Global Trajectory Optimisation: Can we prune the solution space when considering Deep Space Maneuvers?

ARIADNA final presentation

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Plan

1. Introduction
2. Multi gravity Assits - Deep Space Maneuver trajectories
3. Optimization
4. Heuristic optimization
5. GASP-like approach
6. Examples
7. Conclusion

Introduction

MGA trajectories:

- Multi Gravity Assist (MGA) impulsive trajectories
- solution space can be quickly pruned (GASP)[1]

MGADSM trajectory optimization problem:

- Multi Gravity Assist (MGA) with Deep Space Maneuvers (DSM) impulsive trajectories
- Deep Space Maneuvers (DSM) can reduce fuel requirement
- many local minima: requires global techniques
- solution space of great dimension

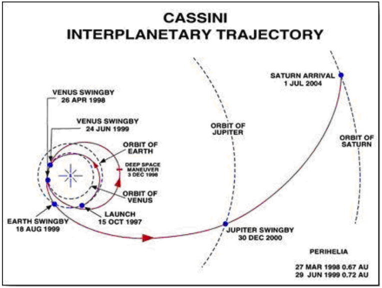
MGADSM

How can we efficiently prune the solution space ?

[1] Myatt, Becerra, Nasuto, Bishop, Advanced global optimisation for mission analysis and design. Final report. Ariadna 03/4100

MGADSM examples

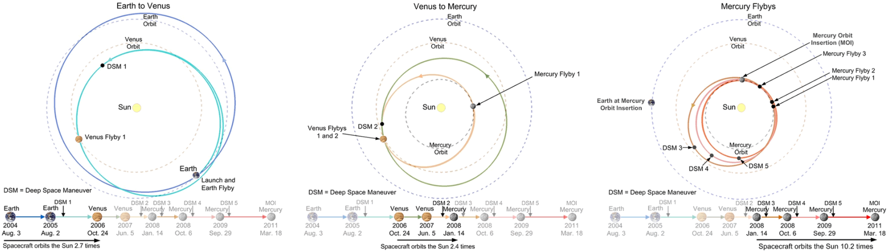
From CASSINI to MESSENGER



- EVVEJS planet sequence
- ΔV gravity assist
- no DSM between JS

MGADSM examples

From CASSINI to MESSENGER



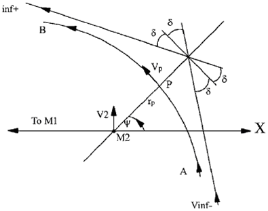
From http://messenger.jhuapl.edu

- To insert a scientific spacecraft into an orbit around Mercury
- Launched on August 3, 2004
- Mercury(Y): High excentricity and inclination, close to the sun, fastest planet ... many technical transfer orbit shape constraints
- Many Gravity Assist to reduce fuel requirement and reduce average speed relative to the mercury.
- Many small DSM to adjust the orbit for the next swingby

Plan

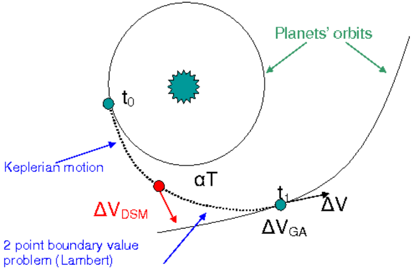
Non powered swingby

hyperbolic excess velocities: ${V_\infty}^+$ and ${V_\infty}^-$
Periapsis radius: r_p



- $\sin\delta = \frac{\mu}{\mu + r_p V_\infty^2}$
- $\Delta V = 2V_\infty \sin\delta$
- $r_p > r_{body}$

DSM models



DSM possible parametrizations

- date, position: $(t, \mathbf{r})$
- date, initial V: $(t, \mathbf{v_0})$
- date, final V: $(t, \mathbf{v_f})$
- swingby + Keplerian propagation

Use forward/backward integration and/or Lambert's problem solver.

Plan

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3. Optimization
   - Lawden Primer Vector theory
   - MGA problems boundary conditions
   - Primer Vector theory and MGA problems
   - Example
   - Summary
4. Heuristic optimization
5. GASP-like approach
6. Examples

Optimization

- The spacecraft is subject to a nonlinear dynamic:

$$\frac{df}{dt} = \begin{cases} \frac{d\mathbf{r}}{dt} = \mathbf{v} \\ \frac{d\mathbf{v}}{dt} = G\left(r;t\right)\mathbf{r} + \frac{qg_0 Isp}{m}\mathbf{u} \\ \frac{dm}{dt} = -q \end{cases} \tag{1}$$

- With the constraints:

$$q(q_{max} - q) \geq 0 \tag{2}$$

$$\|\mathbf{u}\| = 1 \tag{3}$$

- With the initial and final conditions:

$$\begin{cases} r_0 = r(t_0) \\ r_f = r(t_f) \end{cases} \tag{4}$$

Lawden Primer Vector theory[2]

We have to minimize a non linear objective function, defined by the characteristic velocity of the mission (sum of ΔVs) plus the rendezvous maneuver ΔV_f.

$$J = \|\mathbf{\Delta V_f}\| + \sum_{i=0..N} \|\mathbf{\Delta V_i}\| \tag{5}$$

with $N > 0$ the unspecified number of DSM.
ΔV_0 is the departure maneuver.

With the introduction of the co state vector $\Lambda = [\lambda_R, \lambda_V]$ we have:

$$\mathcal{L} = J + \int_{t_0}^{t_f} H(\Lambda, x; t)\, dt \tag{6}$$

$$H(\Lambda, x; t) = \Lambda f(x, u; t) + {\mu_q}^T \left(q(q_{max} - q) - \alpha^2\right) + {\mu_u}^T \left(\|\mathbf{u}\|^2 - 1\right) \tag{7}$$

[2] D. Jezewski, Primer Vector Theory and Applications, NASA Technical report R-454, Nov. 1975

Lawden Primer Vector theory

With calculus of variation we get the TPBVP:

$$\frac{d\lambda_V}{dt} = -\lambda_R \tag{8}$$

$$\frac{d\lambda_R}{dt} = G\left(r;t\right)\lambda_R \tag{9}$$

$$\lambda_V(t_0) = \frac{\mathbf{\Delta V}_0}{\|\mathbf{\Delta V_0}\|} \tag{10}$$

$$\lambda_V(t_f) = \frac{\mathbf{\Delta V}_f}{\|\mathbf{\Delta V_f}\|} \tag{11}$$

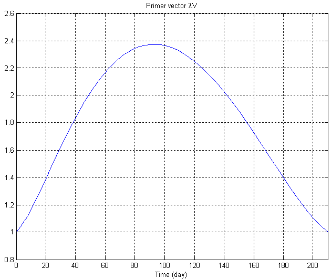
and the control is given by:

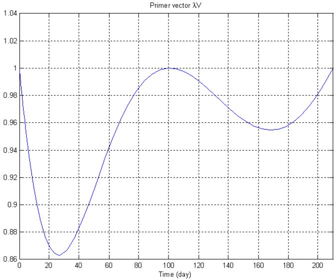
$$\mathbf{u} = \frac{\lambda_V}{\|\lambda_\mathbf{V}\|} \tag{12}$$

$$S(t) = \frac{g_0 I_{sp}}{m}{\lambda_V}^T\mathbf{u} + cnst(q_{max}, g_0, I_{sp}, \mu_q) \tag{13}$$

One major issue: we have all the information on the impulses, but their amplitude $\|\mathbf{\Delta V}\|$!

Lawden Primer Vector theory





Conditions to have an optimal trajectory

- the primer vector and its derivative should be continuous
- if there is an impulse, the primer vector is aligned with the impulse, and its module is 1.
- the primer vector module should not exceed 1.
- the derivative of all intermediate impulses is zero.

Boundary conditions

We can extend the primer vector theory to MGADSM trajectory if we can express the boundary conditions.

Consider a body to body transfer from t_0 to t_f, with one swingby at $t = t^- = t^+$, with the swingby constraints at t:

$$\Phi_\nu(x^-, x^+) = R(x^+) - R(x^-) \tag{14}$$

$$\Phi_\beta(x^-, x^+) = V_\infty(x^+) - Q(\beta)V_\infty(x^-) \tag{15}$$

Use the Lagrangien to express optimality conditions at swing-by:

$$\mathcal{L} = J + \int_{t0}^{t^-} H\left(\Lambda, x; t\right) dt + \int_{t^+}^{t_f} H\left(\Lambda, x; t\right) dt + {\mu_\beta}^T \Phi_\beta(x^-, x^+) + {\mu_\nu}^T \Phi_\nu(x^-, x^+) \tag{16}$$

Boundary conditions

Initial velocity constraint on leg i:

$$\psi(t_i) = \|\mathbf{V(t_i)} - \mathbf{V_{pl}(t_i)}\| - V_\infty \tag{17}$$

$$\lambda_V(t_i) = \mu^T \frac{\mathbf{V(t_i)} - \mathbf{V_{pl}(t_i)}}{\|\mathbf{V(t_i)} - \mathbf{V_{pl}(t_i)}\|} \tag{18}$$

Final velocity constraint on leg i:

$$\psi(t_{i+1}) = \|\mathbf{V(t_{i+1})} - \mathbf{V_{pl}(t_{i+1})}\| - V_\infty \tag{19}$$

$$\lambda_V(t_{i+1}) = -\mu^T \frac{\mathbf{V(t_{i+1})} - \mathbf{V_{pl}(t_{i+1})}}{\|\mathbf{V(t_{i+1})} - \mathbf{V_{pl}(t_{i+1})}\|} \tag{20}$$

μ is the lagrange vector associated to the constraint ψ.

Other results [3][4] are possible depending on the expressions of ψ, Φ_ν or Φ_β.

[3] D.R. Glandorf, Primer Vector Theory for Mateched Conic trajectories, AIAA journal, Technical notes, Vol. 8, No. 1, Jan. 1970

[4] Konstantinov, Fedotov, Petukhov, ACT Global Optimization Workshop, ACT-ESA 2005

Primer Vector theory and MGA problems

Interaction prediction principle[5] gives an efficient algorithm for optimizing MGADSM trajectory without too much sensitivity.

Consider the multiple impulse body to body sub problems i:

$$\begin{aligned}\mathcal{L}_i = \sum_{j_i} \|\mathbf{\Delta V_{i,j}}\| + {c_\nu}^T \Phi_{\nu i}(x(t_i), \xi_i) + {c_{\beta i}}^T \Phi_\beta(x(t_i), \xi_i) + \\ {\mu_\nu}^T \Phi_\nu(\xi_{i+1}, x(t_{i+1})) + {\mu_{\beta i}}^T \Phi_\beta(\xi_{i+1}, x(t_{i+1})) + \\ \int_{t_i}^{t_{i+1}} H_i\left(\lambda, x; t\right) dt\end{aligned} \tag{21}$$

where $H_i\left(\lambda, u; t\right)$ is the Hamiltonian for the sub problem i and $c_{\nu i}$, $c_{\beta i}$ and ξ_i are the coordination variables, and $\mu_{\nu i}$ and $\mu_{\beta i}$ are the lagrange vectors for the constraints ϕ_ν and ϕ_β.

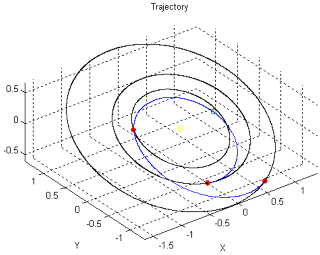
[5] G. Cohen, Optimization by Decomposition and Coordination: a unified approach, IEEE transaction on automatic control, Vol. 23, No. 2, Apr. 1978

Primer Vector theory and MGA problems

Procedure

1. Minimize J_j for all j
2. update the coordination variables c_ν, c_β and ξ using the necessary condition of optimality and the boundary equations.
3. Reconstruct the complete decision vector X and try to optimize the complete problem.
4. if convergence, stop, otherwise goto 1 with update coordination variables.

Example



- EVM planet sequence
- Optimize for given date (no optimization on the phasing)
- Find 1 DSM for the EV leg, and no DSM for the VM leg
- Cost = 10.786 km/s

Primer Vector theory and MGA problems

- Primer Vector theory allows to formulate a TPBVP for impulsive Body to Body transfer problem.
- The number of impulse is optimally found (local).
- We extend the theory to MGA problem through appropriate boundary conditions.
- We introduced a Decomposition-Coordination technique to easily solve the problem.
- Practical results show the efficiency of the method.
- It is however a local optimization method.
- Integration of the state and co-state equations are necessary whatever the resolution method (TPBVP, transition matrix, ...).

Plan

Heuristic algorithms

Increased popularity of heuristic algorithms (PSO[6], DE[7]) for optimization problem because of:

- Easy to code, easy to use
- Few mathematical restrictions on the objective function (no derivatives)
- stochastic: do not get trap into local minima

However:

- lack of theoretical evaluation
- search cost (nb of function evaluation)
- no general rules for parameter tuning

These are fairly good algorithms. Do not always perform well for MGADSM problem.

[6] J. Kennedy, R. Eberhart, Particle Swarm Optimization, Proc. IEEE Intl. Conf. On Neural Networks, 1995.

[7] R. Storn, K. price, Differential Evolution - A simple and efficient adaptive scheme for global optimization over continuous space, Technical Report TR-95-012, ICSI

Heuristic algorithms

Intuition, for those problems, says that there exists a "hard" (or "difficult") and a "soft" (or "easy") part in the decision vector for the optimization and according to the model used.

The choice of the model and the use we make of the variables have a strong influence on the result.

Variables

Our "soft" variables are typically the time and dates.
Our "hard" variables are typically the swingby altitude, B-plane, ...

For the "hard" part, and to improve convergence, we can use a local optimization solver. The remaining "soft" part is left to the evolutionary algorithm.

Heuristics algorithms

PSO[8]: For every particle i, and every coordinate $j \in X_{SOFT}$:

$$\begin{aligned} X_i{}^j(t+1) = X_i{}^j(t) + & \underbrace{\nu\left(X_i{}^j(t) - X_i{}^j(t-1)\right)}_{\text{momentum}} + \\ & \underbrace{\alpha(X_G{}^j - X_i{}^j(t))}_{\text{global best influence}} + \\ & \underbrace{\beta(X_{L,i}{}^j - X_i{}^j(t+1))}_{\text{local previous best influence}} \end{aligned} \tag{22}$$

Newton: For every particle i, and every coordinate $j \in X_{HARD}$:

$$X_i{}^j(t+1) = Newton\left(X_i(t), J(X_i(t))\right) \tag{23}$$

[8] other possible variant. Can also use DE or other Heuristic algorithms

Heuristic / Hybrid SQP - Heuristic comparisons

	PSO /wo SQP	PSO /w SQP
EVM	$M = 11.108$, $StD = 0.183$	$M = 10.821$, $StD = 0.083$
EVEJ	$M = 17.93$, $StD = 3.016$	$M = 12.346$, $StD = 1.800$
EVVEJS	$M = 24.629$, $StD = 6.653$	$M = 16.723$, $StD = 1.856$

what has improved convergence?

- Reduction of the size of the decision vector.
- Convergence less prone to the tuning constants.
- Use of deterministic solvers to quickly find good sub-space.
- Part of the decision vector gets adapted because of the objective function, rather than the history of the particle.

We are however still far above the known solutions[9]!

[9] A. Petropoulos, J. longuski, E. Bonfiglio, Trajectories to Jupiter via Gravity Assists from Venus, Earth and Mars, AIAA JSR, Vol. 37, No. 6

Heuristics algorithm

- We briefly described a "hybrid" Heuristic algorithm[10].
- We extract from the decision vector an "easy to solve" part.
- The remaining part is what we called "hard" or "difficult" as it can be responsible of the "bad qualtiy" of the objective function.
- We mix a standard heuristic algorithm with a Newton based optimization solver.
- "easy" and "hard" part are respectively used in the Heuristic and the Newton based algorithm.
- Practical experiments and results show an improvement from the standard Heuristic algorithm.

[10] hybrid algorithms usually perform Heuristic and SQP optimization sequentially. Not the case here!

Plan

MGA formulation

Consider the decision vector for the MGA problem[11]:

$$X = [t_0, V_0, \alpha_0, \beta_0, t_{DSM(1)}, \ldots, t_i, B_{(i)}, t_{DSM(i)}, \ldots, t_{N_p-2}, B_{(N_p-1)}, t_{DSM(N_p-1)},$$

With:

- t_0 launch date
- t_i intermediate planet encounter date
- t_f arrival date
- $[V_0, \alpha_0, \beta_0]$ launch velocity
- t_{DSM} date of the DSM
- $B = [r_p, \phi]$ swing by description

Where we considered a sequence S_{Body} of N_p planets, with 1 DSM per phase.

[11] M. Vasile, P. DePascale, Preliminary Design of Multiple Gravity Assist Trajectories, AIAA JSR

Decomposition

Split X in n phase, and duplicate missing variable. (phase = Body to Body transfer).

New overall decision vector:

$$X = [\underbrace{t_0, V_0, \alpha_0, \beta_0, t_{DSM(1)}, \tilde{B}_{(1)}, \tilde{t}_1}_{\text{initial leg}}, \dots,$$
$$\underbrace{t_i, B_{(i)}, t_{DSM(i)}, \tilde{B}_{(i)}, \tilde{t}_{i+1}}_{\text{intermediate leg}}, \dots,$$
$$\underbrace{t_{N_p-2}, B_{(N_p-1)}, t_{DSM(N_p-1)}, t_f}_{\text{final leg}}]$$

Decomposition and decision vector

Assumption

Initially, we do not care about the swingby feasibility, so we can replace B with V_∞

Legs are however overdescribed!

$$X_i = [t_i, V_{\infty 0i}, t_{DSM(i)}, V_{\infty fi}, t_{i+1}]$$

It is easier to handle $\|\mathbf{V}_\infty\|$ than V_∞!

sub problems leg description

Intermediate legs are thus well described:

$$X_i = [t_i, \|\mathbf{V}_{\infty \mathbf{0}}\|, t_{DSM(i)}, \|\mathbf{V}_{\infty \mathbf{f}}\|, t_{i+1}]$$

Unique leg ? It is likely that there is no 2 different trajectories of less than 1 revolution with the same X_i (not a proof!)

Global procedure

Decomposition

Now for all i, minimize the characteristic velocity of body to body subproblem i with the decision vector X_i, disregarded the other subproblems.

Patching

To construct complete trajectories, use the constraints:

$$CX = 0 s.t. \left\{ \begin{array}{l} B_{(i)} = \tilde{B}_{(i)} \\ t_i = \tilde{t}_i \end{array} \right. \tag{24}$$

C is sparse, $\tilde{.}$ are called coordination variables.

Subproblem optimization

As we lost information on the complete problem optimality, we need to cover the V_∞ - space with the coordination variable.

Procedure

For each leg $i \in \{1, 2, ..., N_p - 1\}$, for all $X_i = [t_{i-1}, t_i, V_{\infty 0i}, V_{\infty fi}, t_{DSMi}] \in {\mathbb{X}_t}^2 \times {\mathbb{X}_{V_\infty}}^2 \times \mathbb{X}_{t_{DSM}}$, solve the transfer problem from $S_{Body}(i)$ to $S_{Body}(i+1)$:

$$\min_{\alpha,\beta} J_i(X_i, \alpha_0, \beta_0, \alpha_f, \beta_f) = \sum_j \mathbf{\Delta V_j}$$

Where:

$$\mathbf{V_{\infty 0i}} = V_{\infty 0i} \mathbf{u}_{\alpha_0, \beta_0}$$
$$\mathbf{V_{\infty fi}} = V_{\infty fi} \mathbf{u}_{\alpha_f, \beta_f}$$

The minimization formulation permits to get the best trajectory in case the leg formulation is not consistent (unique leg).

Space Pruning

Pruning strategies

1. Initial hyperbolic excess velocity $\|\mathbf{V}_{\infty\mathbf{0}}\|$
2. Maximum ΔV_{DSM}
3. Forward/Backward Constraining [a]
   - The Backward/Forward constraining is applied both to the date t and the velocity V_∞.
4. RendezVous maneuver constraint $\|\mathbf{V}_{\infty\mathbf{f}}\|$
5. Swing by feasibility
   - No discrepancies are allowed on the oncoming and outgoing velocities V_∞.
   - Depending on the V_∞-grid, a minimum tolerance on the angular constraint should be considered.

[a] Becerra et al., An Efficient Pruning technique for the Global Optimisation of Multiple Gravity Assist Trajectories, Proceeding of GO 2005

Complete Algorithm

Algorithm

1. Decompose the problem into simple Body to Body transfers
2. Solve the subproblems on the grid (subproblem optimization procedure)

$$\min_u J_i(X_i, u) \tag{25}$$

3. Prune extremals on max ΔV_{DSM}
4. Patch extremals together using (swingby energetic conservation):

$$CX = 0 \tag{26}$$

   And accordingly with the encounter date.
5. Prune solutions which have $R_p < R_{pmin}$ (unfeasible swingby)

Complexity

Now we have to solve M problems in a search space of dimension 5, whereas in the initial approach we solved 1 problem in a search space of dimension $4M + 2$, where M is the number of phase.

Global search grid: ${\mathbb{X}_t}^2 \times {\mathbb{X}_{V_\infty}}^2 \times \mathbb{X}_{t_{DSM}}$ with:

$$\mathbb{X}_t = [t_0, t_f] \rightarrow \text{n bins}$$
$$\mathbb{X}_{V_\infty} = [V_{\infty min}, V_{\infty max}] \rightarrow \text{m bins}$$
$$\mathbb{X}_{t_{DSM}} = [0.1, 0.2, ..., 0.9] \rightarrow \text{9 bins}$$

Complexity (per leg)

$$C_i = O(n^2)O(m^2)$$

The solver used can strongly penalize the efficiency of the algorithm.

From GASP to DSM-GASP

Comparison:

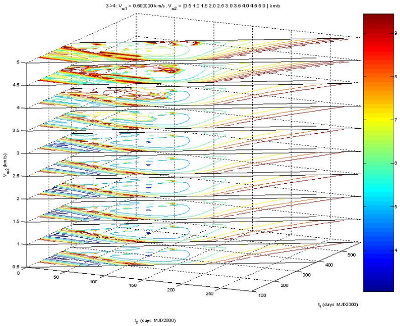
	GASP	DSM-GASP
Dimension	2	5
Time bins	n	n
Velocity bins	0	m
Complexity	$O(n^2)$	$O(n^2)O(m^2)$
Solver	none	SQP (SNOPT)

- Both algorithms use the "cascade" scheme.
- GASP is a particular case where $m = 1$ with appropriate values in $\mathbb{X}_{V_\infty}$ and $\mathbb{X}_{t_{DSM}} = 0$.
- The local solver strongly penalize the computation time (polynomial complexity but with a great multiplicative factor!). (might be a simplified formulation?!)

Plan

Examples

EM



- No easy representation.
- We consider the map $[V_{\infty f} \times t_0 \times t_f]$ for different value of $V_{\infty 0}$
- Constraining the ΔV_{DSM} immediately prunes the search space.

Methodology: some ideas

Multi dimensionnal problem

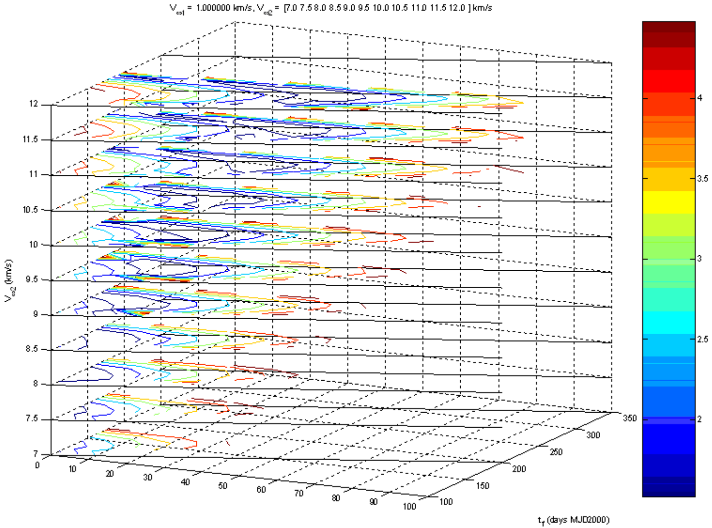
- No easy representation.
- We can use projection in 3D spaces: need a very painstaking job!

Some ideas:

- Consider to prune only the initial phase and the final phase solution space.
  - Forward / Backward propagations give reduced box bounds for intermediate phase.
- Consider only the time and date for the intermediate phasse.
  - The V_∞ in the intermediate phase are difficult to handle.

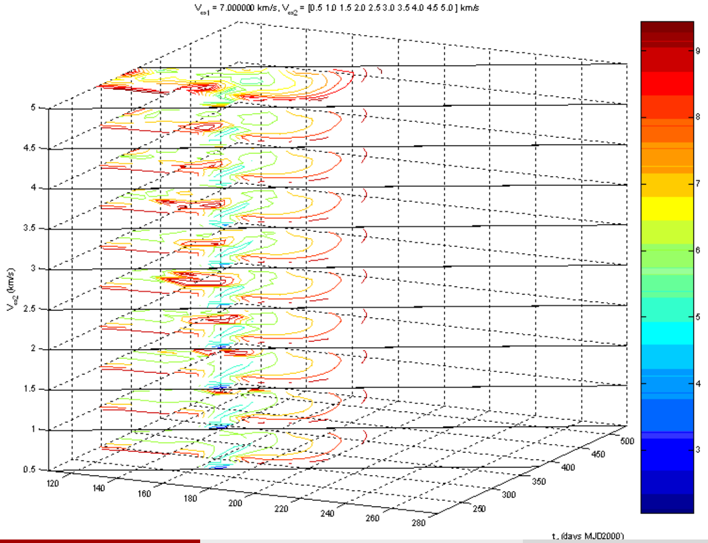
Examples

EVM - EV



Examples

EVM - VM



Plan

Conclusion

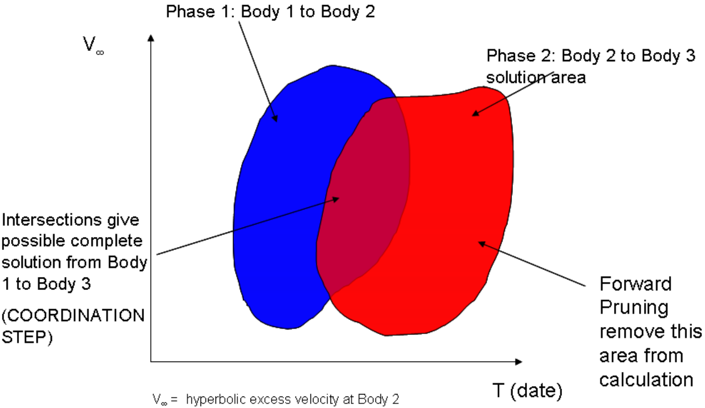
- Introduced the MGADSM problem and different formulations
- Deterministic method for optimizing MGADSM trajectories without constraint on the number of DSM
- Heuristic method to finding optimal solution in the remaining search space
- Deterministic method for pruning out infeasible parts of the search space
- Many possible improvements of ”our GASP” algorithm
  - Speed of convergence of the local solver
  - graphical intuitive representation
  - clear methodology to handle the results

Thanks for your attention!

Thanks to the ACT-team for the experience.

Questions ?

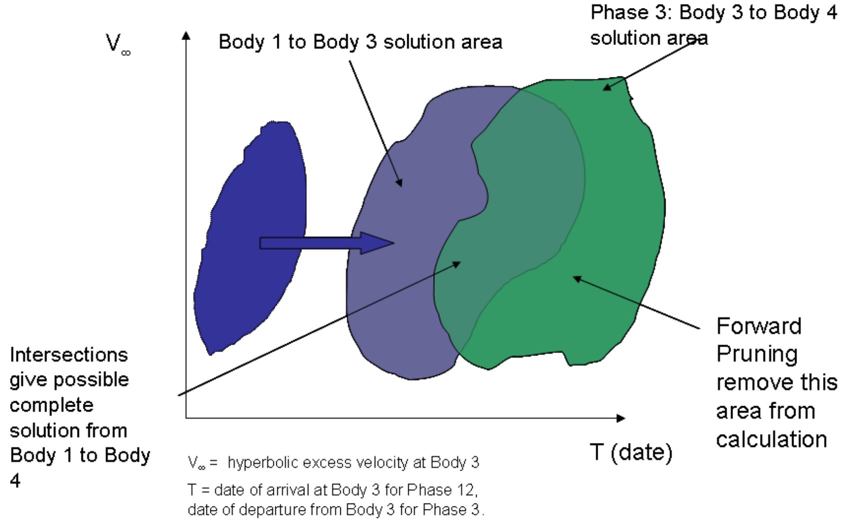
Algorithm



V_∞ = hyperbolic excess velocity at Body 2

T = date of arrival at Body 2 for Phase 1,
date of departure from Body 2 for Phase 2.

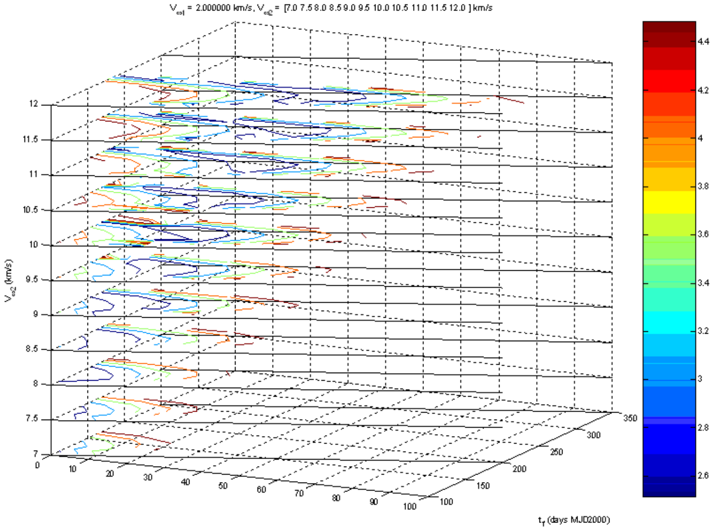
Algorithm



V_∞
Body 1 to Body 3 solution area
Phase 3: Body 3 to Body 4 solution area
Intersections give possible complete solution from Body 1 to Body 4
Forward Pruning remove this area from calculation
T (date)
V_∞ = hyperbolic excess velocity at Body 3
T = date of arrival at Body 3 for Phase 12, date of departure from Body 3 for Phase 3.

Examples

EVM - EV



Examples

EVM - VM