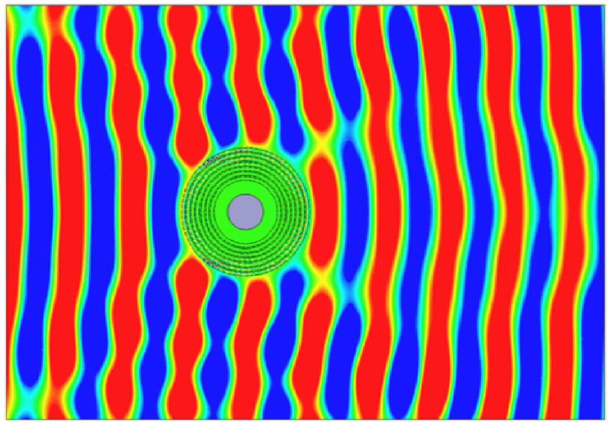


Total H_z field



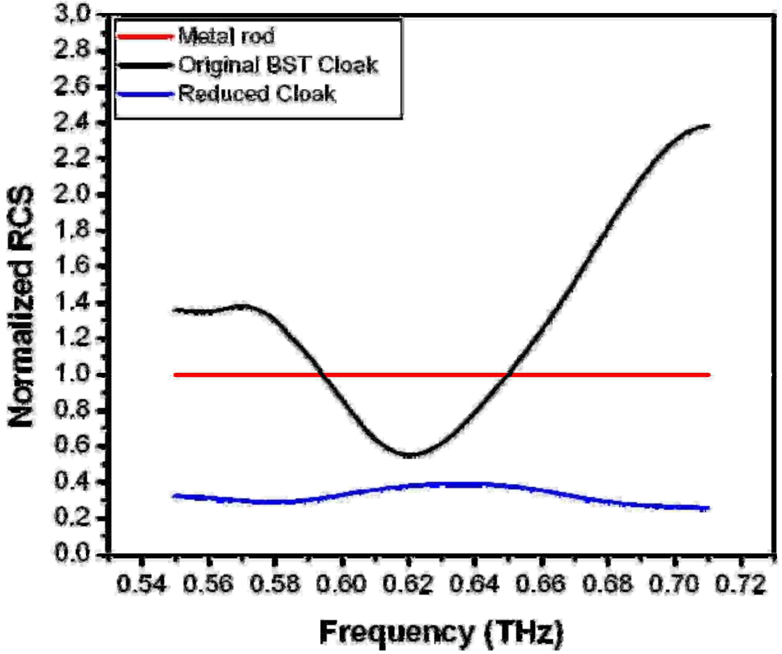
(a)

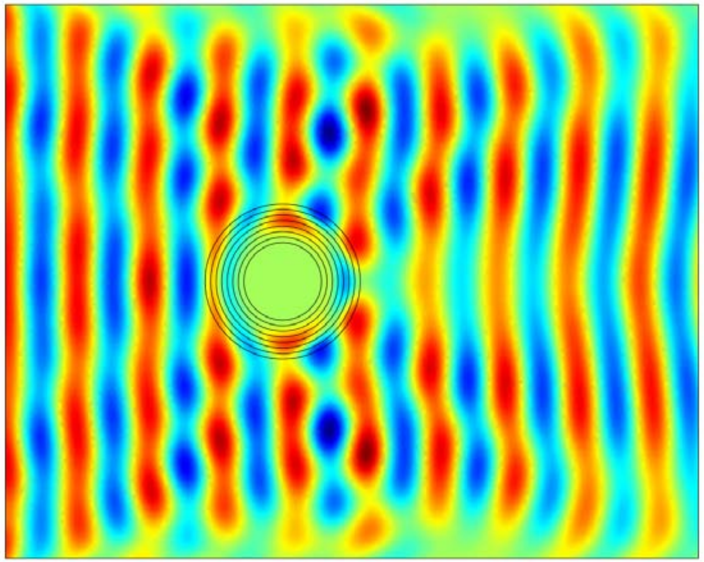


(b)

Gaillot *et al.*, Opt. Exp., (2008)

2D Computations w/ Full Dispersive Parameters : Frequency Robustness

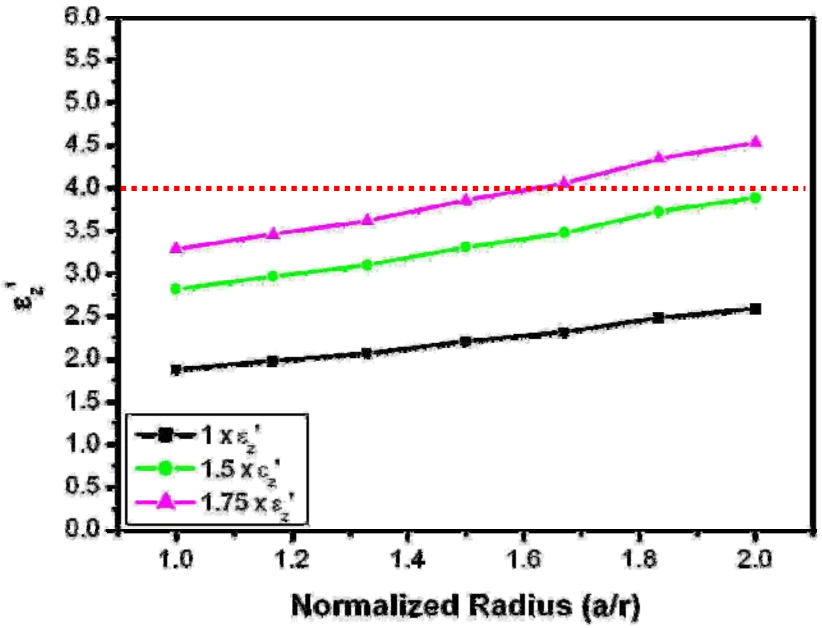


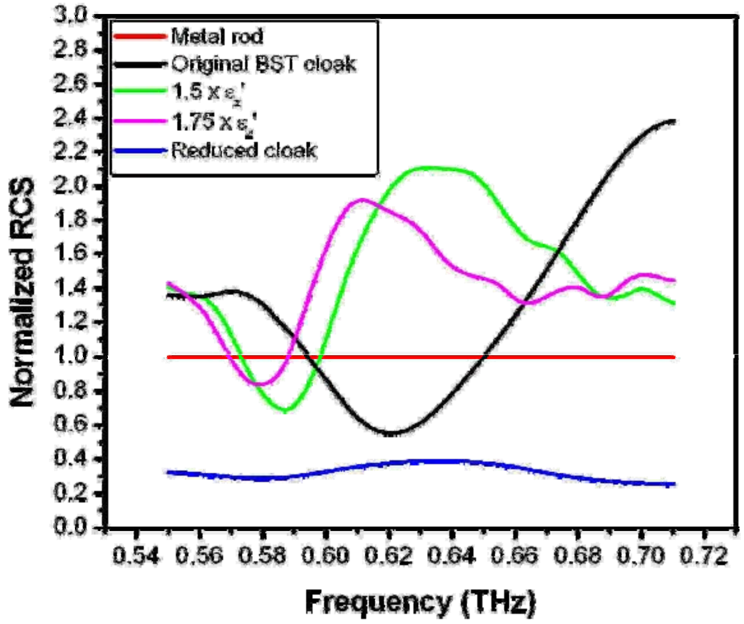


E-field Map at cloaking frequency

- Full lossy dispersive effective parameters incorporated into the 2D FEM model
- Broad cloaking range although phase front reconstruction achieved at single frequency point
  - Mixed lossy and cloaking regime

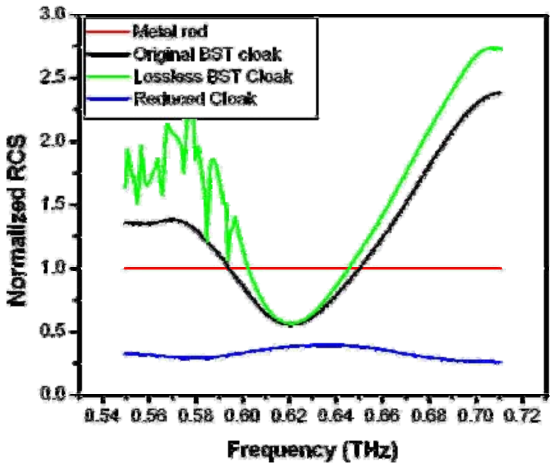
Effect of permittivity mismatch to RCS



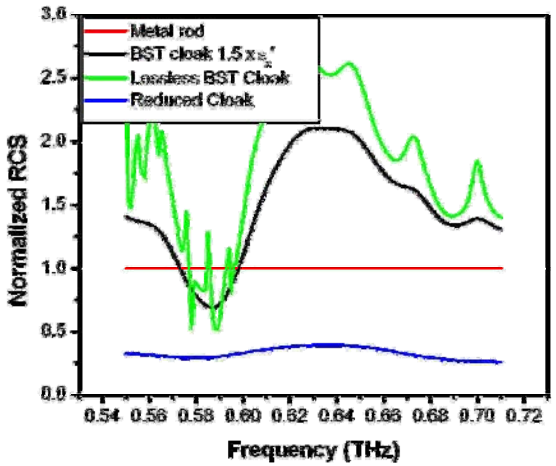


- Original cloak presents mismatch in radial permittivity which is attributed to the low performance of the cloak
- This mismatch is artificially alleviated by shifting permittivity data
  - Allows impedance matching at outer interface of the cloak
- Reduction of the cloaking bandwidth around the expected cloaking frequency

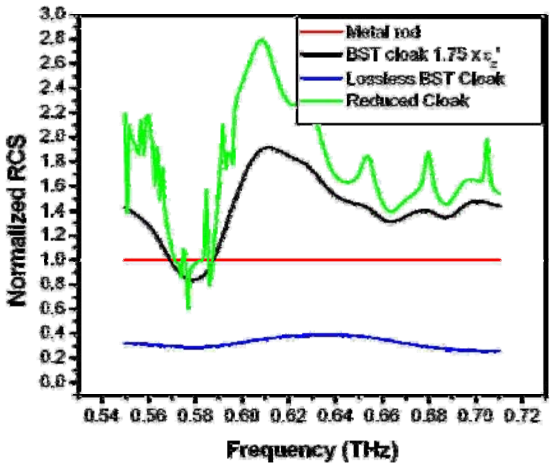
Comparison Lossless Vs. Lossy cloak



$\langle E_z \rangle \sim 2.2$



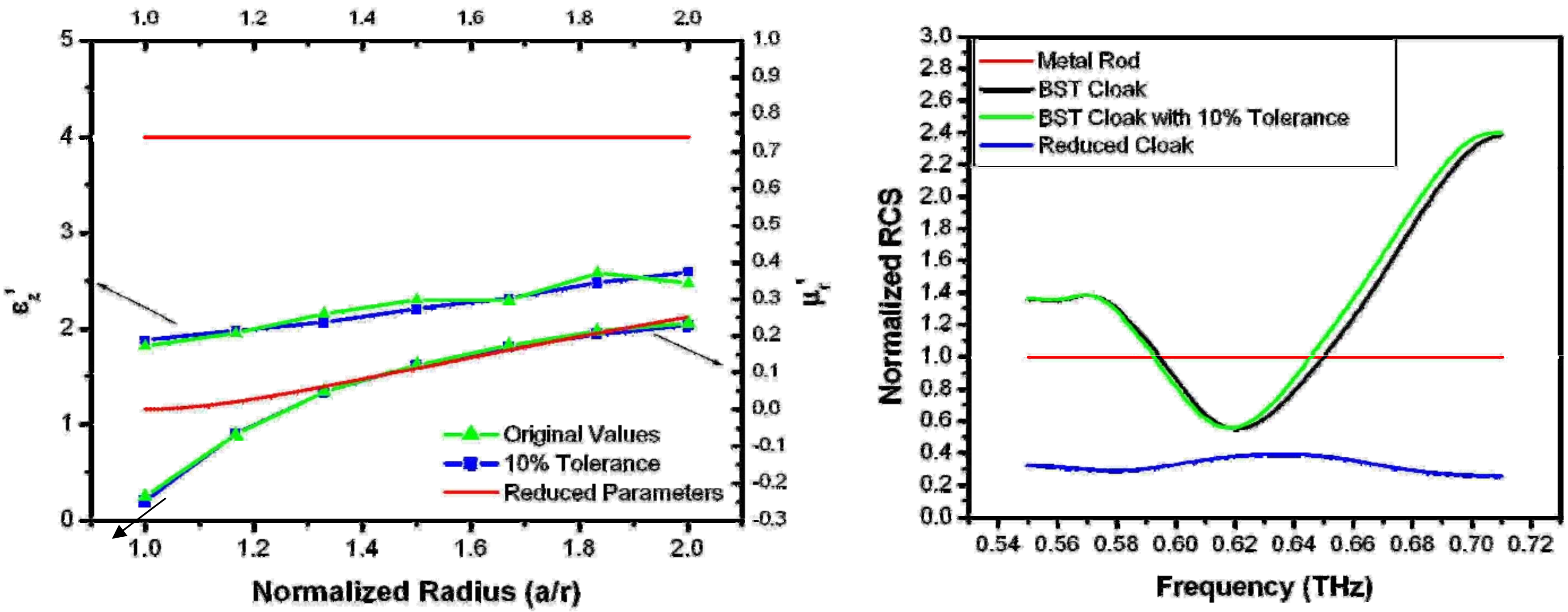
$\langle E_z \rangle \sim 3.3$



$\langle E_z \rangle \sim 4$

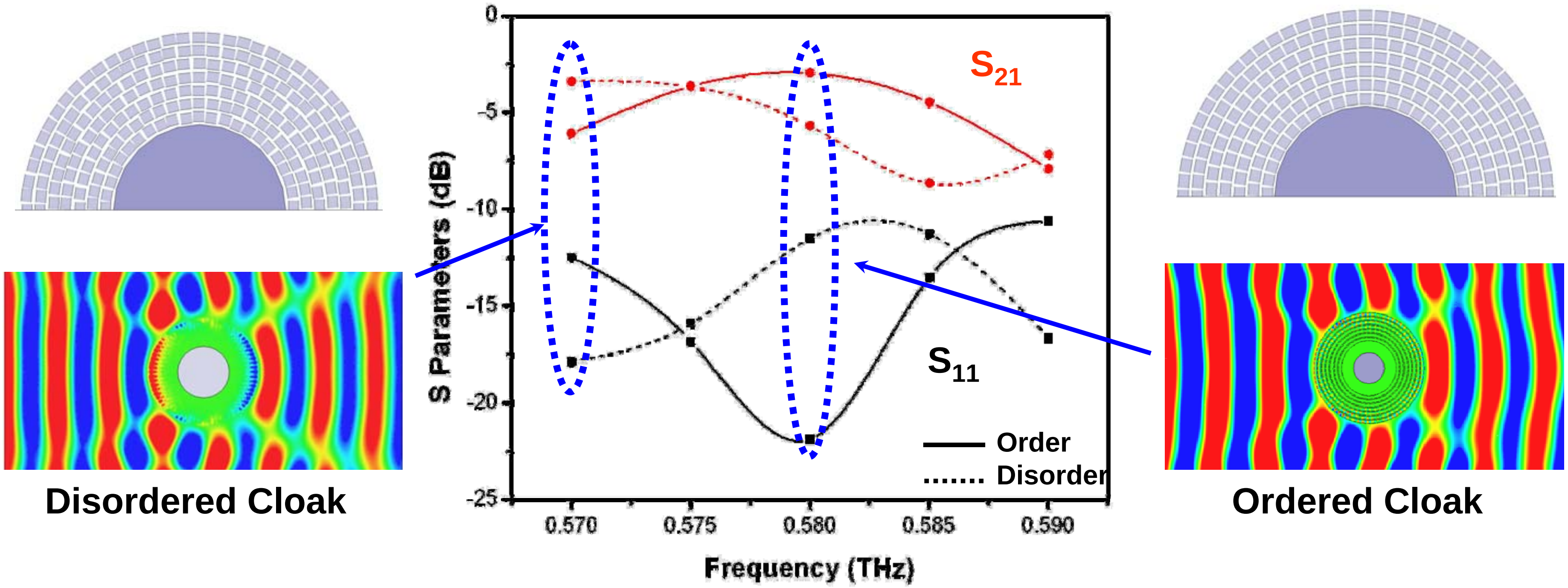
- Losses removed to quantify robustness frequency dependence
- Results show that lossless cloaks exhibit narrower cloaking bandwidths
  - Losses broaden cloaking bandwidth
- Rapid oscillations at lower wavelengths due to numerical artifacts
  - Dampened by losses

Robustness Dependence to the Elements Disorder (2D FEM Solver)



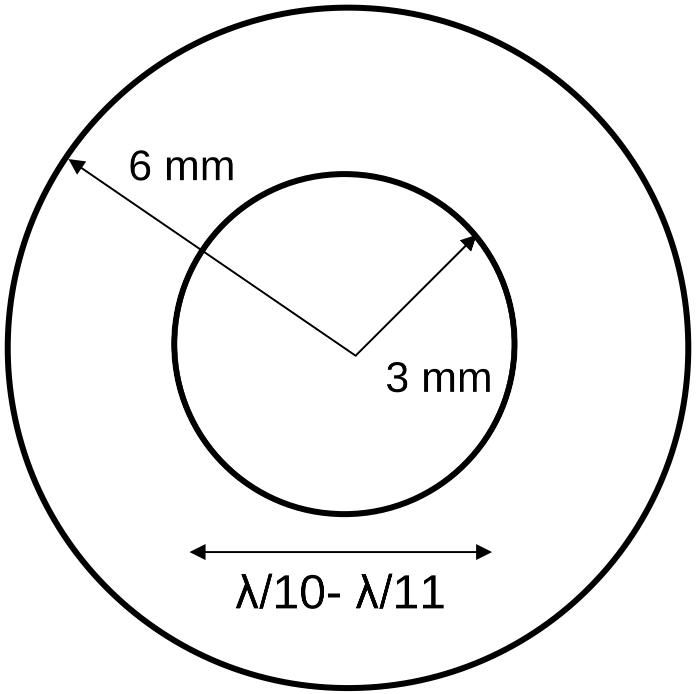
- Disorder is artificially introduced by randomizing effective parameter values with 10% tolerance
  - Simplified approach avoids homogenization of random elements
- Results indicate that the frequency robustness is not very sensitive to the randomization
  - Slight shift of the cloaking bandwidth and cloaking frequency

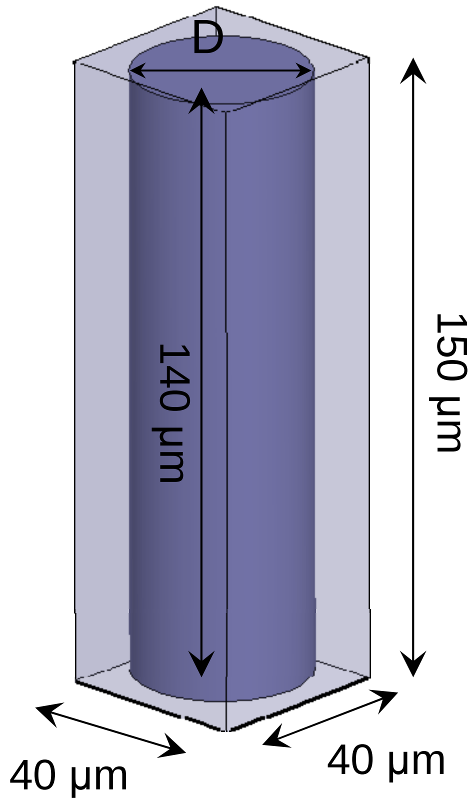
Robustness Dependence to the Elements Disorder (3D FEM Solver)



- Artificially introduced positional (within 5µm) and dimensional disorder (within 1%)
  - Avoids elements from touching each other
- Results show slight shift of the cloaking frequency from 0.58 to 0.57 THz

Design of a Larger BST Cloak



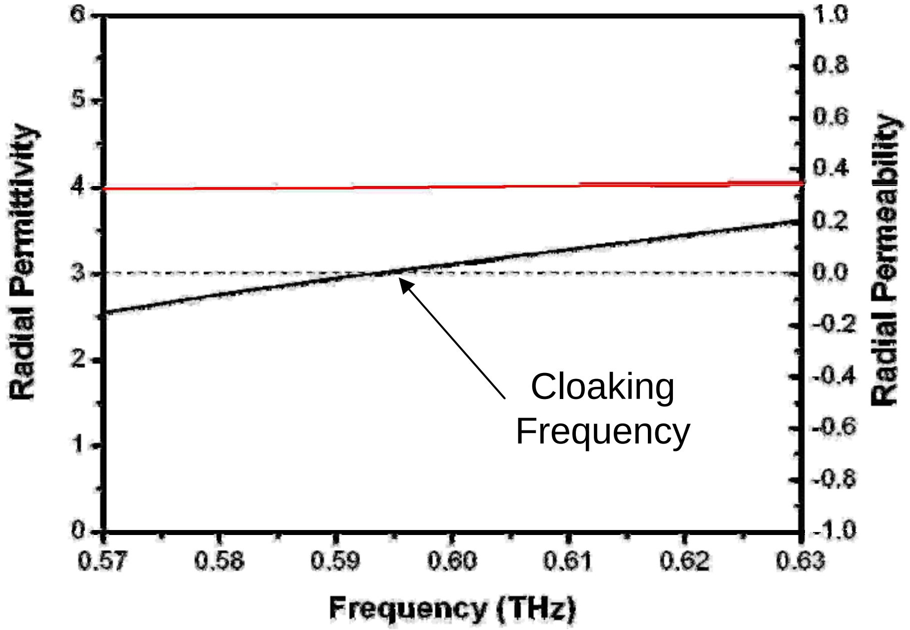


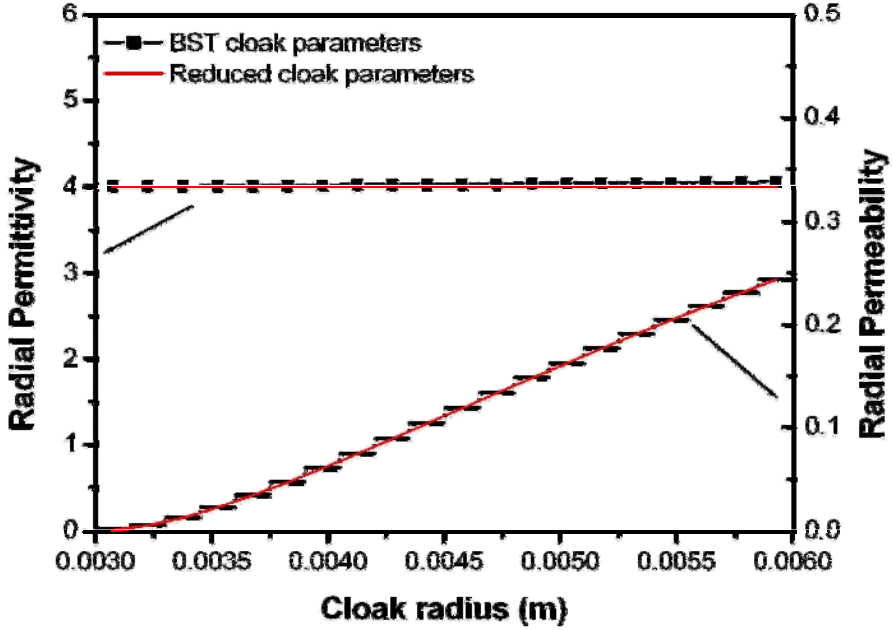
$\varepsilon_{BST} = 200 + 5*j$

- Cloaking Frequency **~550-600 GHz**
- Metallic cylinder diameter > **λ/10**
- **a / b = 0.5; a = 3mm; b = 6mm**
- **$\varepsilon_r = 4$, $\varepsilon_\theta = 1$, $\mu_r(r) = (1-a/r)^2$**
- **Cloaking body presents 20 layer-by-layer shells**

Cloak Parameters

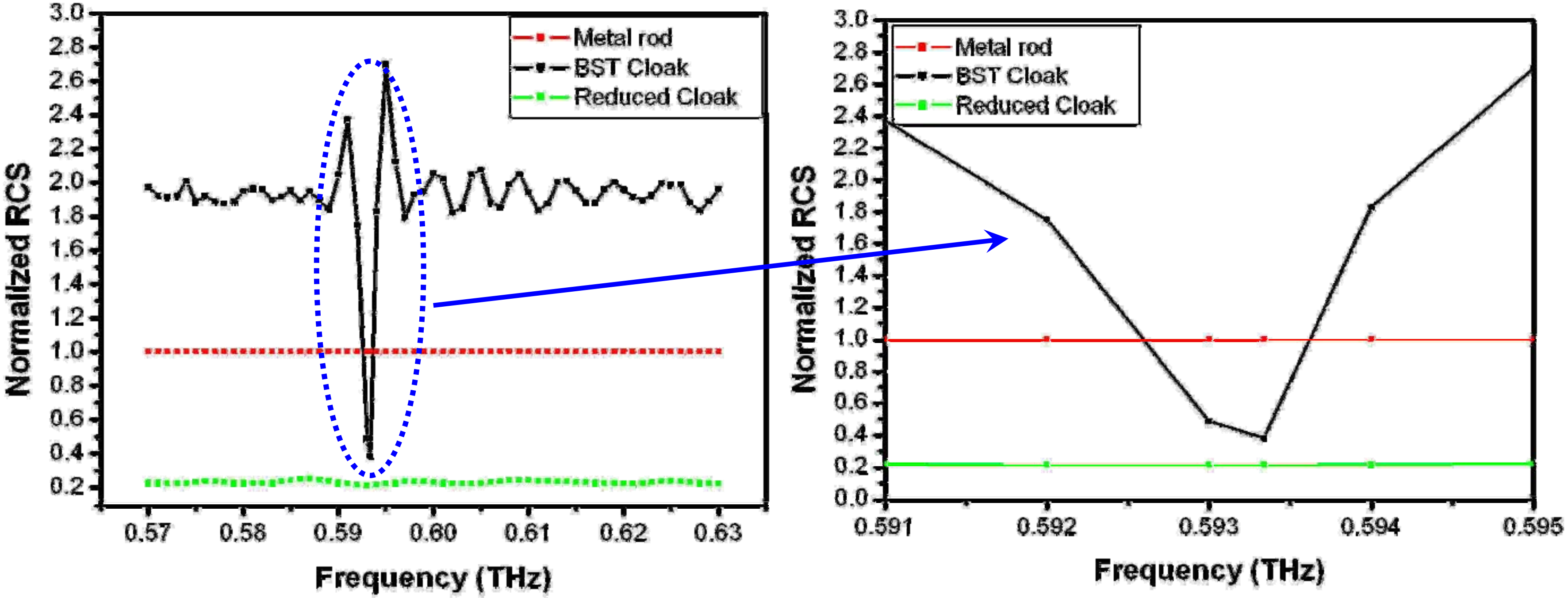
1st Shell Parameters





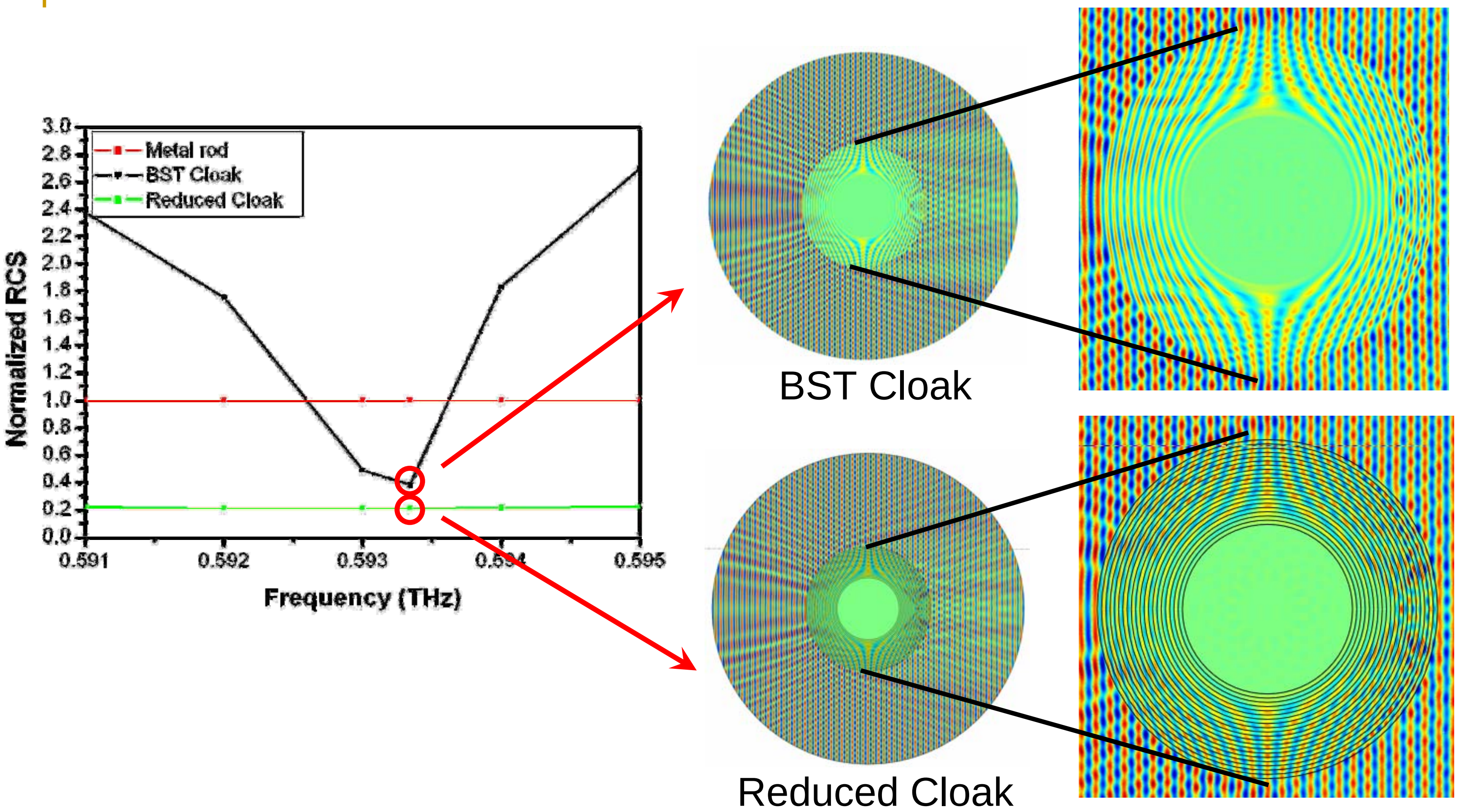
- Cloaking frequency expected at 593.4 GHz
  - μ_r (a = 3mm) = 0
- Matched fit to the reduced Eqns for optimum performance
  - Slight permittivity mismatch at outer interface

Frequency Dependence of Cloaking Performance with Lossless Parameters



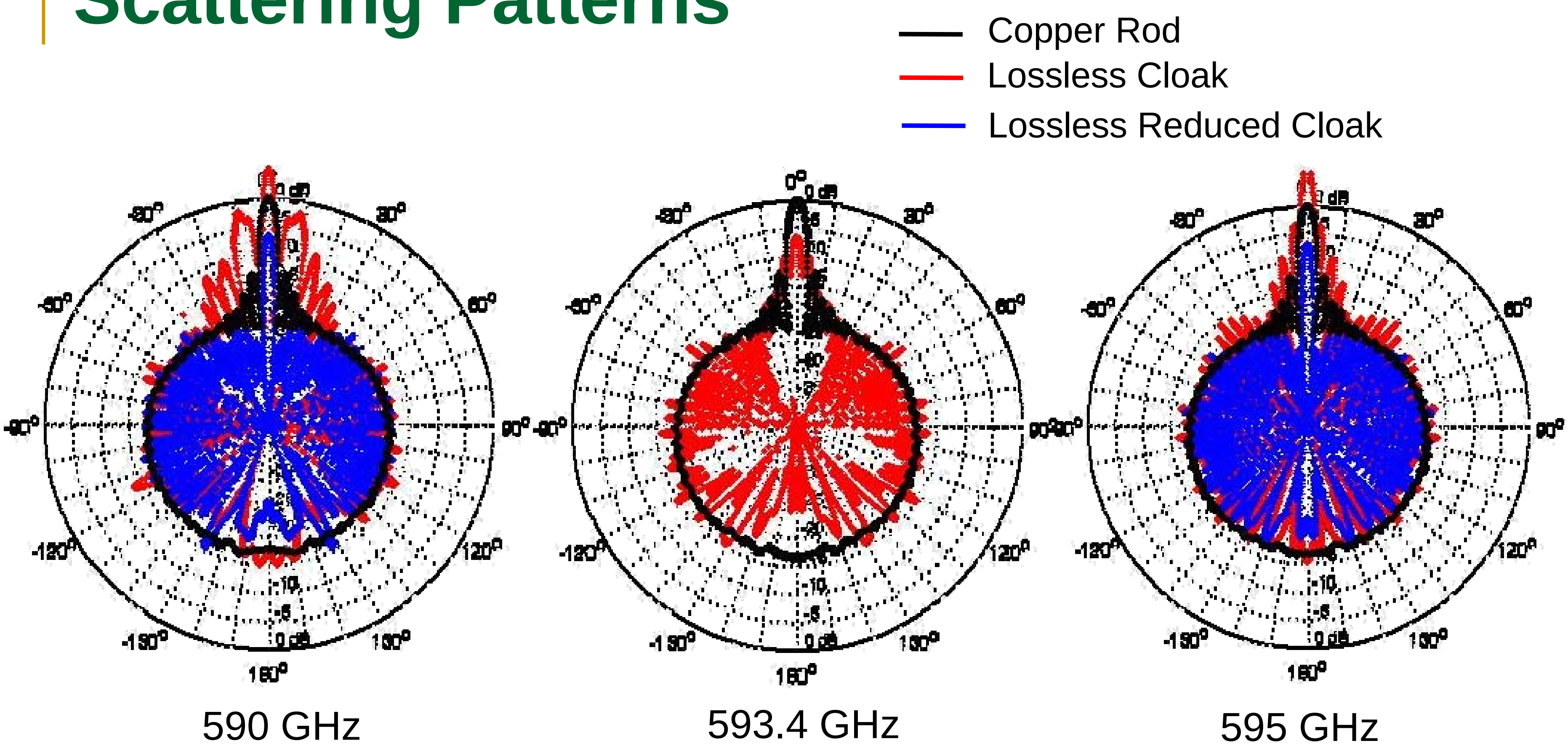
- Full dispersive lossless parameters entered in 2D FEM model
- Cloaking achieved around 593.4 GHz as expected
- Narrow cloaking bandwidth ~1%

E-field Map : Reduced Vs. BST Cloak



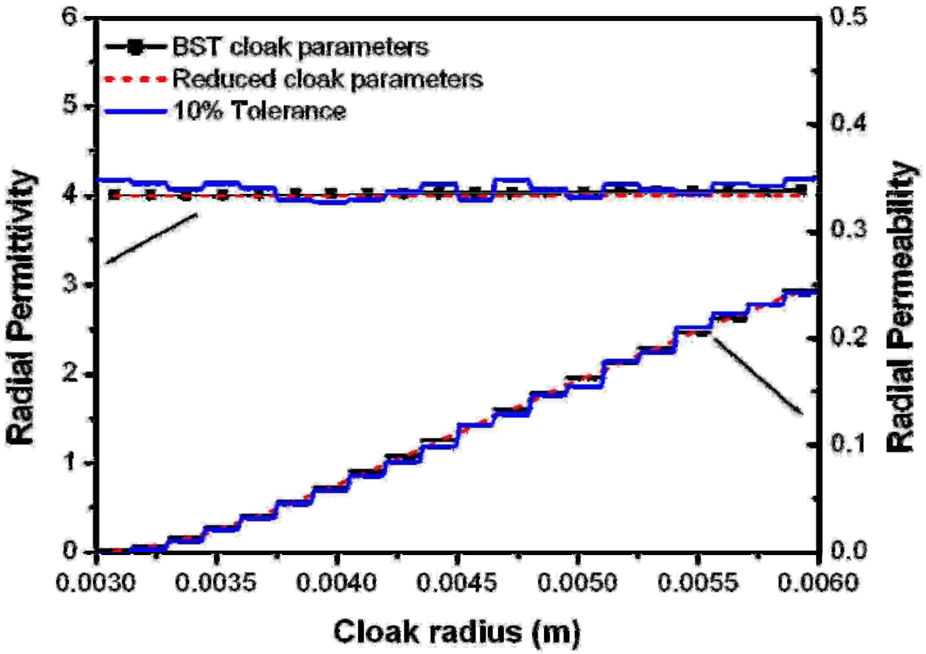
- Reduced and BST cloak perform almost equally at cloaking frequency
  - E-field maps close to each other

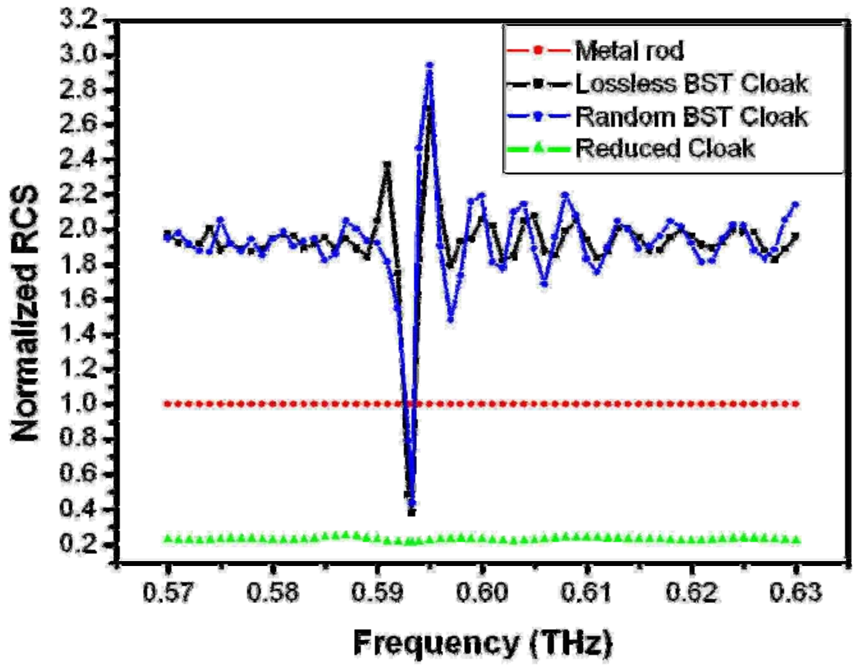
Scattering Patterns



- Minimized forward and backward scattering at cloaking frequency
  - Pattern similar to that of the reduced cloak
- -7.5dB forward scattering reduction
- -25dB backward scattering reduction

Cloaking performance dependence to the disorder





- Positional and dimensional disorder can be understood in effective parameters mismatch
  - Avoid complex homogenization procedures
  - Enables use of 2D homogenous layer-by-layer models
- 10% randomization has little effect on the performance of the cloak
  - Slight reduction of the cloaking bandwidth

Summary : Electric Cloaking Devices

- Effective Medium Theory (EMT) algorithms used to compute matched effective parameters of ellipsoidal metallic particles for electric cloaking device with
  - Bruggeman's formula
    - Original Cai's approach who assumed artificial $f(r) = f_a(a/r)$
    - Jose's approach w/o assumptions on filling fraction
  - Maxwell-Garnett's formula
    - Local optimization subroutine to minimizing both permittivity components
- Cloaking performance is strongly dependent to the
  - Effective parameters fit to the reduced Eqns.
  - Electric losses
- Need to develop optimization routine to integrate losses strong influence to the cloaking performance

Summary : Magnetic Cloaking Devices

- Mie resonance in high-κ dielectrics engineered to adjust magnetic plasma frequency
  - Design of wavelength-scaled and large magnetic cloak
- Cloaking ability and performance simulated with 2D homogeneous and 3D microstructured models
  - Lossless models close to single-frequency cloak with reduced Eqns.
  - Cloaking bandwidth broaden by
    - Losses
    - Permittivity mismatch
- Studied structure disorder in homogeneous and microstructured models
  - Limited / reasonable disorder does not fundamentally affect cloaking performance

Transformation Optics:

Starting point:

Engheta’s channeling slab with values:

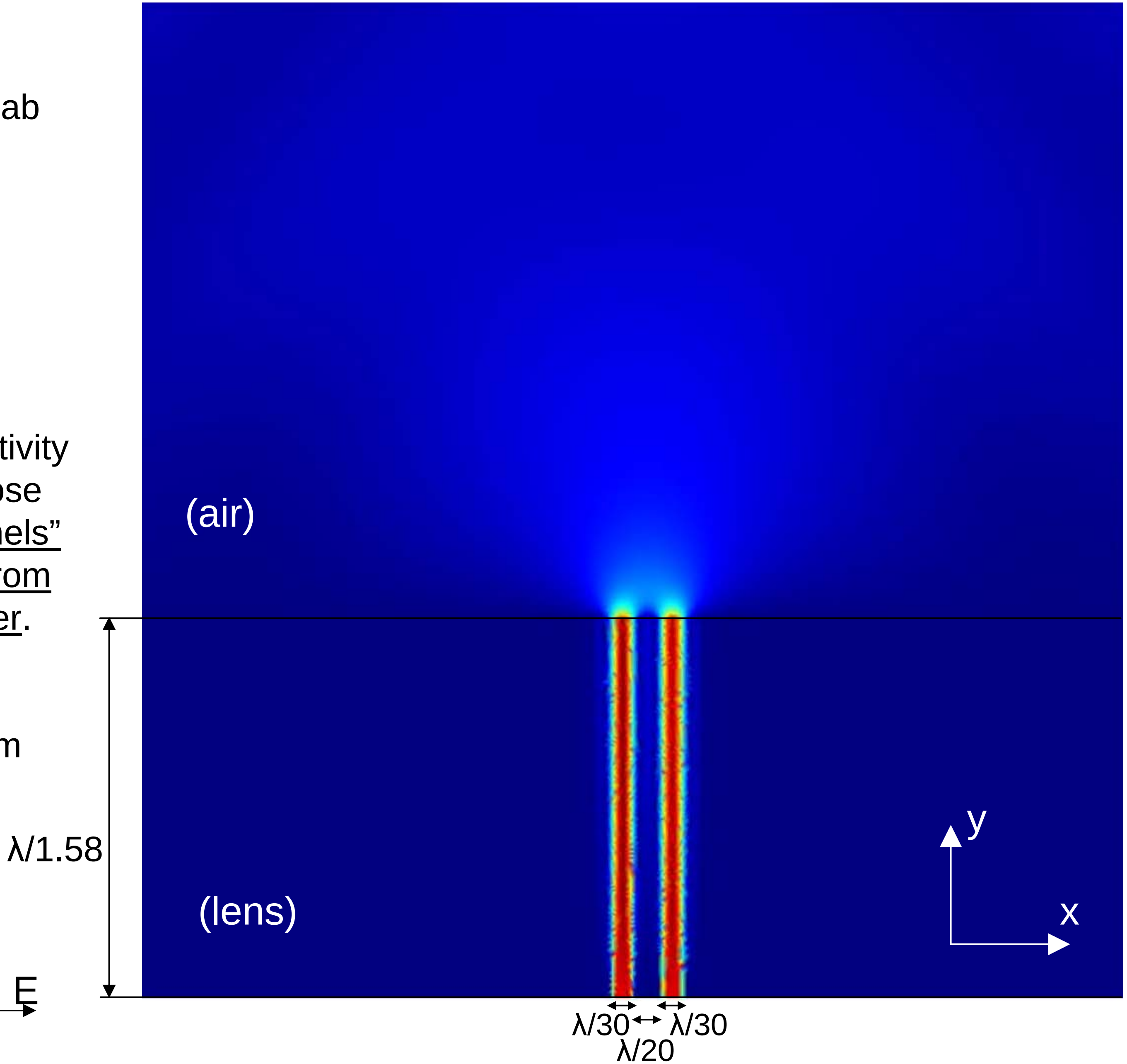
$\varepsilon_{xx} = 0.001$

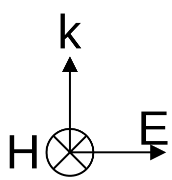
$\varepsilon_{yy} = 2.5$

$\mu_{zz} = 1$

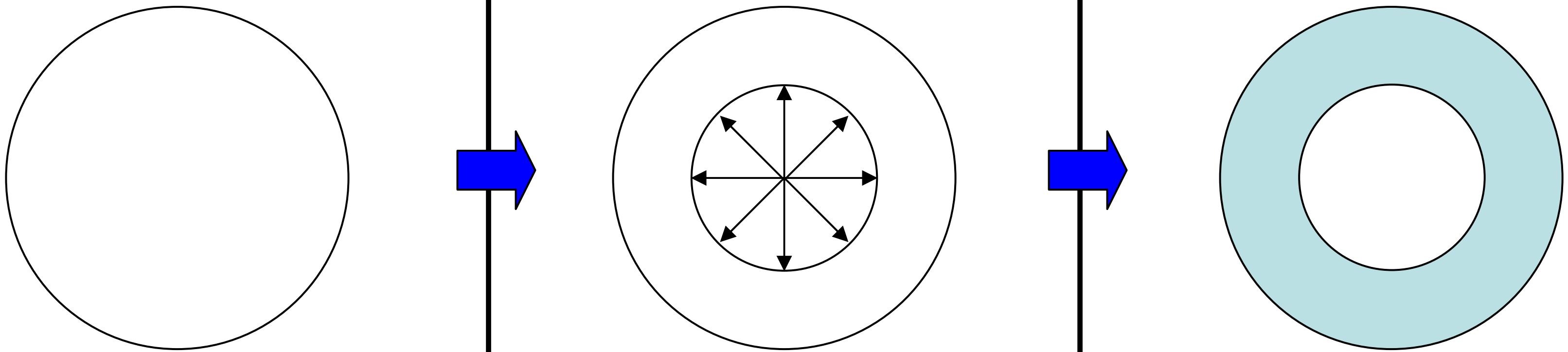
Thanks to the strong anisotropy of the permittivity (with a parallel value close to 0), this device “channels” arbitrary field patterns from one interface to the other.

Is it possible to transform this device to obtain magnification ?

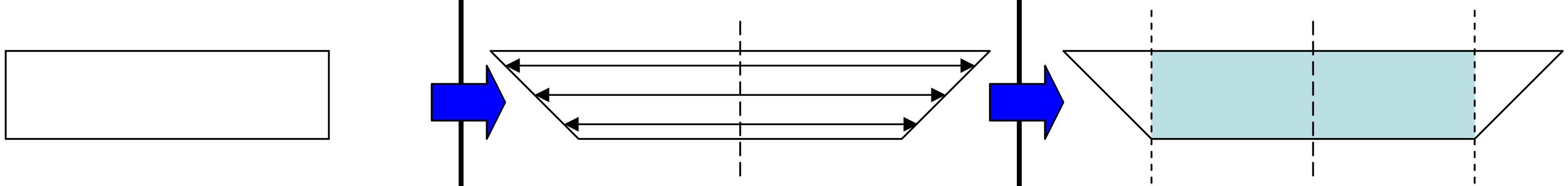




Original space	Transformed space	"Field controlling" device

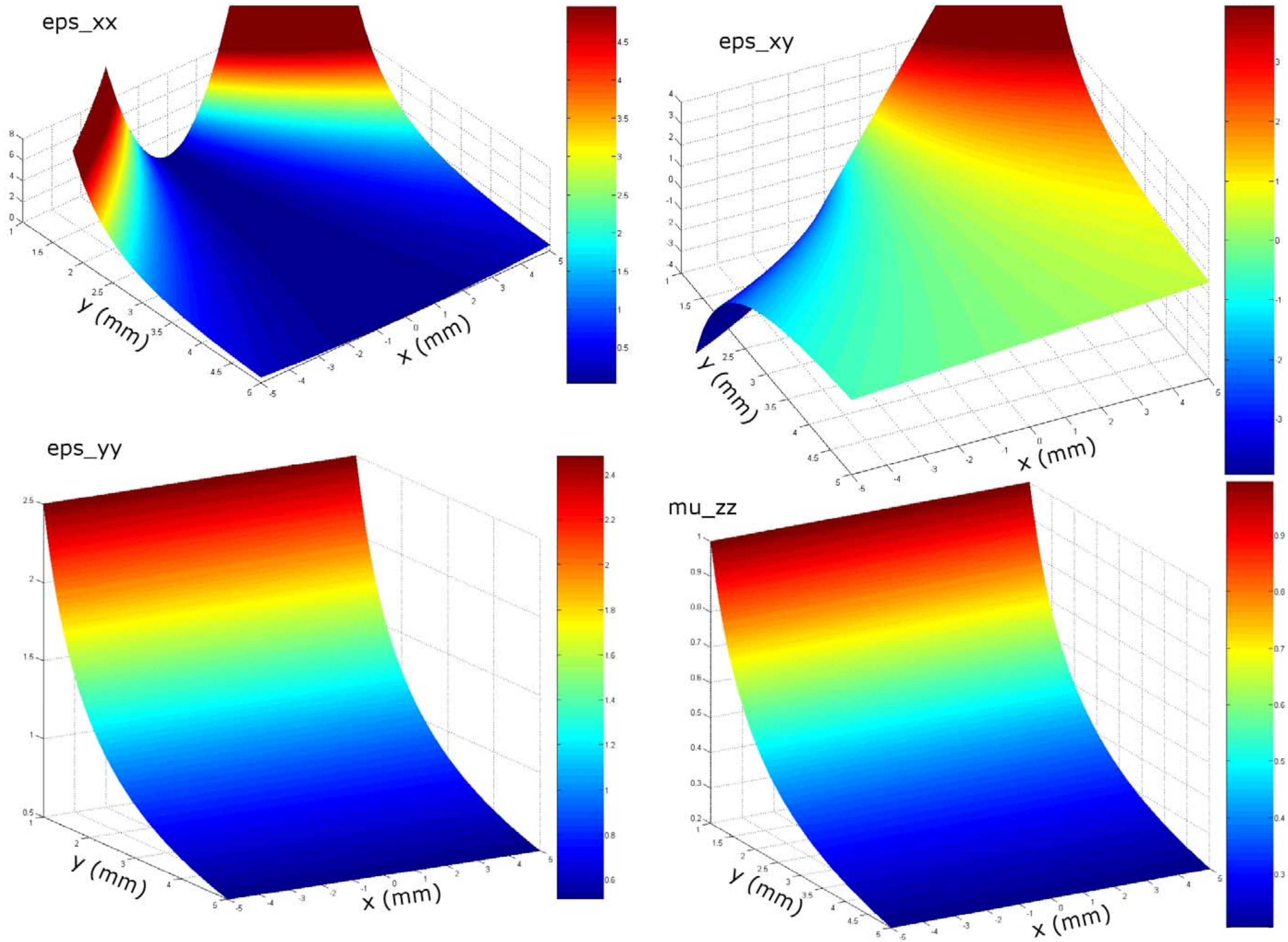


ε=1, μ=1	$r' = \frac{b-a}{b} r + a$	ε(x,y,z)=… μ(x,y,z)=…
(vacuum sphere)		(Pendry's cloak)

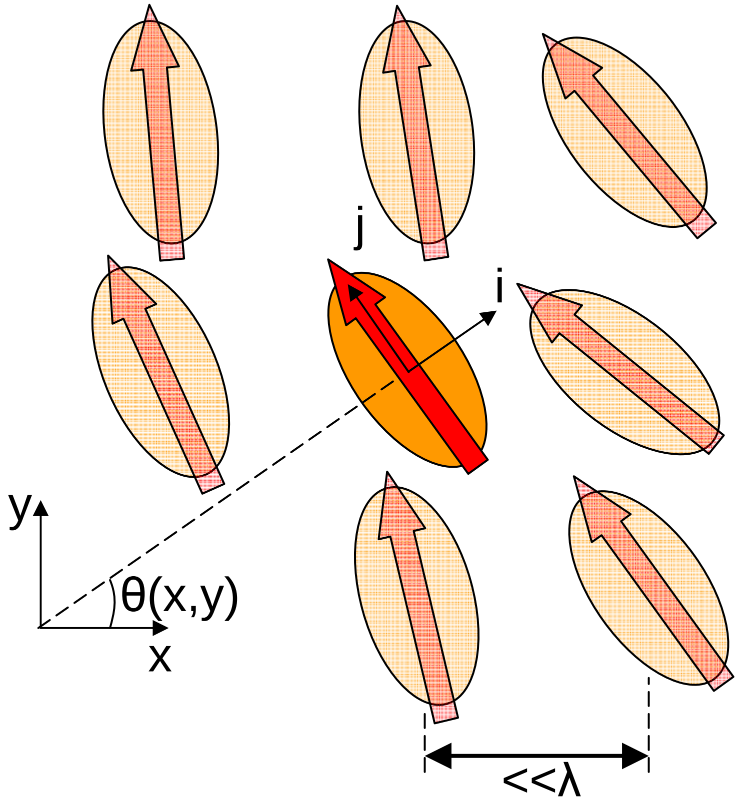


$\varepsilon_{xx} << 1$, $\varepsilon_{yy} > 1$, $\mu_{zz} = 1$	$x' = \left[\left(\frac{y-a}{b-a}\right)(t-1)+1\right]x$	ε(x,y,z)=… μ(x,y,z)=…
(channeling slab for TM polarization)		(a lens that "magnifies" near-field patterns)

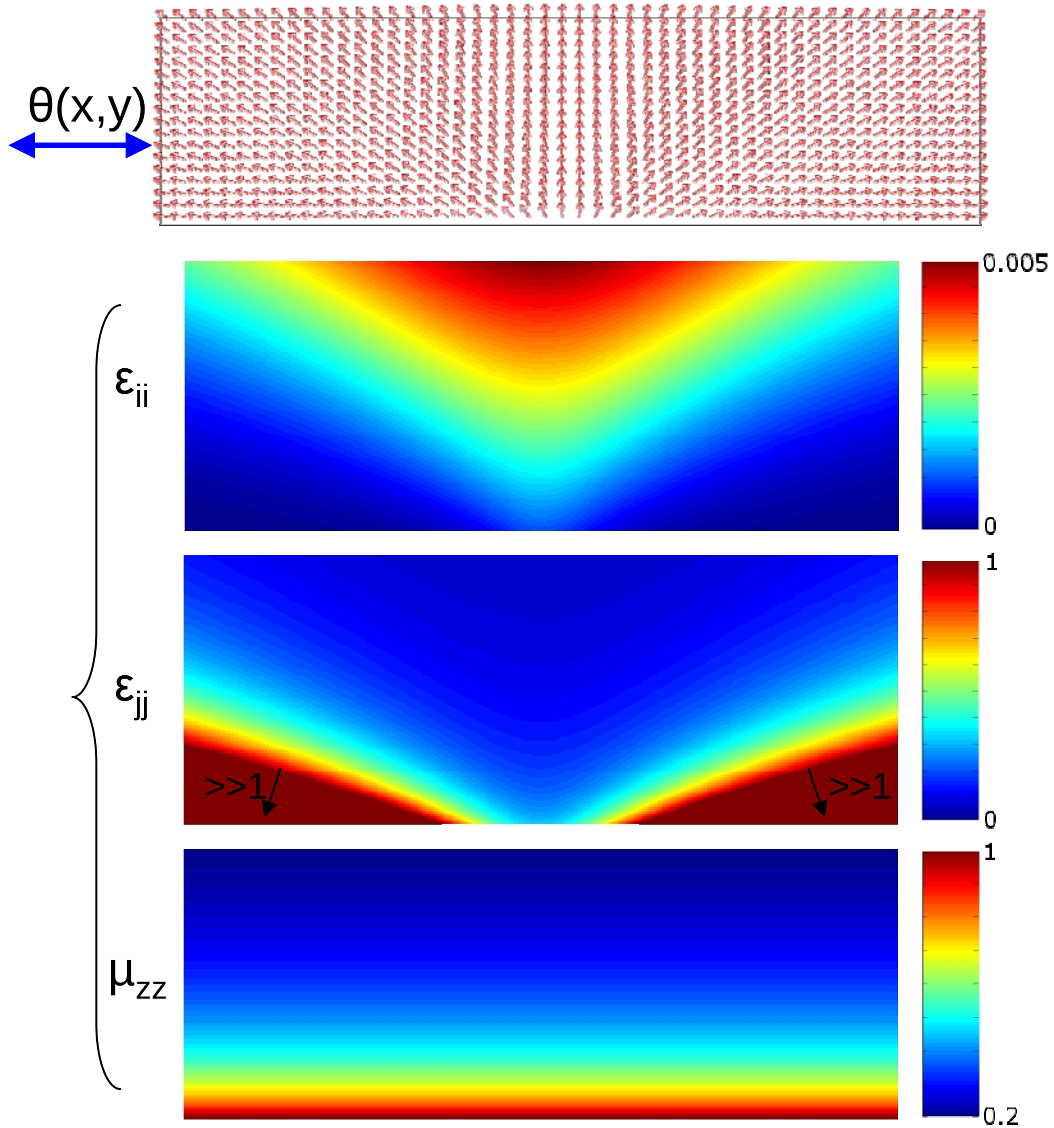
Effective parameters inside the magnifying lens:

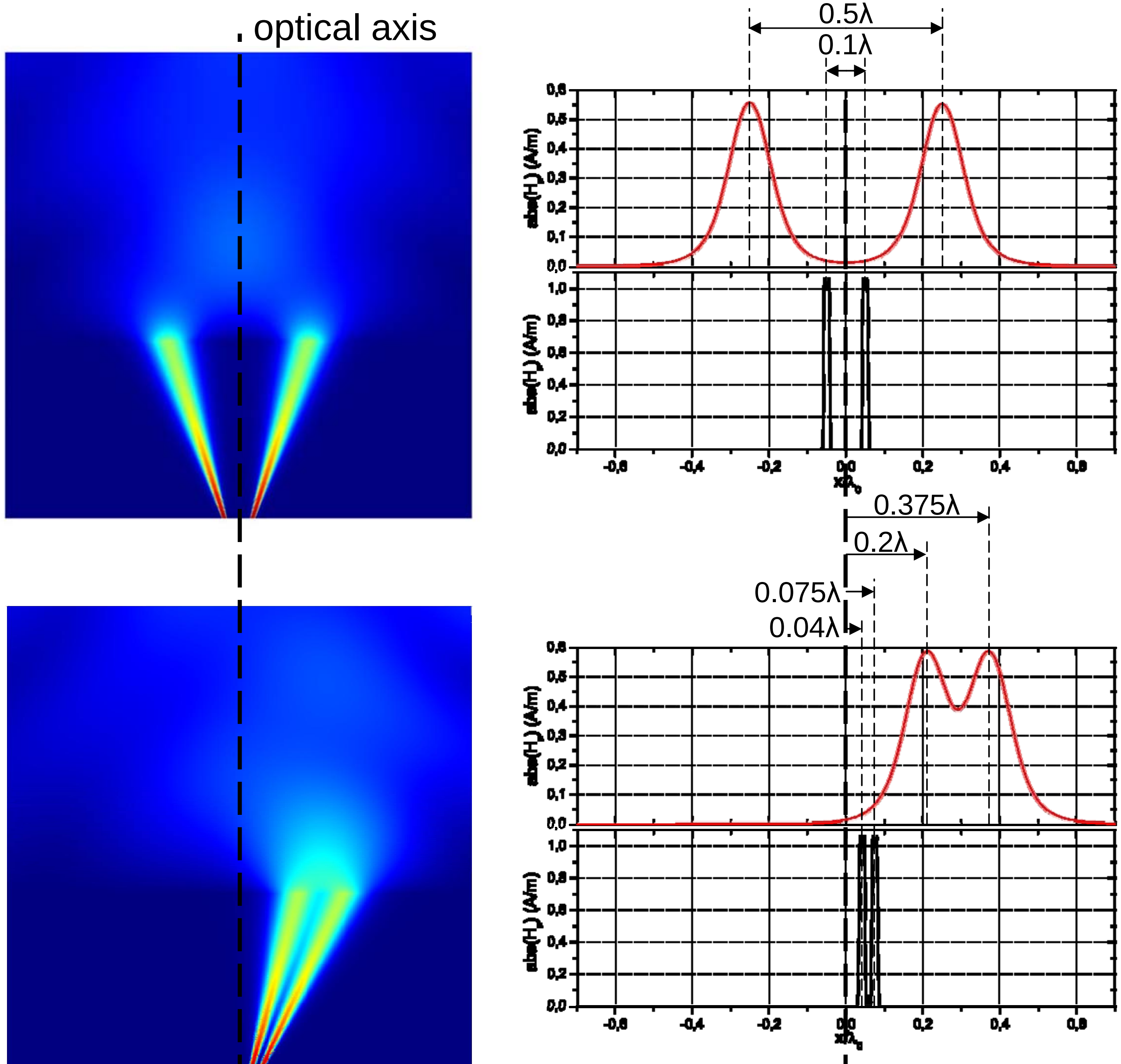


Let us now consider a local rotation of the coordinate system inside the lens:



With a suitable function for the local rotation, it is possible to avoid the off-diagonal term in ε(x,y):





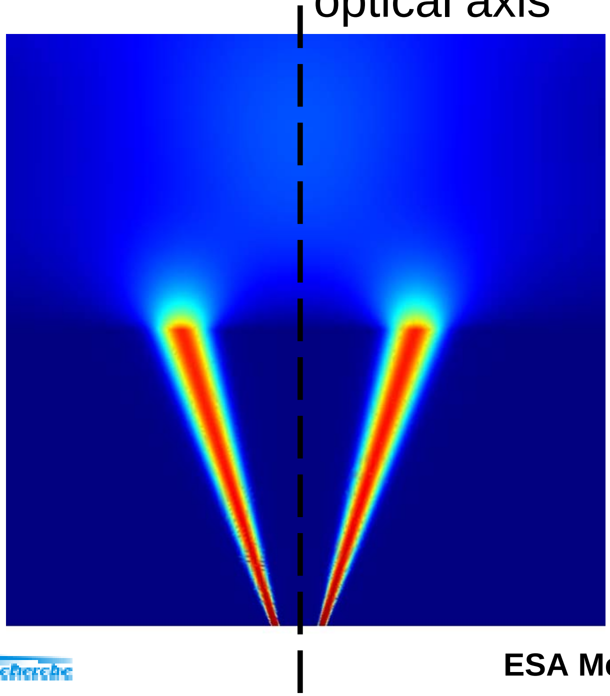
optical axis
0.5λ
0.1λ
0.375λ
0.2λ
0.075λ
0.04λ
abs(H_y) (A/m)
x/λ_0

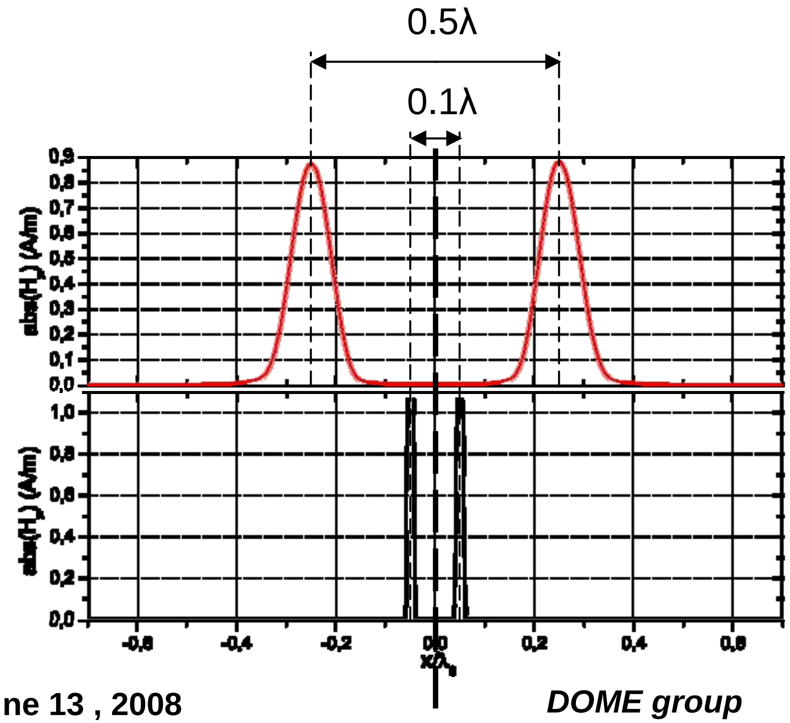
Main disadvantage: we still have strong gradients for all three *local effective* parameters.
We can drop some of the complexity while retaining the main effect if we keep the local rotation but without any local gradient. In summary:

ε_{ii}=0.001 ; ε_{jj}=2.5 ; μ_{zz}=1 in the coordinate system given by

$$\theta = \frac{1}{2}\tan^{-1}\left(\frac{2\varepsilon^{xx}(b-a)^2(t-1)xf(y)}{\varepsilon^{xx}f(y)^4+\varepsilon^{yy}(b-a)^2(f(y)+(t-1)x)(f(y)-(t-1)x)}\right) \quad \text{with} \quad f(y)=((b-at)+(t-1)y)$$

optical axis



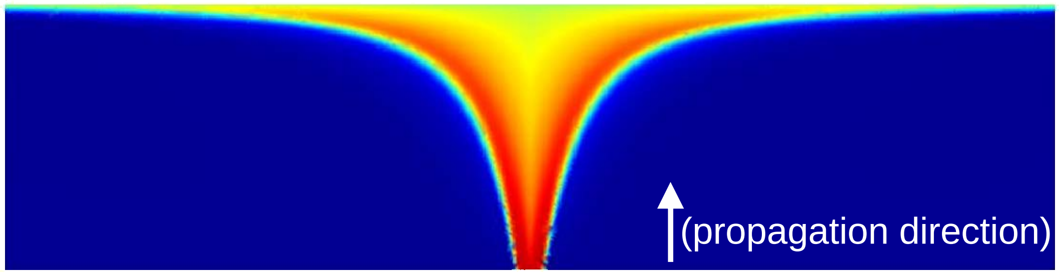


At this point we can choose freely our θ(x,y) distribution.

Let us look for distributions that perform other functions on the incident field pattern.

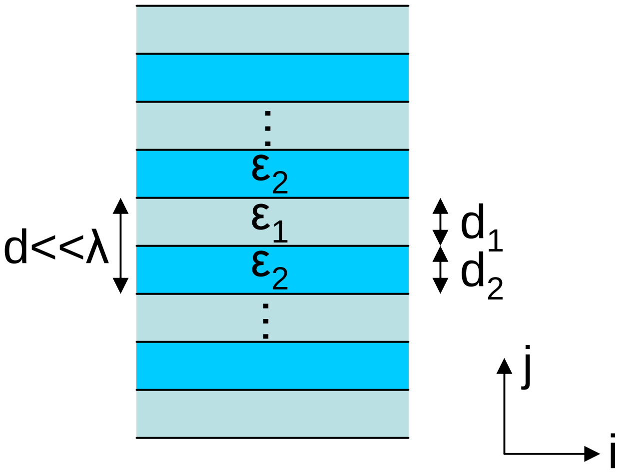
Example: $$\theta(x,y) = sign(x)\left[\tan^{-1}\left(-c_1\left(|x| + c_2\right)\frac{b-a}{b-y}\right)\right]$$

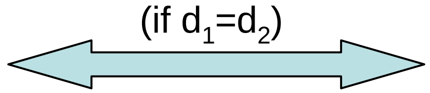
It « spreads » a point source along the transverse direction, transforming it into a line source.



How can we implement the required anisotropy ?

One possibility is a stack alternating two different layers:





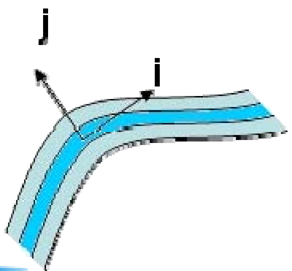
$$\varepsilon_{ii} = \frac{\varepsilon_1 + \varepsilon_2}{2}$$

$$\varepsilon_{jj} = \frac{2\varepsilon_1\varepsilon_2}{\varepsilon_1 + \varepsilon_2}$$

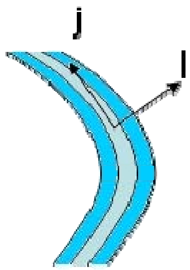
The limits of the layers form a family of curves.

At every point, those curves must be either normal or tangential to the local *j* vector of the rotated coordinate system.

case (a) : locally normal :



case (b) : locally tangential :

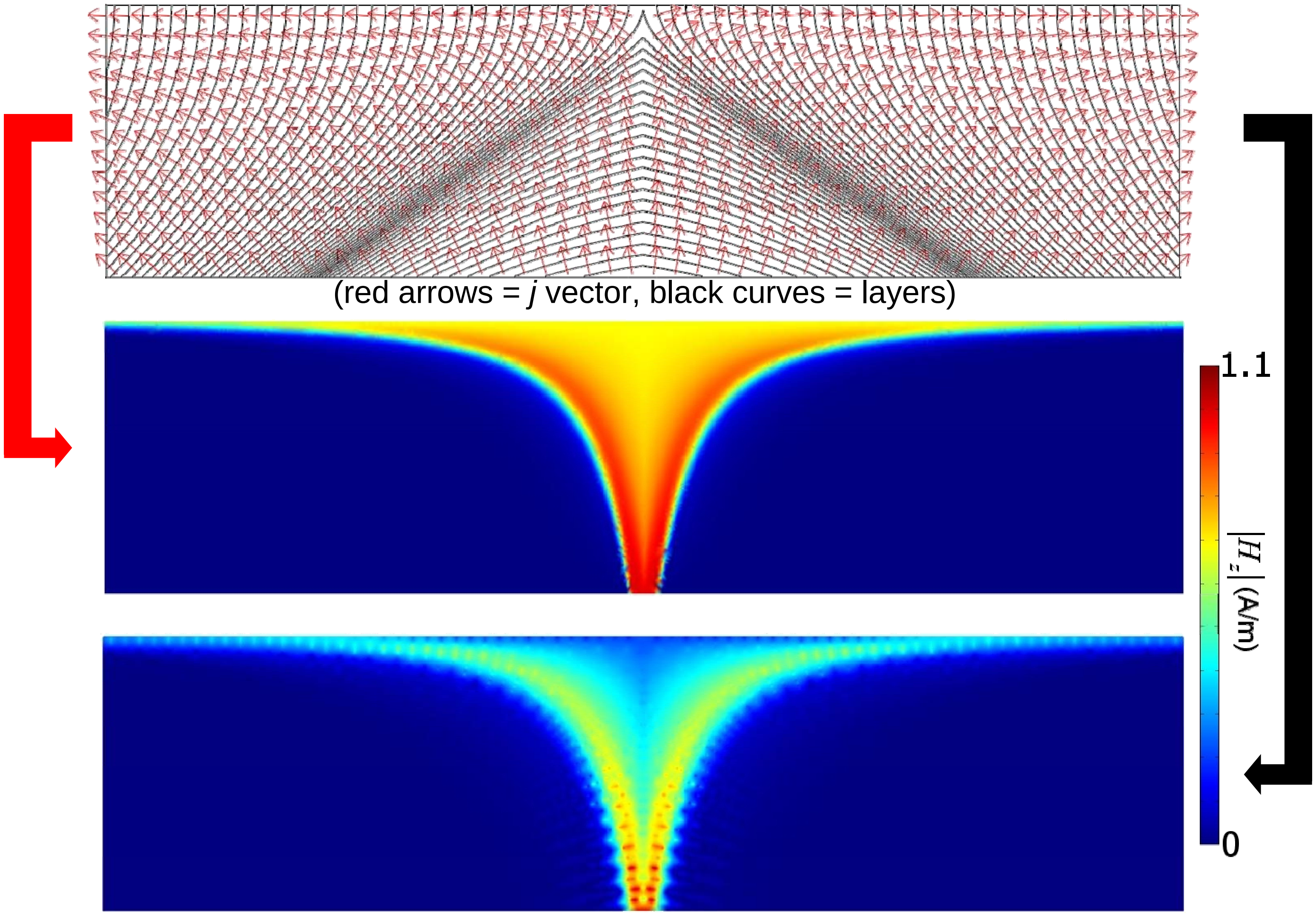


case (a): locally normal:

$$\theta(x,y) = sign(x)\left[\tan^{-1}\left(-c_1\left(|x|+c_2\right)\frac{b-a}{b-y}\right)\right]$$

$$\left.\begin{matrix}\varepsilon_{ii} = 0.001 \\ \varepsilon_{jj} = 2.5\end{matrix}\right\} \Leftrightarrow \left\{\begin{matrix}\varepsilon_1 = 0.001 - i0.05 \\ \varepsilon_2 = 0.001 + i0.05\end{matrix}\right.$$

θ(x,y)
+ ε_{ii}
+ ε_{jj}



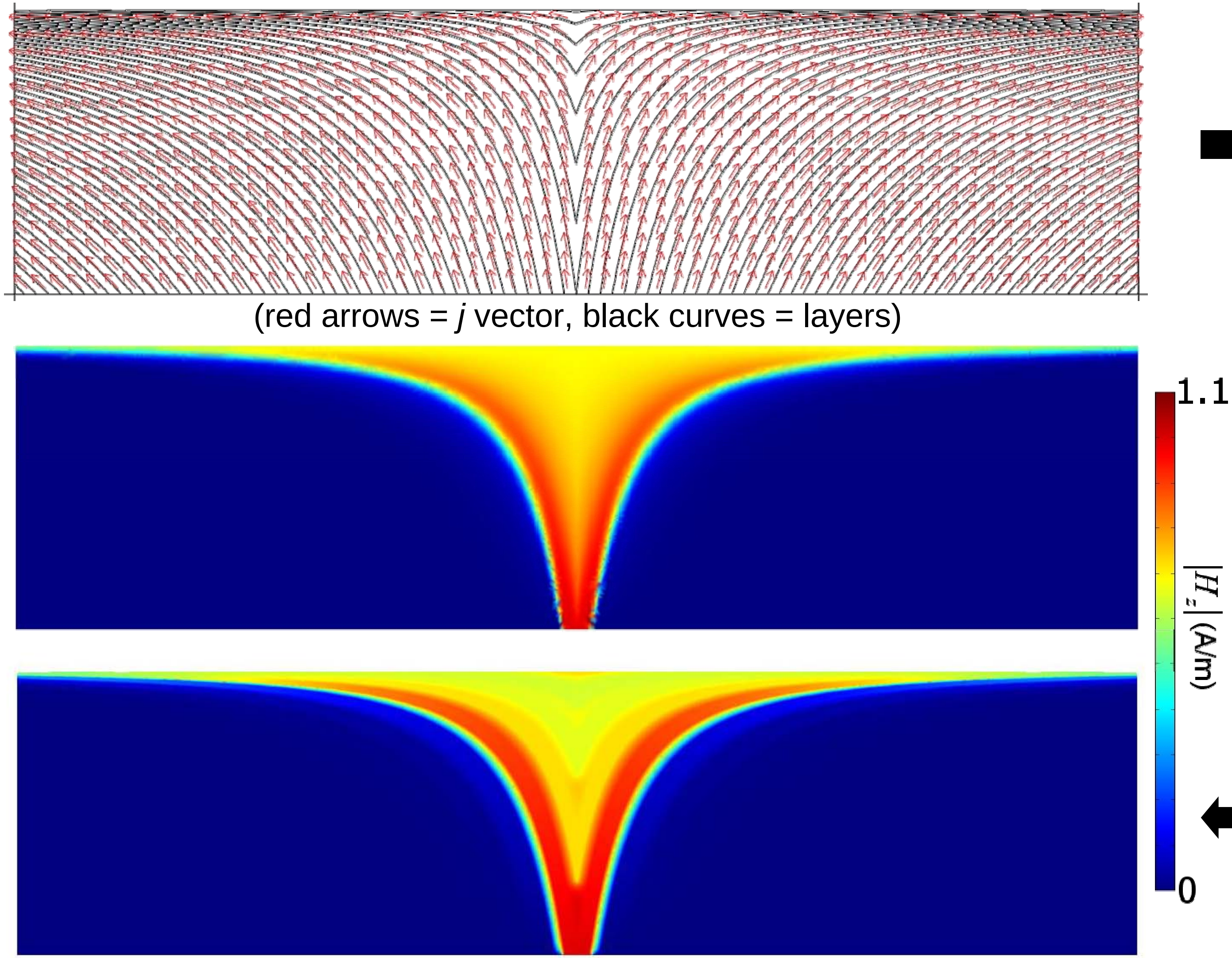
layers
+ ε_1
+ ε_2

case (b): locally tangential:

$$\theta(x,y) = sign(x)\left[\tan^{-1}\left(-c_1\left(|x|+c_2\right)\frac{b-a}{b-y}\right)\right]$$

$$\left.\begin{aligned}\varepsilon_{ii} &= 0.001 \\ \varepsilon_{jj} &= 2.5\end{aligned}\right\} \Leftrightarrow \left\{\begin{aligned}\varepsilon_1 &= 4.9995 \\ \varepsilon_2 &= 0.0005\end{aligned}\right.$$

θ(x,y)
+ ε_{ii}
+ ε_{jj}



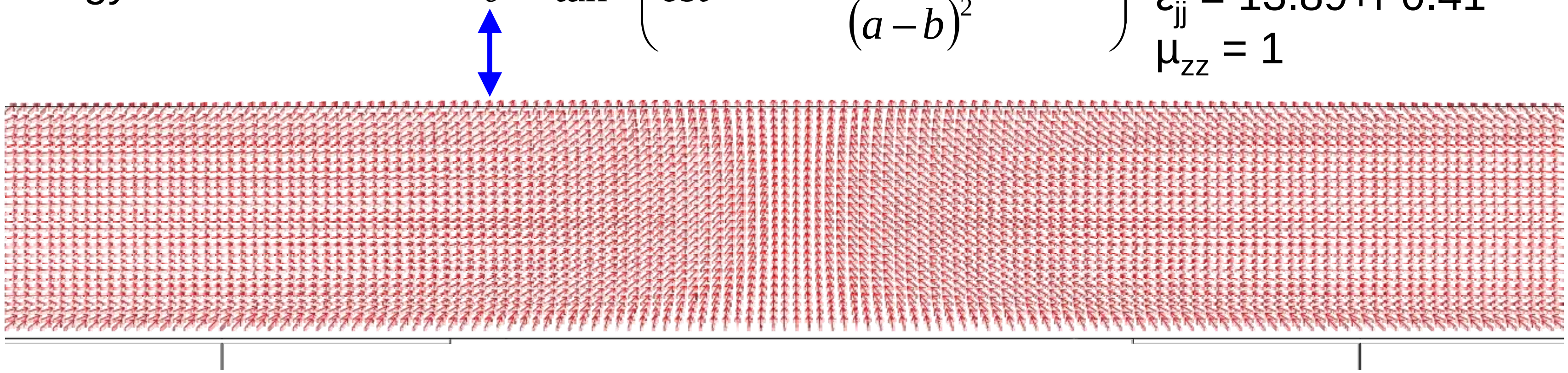
(red arrows = *j* vector, black curves = layers)

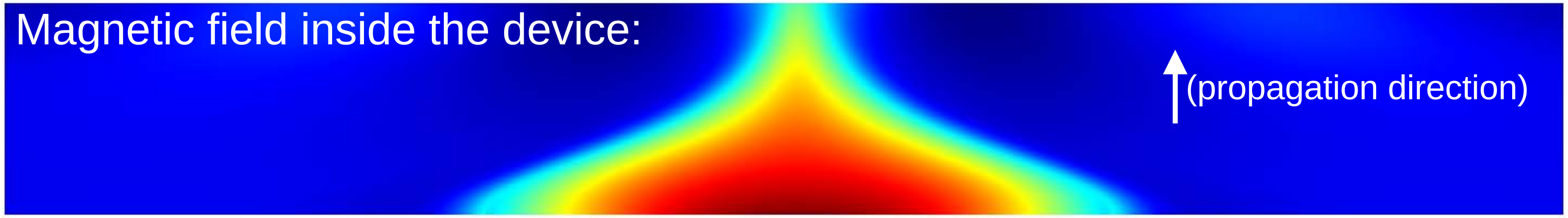
layers
+ ε_1
+ ε_2

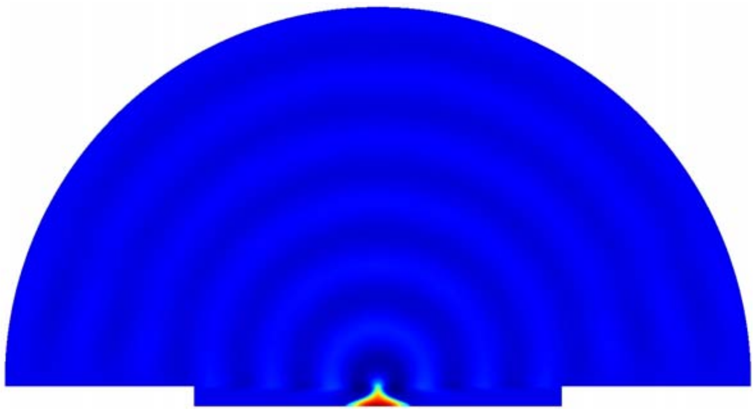
Another example:
an energy concentrator

$$\theta = \tan^{-1}\left(cst \frac{x\left(-y^2 + (a+b)y - ab\right)}{(a-b)^2} \right)$$

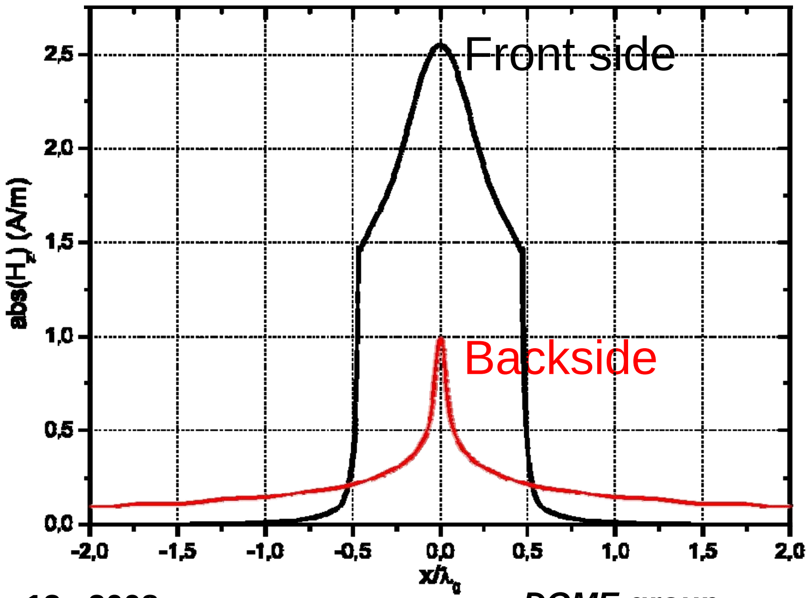
$\varepsilon_{ii} = 0.036+i*0.76$
$\varepsilon_{jj} = 13.89+i*0.41$
$\mu_{zz} = 1$







Magnetic field in air behind the device



Summary

Two different approaches:

- full conformal mapping leading to a field controlling device
- direct use of an empirical function to tune the channeling direction

A large range of functions:

- hyperlens: convert evanescent waves into propagative ones for super resolution
- channeling / collimating / diverging systems…

Two different implementations:

-stack of two different layers with a specific shape (important technological challenge)
- array of particles individually oriented (very high requirement on the anisotropy)