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Global Trajectory Optimisation: Can We Prune the Solution Space when Considering Deep Space Maneuvers?

Final Presentation

Politecnico di Milano - Michigan State University

ESA/ESTEC, Noordwijk, The Netherlands
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Team Composition

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- **Michigan State University**
  - Prof. Martin Berz

    DA computation and hints on the implementation of GASP-DA
- **Acknowledgments**

We want to stress the fruitful support of Dr. Kyoko Makino (MSU) for the validated global optimization

Outline

- **Introduction to Differential Algebra (DA)**
- **Pruning of MGA transfers using DA (GASP-DA)**
  - Time of flight approach
    - discontinuities analysis
    - dependency problem
  - Absolute time approach
  - Objective function semi-analytical approximation
  - Non-validated quadratic bounder
  - Test cases
- **Introduction of DSM in GASP-DA**
  - Forward propagation approach
  - Absolute variables approach
  - Test cases

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Outline

- **Alternative strategy: sequential GASP-DA + DSM**
  - Post-processing GASP results
    - Solution set selection
    - DSM modeling
  - Test cases
- **Conclusions on DA-based MGA transfers pruning**
- **Extra-Schedule Application: Validated Optimization of MGA Transfers**
  - Introduction to Taylor Models
  - Main problem
  - Verified optimization of planet-to-planet transfers: Earth-Mars
  - Verified optimization of MGA transfers: Earth-Venus-Mars
  - Conclusions and future works

Differential Algebra: Some History

- **Differential Algebra (DA)** is an **automatic differentiation** technique
- DA was first developed by Martin Berz in the late '80s:
  - **1986**
    Definition of the algebra of Taylor Polynomials in the so-called Truncated Power Series Algebra (TPSA).
  - **1987**
    Introduction of methods to treat common elementary functions and the operations on them
  - **1989**
    Introduction of the analytic operations of differentiation and integration (Differential Algebra)
  - **1998**
    Validated Remainder Enhanced Differential Algebra (Taylor Models)

(1989 and 1998:) Implementation in **COSY-Infinity**

Introduction to Differential Algebra

- The basic idea is to bring the treatment of **functions** and the operations on them to the computer in a similar way as the treatment of **numbers**

$$\begin{array}{ccc} a,b\in\Re & \xrightarrow{\;\mathcal{T}\;} & \bar{a},\bar{b}\in FP \\ \downarrow{*} & & \downarrow{\circledast} \\ a*b & \xrightarrow[\;\mathcal{T}\;]{} & \bar{a}\circledast\bar{b} \end{array} \quad \Longrightarrow \quad \begin{array}{ccc} f,g & \xrightarrow{\;\mathcal{T}\;} & F,G \\ \downarrow{*} & & \downarrow{\circledast} \\ f*g & \xrightarrow[\;\mathcal{T}\;]{} & F\circledast G \end{array}$$

- Real numbers are approximated by floating point numbers
- For each $*$, adjoint $\circledast$ can be crafted on floating point numbers

- $\mathcal{T}$ is the extraction of Taylor coefficients (equivalence relation)
- The new space can be endowed with corresponding operations

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Minimal Differential Algebra

First order Differential Algebra

- Consider the set of all ordered pairs of reals (a_0, a_1)
- Define the operations:

$$\left.\begin{aligned}(a_0, a_1) + (b_0, b_1) &:= (a_0 + b_0, a_1 + b_1) \\ t \cdot (a_0, a_1) &:= (t \cdot a_0, t \cdot a_1) \\ (a_0, a_1) \cdot (b_0, b_1) &:= (a_0 \cdot b_0, a_0 \cdot b_1 + a_1 \cdot b_0)\end{aligned}\right\} \begin{matrix} \textit{Algebra} \\ {}_1D_1 \end{matrix}$$

$$(a_0, a_1)^{-1} := (1/a_0, -a_1/a_0^2)$$

- The previous algebra allows the automatic computation of derivatives. E.g.:
  - Assume to have f and g, and to put their values and derivatives at the origin in ${}_1D_1$: $(f(0), f'(0))$ and $(g(0), g'(0))$
  - Evaluate:

$$(f(0), f'(0)) \cdot (g(0), g'(0)) = (f(0) \cdot g(0), f'(0) \cdot g(0) + f(0) \cdot g'(0))$$

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Minimal Differential Algebra

- This observation can be used to compute derivatives of many functions starting from the ordered pair corresponding to the identity function $x + x_0$ ► $(x_0, 1)$

  E.g.:

$$f(x) = \frac{1}{x + 1/x} \qquad f'(x) = \frac{1/x^2 - 1}{(x + 1/x)^2}$$

$$f(3) = \frac{3}{10} \qquad f'(3) = -\frac{2}{25}$$

Symbolic manipulators approach

Evaluating f in $x + 3$ ► $(3, 1)$ using the previous algebra:

$$f((3,1)) = \frac{1}{(3,1) + 1/(3,1)} = \frac{1}{(3,1) + (1/3, -1/9)} = \frac{1}{(10/3, 8/9)} = \left(\frac{3}{10}, -\frac{2}{25}\right)$$

- Important implementation advantages:
  - ordered pairs ► new variable type
  - algebra ► operator overloading

General Differential Algebra ${}_nD_v$

- ${}_1D_1$ can be generalized to ${}_nD_v$ for function of v variables and the **arbitrary** order n

$$\underbrace{(a_0, a_1)}_{{}_1D_1} \;\rightarrow\; \underbrace{(\ldots, c_{j_1,\ldots,j_v}, \ldots)}_{{}_nD_v} \qquad j_1 + \ldots + j_v \leq n$$

a vector in ${}_nD_v$ is a collection of all the **Taylor coefficients** of the function f w.r.t the v variables up to the order n

- ${}_nD_v$ can be further extended to treat any **transcendental function** (sin, cos, exp, log, etc.)

▼

- Real algebra is substituted by Taylor polynomial algebra
- Starting from the Taylor polynomial of the identity function, the DA computation of f returns its Taylor expansion

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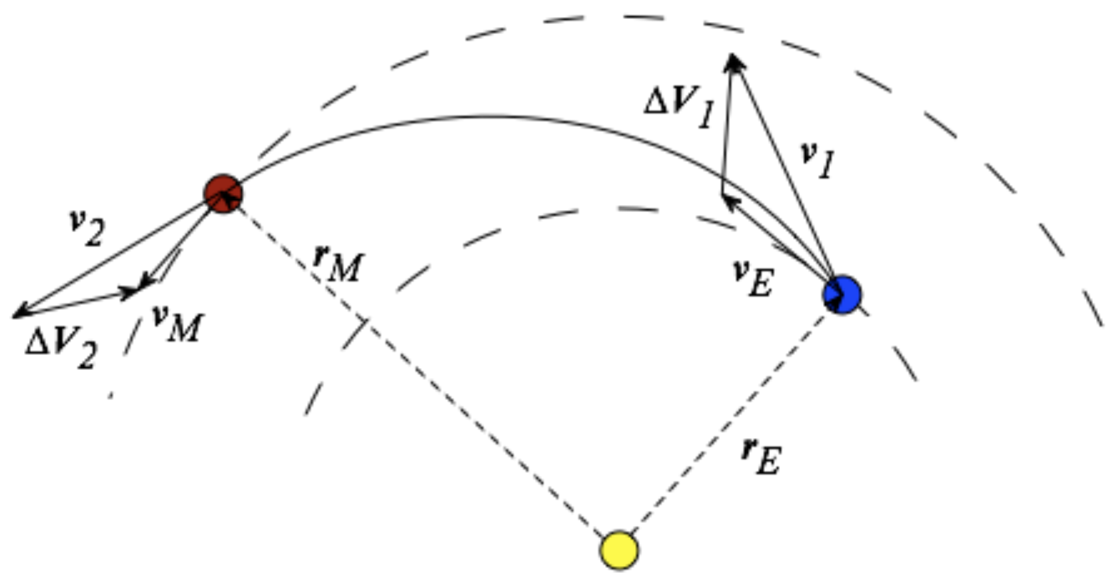
Pruning of MGA Transfers: Problem Formulation

- Example: Direct Earth-Mars transfer

  - The positions of the starting and arrival planets are computed through the ephemerides evaluation:
    $$(\mathbf{r}_E, \mathbf{v}_E) = eph(T_E, \text{Earth}) \qquad (\mathbf{r}_M, \mathbf{v}_M) = eph(T_M, \text{Mars})$$
  - The starting velocity $\mathbf{v}_1$ and the final one $\mathbf{v}_2$ are computed by solving the Lambert's problem
  - Objective function:
    $$\Delta V = \Delta V_1 + \Delta V_2$$
  - Pruning constraints:
    $$\Delta V_1 < \Delta V_{1,max}$$
    $$\Delta V_2 < \Delta V_{2,max}$$

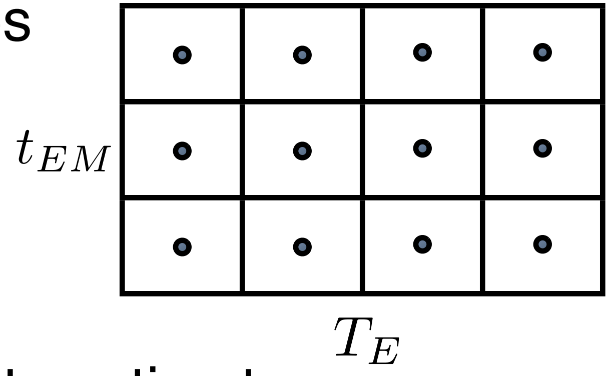


Differential Algebra - Based GASP

Main Idea:

The point-wise evaluation of the objective function in GASP is substituted by a DA-based evaluation

- The search space is subdivided in boxes
- The Taylor expansion of the objective function is computed within each box (the center is used as reference point)



- The resulting polynomials are bounded to estimate the range of the objective function over each box
- The resulting range is used in the pruning process

Advantages:

- Wider sampling of the search space
- Lipschitz's constant avoidance
- Availability of analytical information

Objective Function Evaluation

- A set of nonlinear equations must be solved
- Planet-to-planet transfers
  - **Ephemeris evaluation**
    An analytical model is available to obtain e and M as a function of the epoch. The Kepler's equation must be solved:
    $$f(E) = E - e\sin E - M = 0$$
  - **Lambert problem**
    An algorithm developed by Izzo has been used. The Lagrange equation for the time of flight must be solved:
    $$f(x) = \log(A(x)) - \log(t_{tof}) = 0$$
- MGA transfers
  - **Bending angle equation**
    $$f(r_p) = \arcsin\frac{a^-}{a^- + r_p} + \arcsin\frac{a^+}{a^+ + r_p} - \alpha = 0$$

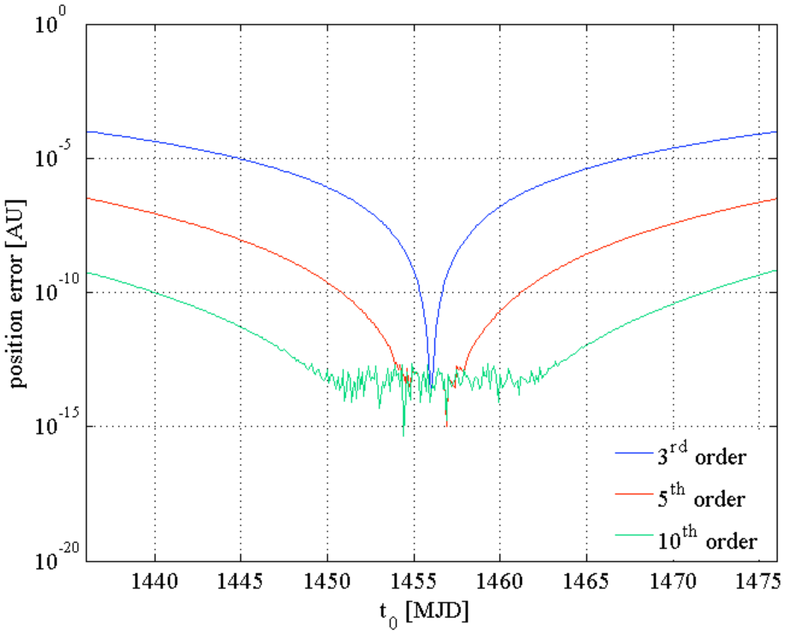
Parametric Implicit Equations

- Taylor expanding the objective function leads to the necessity of expanding the solution of the implicit equations
- E.g.: Kepler's equation. Given a reference epoch t^0
  - The DA evaluation of the analytical model gives: $e(\delta t)$, $M(\delta t)$
  - Kepler's equation becomes a parametric implicit equation:

$$f(E, \delta t) = E - e(\delta t) \sin E - M(\delta t) = 0$$

  - We need to solve the previous equation for $E(\delta t)$

- **Algorithm overview**:
  - A point solution is computed at the reference point (Newton method)
  - The solution is expanded around the computed one



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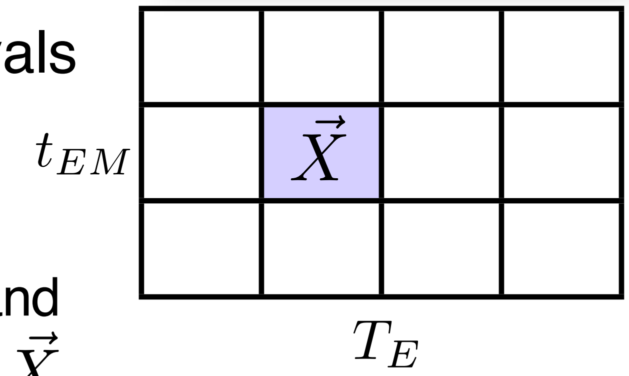
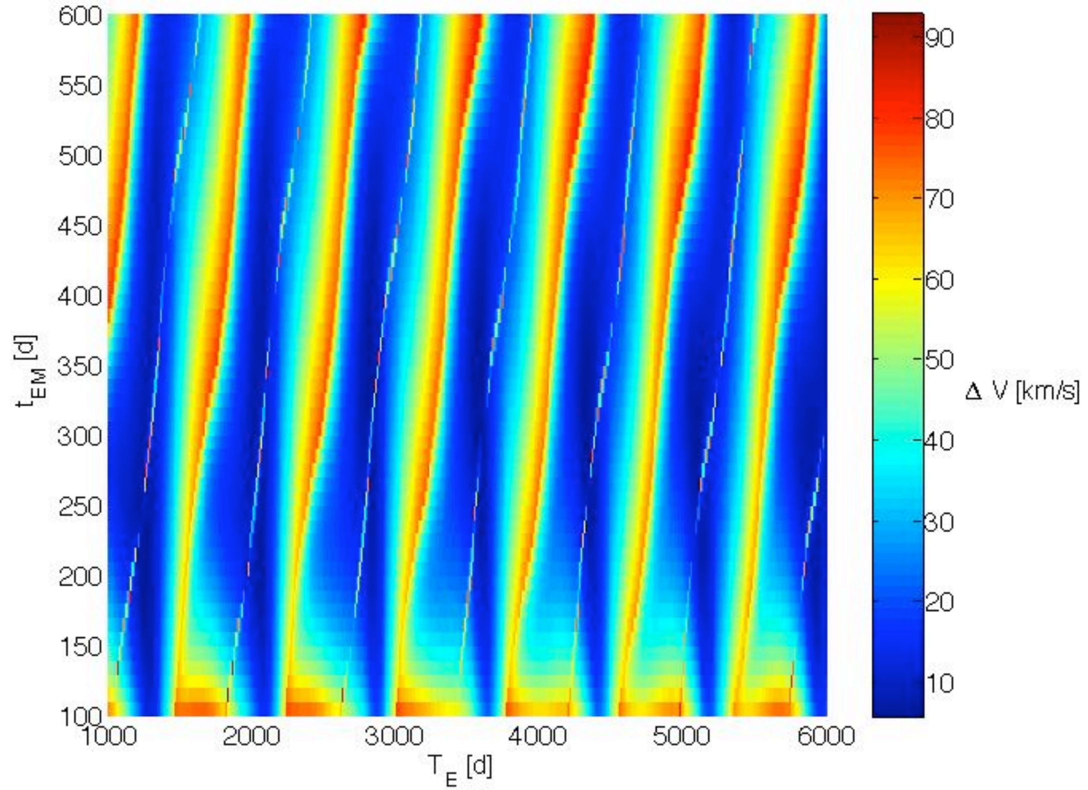
DA Based Pruning: ToF Approach



- Subdivide the search space in subintervals
- For each subinterval
  - Initialize T_E and t_{EM} as DA variables and compute the Taylor expansion of ΔV_1 on $\vec{X}$
  - Bound the polynomial expansion of ΔV_1 on $\vec{X}$
  - IF $\min \Delta V_1 > \Delta V_{1,max} \longrightarrow$ discard $\vec{X}$ and analyze the next subinterval
  - Compute the Taylor expansion of ΔV_2 on $\vec{X}$
  - Bound the polynomial expansion of ΔV_2 on $\vec{X}$
  - IF $\min \Delta V_2 > \Delta V_{2,max} \longrightarrow$ discard $\vec{X}$ and analyze the next subinterval

Earth-Mars Transfer: ToF Approach

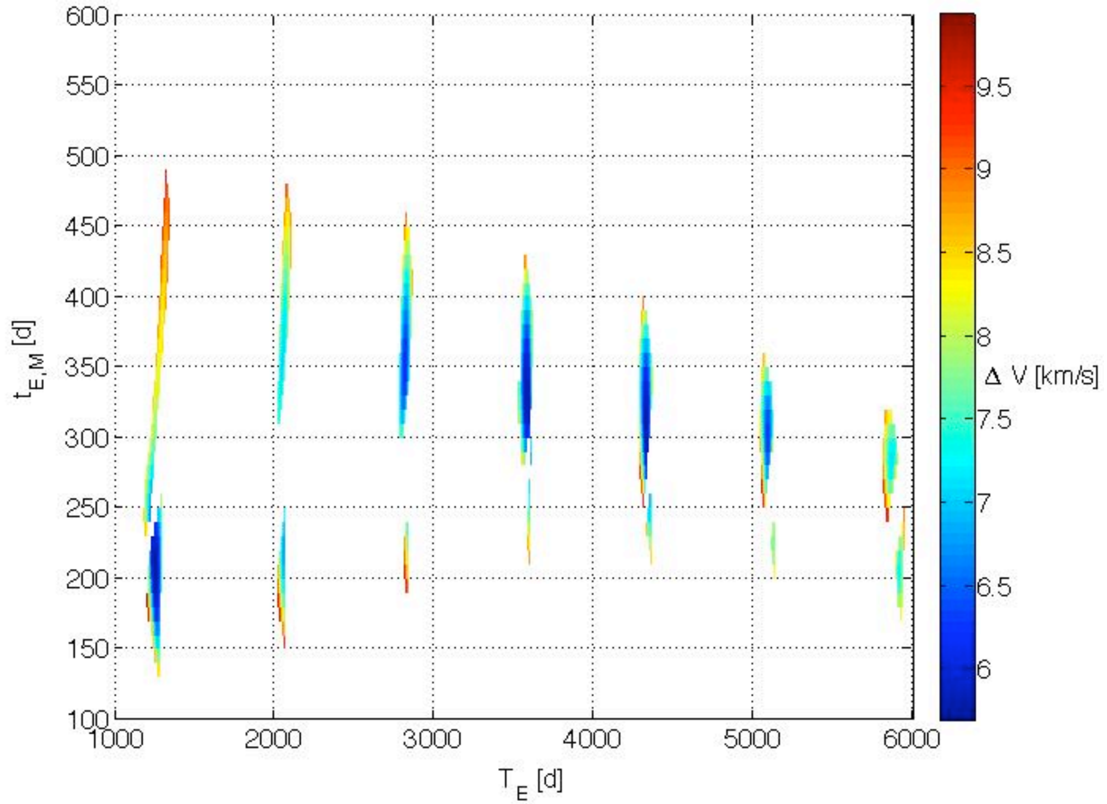
- Example: Direct Earth-Mars transfer
  - The optimization variables are the departure epoch T_E and the time of flight $t_{EM} = T_M - T_E$
  - Search space: $[1000, 6000] \times [100, 600]$



Objective function structure

Earth-Mars Transfer: ToF Approach

- Example: Direct Earth-Mars transfer
  - Pruning constraints: $\Delta V_1, \Delta V_2 < 5$ km/s



Pruned search space

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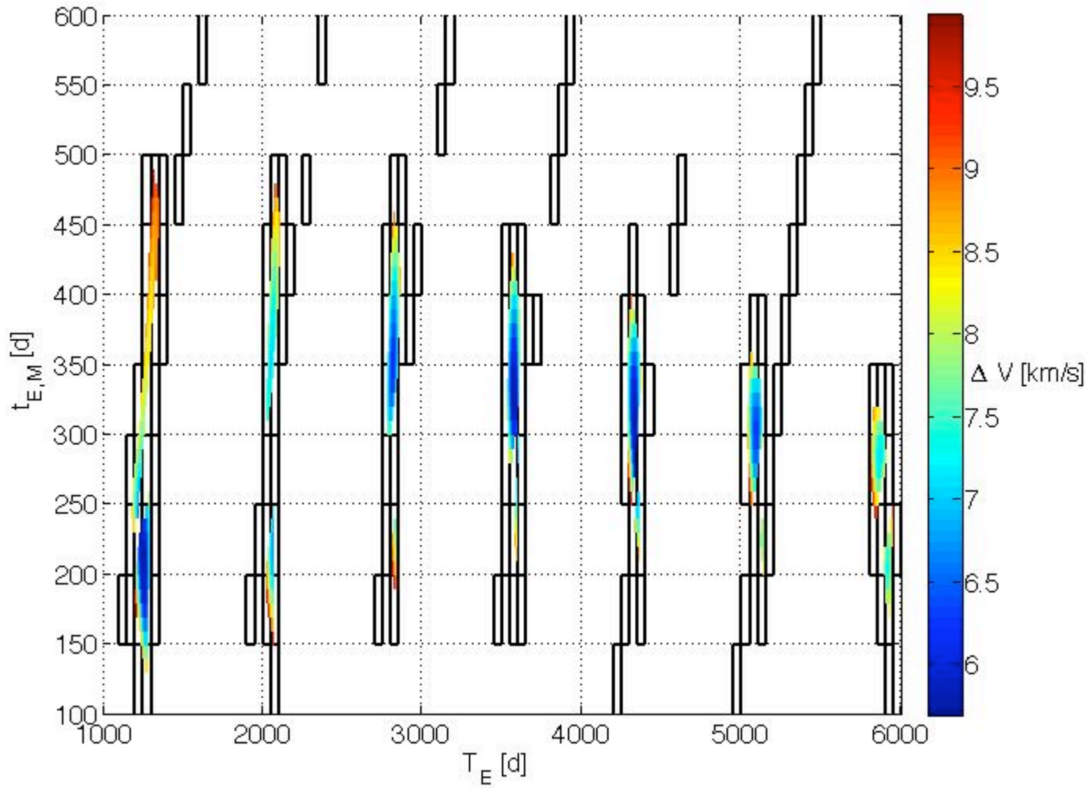
Earth-Mars Transfer: ToF Approach

- Example: Direct Earth-Mars transfer
  - Pruning constraints: $\Delta V_1, \Delta V_2 < 5$ km/s
  - Box size: 50 x 50 days



Remaining boxes

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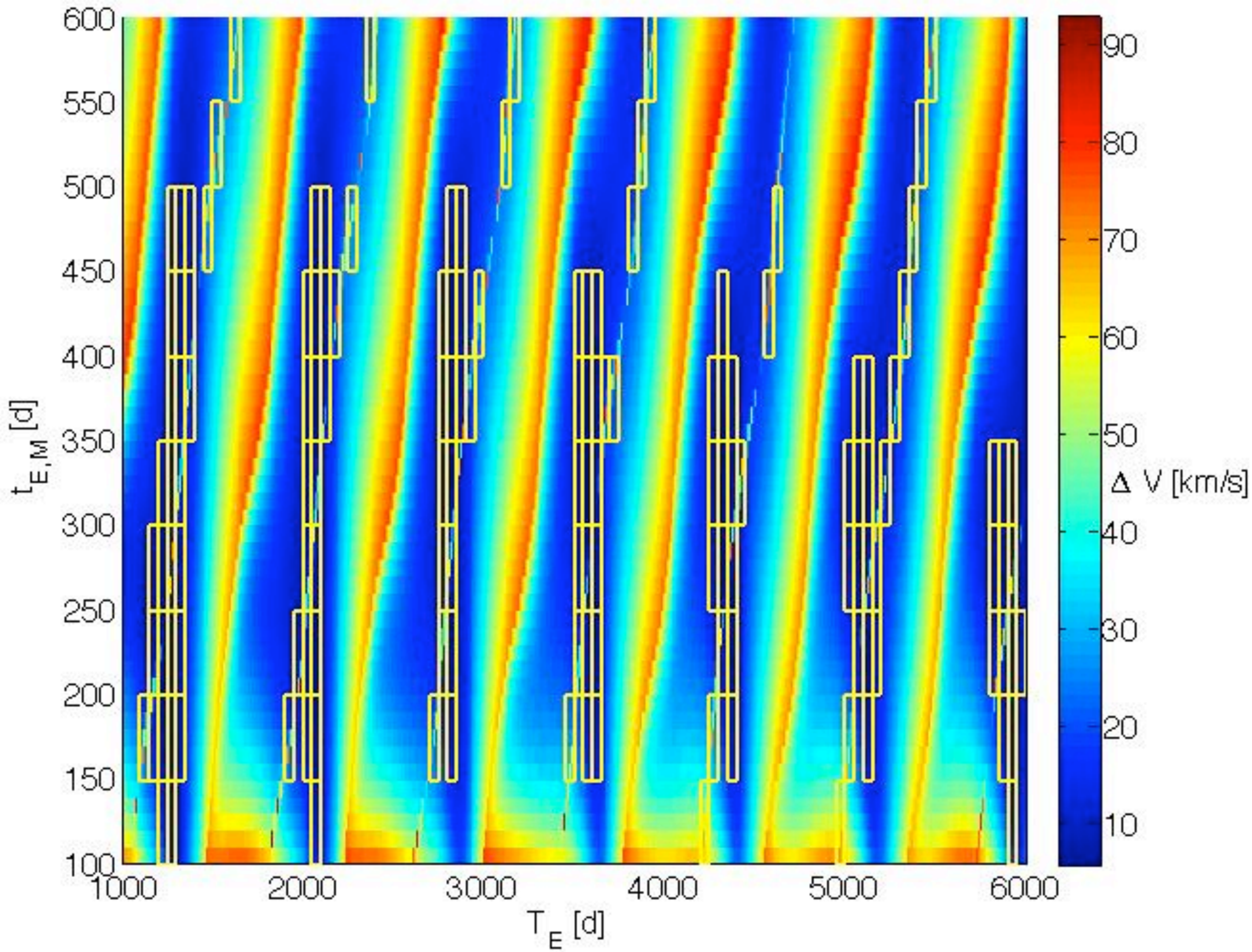
Earth-Mars Transfer: ToF Approach

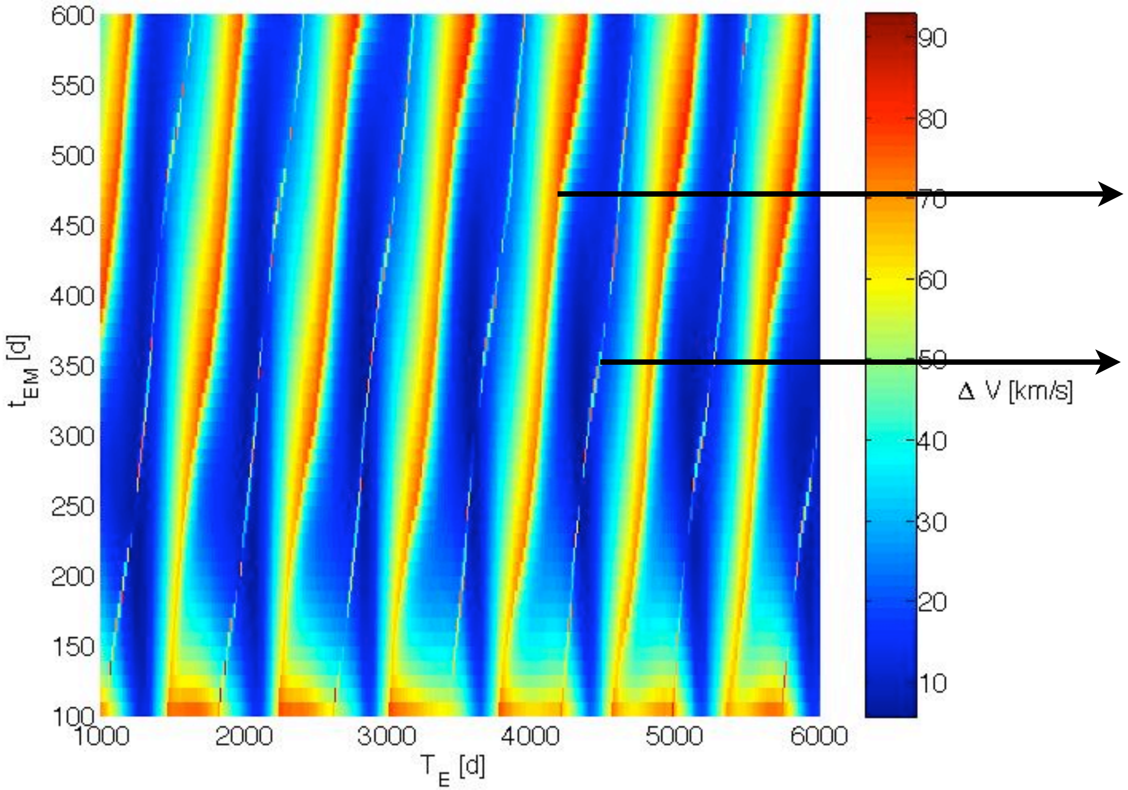
- Example: Direct Earth-Mars transfer



Remaining boxes

Earth-Mars Transfer: Discontinuities

- Example: Direct Earth-Mars transfer
  - The optimization variables are the departure epoch T_E and the time of flight $t_{EM} = T_M - T_E$
  - Search space: $[1000, 6000] \times [100, 600]$



Objective function structure

Discontinuities:

From "long way" to "short way"

From "short way" to "long way"

- The transfer orbit is perpendicular to the ecliptic

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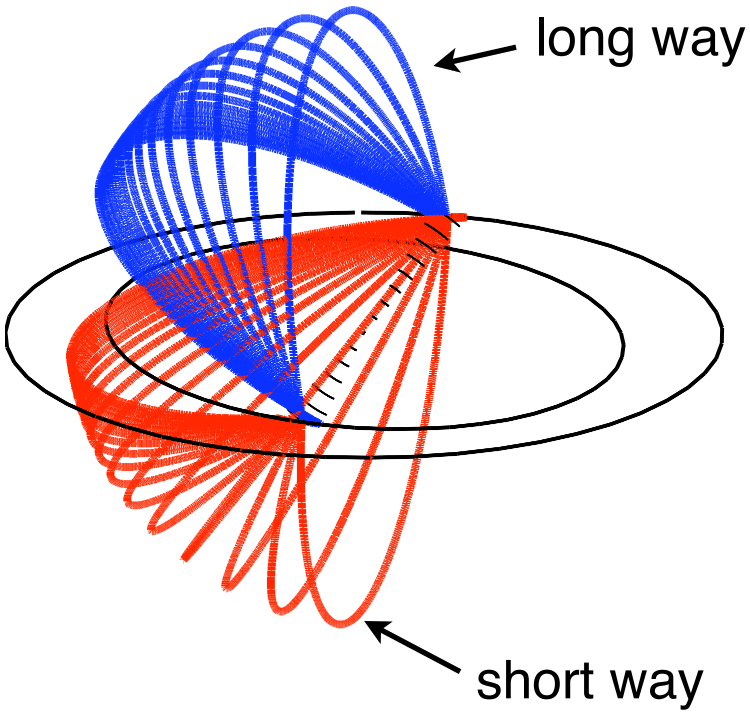
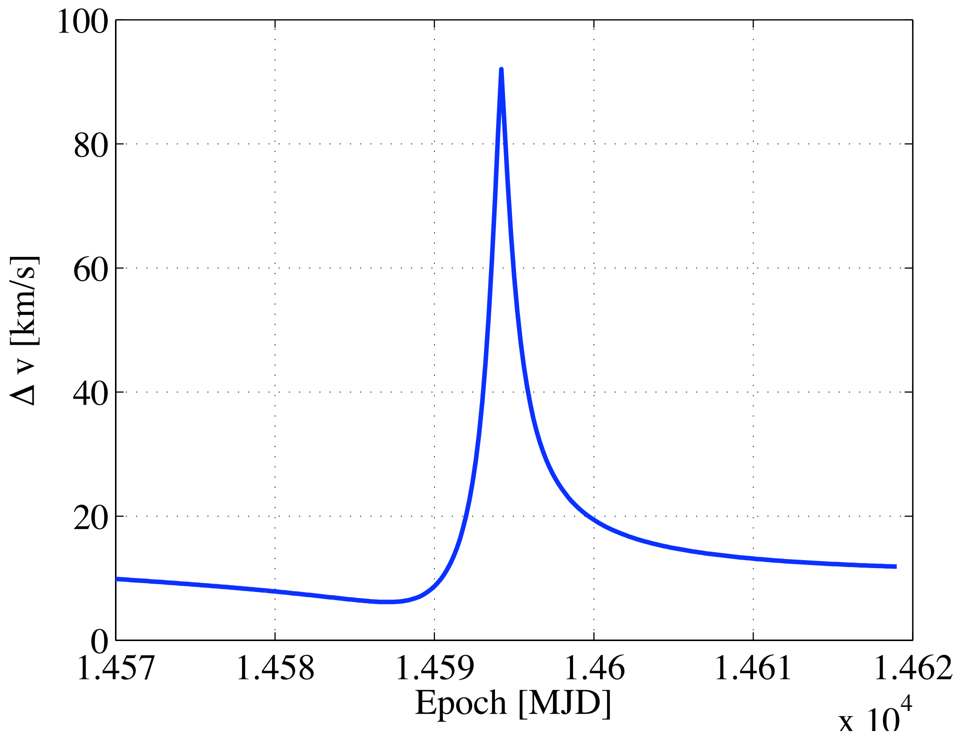
Discontinuity Problem

- Example: Direct Earth-Venus transfer
- “Short way” to “long way” transition:

Geometrical View



Objective function discontinuity



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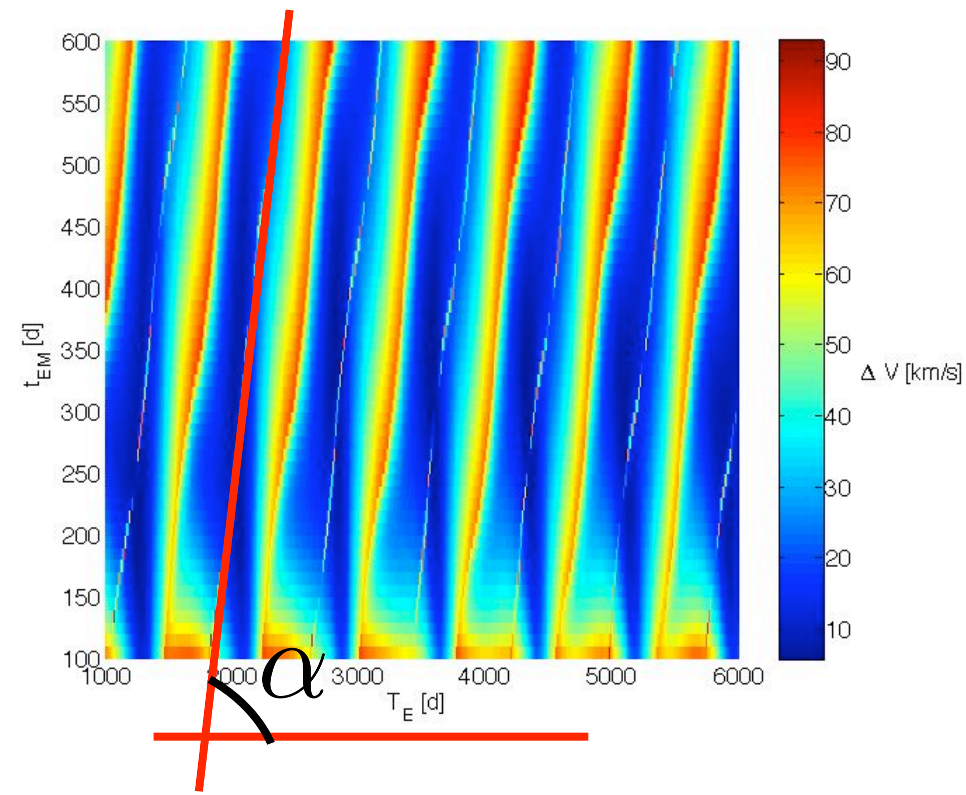
Discontinuity Problem: Box Reshaping

- The discontinuities correspond to the transition from "short way" to "long way" and vice versa in the Lambert solver
- In case of circular and coplanar orbits:
  - they would be straight lines
  - their slope could be easily computed using the orbital periods:

$$\tan\alpha = \frac{P_M}{P_E} - 1$$

- Based on the previous observations we can suitably reshape the boxes



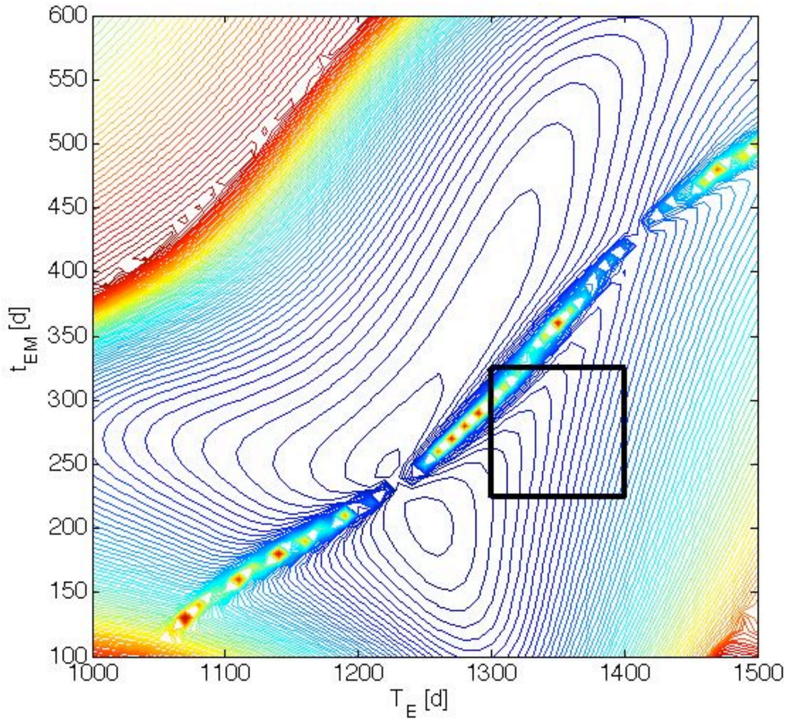
Discontinuity Problem: Box Reshaping

- Rectangular box:

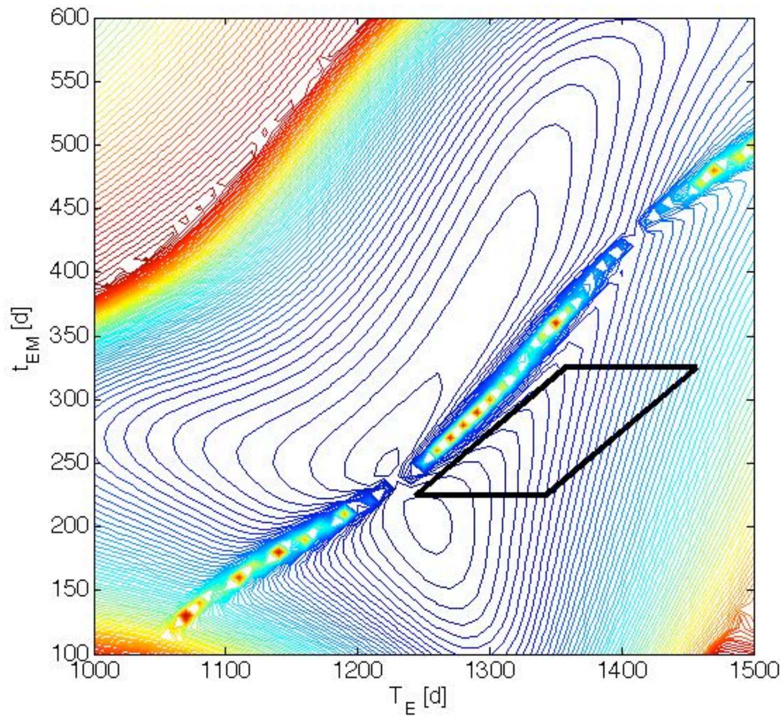
$$\vec{X} = [T_E + \Delta T_E, t_{EM} + \Delta t_{EM}]$$

- Reshaped box:

$$\vec{X} = [T_E + \Delta T_E + (1/\tan\alpha) \cdot \Delta t_{EM}, t_{EM} + \Delta t_{EM}]$$



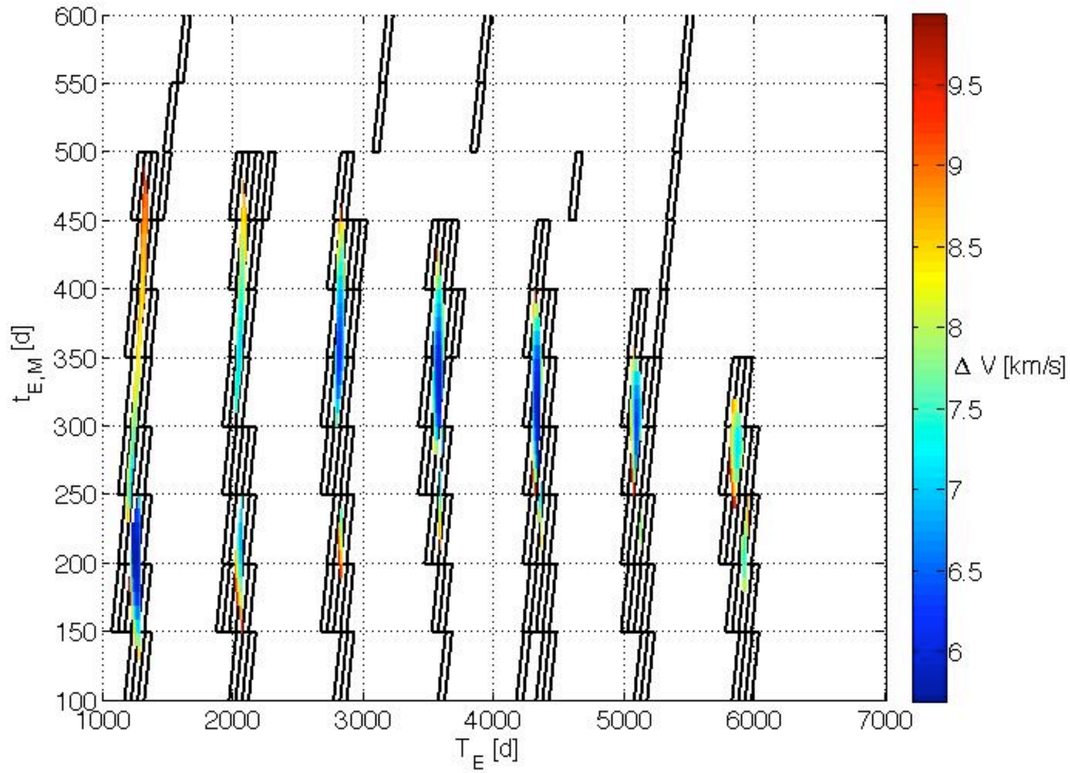
Rectangular box



Reshaped box

Discontinuity Problem: Box Reshaping

- Example: Direct Earth-Mars transfer
  - Pruning constraints: $\Delta V_1, \Delta V_2 < 5$ km/s
  - Box size: 50 x 50 days



Remaining boxes

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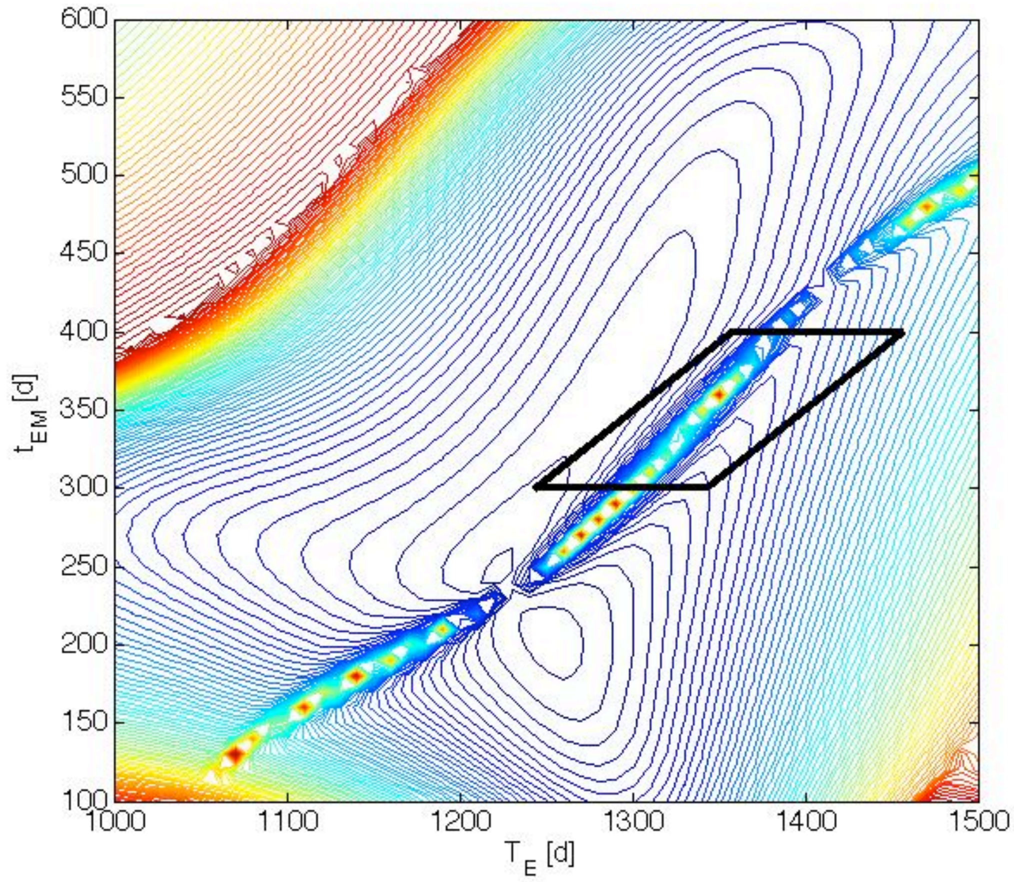
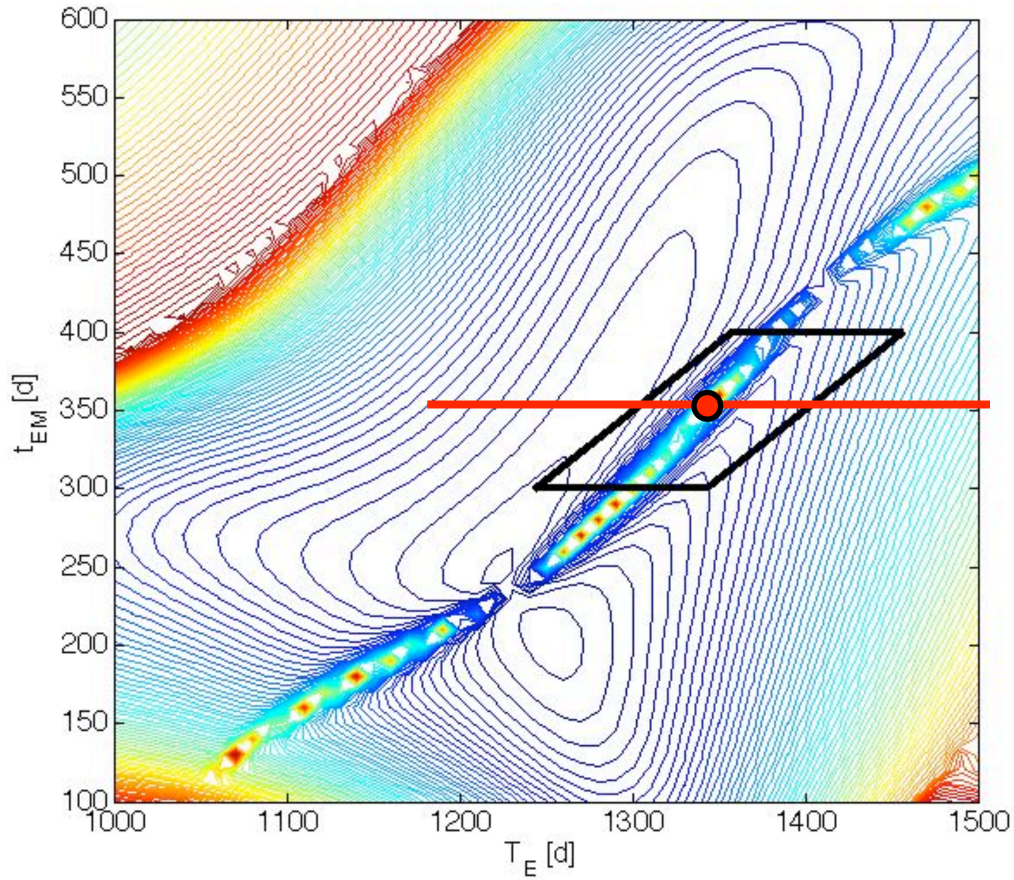
Discontinuity Problem: Box Splitting

- Suppose a box lying on the discontinuity is being analyzed



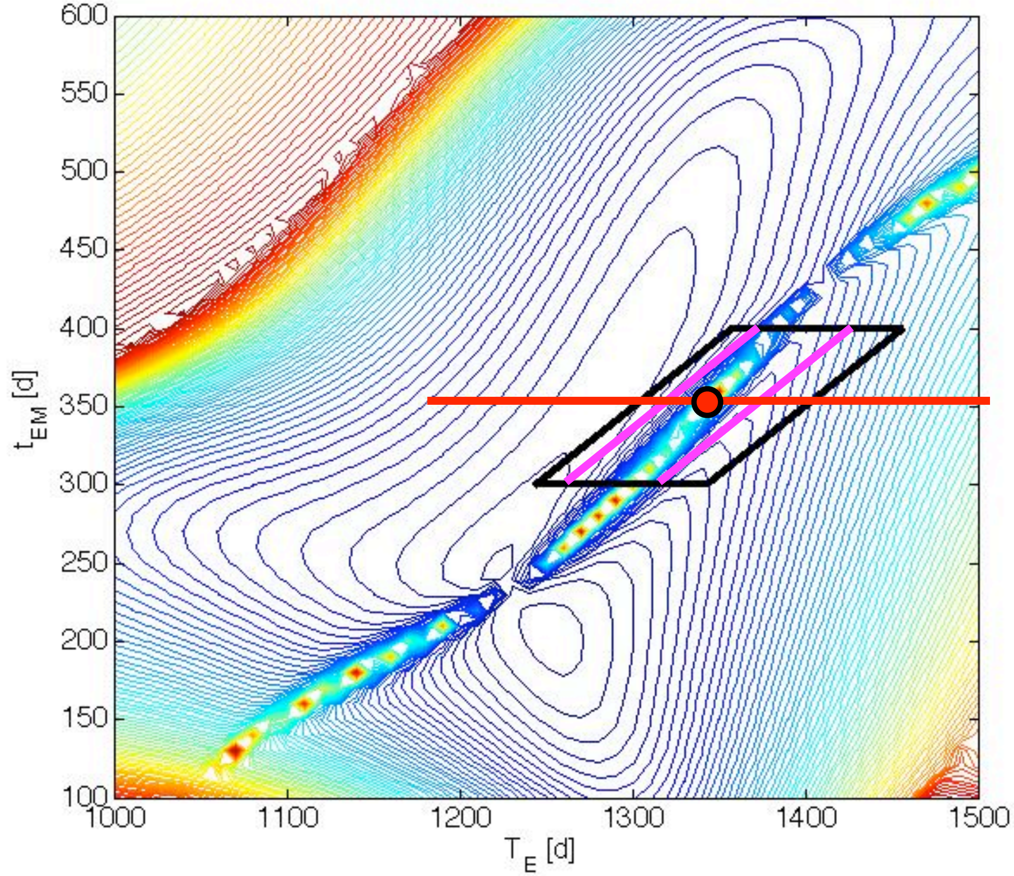
Discontinuity Problem: Box Splitting

- Suppose a box lying on the discontinuity is being analyzed
- Moving on a horizontal line, identify a point lying on the discontinuity



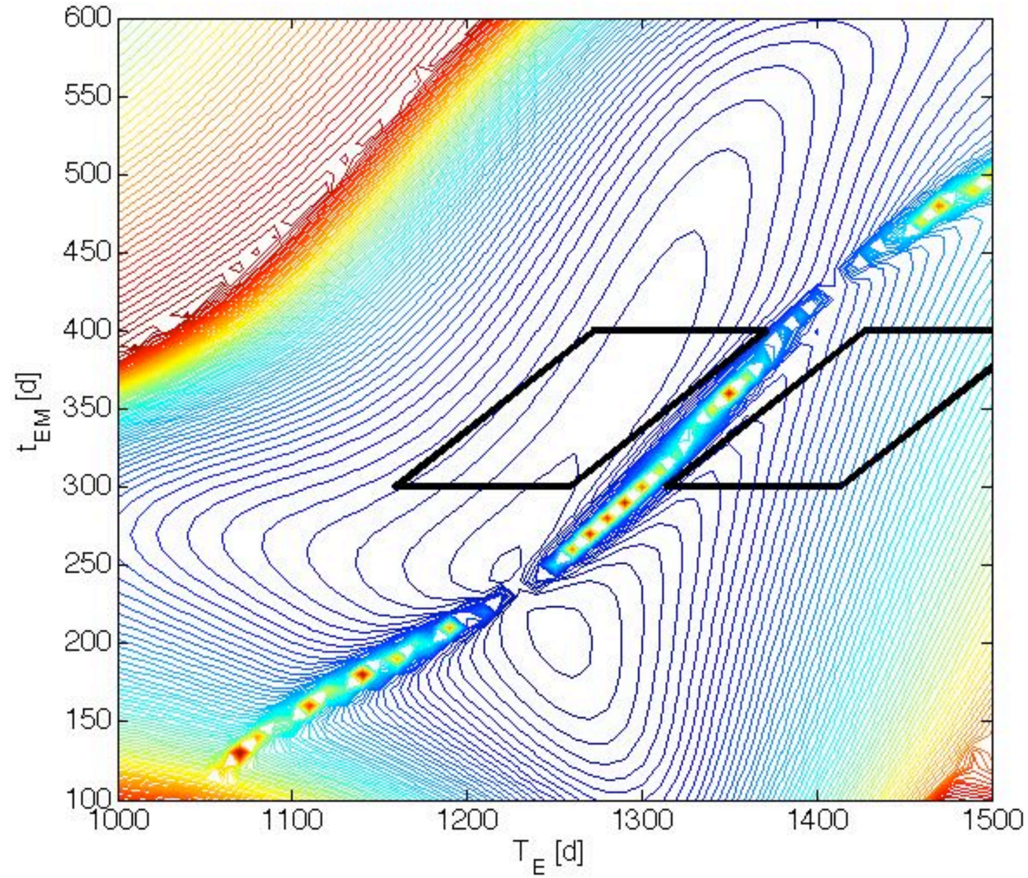
Discontinuity Problem: Box Splitting

- Suppose a box lying on the discontinuity is being analyzed
- Moving on a horizontal line, identify a point lying on the discontinuity
- Enclose the discontinuity in a strip



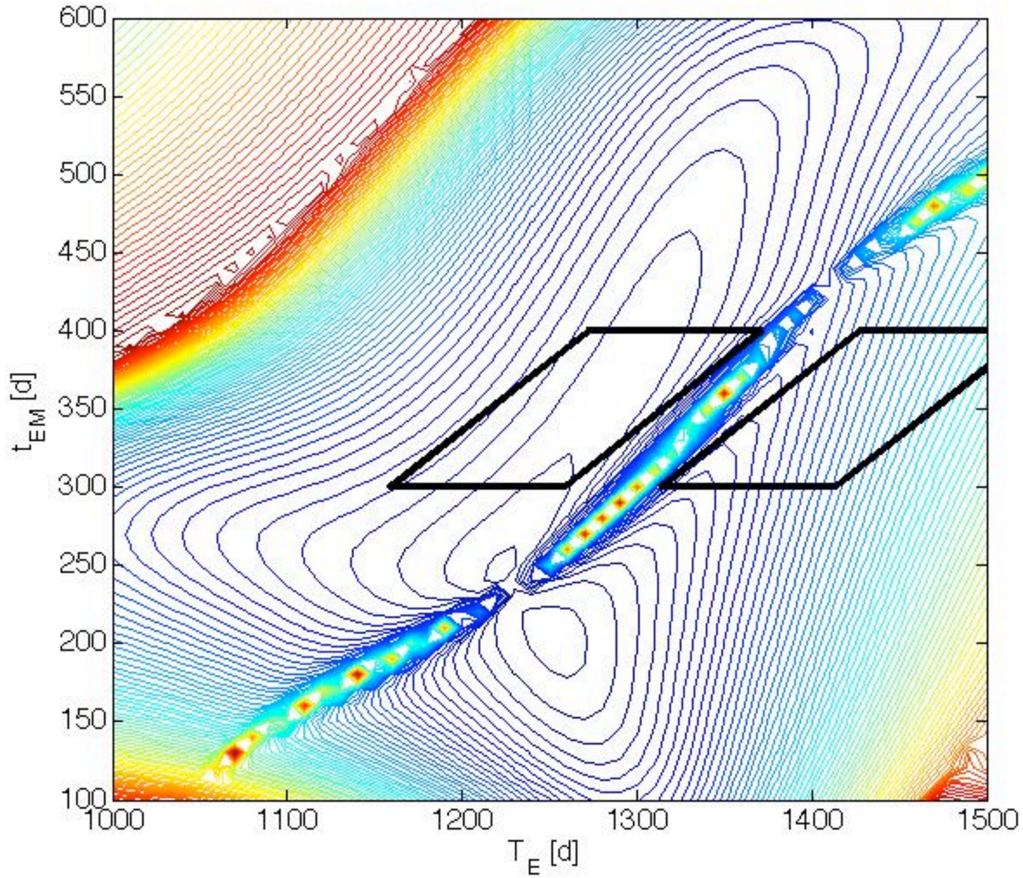
Discontinuity Problem: Box Splitting

- Suppose a box lying on the discontinuity is being analyzed
- Moving on a horizontal line, identify a point lying on the discontinuity
- Enclose the discontinuity in a strip
- Identify and process two "discontinuity-free" boxes



Discontinuity Problem: Box Splitting

- Suppose a box lying on the discontinuity is being analyzed
- Moving on a horizontal line, identify a point lying on the discontinuity
- Enclose the discontinuity in a strip
- Identify and process two "discontinuity-free" boxes
- Problems and Drawbacks:
  - Difficult assessment of lying conditions
  - Computational time increase



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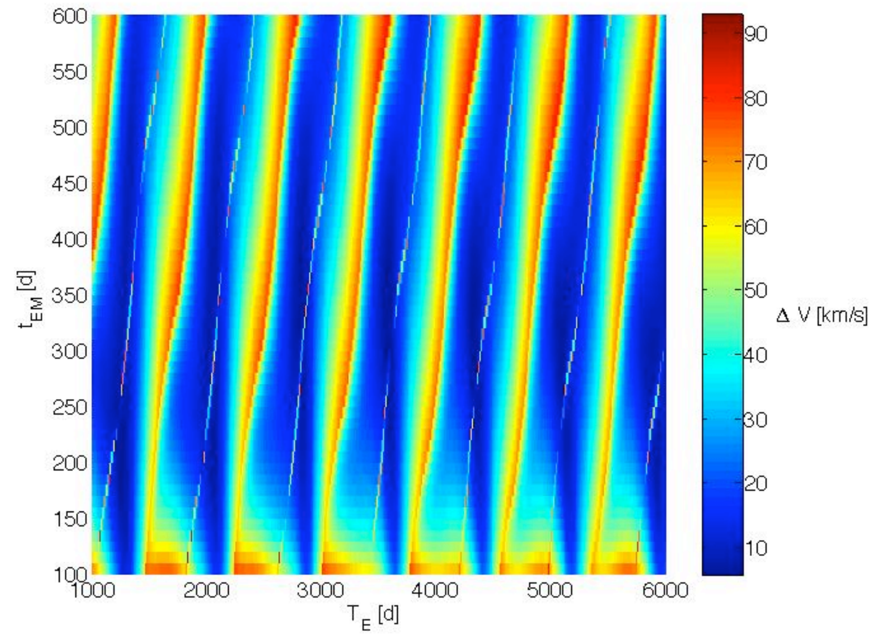
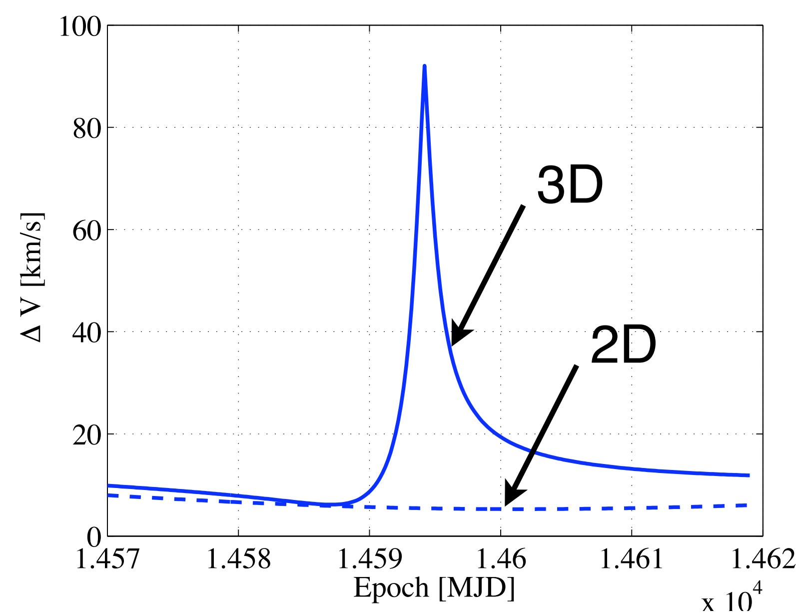
Discontinuity Problem: Planar Model

- The unfavorable discontinuity lines correspond to the transition from "short way" to "long way" in the Lambert solver



- The previous discontinuity does not occur in a **planar** planetary model (the orbital plane of the Lambert's arc is uniquely determined)



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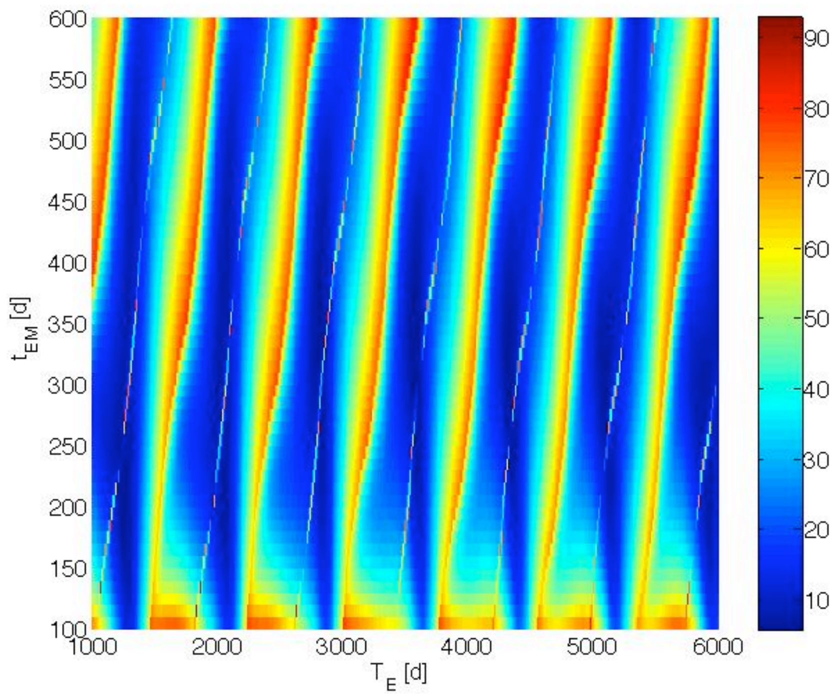
Discontinuity Problem: Planar Model

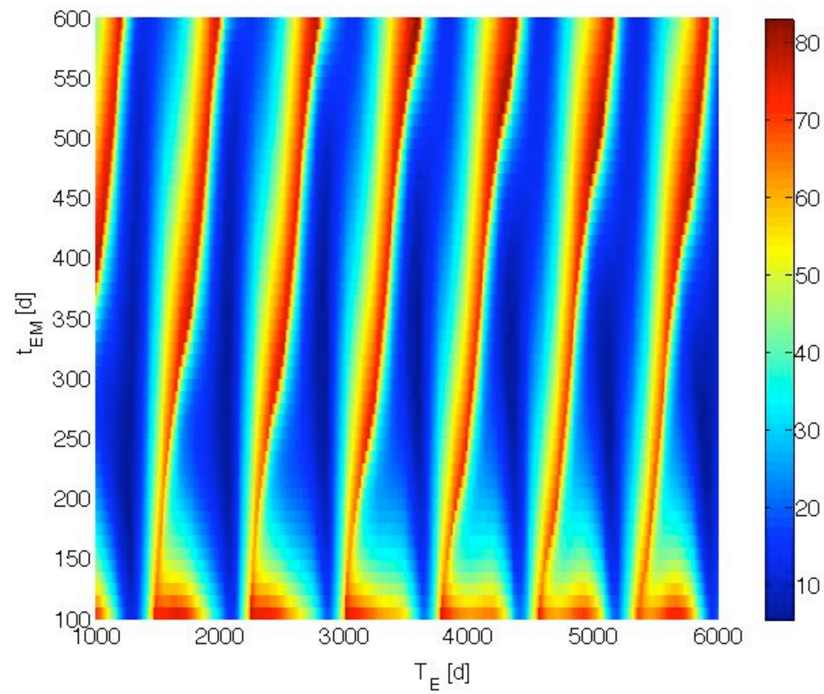
- **Adopted solution**:

  Given the previous consideration and the low inclination of all planetary orbits, a planar Solar System model has been adopted to perform the **pruning process**

Objective Function Comparison



3D-Model



2D-Model

Discontinuity Problem: Planar Model

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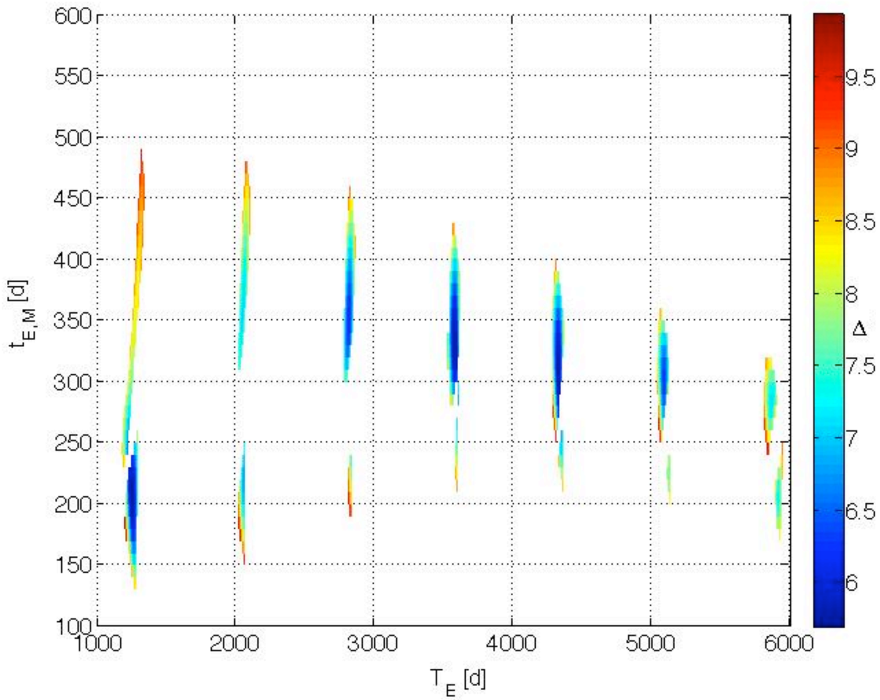
- **Main observation**:

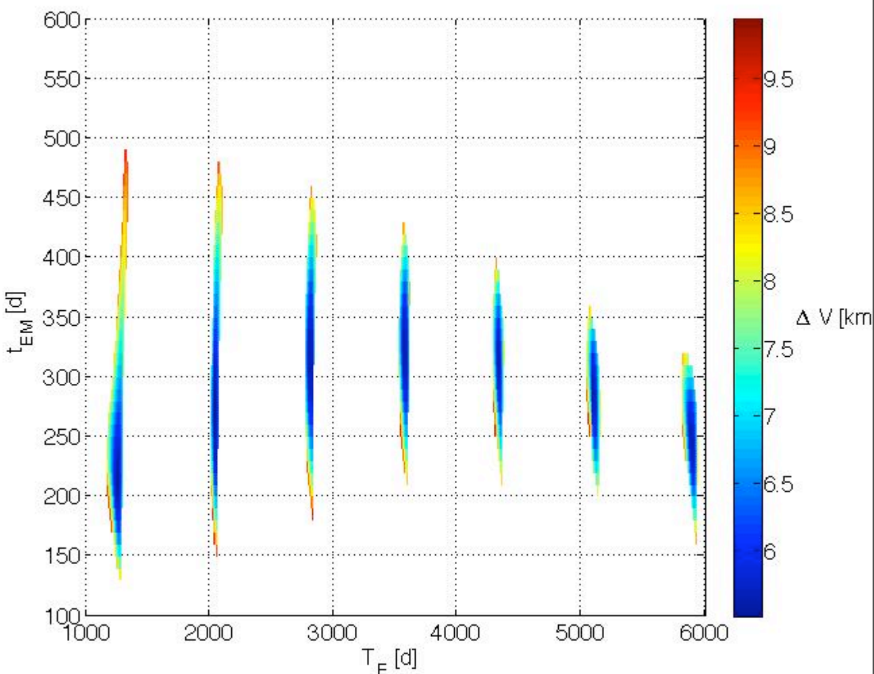
$$\Delta V_{err} = \Delta V_{3D} - \Delta V_{2D} \geq 0 \quad \rightarrow \quad \Delta V_{3D} \geq \Delta V_{2D}$$

The pruned search space in the 2D-Model encloses the pruned search space in the 3D-Model

Pruned Search Spaces



3D-Model

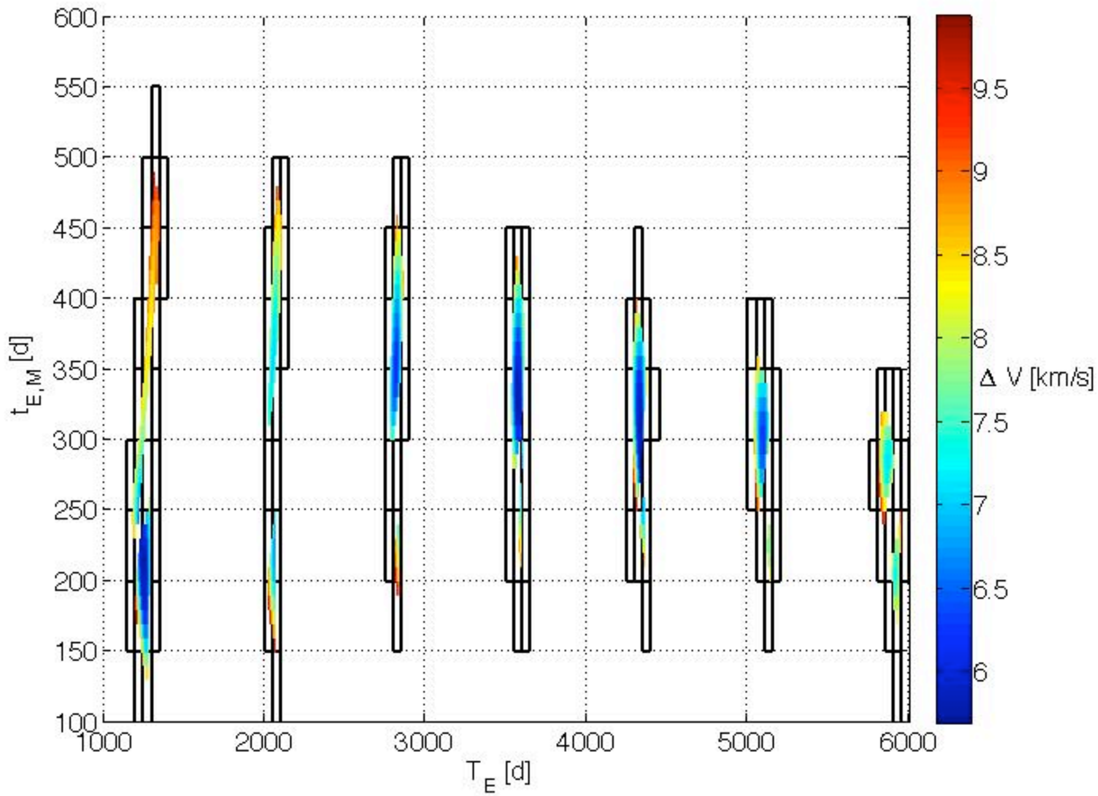


2D-Model

Discontinuity Problem: Planar Model

- The pruned search space in the 2D-Model encloses the pruned search space in the 3D-Model

DA Based Pruning



3D-Model

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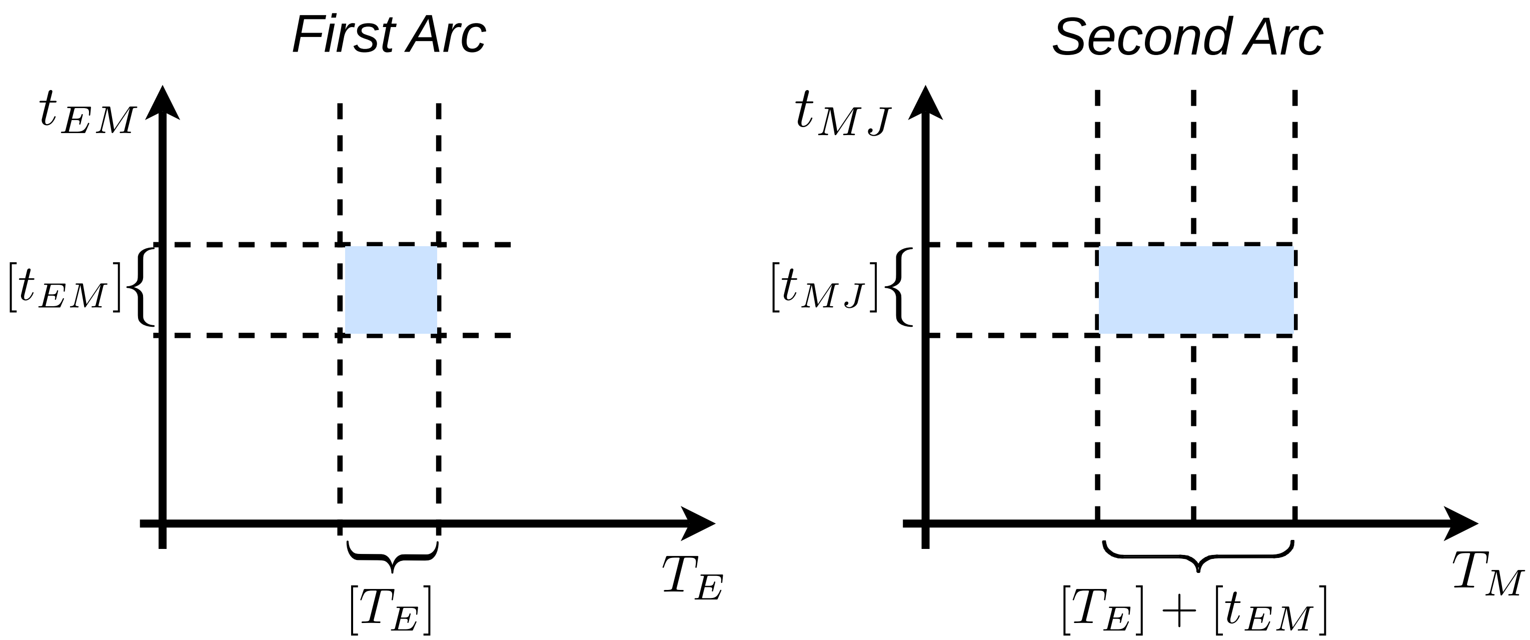
Dependency Problems

- Consider a Earth-Mars-Jupiter (EMJ) transfer



- The box size increases along the transfer

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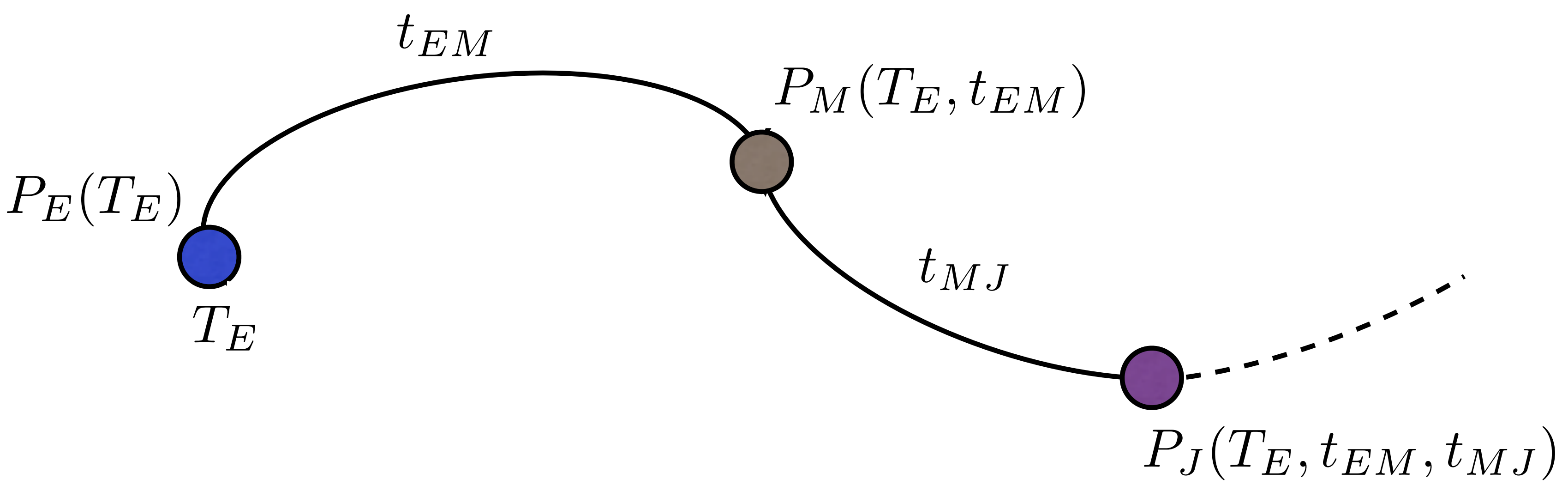
Dependency Problems

- Consider a Earth-Mars-Jupiter (EMJ) transfer

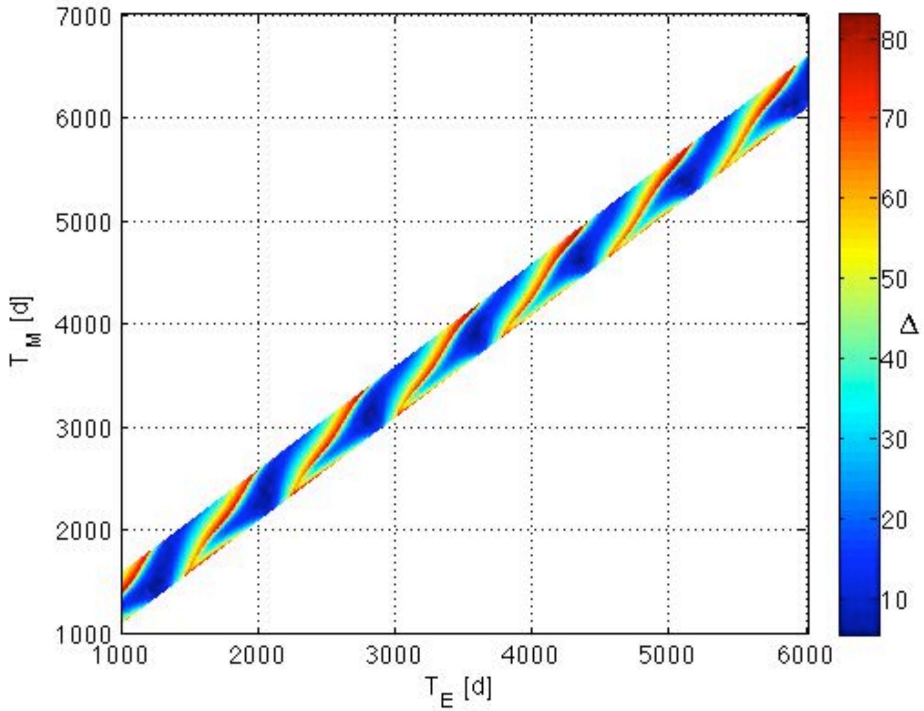


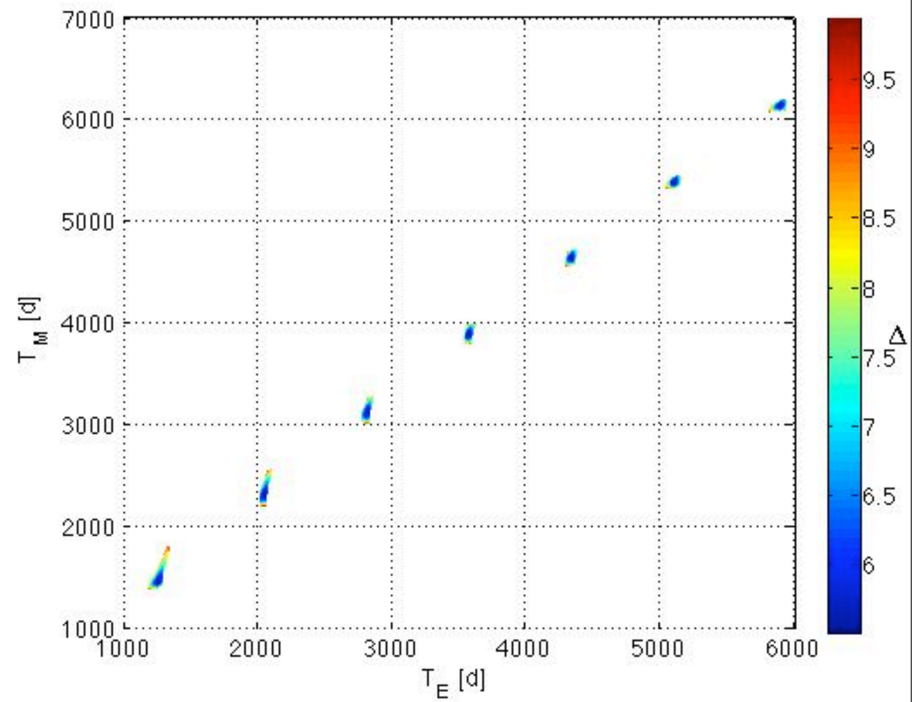
- The box size increases along the transfer
- The dependency on the design variables increases along the transfer

→ The Absolute Times formulation to the problem is adopted

GASP-DA Absolute Time Approach

- GASP is fully translated in the DA environment
- The 2D model is used to avoid discontinuities
- The computed ΔVs and r_p are Taylor expansions about the center of the boxes
- The absolute time approach allows to limit the maximum number of dependencies to 3 (for ΔV_{GA} and r_p)





Semi-Analytical Solution of the Implicit Eqs.

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- The use of semi-analytical solutions of the involved implicit equations has been introduced to:
  - Avoid the iterative process of Newton's method
  - Avoid the use of the dedicated auxiliary DA variable

  } Computational time savings (20%)

- The evaluation of the objective function requires the solution of three scalar nonlinear equations
  - Ephemerides function → Kepler's eq
  - Lambert´s problem → Lagrange's eq
  - Powered gravity assist → bending angle eq

Analytical Ephemerides

- Kepler´s equation $M = E - e\sin E$ is replaced by a **third order expansion** in the eccentricity

$$E = M + \frac{e\sin M}{1 - e\cos M} - \frac{1}{2}\left(\frac{e\sin M}{1 - e\sin M}\right)^3 + O(e^4)$$

- A **first order expansion** has been derived for:
  - Lagrange equation (Lambert's problem):

$$f(x) = \log(A(x)) - \log(t_{tof}) = 0$$

using the variable change $t = \log(1 + x)$ [Izzo]

  - Bending angle equation (powered GA)

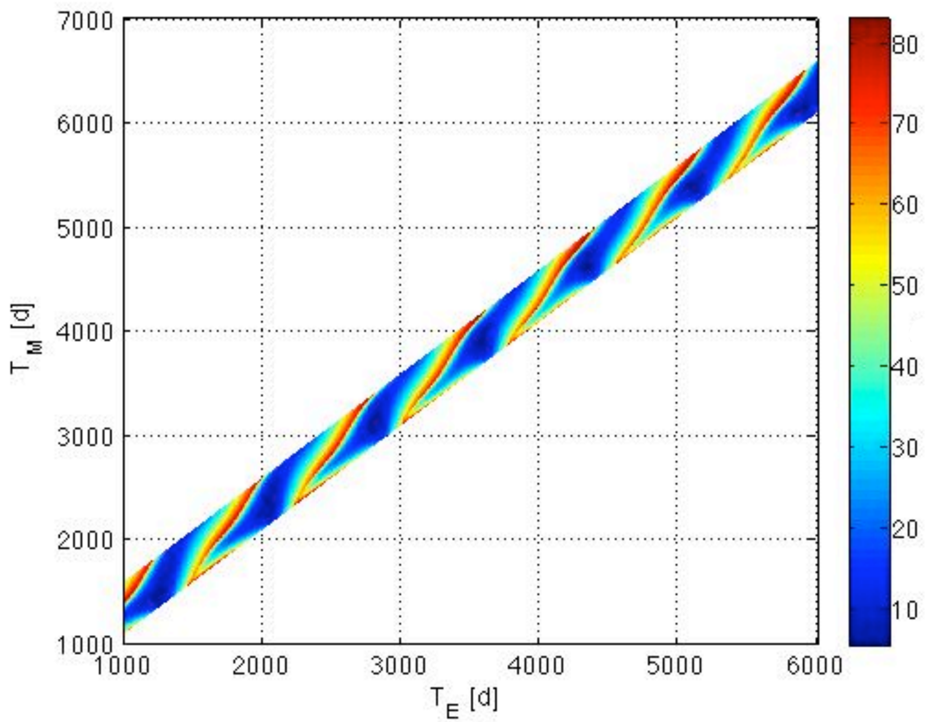
$$f(r_p) = \arcsin\frac{a^-}{a^- + r_p} + \arcsin\frac{a^+}{a^+ + r_p} - \alpha = 0$$

using the variable change $t = 1/r_p$

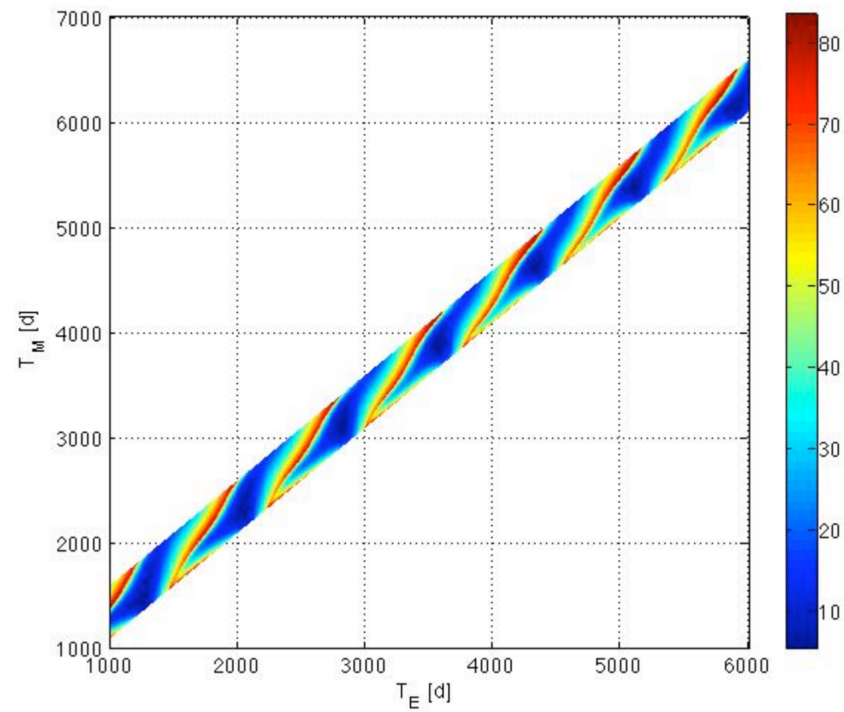
Semi-Analytical Solution: Objective Function

- **Purely Numerical** vs. **Semi-Analytical** approach

Objective Function Comparison



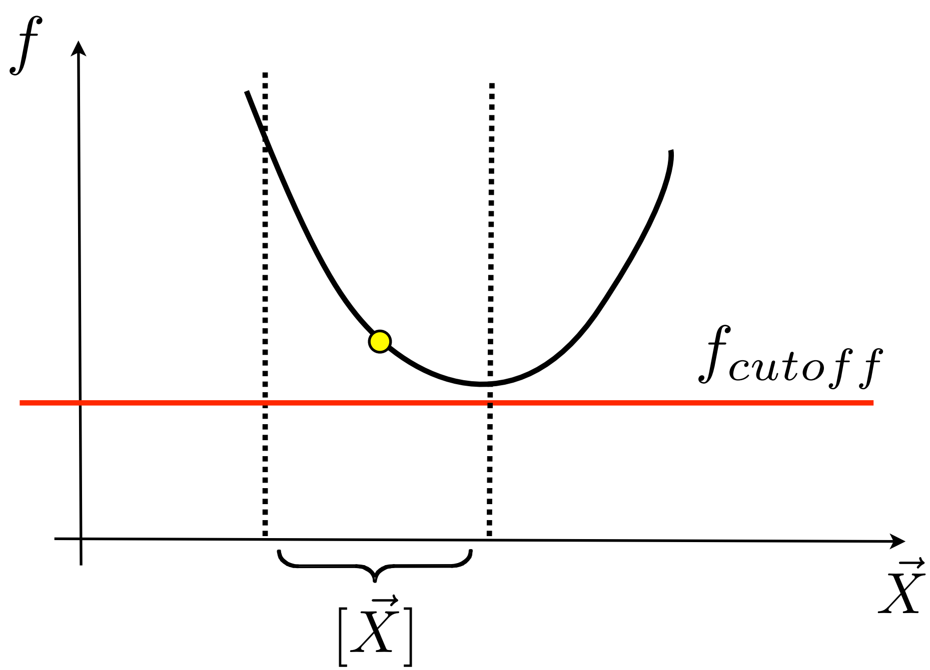
Purely Numerical



Semi-Analytical

Validated Linear Bounder

- **Problem**: careful analyses show that the pruning process is not as effective as expected
- Suppose a box $\vec{X}$ can be pruned away if: $\min_{\vec{X}} f > f_{cutoff}$
- The validated linear bounder uses the **linear part** of the Taylor expansion to get **validated bounds** of the Taylor expansion



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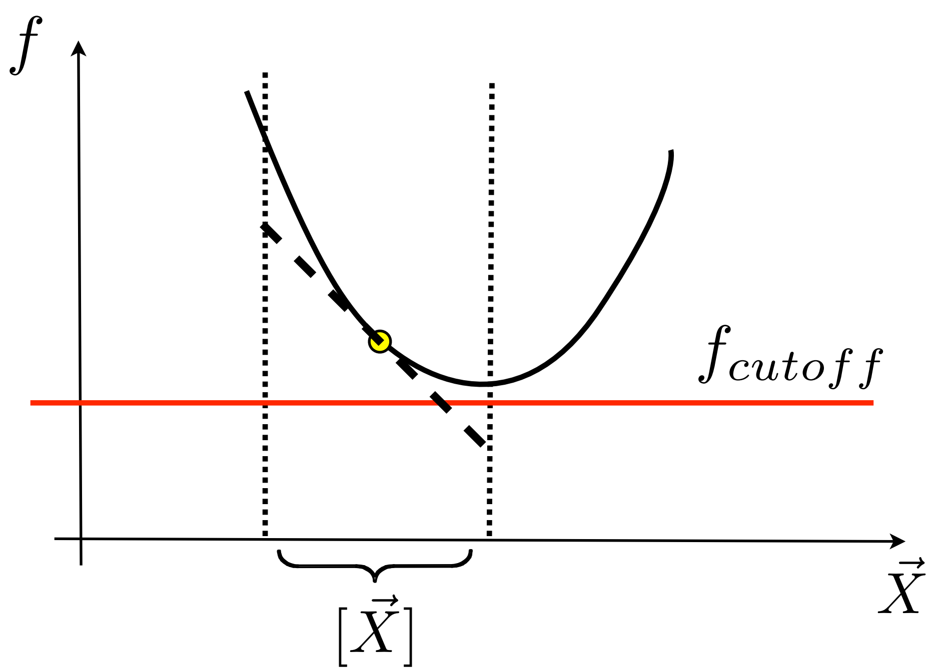

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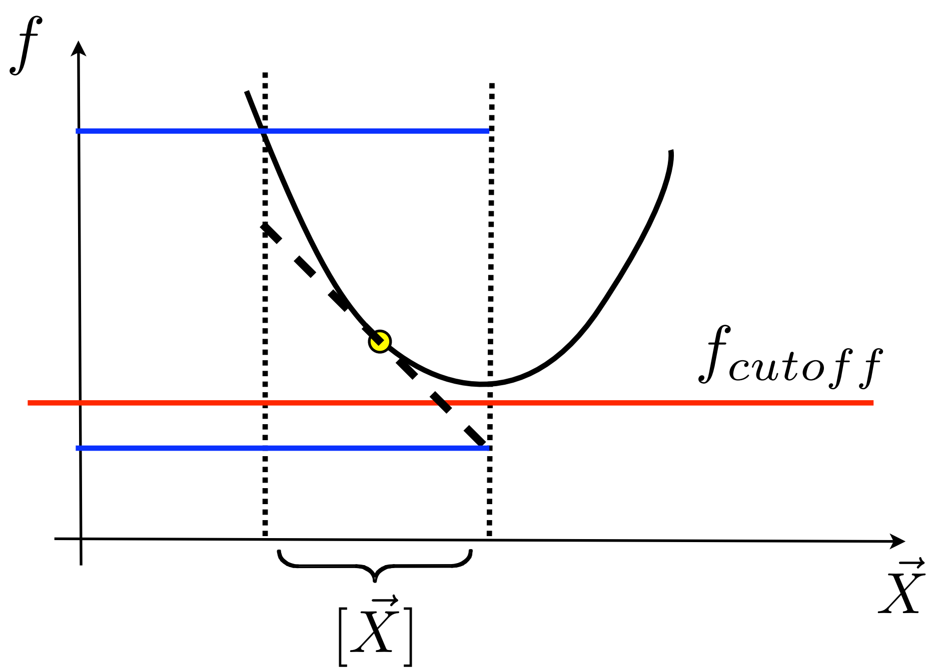

- The computed bounds overestimate the exact range

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Validated Linear Bounder

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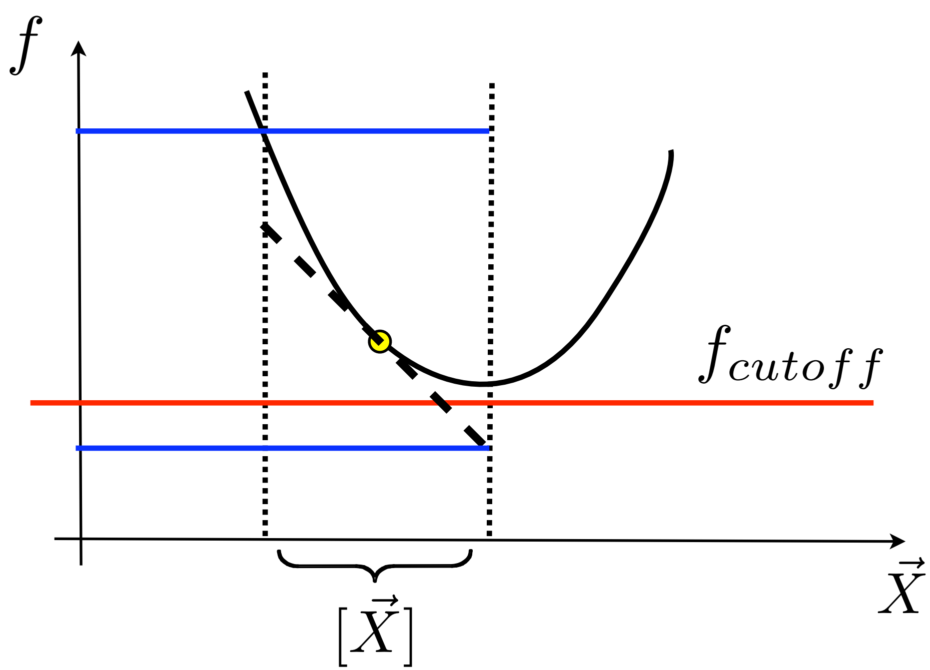
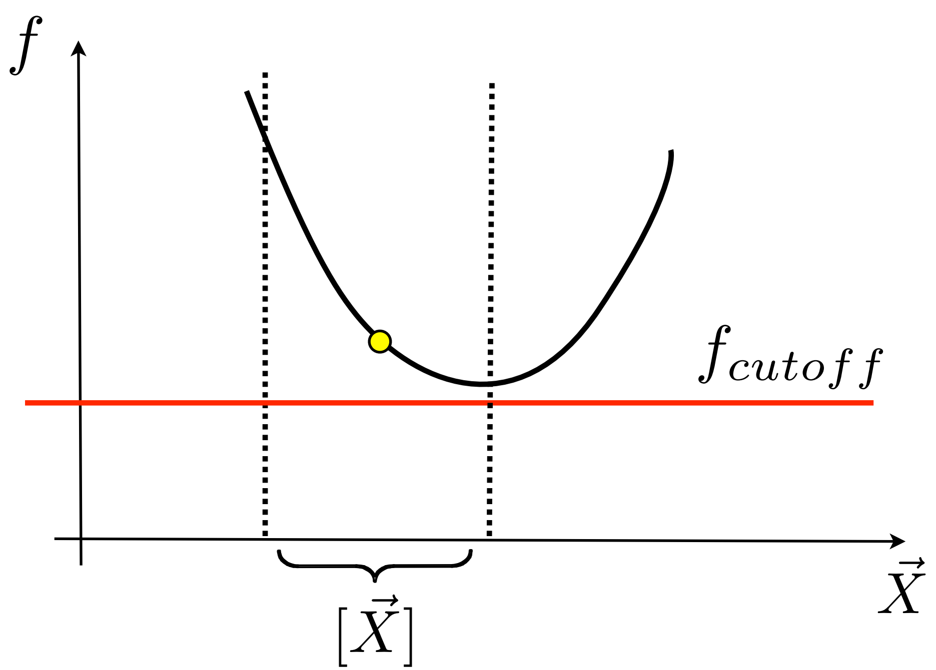


- The computed bounds overestimate the exact range

The linear bounder **would not prune** away the box

Non-Validated Quadratic Bounder

- The non-validated quadratic bounder uses the **quadratic part** of the Taylor expansion to get **non-validated bounds** of the Taylor expansion



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Non-Validated Quadratic Bounder

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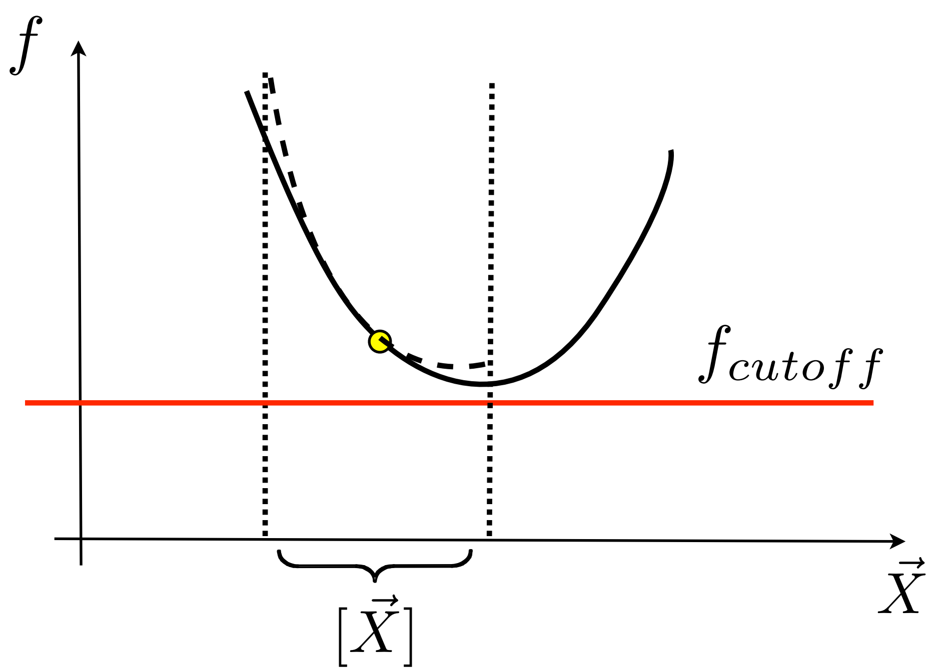

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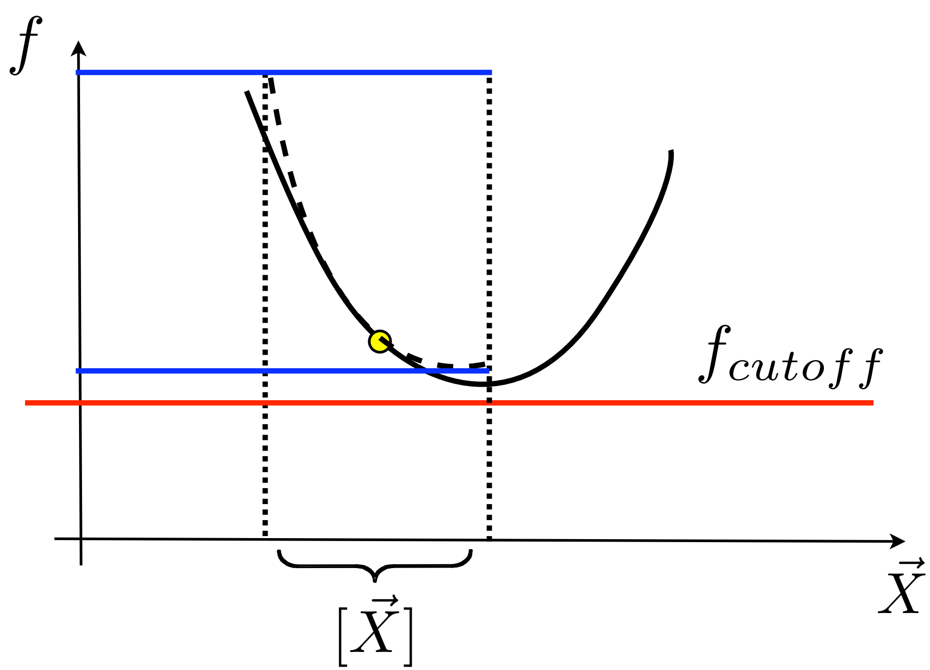
Non-Validated Quadratic Bounder

- The non-validated quadratic bounder uses the **quadratic part** of the Taylor expansion to get **non-validated bounds** of the Taylor expansion

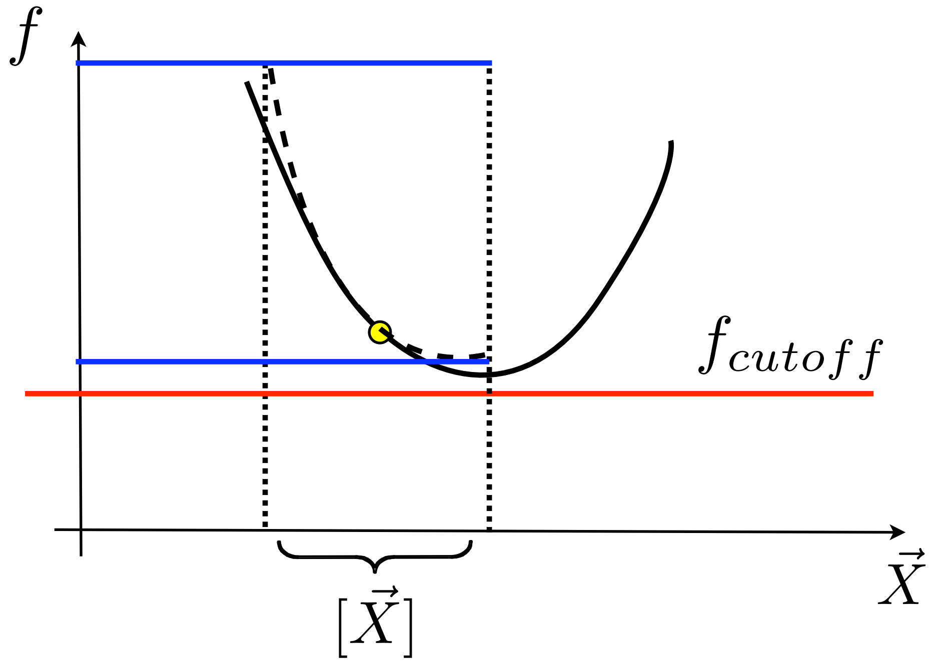


➔ The non-validated quadratic bounder **prunes** away the box

Non-Validated Quadratic Bounder

- The non-validated quadratic bounder uses the **quadratic part** of the Taylor expansion to get **non-validated bounds** of the Taylor expansion

- Drawback: the computed bounds might **underestimate** the exact range

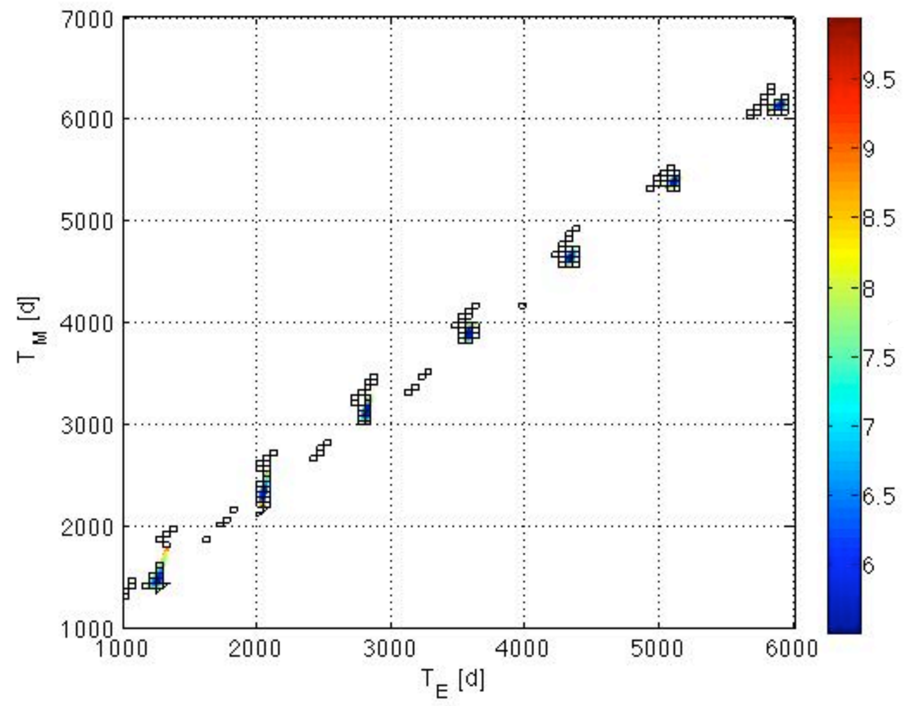


The non-validated quadratic bounder **prunes** away the box

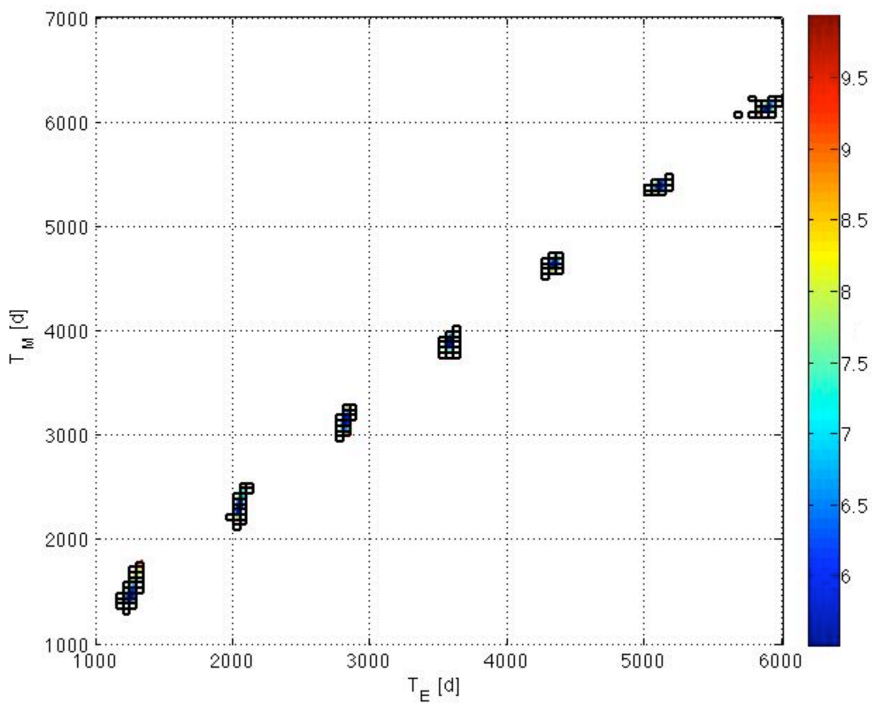
DA-Based Pruning: Earth-Mars Transfer

- Example: Direct Earth-Mars transfer
  - Pruning constraints: $\Delta V_1, \Delta V_2 < 5$ km/s

Remaining Boxes



Validated Linear Bounder



Non-Validated Quadratic Bounder

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DA-Based Pruning: Earth-Mars Transfer

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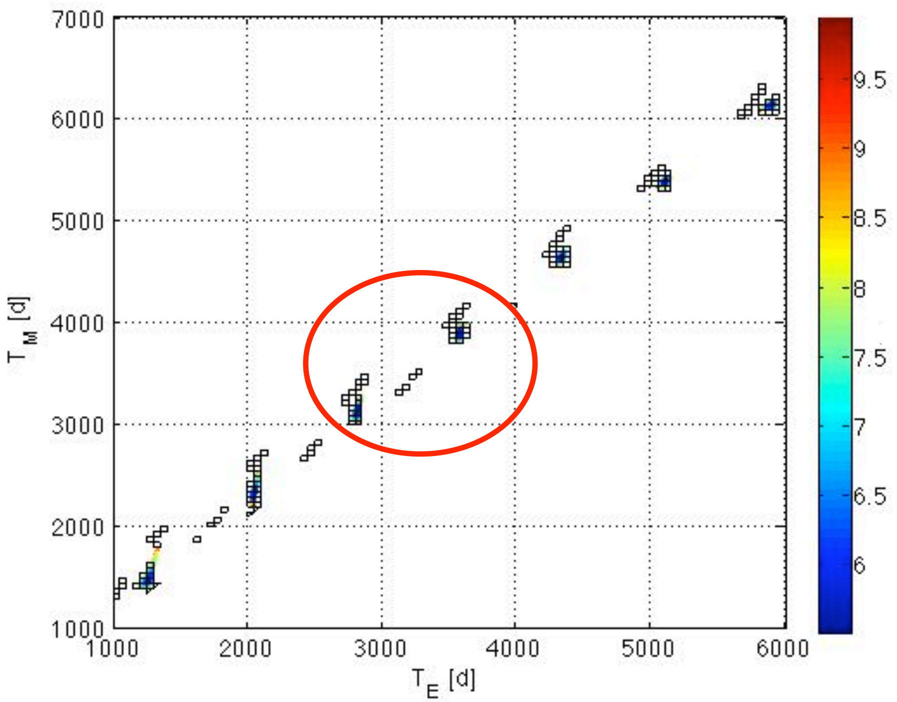
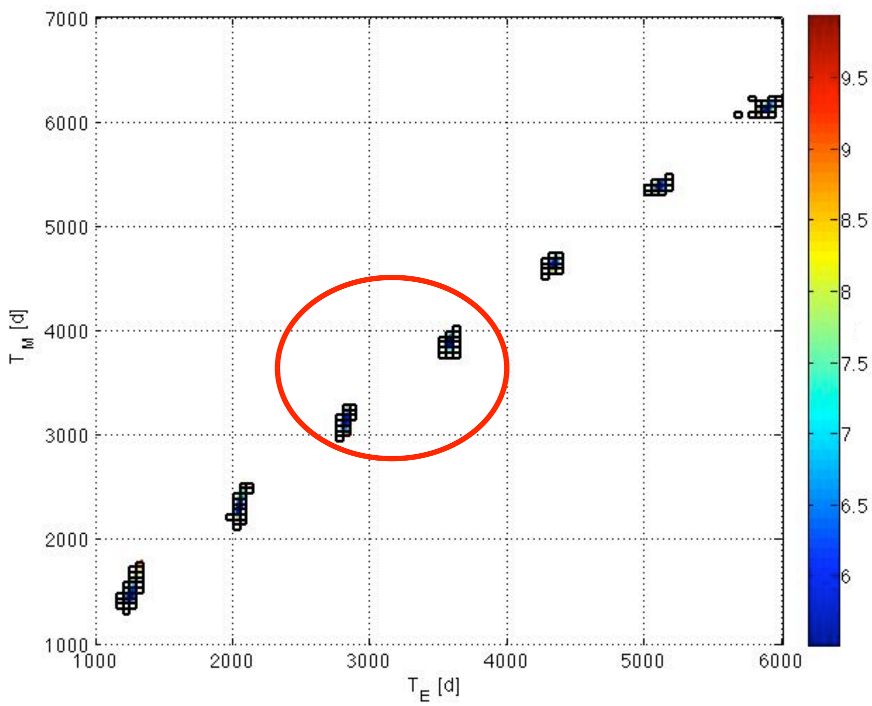
Remaining Boxes



Validated Linear Bounder



Non-Validated Quadratic Bounder

DA-Based Pruning: Earth-Mars Transfer

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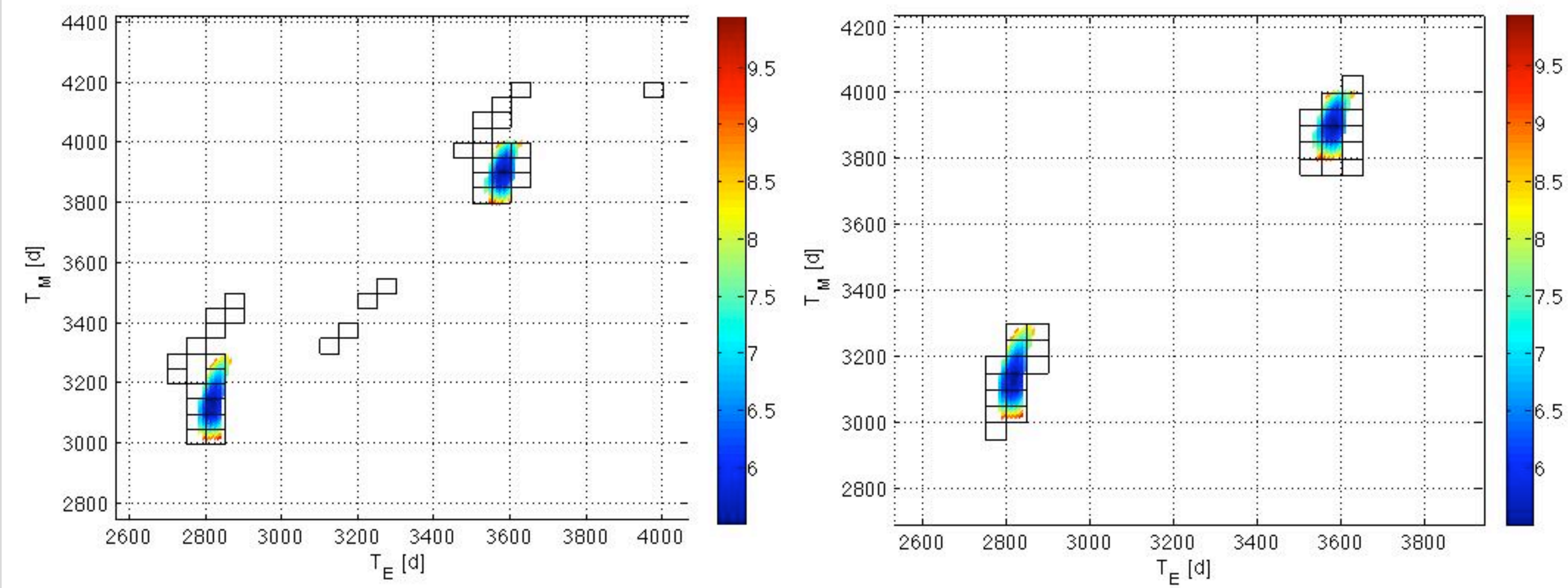
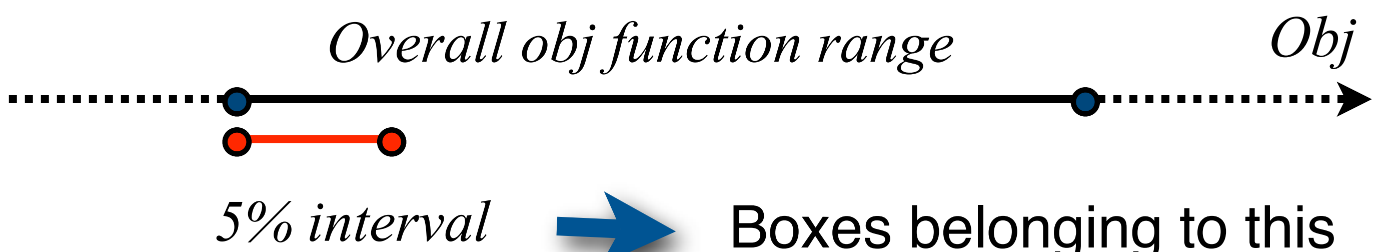
Remaining Boxes



Validated Linear Bounder *Non-Validated Quadratic Bounder*

Optimization Process

- An optimization process follows the pruning process
- Two approaches have been studied:
  - **COSY-GO:** Cosy language based Validated Global Optimizer
  - **Multiple runs of a local optimizer**
    - The non-validated quadratic bounder is used to estimate the minimum within each box (used as starting point)
    - The boxes are sorted based on the objective function
    - Boxes selection heuristic



Boxes belonging to this interval are selected

    - Local runs are performed within each selected box

Performance Analysis

- All tests have been performed on a standard pc with
  - 2.01 GHz CPU
  - 512 Mb RAM
  - Microsoft Windows XP Professional Service Pack 2
- All tests used expansion order = 2
- Problem dimension = number of planets involved
- No Deep Space Maneuver considered

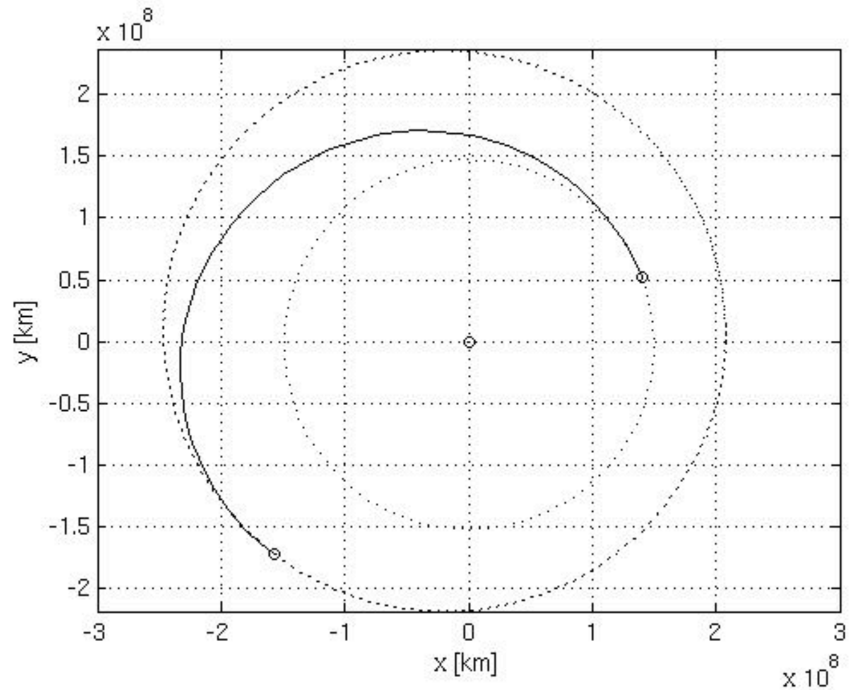
EM Transfer

- Problem Definition:

Problem	Search Space		Int. size	Cut-off (10)
E	Dep. Epoch	1000 - 6000	50	5
M	tof_1	100 - 600	50	5

- Results:

Tot. boxes	1000
Remaining	64 (6.4%)
CPU Time	0.22
Obj Fcn	5.6673



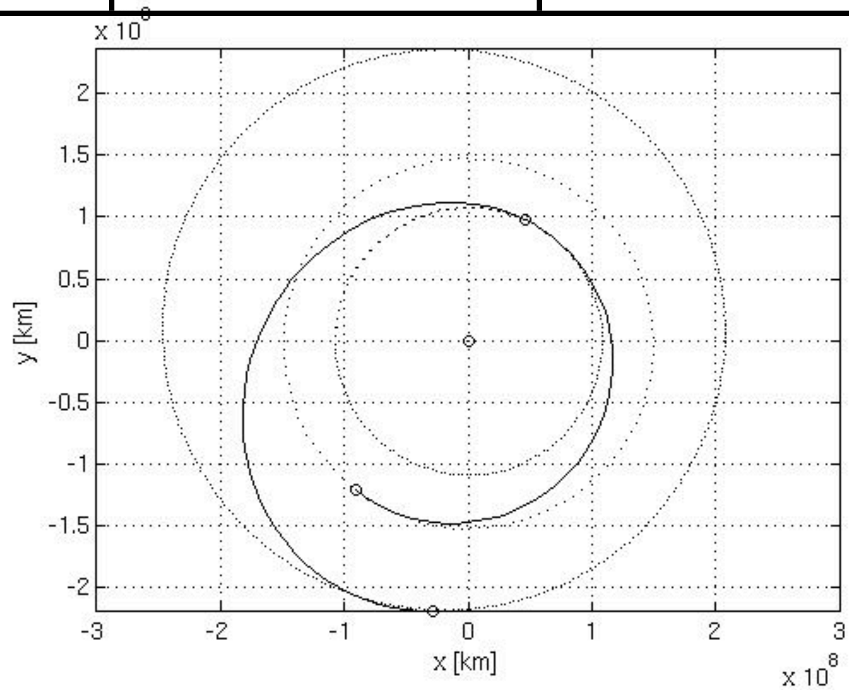
EVM transfer

- Problem Definition:

Problem	Search Space		Int. size	Cut-off (12)
E	Dep. Epoch	1000 - 6000	50	5
V	tof_1	100 - 500	50	2
M	tof_2	100 - 1000	50	5

- Results:

Tot. boxes	14400
Remaining	165 (1.1%)
CPU Time	0.65
Obj Fcn	8.5226



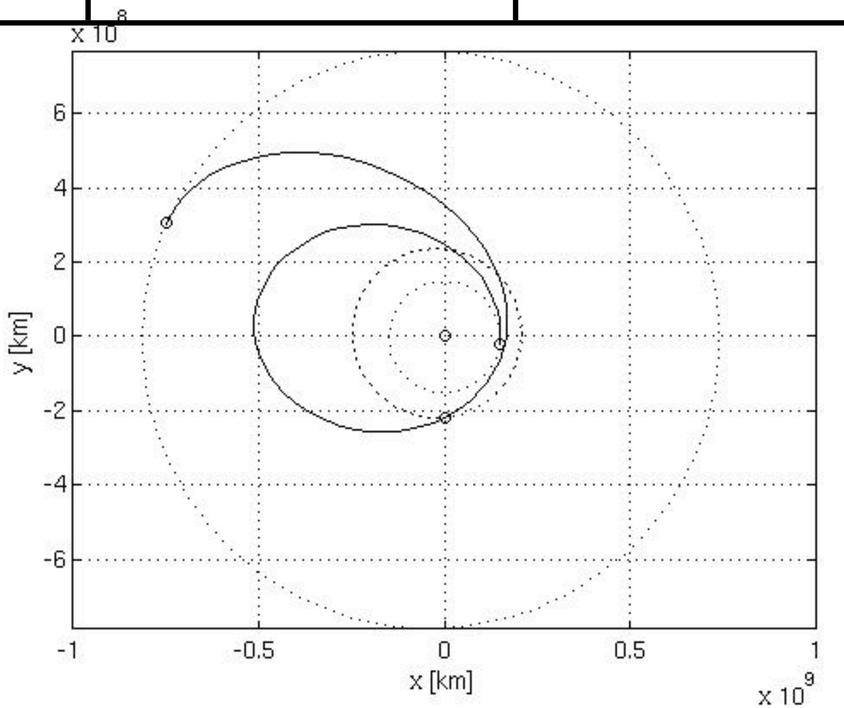
EMJ transfer

- Problem Definition:

Problem	Search Space		Int. size	Cut-off (20)
E	Dep. Epoch	1000 - 6000	50	10
M	tof_1	100 - 1200	50	5
J	tof_2	100 - 2000	100	10

- Results:

Tot. boxes	41800
Remaining	329 (0.7%)
CPU Time	2.64
Obj Fcn	13.4165



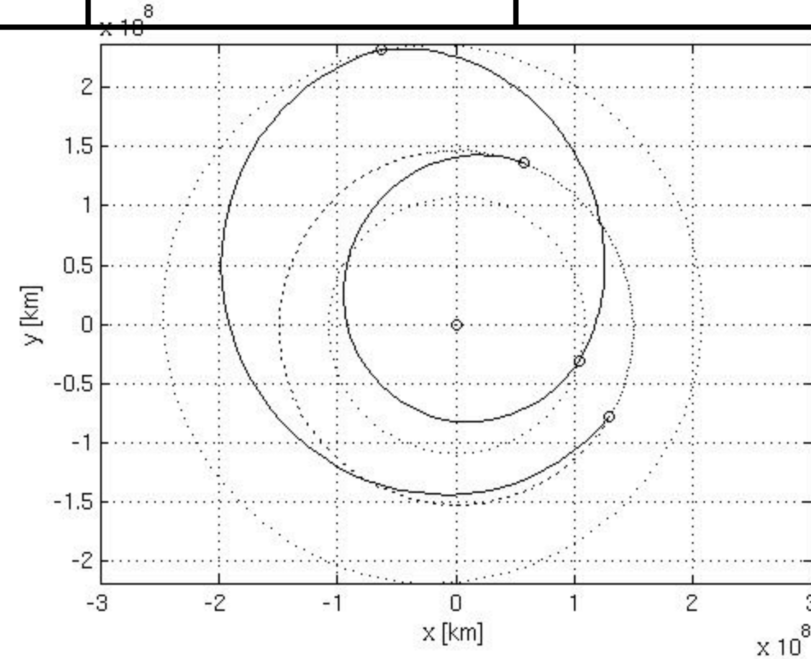
EVME transfer

- Problem Definition:

Problem	Search Space		Int. size	Cut-off (15)
E	Dep. Epoch	3000 - 4000	50	6
V	tof_1	25 - 525	50	2
M	tof_2	20 - 520	50	2
E	tof_3	25 - 525	50	6

- Results:

Tot. boxes	20000
Remaining	26 (0.1%)
CPU Time	0.20
Obj Fcn	12.4431



Cassini-like transfer

- Problem Definition:

Problem	Search Space		Int. size	Cut-off (12)
E	Dep. Epoch	-1000 - 0	50	4
V	tof_1	10 - 410	25	2
V	tof_2	100 - 500	25	2
E	tof_3	10 - 410	25	2
J	tof_4	400 - 2000	200	2
S	tof_5	1000 - 6000	200	6

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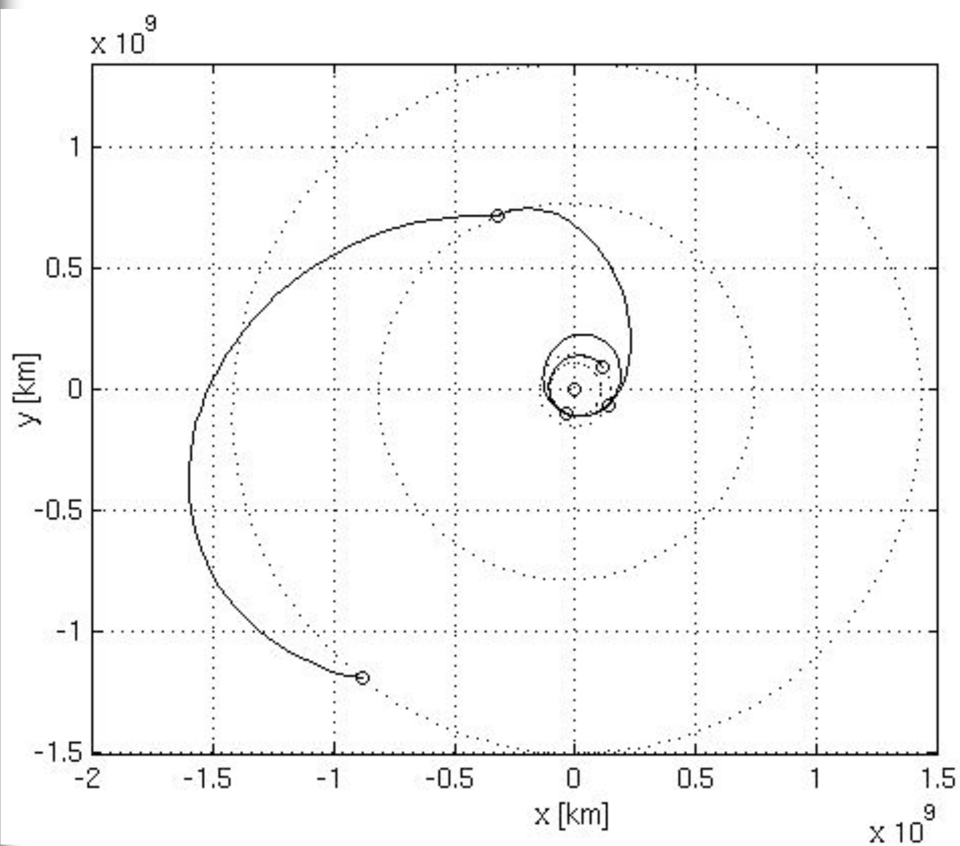
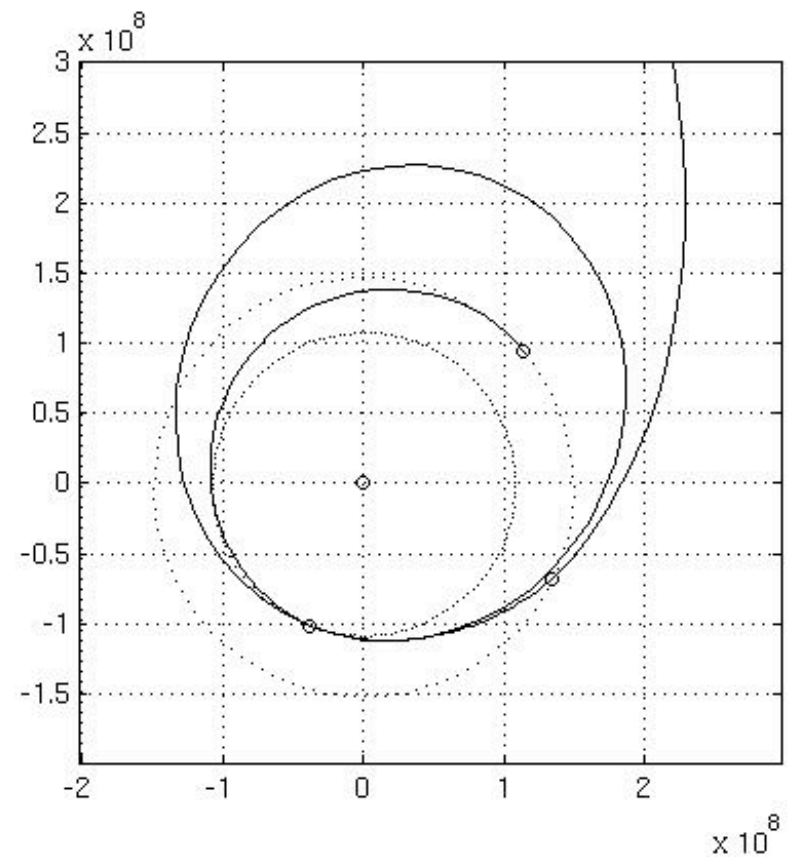
Cassini-like transfer

Tot. boxes	32768000
Remaining	1085 (0.003%)
CPU Time	1.93
Obj Fcn	4.9357

The “black-box” objective function provided by the ACT has been used





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Introduction of DSM in GASP-DA

- The strategy for the introduction of the DSM is not unique
- In the DA frame, the problem formulation strongly affects the results!
- If the number of optimization variables grows, the number of Taylor monomials increases with factorial law
- The problem complexity increases (nonlinear functions of many variables to approximate)

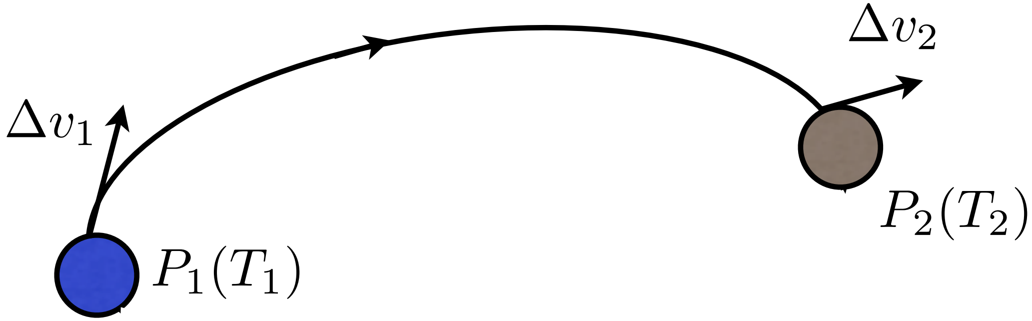
→ In the DA environment, it is important to carefully formalize the DSM problem in order to reduce the polynomial dimension

DSM in GASP-GA

Characteristics of GASP

- cascade of planet-to-planet problems
- definition of cut-off pruning values

It would be useful to exploit the structure of GASP for the DSM introduction. Let's consider the following planet-to-planet transfer



1 Lambert's problem

T_1, T_2 : epochs

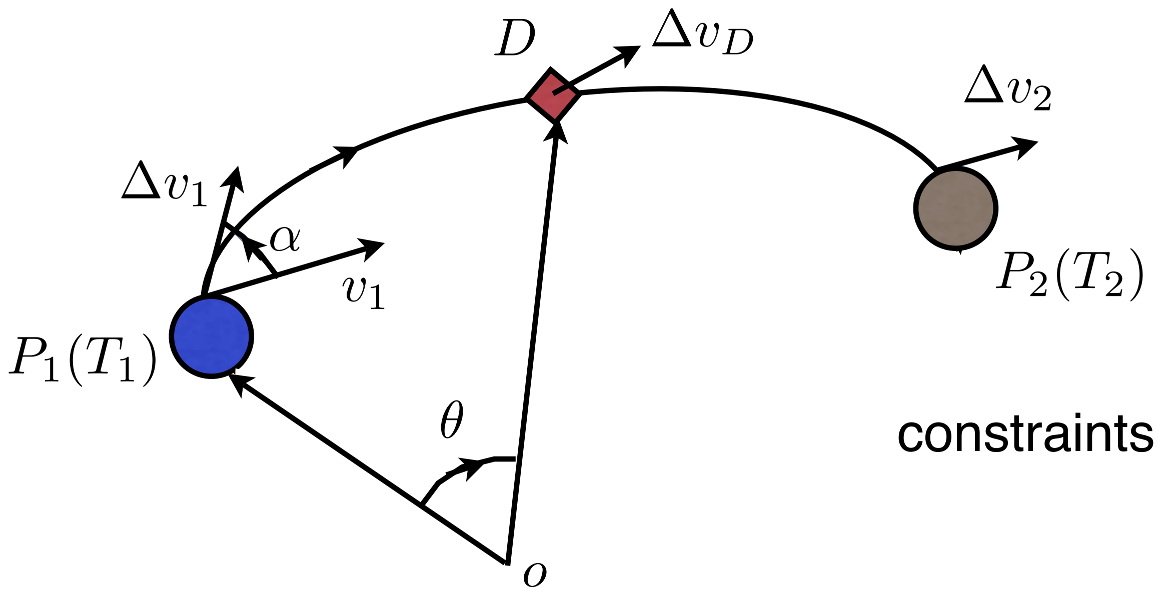
$\Delta v_1^{max}, \Delta v_2^{max}$: cut-off values

constraints: $\Delta v_1 \leq \Delta v_1^{max}$

$\Delta v_2 \leq \Delta v_2^{max}$

DSM formulation: forward propagation

- In the planar case, the DSM point is uniquely specified once three scalars are given
- The DSM point, $D = D(T_1, \Delta v_1, \alpha, \theta)$, can be obtained by forward propagation (with Lagrange coefficients).
  - angle: $\alpha \in [0, 2\pi]$
  - anomaly: $\theta \in [0, 2\pi]$
  - maneuver magnitude: $\Delta v_1 \in [0, \Delta v^{max}]$



1 Propagation

1 Lambert's problem

constraints: $\Delta v_1 \leq \Delta v_1^{max}$

$\Delta v_D \leq \Delta v_D^{max}$

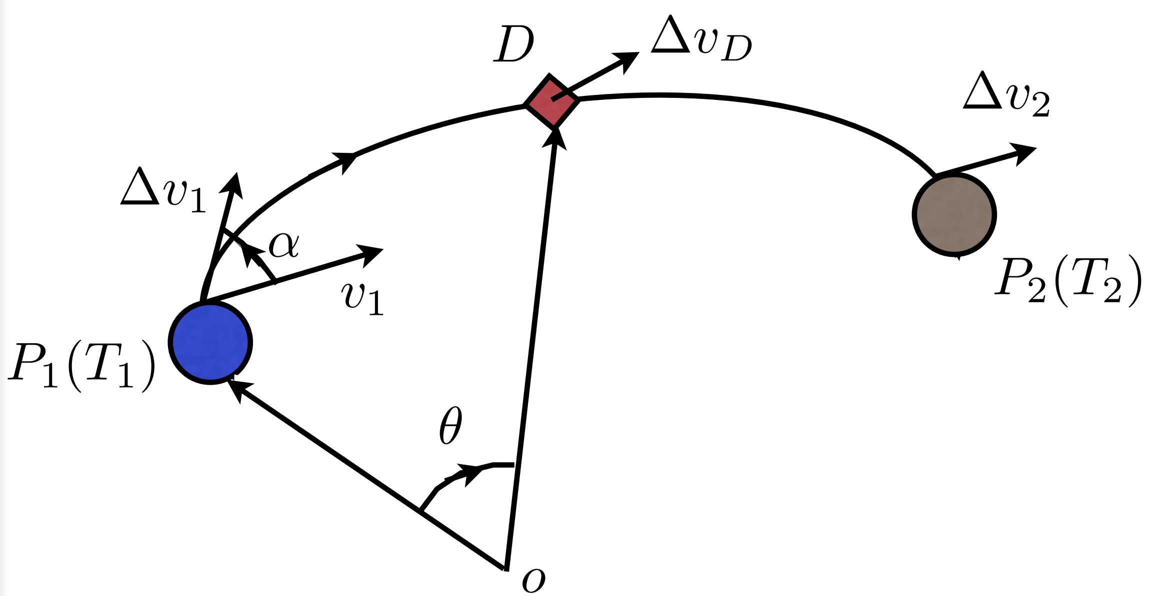
$\Delta v_2 \leq \Delta v_2^{max}$

DSM formulation: forward propagation

- The forward propagation + Lambert's problem fits the cascade structure of GASP, but ...

optimization variables: $\mathbf{x} = \{T_1, T_2, \Delta v_1, \alpha, \theta\}^T$

functions to approximate: $\begin{cases} \Delta v_1 = \Delta v_1 \\ \Delta v_D = \Delta v_D(T_1, T_2, \Delta v_1, \alpha, \theta) \\ \Delta v_2 = \Delta v_2(T_1, T_2, \Delta v_1, \alpha, \theta) \end{cases}$



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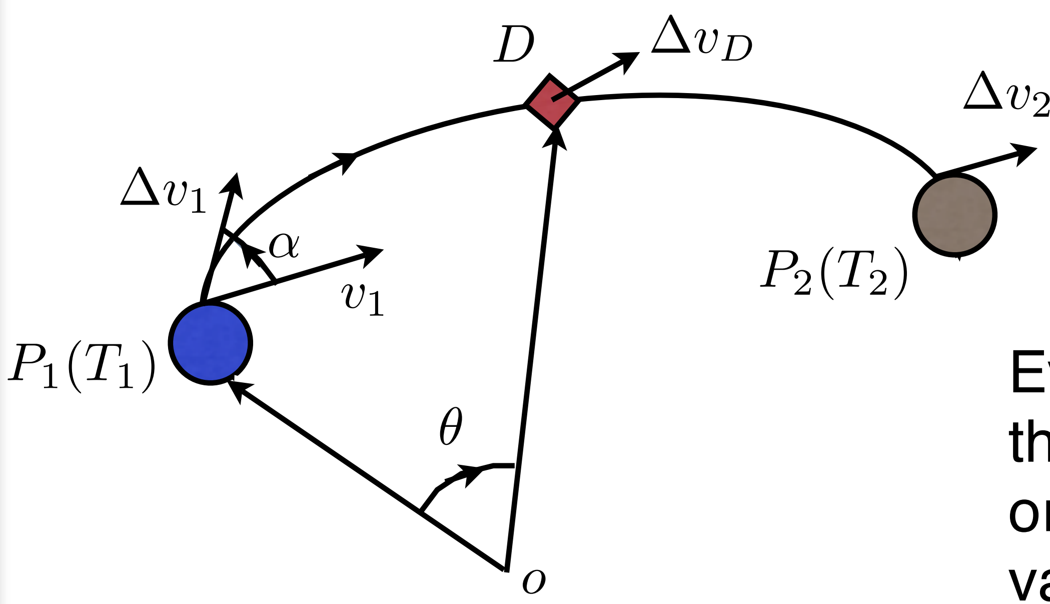

Even in this very simple case, these two functions depend on all the optimization variables!

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DSM formulation: forward propagation

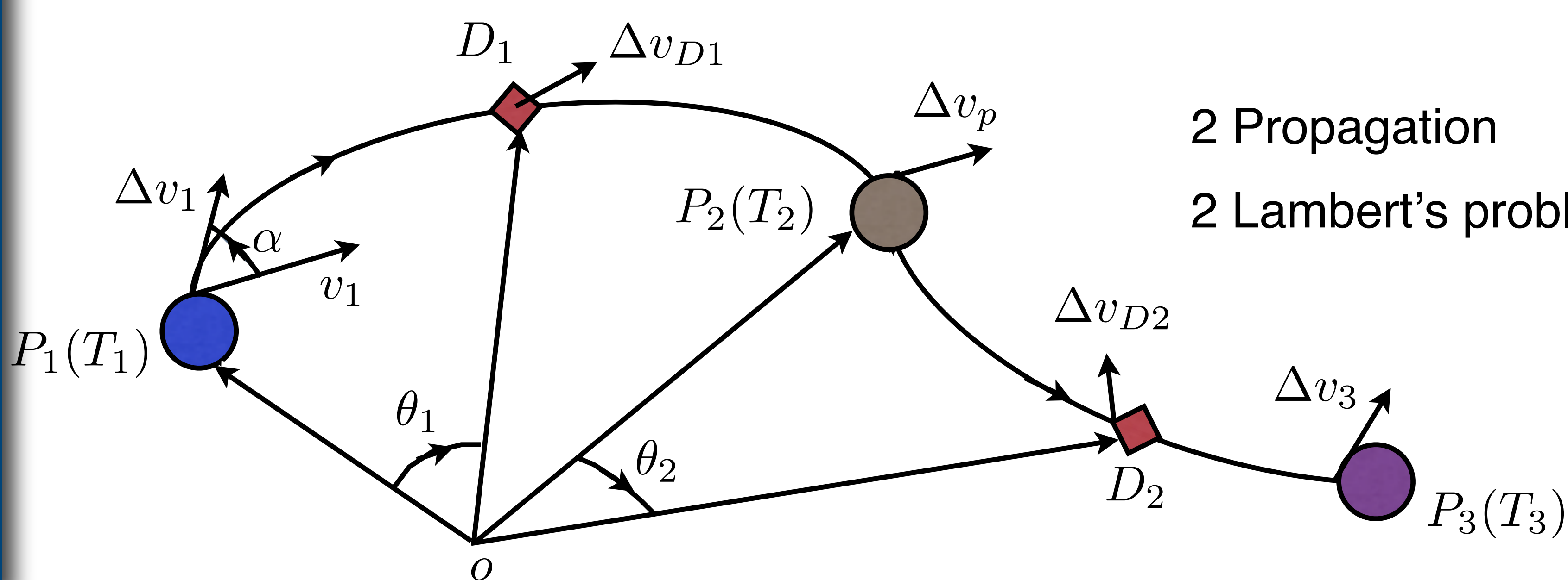
- If we simply add another planet and another DSM ...

optimization variables: $\mathbf{x} = \{T_1, T_2, T_3, \Delta v_1, \alpha, \theta_1, \underbrace{\Delta v_p, r_p,}_{GA} \theta_2\}^T$

functions to approximate:

$$\begin{cases} \Delta v_1 &= \Delta v_1 \\ \Delta v_{D1} &= \Delta v_D(T_1, T_2, \Delta v_1, \alpha, \theta_1) \\ \Delta v_p &= \Delta v_p \\ \Delta v_{D2} &= \Delta v_2(T_1, T_2, T_3 \Delta v_1, \alpha, \theta_1, \Delta v_p, r_p, \theta_2) \\ \Delta v_3 &= \Delta v_3(T_1, T_2, T_3 \Delta v_1, \alpha, \theta_1, \Delta v_p, r_p, \theta_2) \end{cases}$$

$D_1(T_1, \Delta v_1, \alpha, \theta_1)$ $D_2(T_1, T_2, \Delta v_1, \alpha, \theta_1, \Delta v_p, r_p, \theta_2)$



2 Propagation

2 Lambert's problems

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DSM formulation: forward propagation

- If we simply add another planet and another DSM ...

optimization variables: $\mathbf{x} = \{T_1, T_2, T_3, \Delta v_1, \alpha, \theta_1, \underbrace{\Delta v_p, r_p}_{GA}, \theta_2\}^T$

functions to approximate:

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function of $\mathbf{n_{planets} + 3n_{DSM}}$ →

- The more planets and DSM, the **more complicated functions to approximate**.
- **Bounding** functions of many variables **is very difficult** (and leads to incorrect results).
- **Re-formulate the problem** in suitable variables such that the “dependency chain” is broken (analogously in the absolute time formulation of GASP).

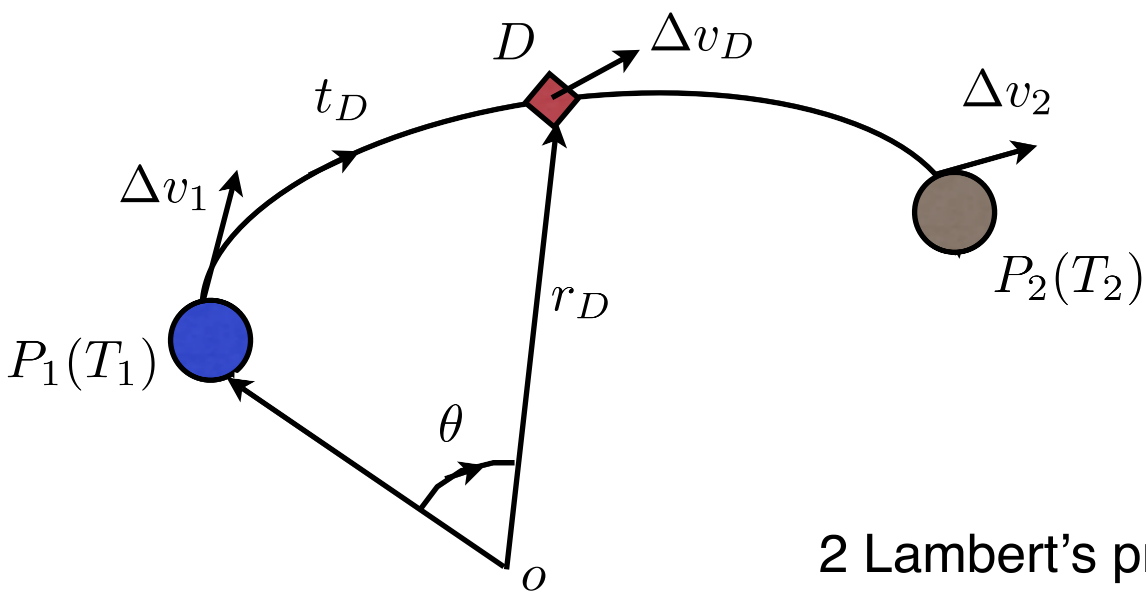
DSM formulation: absolute variables

- The DSM point is specified by a radius and an angle
- The problem is solved by two Lambert's arcs

$$r_D \in [r_D^{min}, r_D^{max}]$$
$$\theta \in [0, 2\pi]$$
$$t_D \in [0, T_2 - T_1]$$

variables: $\mathbf{x} = \{T_1, T_2, r_D, \theta, t_D\}^T$

functions: $\begin{cases} \Delta v_1 = \Delta v_1(T_1, r_D, \theta, t_D) \\ \Delta v_D = \Delta v_D(T_1, T_2, r_D, \theta, t_D) \\ \Delta v_2 = \Delta v_2(T_1, T_2, r_D, \theta, t_D) \end{cases}$



Apparently there is no improvement, but ...

2 Lambert's problems

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DSM formulation: absolute variables

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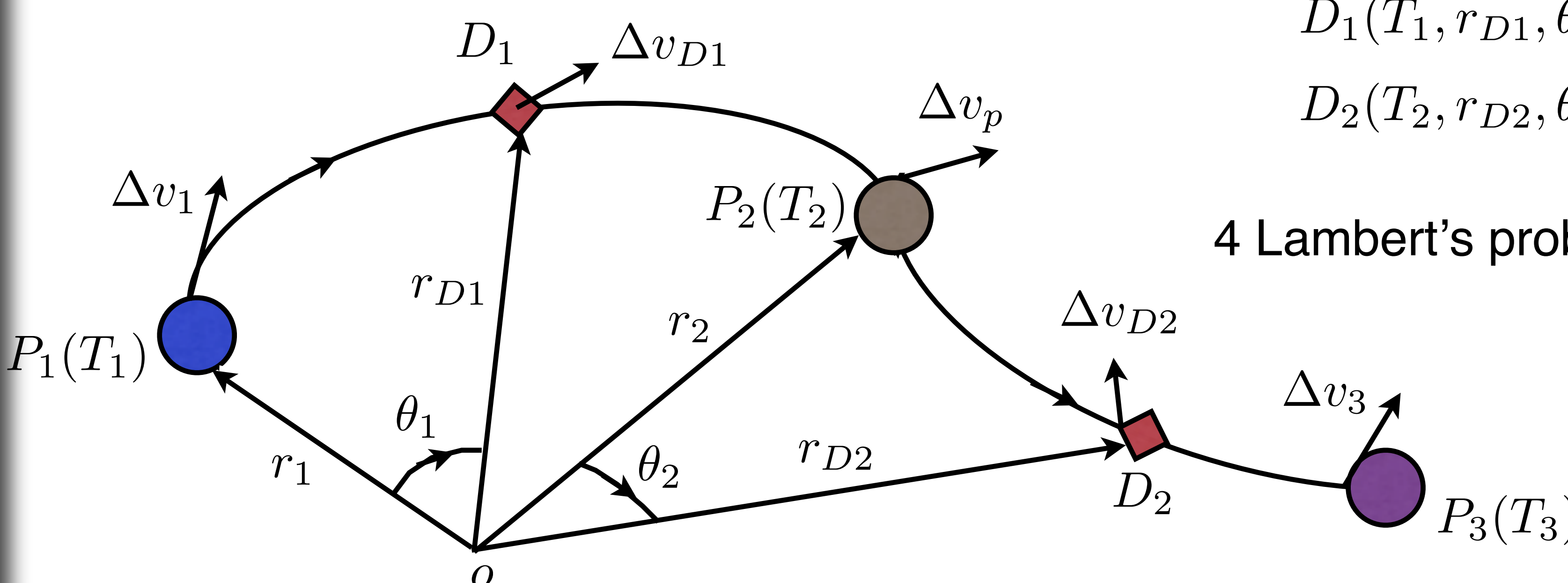
- If both one planet and one DSM are added ...

variables: $\mathbf{x} = \{T_1, T_2, T_3, \underbrace{r_{D1}, \theta_1, t_{D1}}_{DSM1}, \underbrace{r_{D2}, \theta_2, t_{D2}}_{DSM2}\}^T$

functions:
$$\begin{cases} \Delta v_1 &= \Delta v_1(T_1, r_{D1}, \theta_1, t_{D1}) \\ \Delta v_{D1} &= \Delta v_{D1}(T_1, T_2, r_{D1}, \theta_1, t_{D1}) \\ \Delta v_p &= \Delta v_p(T_1, T_2, r_{D1}, \theta_1, t_{D1}, r_{D2}, \theta_2, t_{D2}) \\ \Delta v_{D2} &= \Delta v_{D2}(T_2, r_{D2}, \theta_2, t_{D2}) \\ \Delta v_3 &= \Delta v_3(T_2, T_3, r_{D2}, \theta_2, t_{D2}) \end{cases}$$



$D_1(T_1, r_{D1}, \theta_1)$

$D_2(T_2, r_{D2}, \theta_2)$

4 Lambert's problems

DSM formulation: absolute variables

- If both one planet and one DSM are added ...

variables: $\mathbf{x} = \{T_1, T_2, T_3, \underbrace{r_{D1}, \theta_1, t_{D1}}_{DSM1}, \underbrace{r_{D2}, \theta_2, t_{D2}}_{DSM2}\}^T$

functions:

$$\begin{cases} \Delta v_1 = \Delta v_1(T_1, r_{D1}, \theta_1, t_{D1}) \\ \Delta v_{D1} = \Delta v_{D1}(T_1, T_2, r_{D1}, \theta_1, t_{D1}) \\ \Delta v_p = \Delta v_p(T_1, T_2, r_{D1}, \theta_1, t_{D1}, r_{D2}, \theta_2, t_{D2}) \\ \Delta v_{D2} = \Delta v_{D2}(T_2, r_{D2}, \theta_2, t_{D2}) \\ \Delta v_3 = \Delta v_3(T_2, T_3, r_{D2}, \theta_2, t_{D2}) \end{cases}$$

- Independently from the problem structure, the **absolute variables** formulation involves **functions of at most 8 variables.**
- In the **forward propagation** method, the dependencies blow up with $\mathbf{n_{planets}+3_{DSM}}$.

DSM formulation: absolute variables

Absolute time formulation

- The "dependency chain" is broken
- Reduced number of variable dependencies
- Better functions to approximate and bound
- More Lambert's problems to solve (but for us it is a mere function evaluation)

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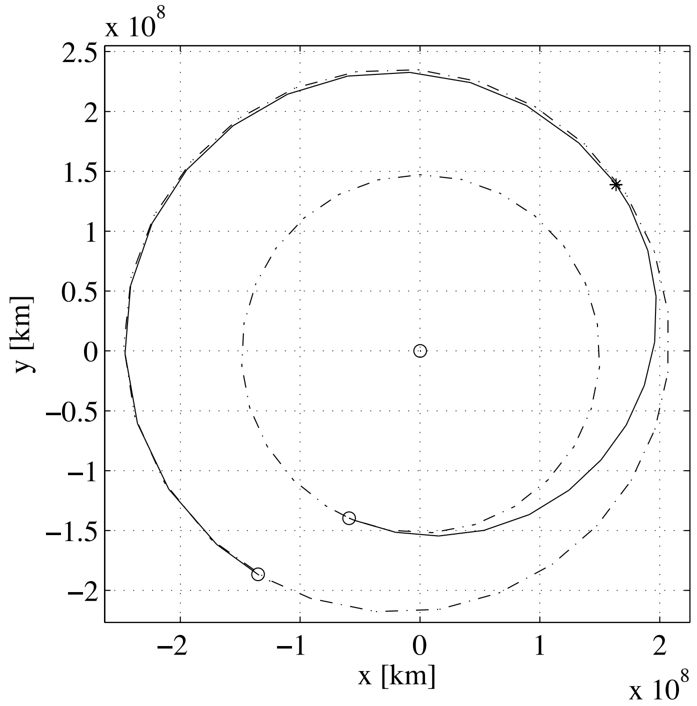
Roberto Armellin

Results: EdM problem

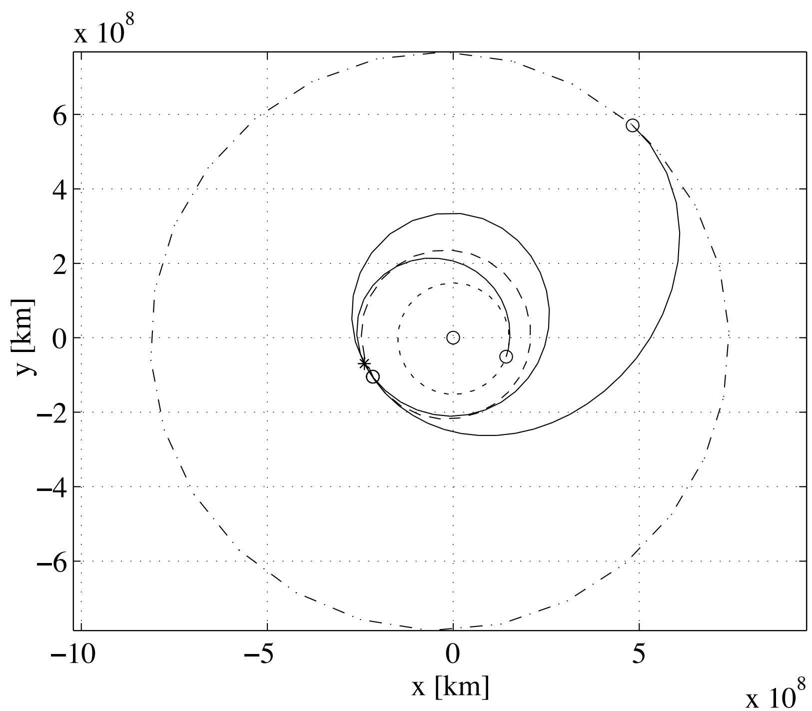
Problem	Search Space	Int. size	Cut-off (7)
E	1000 - 2000	50	3
d	$0.9r_E$ - $1.1r_M$ 0 - $(T_M$-$T_E)$ 0 - 360	0.1 50 10	3
M	200 - 650	50	3



- Total n. of boxes = 388800
- Feasible boxes = 1603 (0.41%)
- CPU time = 253.2 s
- Best ObjFcn = 5.632
- Best ObjFcn = 2.77 + 2.77 + 0.07
- Best ObjFcn (GASP) = 5.667

Results: EMdJ problem

Problem	Search Space	Int. size	Cut-off (15)
E	1000 - 3000	50	5
M	300 - 700	50	0
d	$0.9r_M$ - $1.1r_J$ 0 - $(T_J$-$T_M)$ 0 - 360	0.1 50 10	5
J	1000 - 2000	100	5



- Total n. of boxes = 8.52e7
- Feasible boxes = 323 (3.79e-4%)
- CPU time = 451 s
- Best ObjFcn = 12.481
- Best ObjFcn = 3.93 + 0 + 4.16 + 4.39
- Best ObjFcn (GASP) = 13.416

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Results: EMdMJ problem

Problem	Search Space	Int. size	Cut-off (12)
E	3650 - 7300	50	4
M	30 - 430	100	0
d	$0.9r_M$ - $1.1a_{(res\ 2:1)}$ 0 - $T_{(res\ 2:1)}$ 0 - 360	0.3 50 20	3
M	330 - 830	100	0
J	600 - 2000	200	7

- Total n. of boxes = 9.19e7
- Feasible boxes = 717 (7.8e-3%)
- CPU time = 144.26 s
- Best ObjFcn = 10.843
- Best ObjFcn = 3.18 + 0 + 1.01 + 2.42 + 4.21
- Best ObjFcn (GASP) = 12.864

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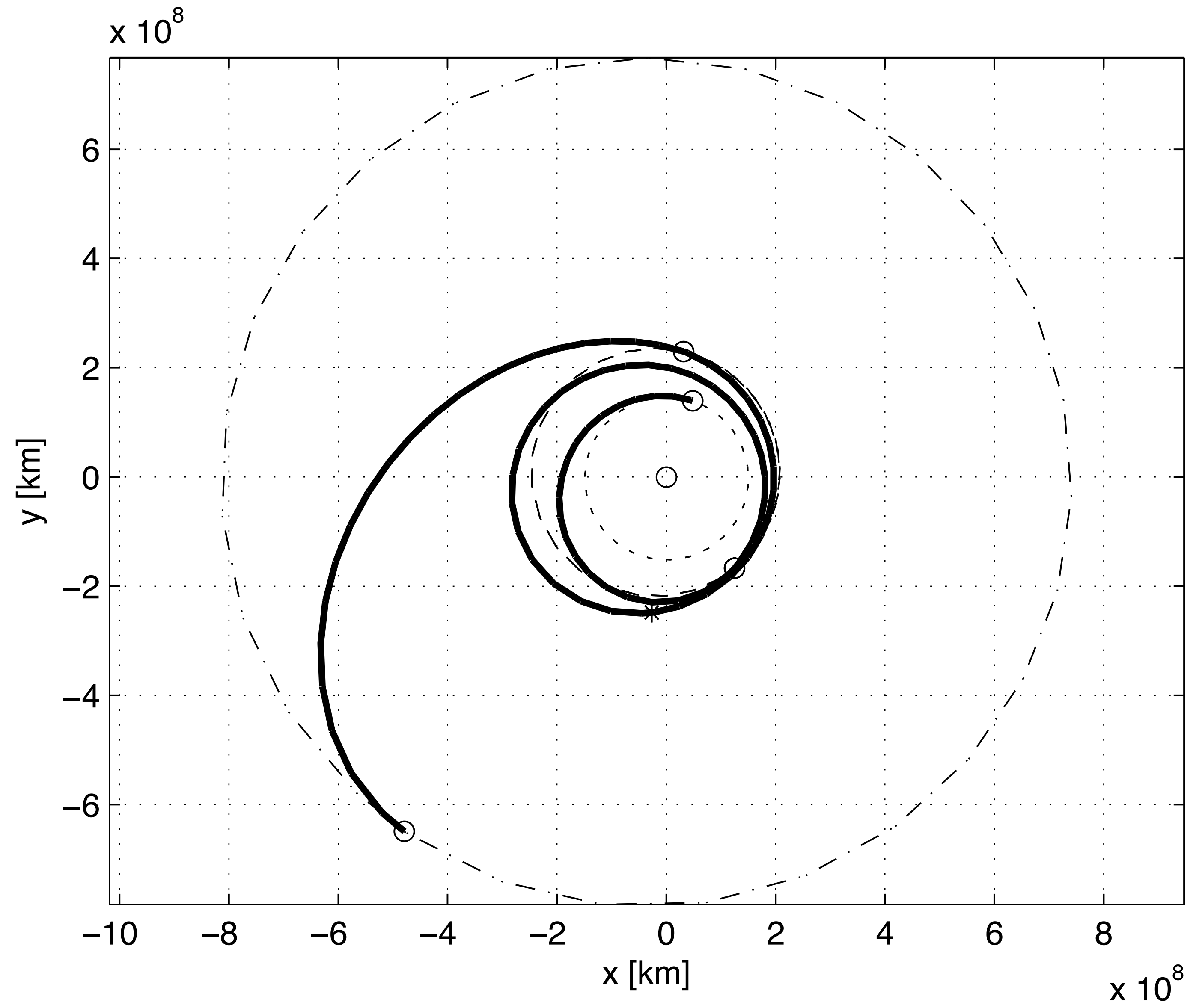
Results: EMdMJ problem



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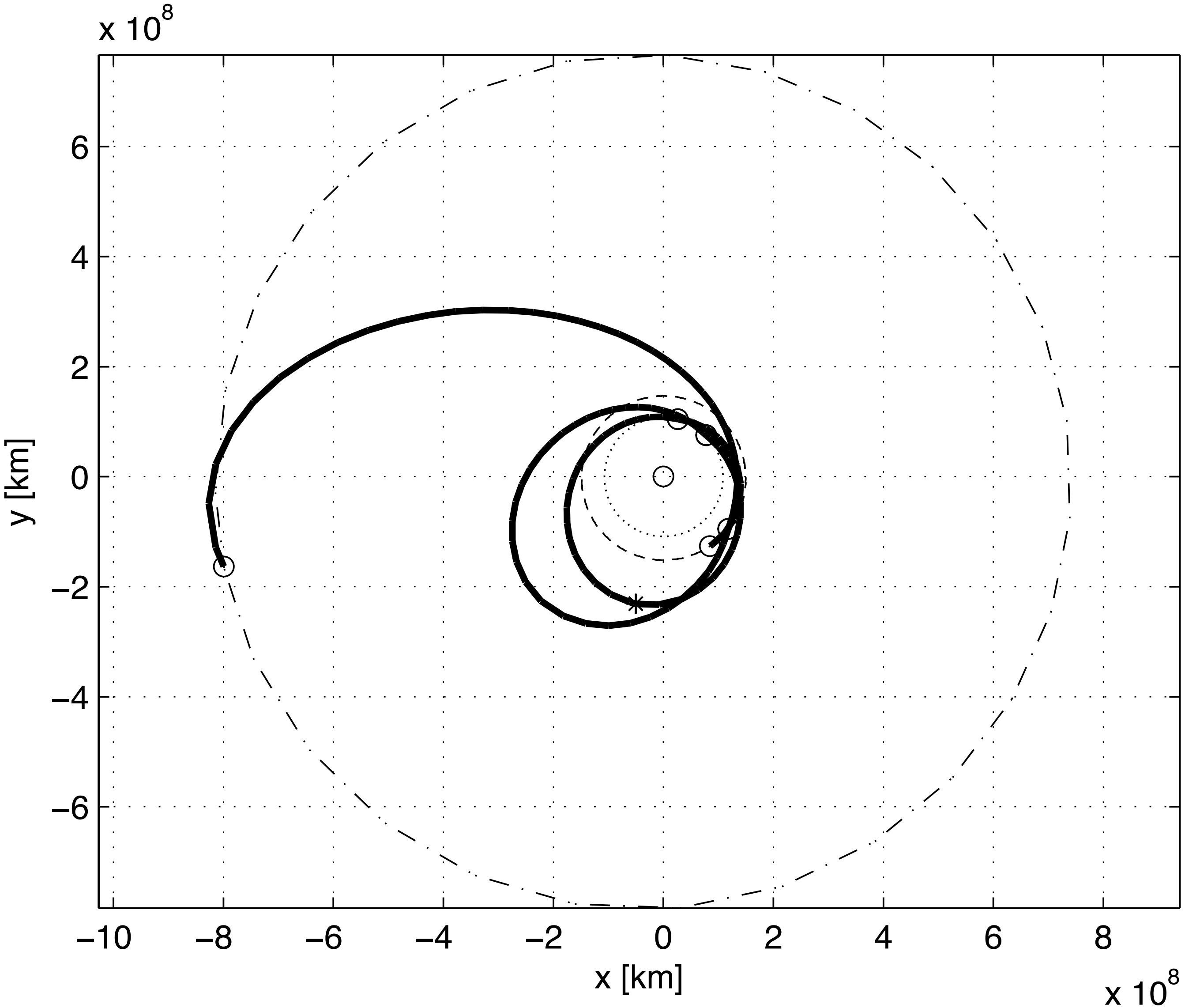
Results: EVdVEJ problem

Problem	Search Space	Int. size	Cut-off (12)
E	3650 - 7300	50	4.5
V	80 - 430	25	0
d	$0.9 r_V$ - $1.1 a_{(\text{res } 2:1)}$ 0 - $T_{(\text{res } 2:1)}$ 0 - 360	0.3 50 10	0.5
V	80 - 830	25	0
E	80 - 830	50	0
J	600 - 2000	200	7

- Total n. of boxes = 1.85e10
- Feasible boxes = 38025 (2.06e-4%)
- CPU time = 2770 s
- Best ObjFcn = 9.304 (Ref. solution†: 10.503)
- Best ObjFcn = 3.04 + 0 + 0.26 + 0 + 0 + 5.99

† Vasile and De Pascale, 2006

Results: EVdVEJ problem



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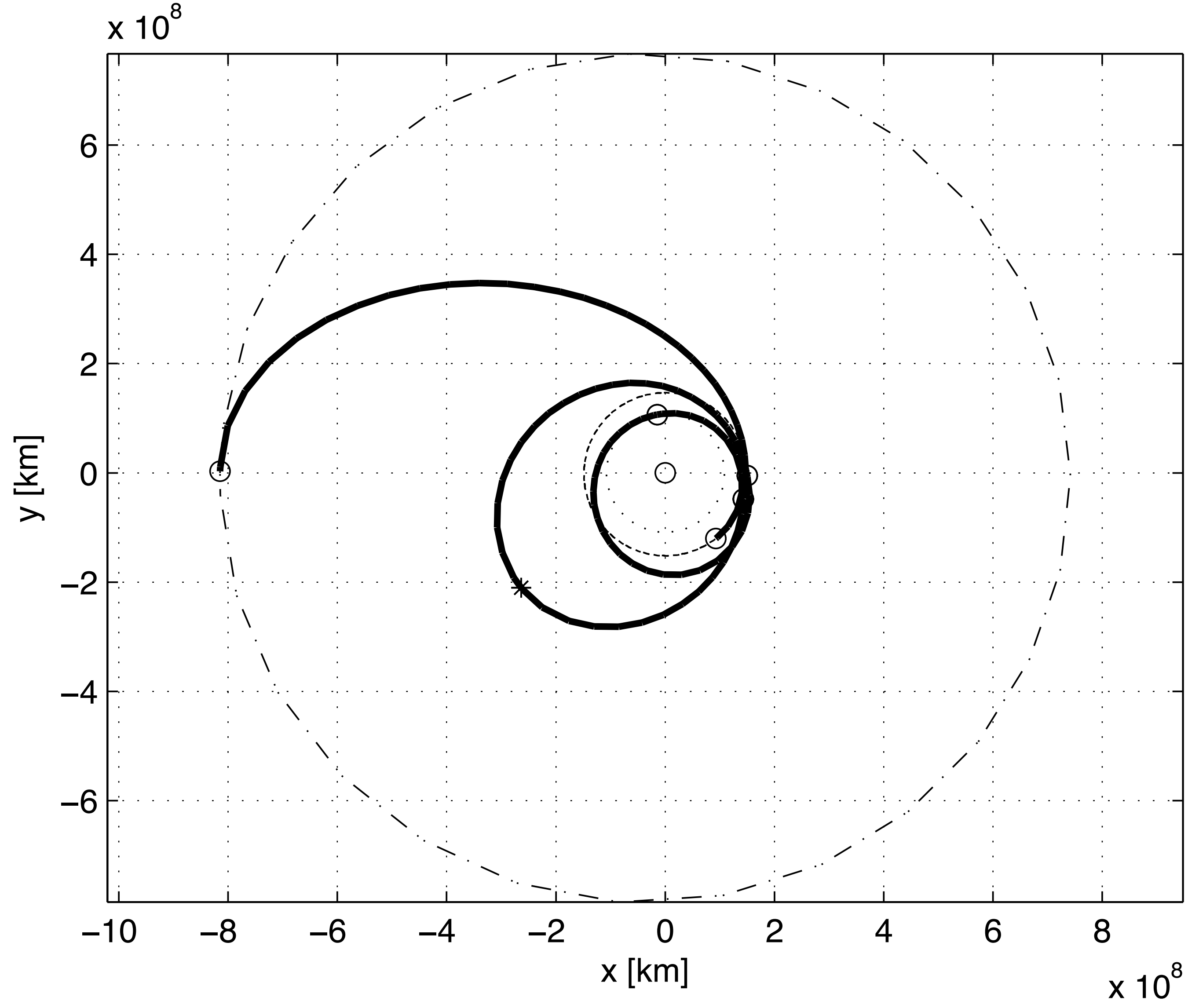
Results: EVEdEJ problem

Problem	Search Space	Int. size	Cut-off (12)
E	3650 - 7300	50	4
V	80 - 430	25	0
E	80 - 830	50	0
d	$0.9r_V$ - $1.1a_{(res\ 2:1)}$ 0 - $T_{(res\ 2:1)}$ 0 - 360	0.3 50 10	3
E	80 - 830	50	0
J	600 - 2000	200	7

- Total n. of boxes = 9.25e9
- Feasible boxes = 48461 (5.23e-4%)
- CPU time = 2392 s
- Best ObjFcn = 8.670 (Ref. solution†: 8.680)
- Best ObjFcn = 2.84 + 0 + 0 + 0.39 + 0 + 5.42
- Best ObjFcn (GASP) = 10.09

† Vasile and De Pascale, 2006

Results: EVEdEJ problem



Results: EdVdM problem

Problem	Search Space	Int. size	Cut-off (10)
E	0 - 1000	50	4
d	$0.9r_V$ - $1.1r_E$ 0 - (T_V-T_E) 0 - 360	0.1 50 10	1
V	50 - 400	25	0
d	$0.9r_V$ - $1.1r_M$ 0 - (T_M-T_V) 0 - 360	0.1 50 10	5
M	200 - 1000	50	3

- Total n. of boxes = 8.70e9, Feasible boxes = 1.13e4 (1.24e-4%)
- CPU time = 2324 s
- Best ObjFcn = 8.167
- Best ObjFcn = 2.89 + 0 + 0 + 4.37 + 0.90
- Best ObjFcn (GASP) = 8.522

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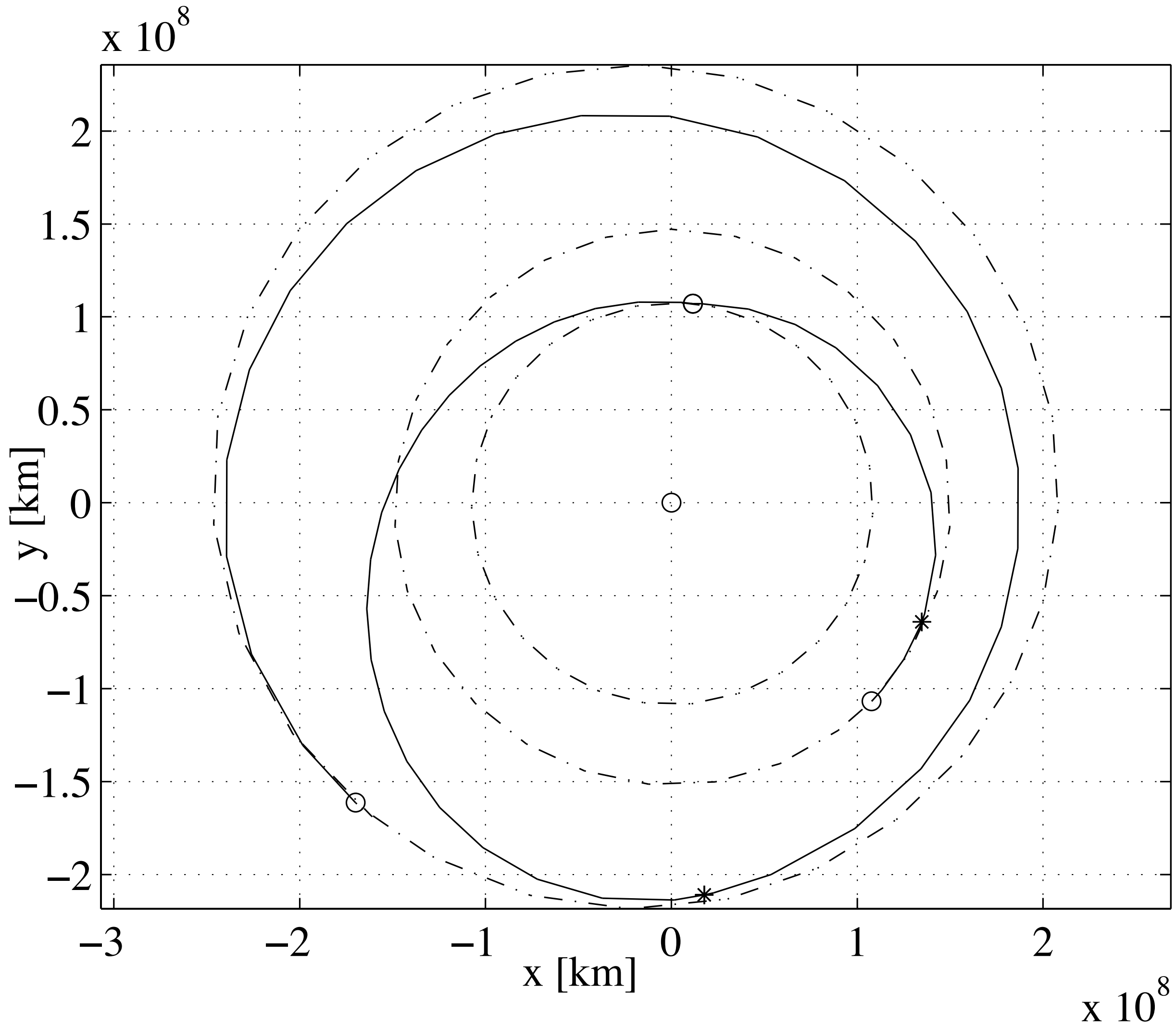
Results: EdVdM problem



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Results: EVdMdE problem

Problem	Search Space	Int. size	Cut-off (13)
E	3000 - 4000	50	5
V	25 - 425	25	0
d	$0.9r_M$ - $1.1r_M$ 0 - $(T_M$-$T_J)$ 0 - 360	0.2 50 10	2
M	20 - 420	50	0
d	$0.9r_E$ - $1.1r_M$ 0 - $(T_M$-$T_E)$ 0 - 360	0.2 50 10	4
E	25 - 525	50	5

- Total n. of boxes = 4.97e10, Feasible boxes = 2.05e4 (4e-5%)
- CPU time = 2456 s
- Best ObjFcn = 10.931
- Best ObjFcn = 5.38 + 0 + 0 + 0.97 + 3.64 + 0.93
- Best ObjFcn (GASP) = 12.443

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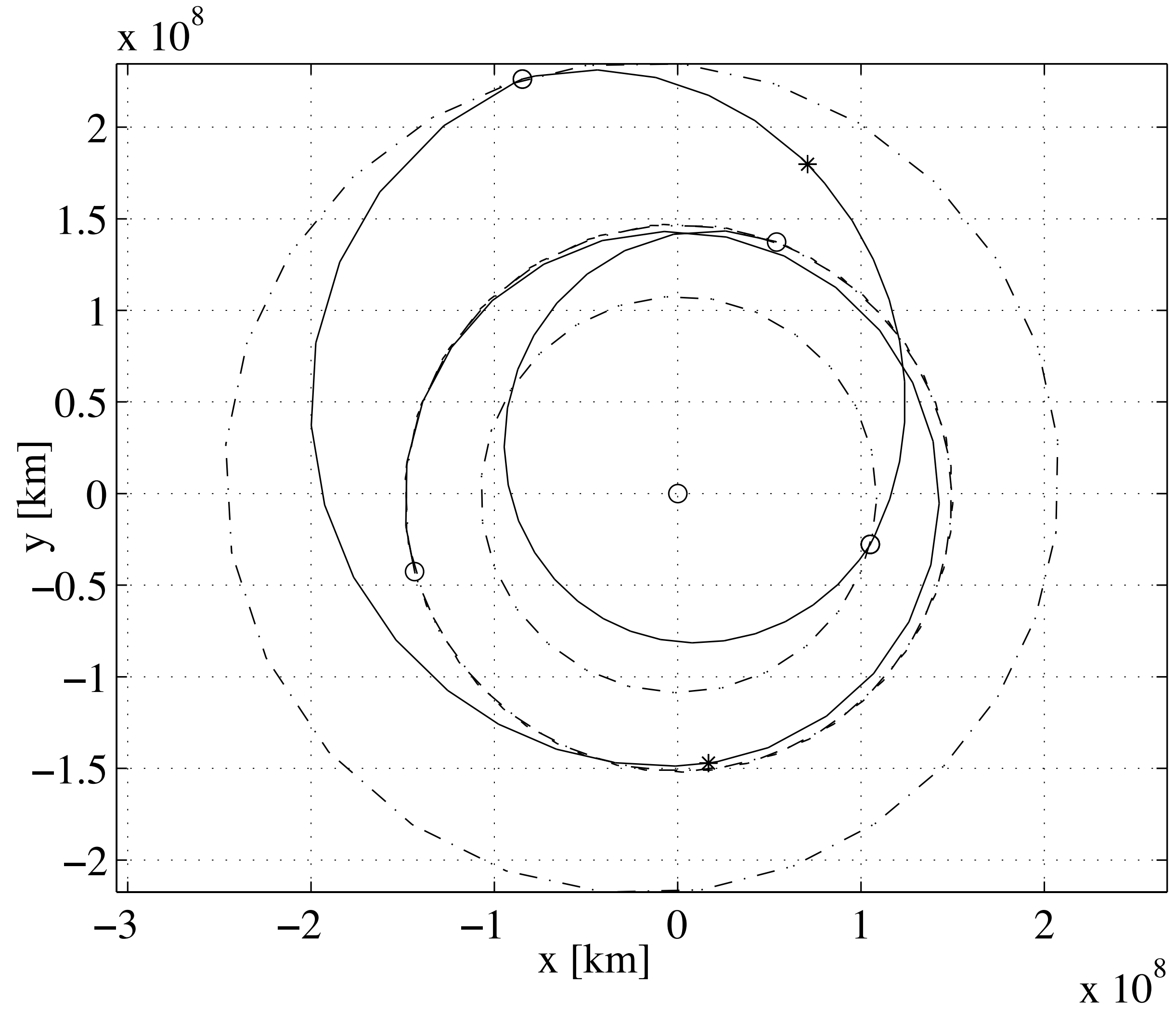
Results: EVdMdE problem



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Results: EVdVEJS problem

Problem	Search Space	Int. size	Cut-off (11)
E	-1000 - 0	50	4
V	80 - 430	25	1
d	$0.9r_V$ - $a_{(res\ 2:1)}$ 0 - $T_{(res\ 2:1)}$ 0 - 360	0.3 50 10	1
V	200 - 500	25	0
E	30 -180	50	0
J	400 - 1600	200	0
S	800 - 2200	200	5

- Total n. of boxes = 3.92e8
- Feasible boxes = 2281 (1e-3%)
- CPU time = 210 s

- Best ObjFcn = 8.299
- Best ObjFcn = 2.933 + 0.693 + 0.428 + 0 + 0 + 0 + 4.243
- Best ObjFcn (GASP) = 8.619

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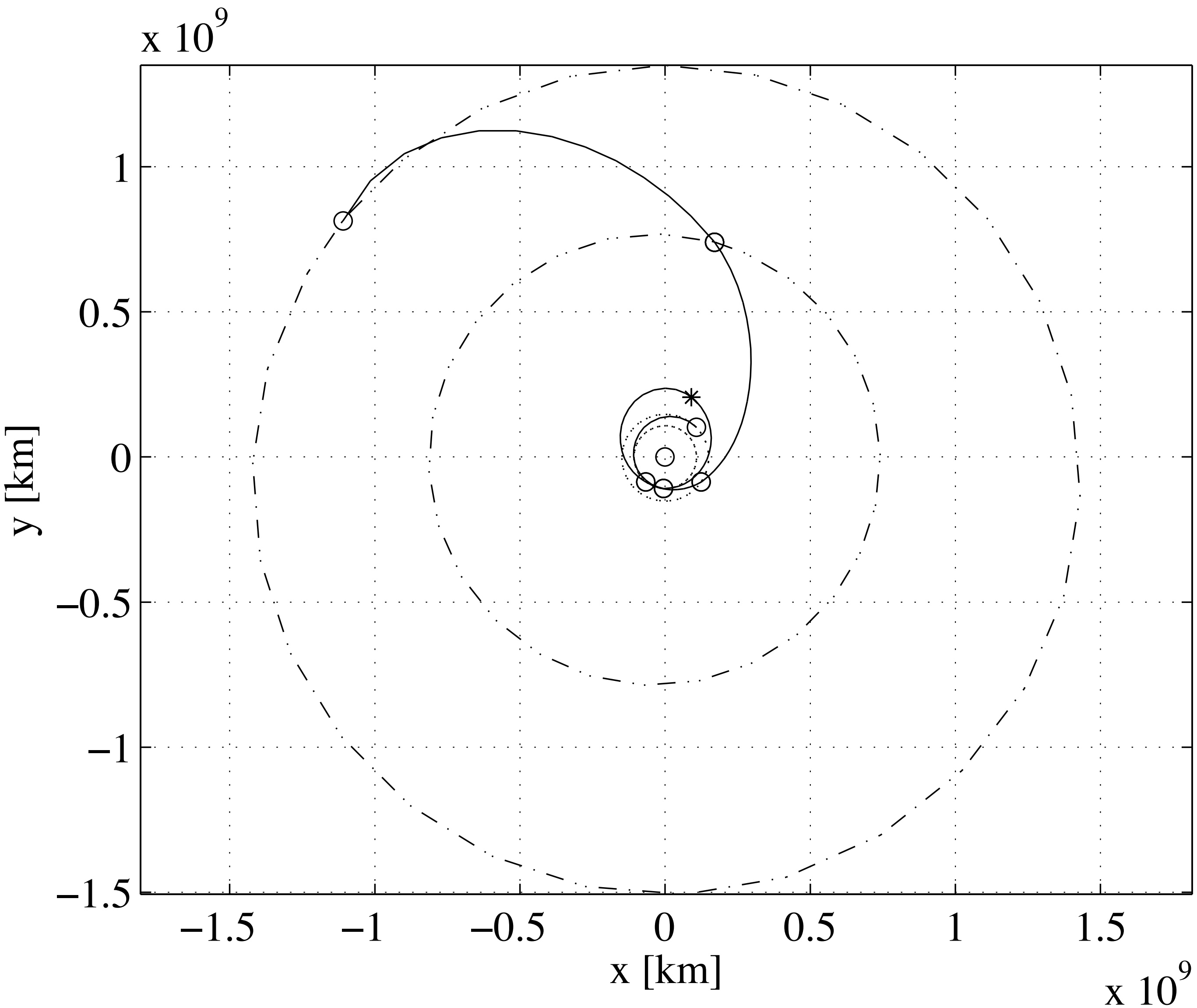
Results: EVdVEJS problem



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Results: EVdVEJdS problem

Problem	Search Space	Int. size	Cut-off (11)
E	-1000 - 0	50	4
V	80 - 430	25	1
d	$0.9r_V$ - $a_{(res\ 2:1)}$ 0 - $T_{(res\ 2:1)}$ 0 - 360	0.3 50 10	1
V	200 - 500	25	0
E	30 -180	50	0
J	400 - 1600	200	0
d	$0.9r_J$ - $1.1r_S$ 0 - $(T_S - T_J)$ 0 - 360	0.6 50 30	1
S	1600 - 3000	200	5

- Total n. of boxes = 1.4109e+12
- CPU time = 2000 s
- Best ObjFcn = 2.78 + 0.89 + 0.38 + 0 + 0 + 0 + 0 + 4.2268

Feasible boxes = 22350 (1.6e-6%)

Best ObjFcn = 8.276

Best ObjFcn (GASP) = 8.619

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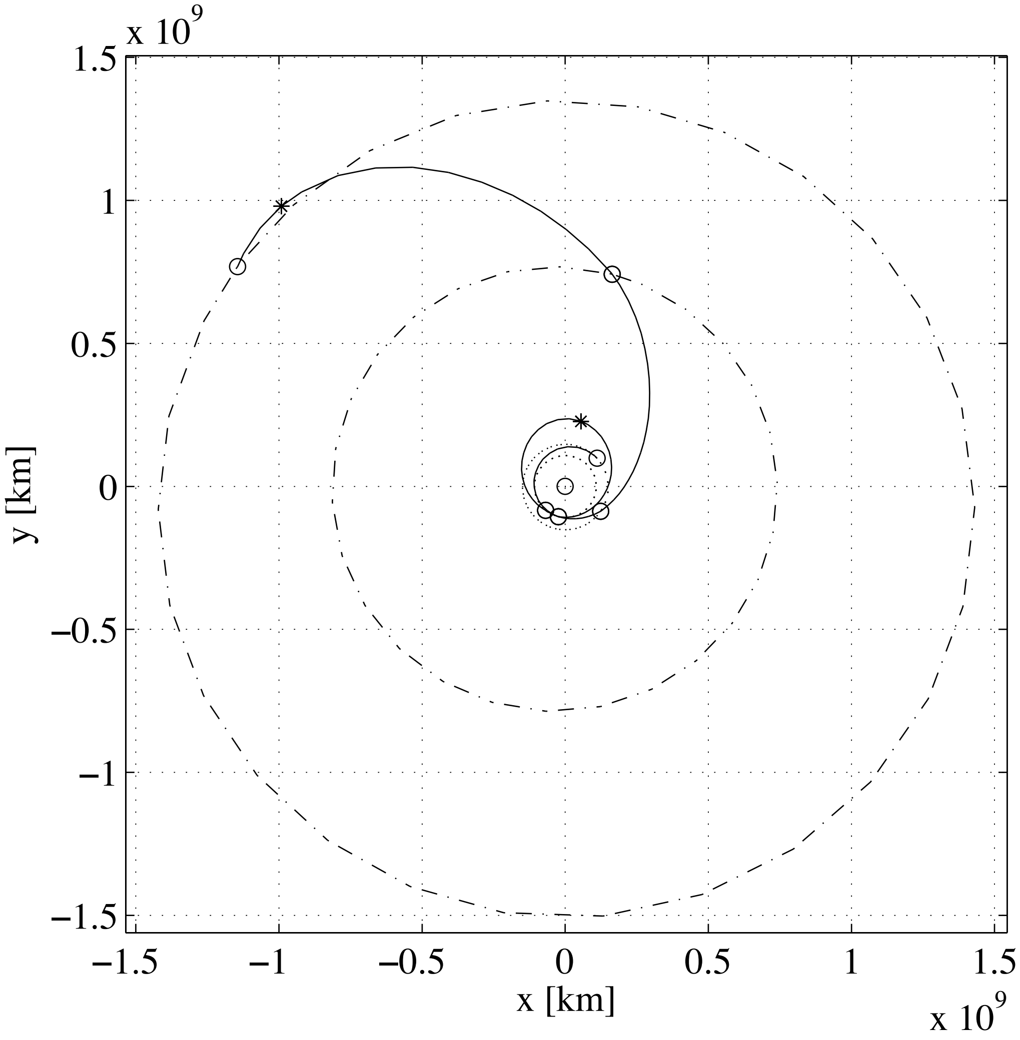
Results: EVdVEJdS problem



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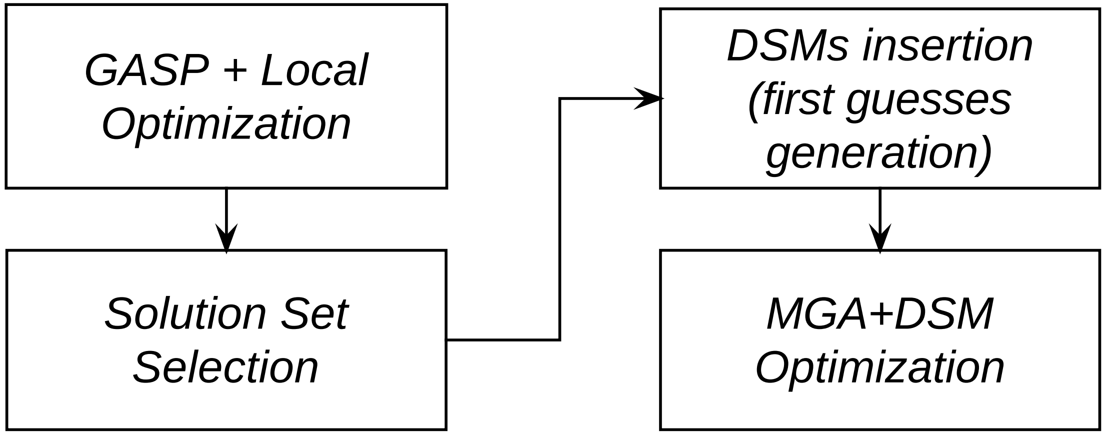
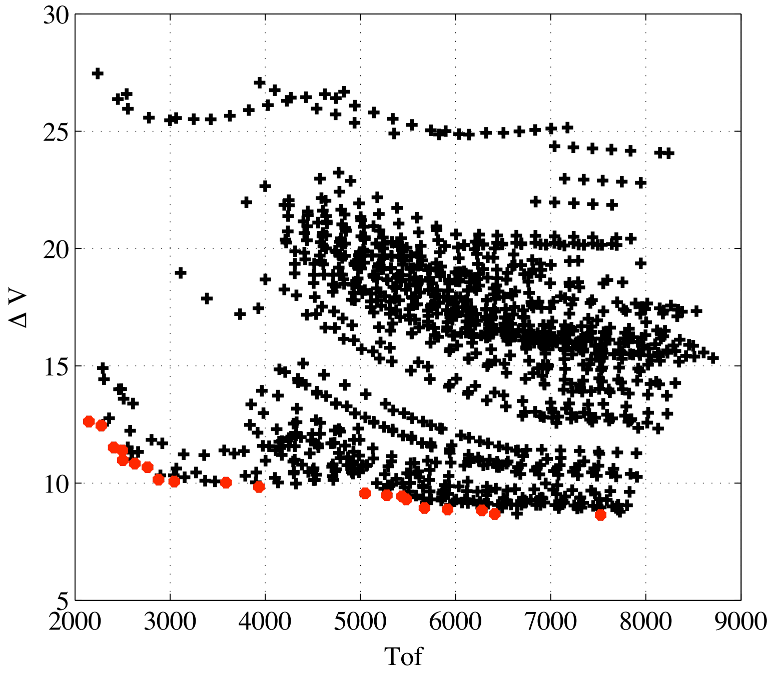
Sequential GASP+DSM

- The method is based on the consideration that DSM transfers can be obtained by modifying MGA ones
- MGA-DSM trajectories are obtained by inserting DSM in desired transfer legs
- The need of a DSM is suggested by the magnitude of the Δ*V*s required for the powered GAs
- The overall optimization architecture is



Solution Set Selection

- GASP delivers a set of solutions as a result of local optimizations inside the remaining boxes
- The non-dominated solutions in the *Tof*-Δ*V* plot are selected for DSMs insertion
- Solutions close to the Pareto set can be included to increase the number of cases analyzed (by dominance level)



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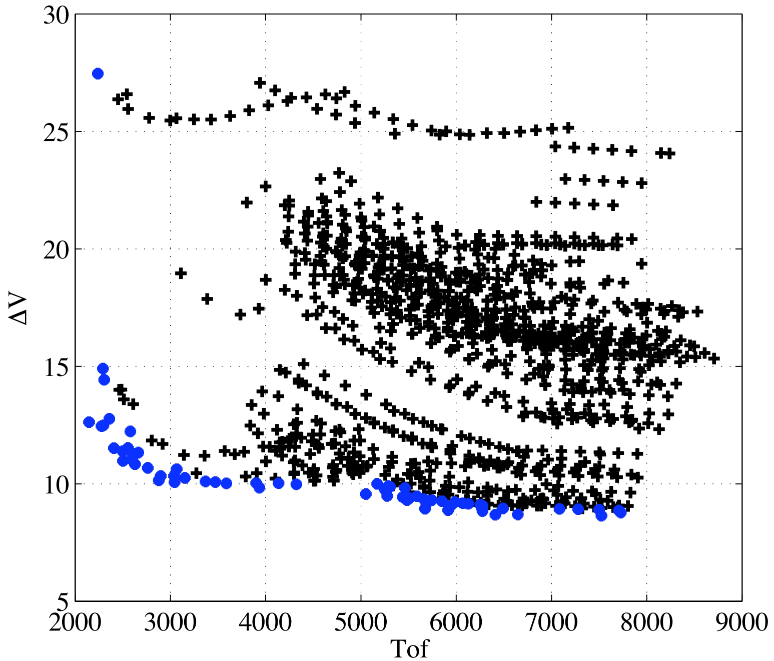
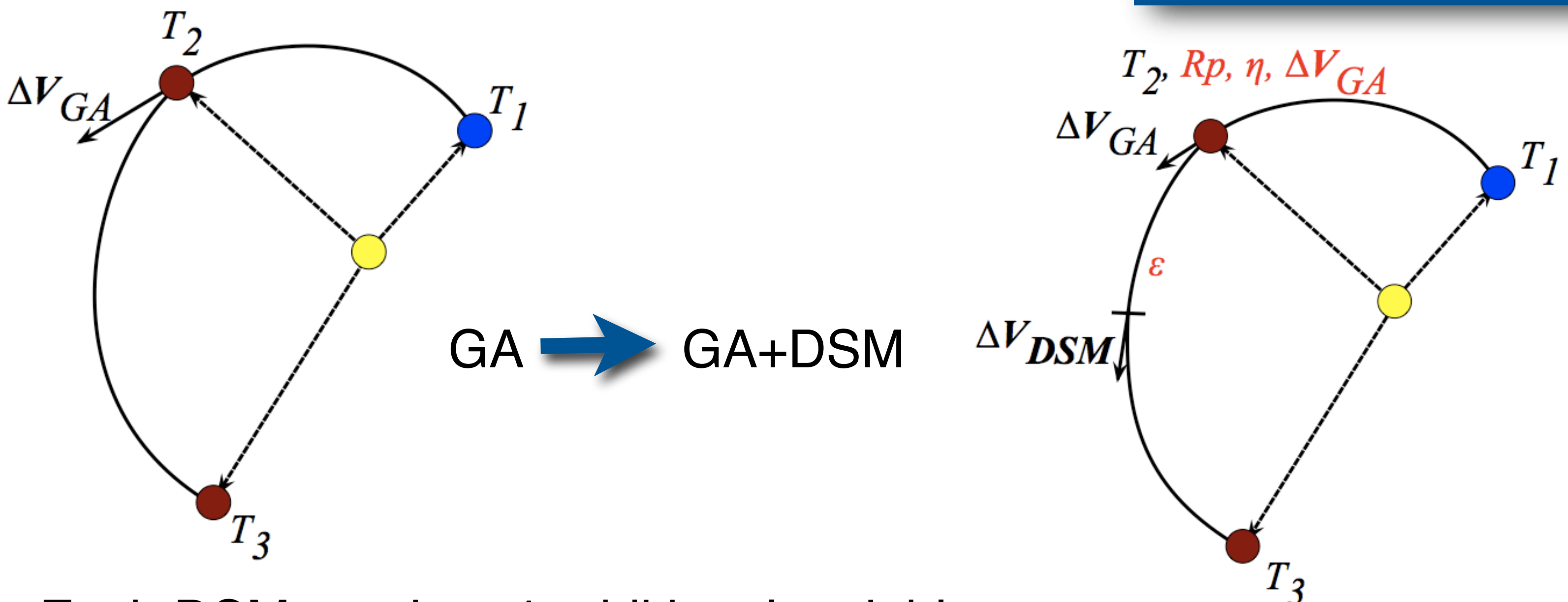


DSM modeling



- Each DSM requires 4 additional variables
  - The pericenter radius r_p is no more computed trough the bending angle equation
  - The GA plane is determined by the additional variable η
  - A tangential ΔV is allowed at the hyperbola pericenter
  - The first part of a leg including a DSM is analytically propagated for the fraction of the time of flight ϵ
- The ΔV_{DSM} is computed by solving a Lambert´s problem

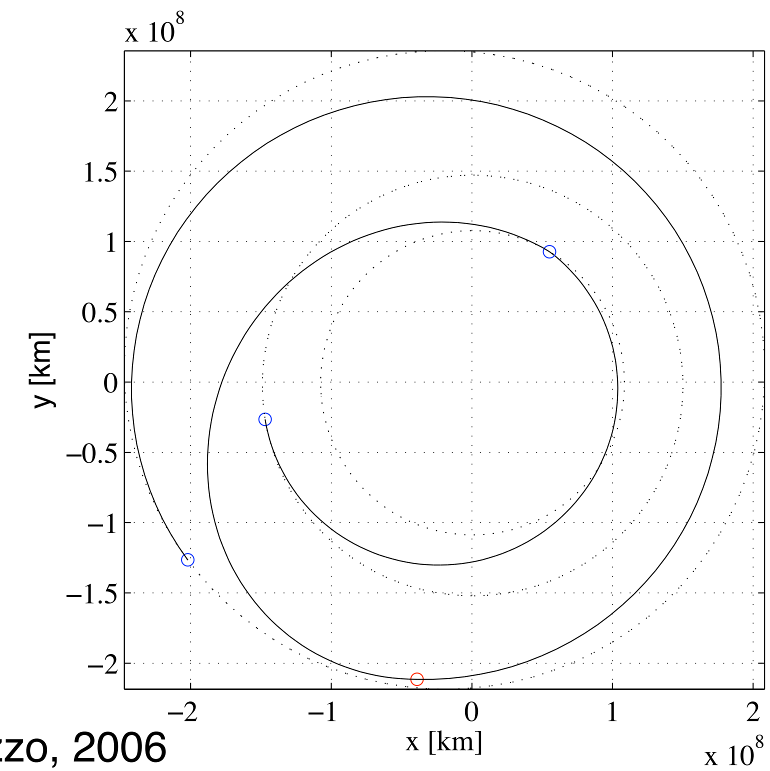
First Guesses and Optimization

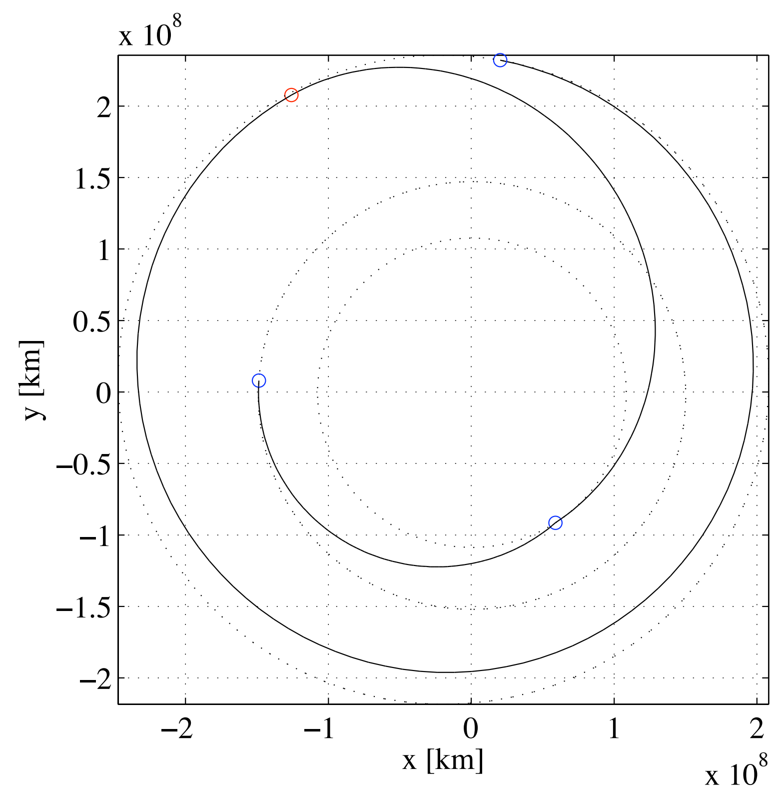
- The number and the arcs where DSMs are introduced is given by setting a limit on ΔV_{GAs}
- No DSM in the first leg is allowed (the ΔV at launch has been already bounded in GASP)
- Each selected MGA solution is modified to obtain a first guess for a DSM transfer setting
  - $\Delta V_{GA} = 0$, $\epsilon = 0.5$
  - R_p and η are those of the MGA transfer
- The problem is formulated as nonlinear programming problem with box constraints
- A SQP optimizer is used to obtain optimal DSM transfers
- The non dominated solutions in the plot *ΔV-Tof* are identified after the optimization

Results: EVM problem

- Optimal structure: EVdM
  - Search space [days]: [0, 2000]x[100, 1200]x[100, 2000]
  - Best obj [km/s] = 7.75
  - ΔVs [km/s] = 3.43 + 0 + 3.52 + 0.80
  - GASP sol [km/s] = 9.03
  - Ref sol† [km/s] = 8.15

  - Search space [days]: [1000, 6000]x[100, 500]x[100, 1000]
  - Best obj [km/s] = 8.07
  - ΔVs [km/s] = 2.95 + 0 + 3.90 + 1.22
  - GASP sol [km/s] = 8.52





† Izzo, 2006

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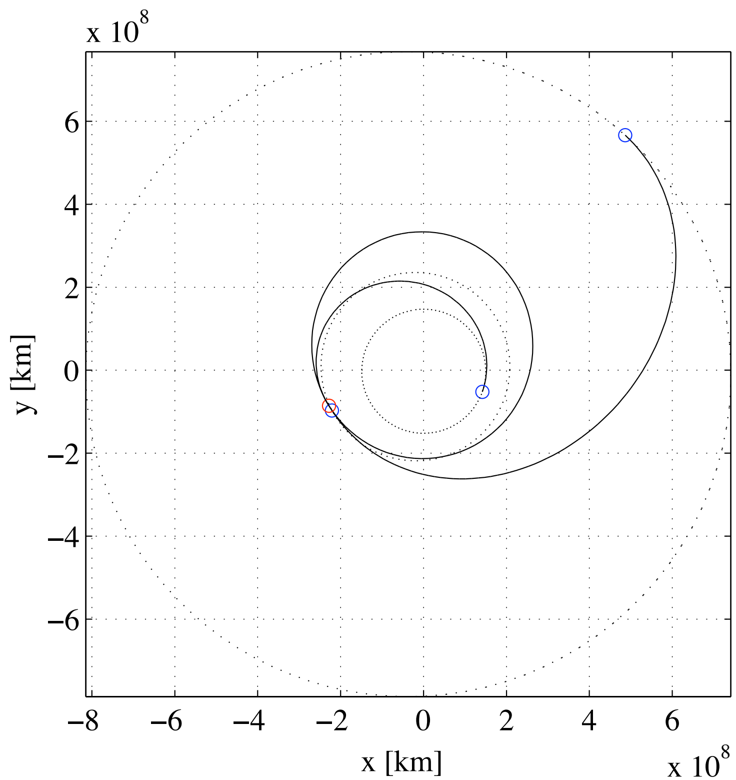
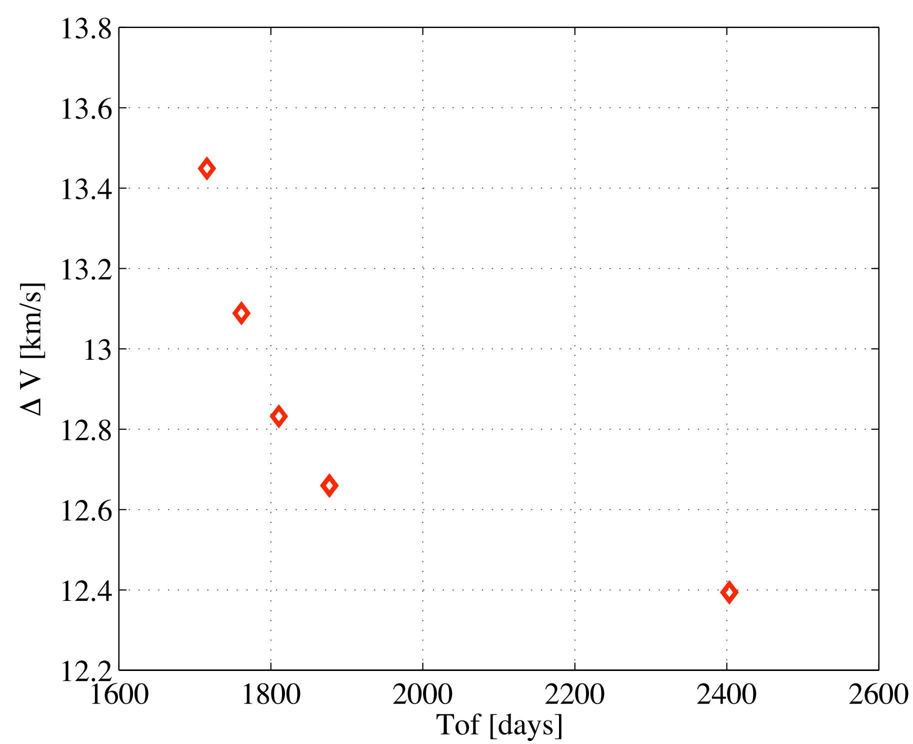
Results: EMJ problem

- Optimal Structure: EMdJ
  - Search space [days]: [1000, 6000]x[100, 1200]x[100, 2000]
  - Best obj [km/s] = 12.39 (12.48 GASP DSM)
  - ΔV dep [km/s] = 3.93
  - $\Delta V_{GA,\ DSM}$ [km/s] = 0 + 4.04
  - ΔV arr [km/s] = 4.41
  - GASP sol [km/s] = 13.41





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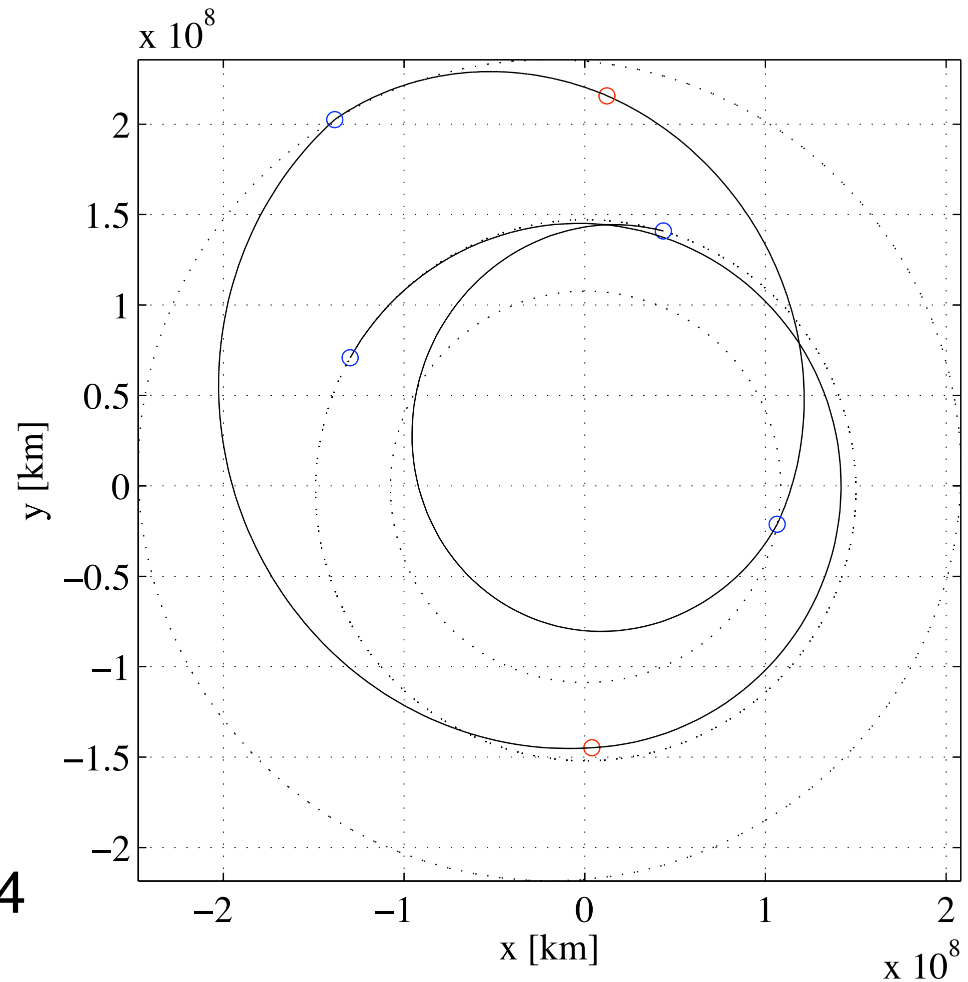
Results: EVME problem

- Optimal structure: EVdMdE
  - Search space [days]:
    [3000, 4000]
    [25, 525]
    [20, 520]
    [25, 525]
  - Best obj [km/s] = 10.87
    (10.93 GASP DSM)
  - ΔV dep [km/s] = 5.47
  - $\Delta V_{GA,DSM}$ [km/s] =
    0 + 0.70 + 0 + 4.21
  - ΔV arr [km/s] = 0.49
  - GASP sol [km/s] = 12.44



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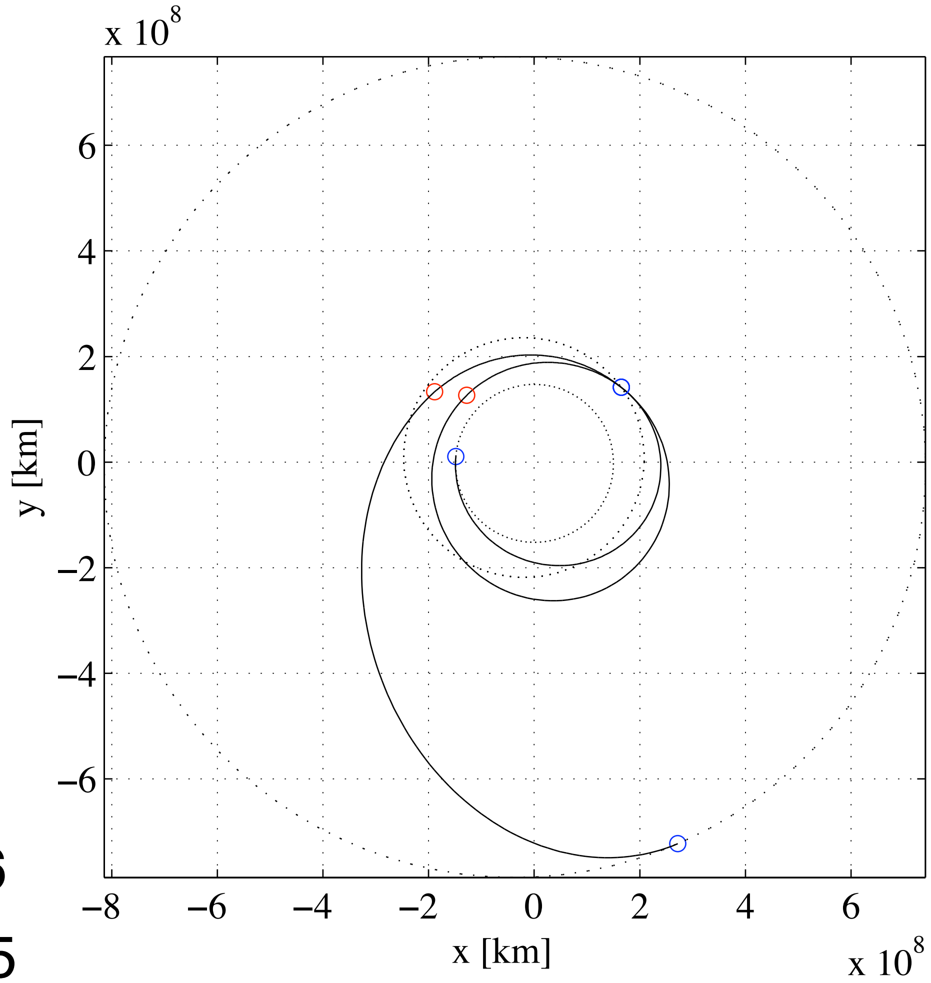
Results: EMMJ problem

- Optimal structure: EMdMdJ
  - Search space [days]:
    [3650, 5500]
    [80, 430]
    [330, 830]
    [1000, 2000]
  - Best Obj [km/s] = 10.97
    (10.84 GASP DSM)
  - ΔV dep [km/s] = 3.26
  - $\Delta V_{GA,DSM}$ [km/s] =
    0 + ≈ 0 + 0 + 3.20
  - ΔV arr [km/s] = 4.51
  - GASP sol [km/s] = 12.86
  - Ref sol† [km/s] = 11.05



† Vasile and De Pascale, 2006

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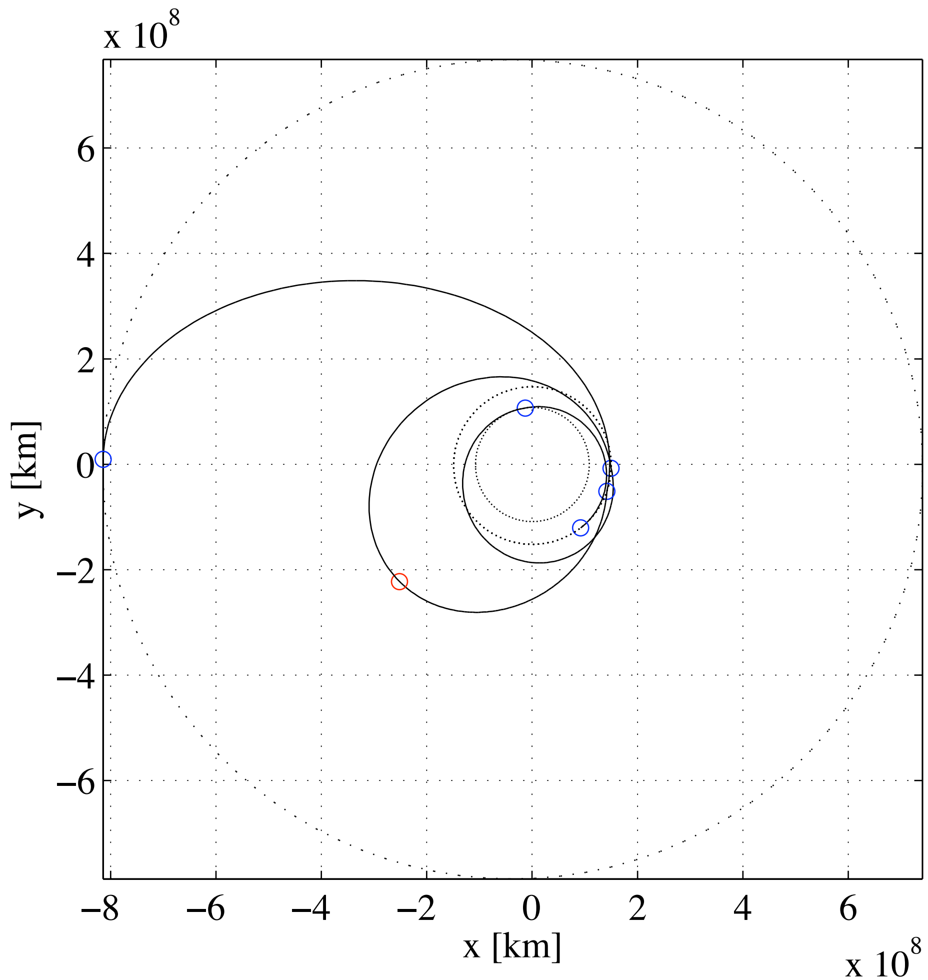
Results: EVEEJ problem

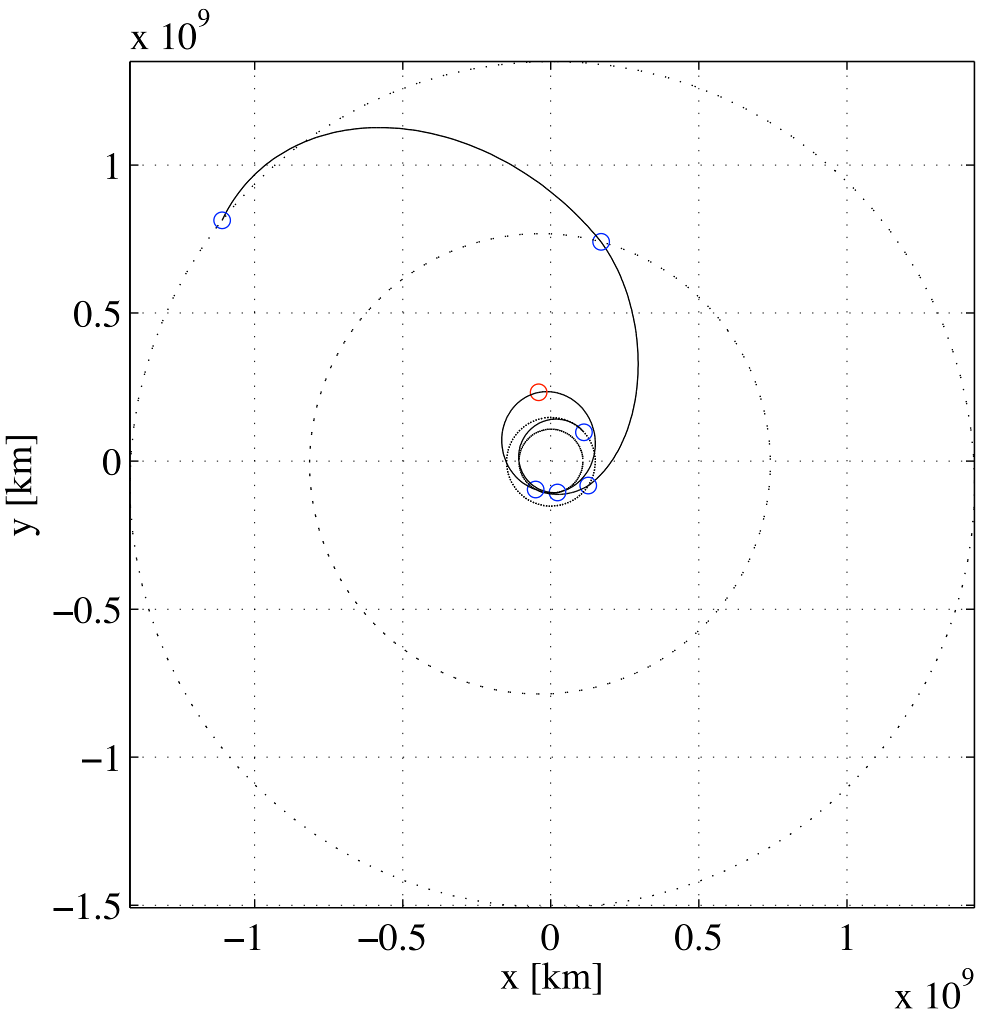
- Optimal structure: EVEdEJ
  - Search space [days]:
    [3650, 7300]
    [80, 430]
    [80, 830]
    [80, 830]
    [600, 2000]
  - Best obj [km/s] = 8.68 (8.67 GASP DSM)
  - ΔV dep [km/s] = 2.83
  - $\Delta V_{GA,\ DSM}$ [km/s] = 0 + 0 + 0.42 + 0
  - ΔV arr [km/s] = 5.43
  - GASP sol [km/s] = 9.72
  - Ref sol† [km/s] = 8.68



† Vasile and De Pascale, 2006

Results: EVVEJS problem

- Optimal structure: EVdVEJS
  - Search space [days]
    [-1000, 0]
    [30, 430]
    [100, 500]
    [30, 330]
    [400, 1600]
    [1000, 2200]
  - Best obj [km/s] = 8.59 (8.29 GASP DSM)
  - ΔV dep [km/s] = 3.57
  - $\Delta V_{GA,DSM}$ [km/s] = 0.40 + 0.58 + 0 + 0 + 0
  - ΔV arr [km/s] = 4.24
  - GASP sol [km/s] = 8.68
  - Ref sol† [km/s] = 9.06

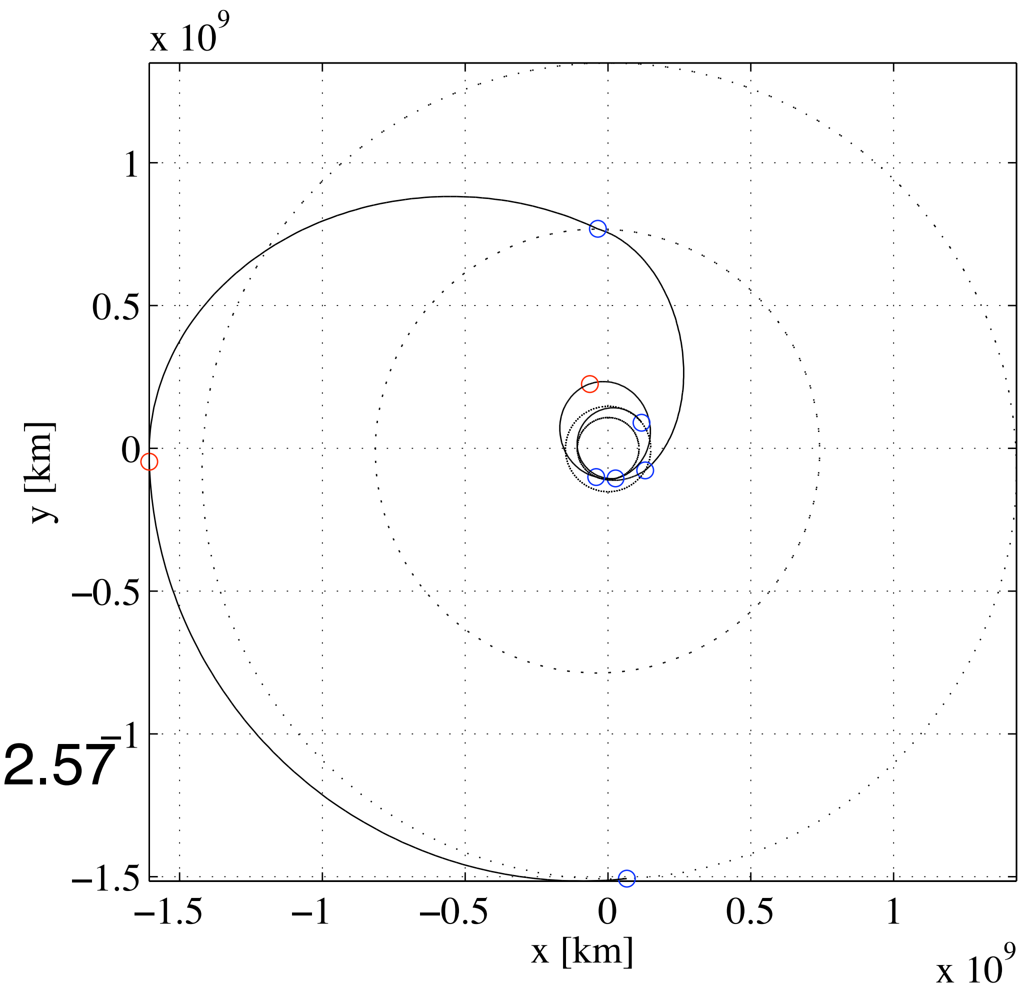


† Vasile and De Pascale, 2006

Results: EVVEJS problem

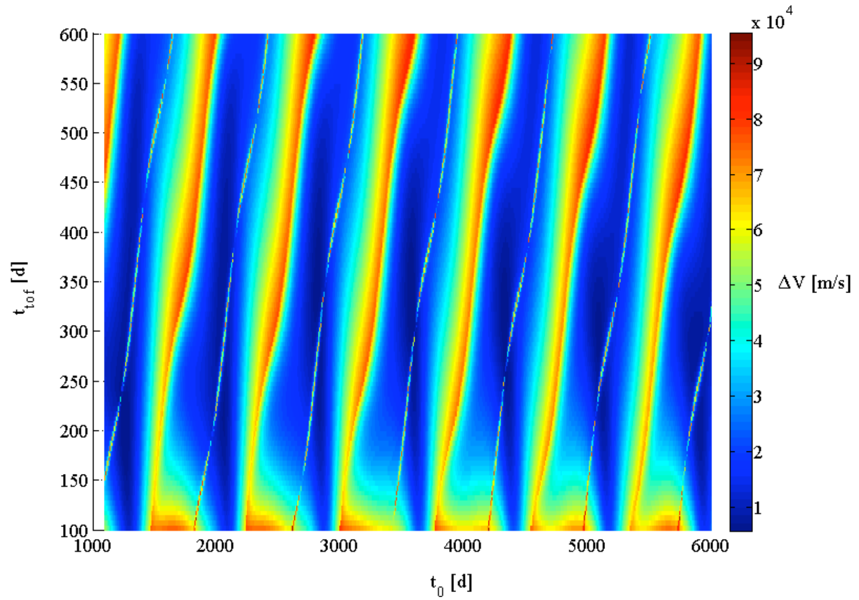
- Optimal Structure EVdVEJdS

  - Search space [days]
    [-1000, 0]
    [30, 430]
    [100, 500]
    [30, 330]
    [400, 2000]
    [1000, 6000]
  - Best obj [km/s] = 7.60
  - ΔV dep [km/s] = 3.57
  - $\Delta V_{GA,DSM}$ [km/s] =
    0.33 + 0.33 + 0 + 0 + 0 + 2.57
  - ΔV arr [km/s] = 0.80
  - GASP sol [km/s] = 8.61



Conclusions: General Considerations

- A DA-Based version of GASP has been implemented
  - Significant work was devoted to solve the **discontinuity and dependency problems**
  - The resulting algorithm can effectively optimize MGA transfers with no DSMs



- The DA-Based GASP has been **extended to include DSMs**
  - Dependency problem leads to the choice of modeling the whole transfer as a sequence of Lambert´s arcs
  - Test cases show the effectiveness of the algorithm

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Conclusions: GASP-DA with DSM

Main problems limiting the number of DSMs:

- **Memory**:
  - A **DA number is a vector** of $(n+v)!/(n!v!)$ coefficients where n is the order and v the number of variables
  - **Many coefficients** are usually equal to zero
  - But COSY does not allow dynamic memory allocation

A **great amount of memory** must be allocated anyway

- **Dependency**:
  - 3 variables must be added for each DSM
  - **Multimodality** increases with DSM number
    → Taylor expansions might **lose** their **accuracy**

Conclusions: Alternative Approach

- An **alternative approach** has been developed
  - The **DA based GASP** with no DSM is used to gain suitable **first guess solutions** (Pareto optimality)
  - An **optimization process** is performed to **insert DSMs**
- **Main advantages**:
  - **Larger search spaces** can be processed
  - The **resulting solutions** are **comparable** with those achieved by the pruning based algorithm
  - The **computational time decreases** significantly
- **Future steps**:
  - Insertion of a **global component** at the beginning of the optimization process
  - Insertion of **more than 1 DSM** per arc

Taylor Models

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Differential Algebra

- Describe the bulk of the functional dependence through a Taylor polynomial

Interval Arithmetic

- Bound the deviation of the original function from the polynomial by an interval

Taylor Model

- Theorem (Taylor):
  - Given $f : [\boldsymbol{a}, \boldsymbol{b}] \subset \mathbb{R}^v \rightarrow \mathbb{R}, \quad f \in \mathcal{C}^{n+1}([\boldsymbol{a}, \boldsymbol{b}])$
  - Given $\boldsymbol{x}_0 \in [\boldsymbol{a}, \boldsymbol{b}]$
  - Then, $\forall \boldsymbol{x} \in [\boldsymbol{a}, \boldsymbol{b}]$, there is $\theta \in \mathbb{R}, \; 0 < \theta < 1,$ such that:

$$f(\boldsymbol{x}) = P_{n,\boldsymbol{x}_0,f} + \frac{1}{(n+1)!}((\boldsymbol{x} - \boldsymbol{x}_0) \cdot \boldsymbol{\nabla})^{n+1} f(\boldsymbol{x}_0 + (\boldsymbol{x} - \boldsymbol{x}_0)\theta)$$

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Taylor Models

- Consequently, the function f can be written as:

$$f(\boldsymbol{x}) = P_{n,\boldsymbol{x}_0,f}(\boldsymbol{x}-\boldsymbol{x}_0) + \varepsilon_{n,\boldsymbol{x}_0,f}(\boldsymbol{x}-\boldsymbol{x}_0)$$

- Let the interval $I_{\alpha,f}$, $\alpha = (n, \boldsymbol{x}_0, [\boldsymbol{a}, \boldsymbol{b}])$, be such that:

$$\forall \boldsymbol{x} \in [\boldsymbol{a}, \boldsymbol{b}], \quad \varepsilon_{n,\boldsymbol{x}_0,f}(\boldsymbol{x}-\boldsymbol{x}_0) \in I_{\alpha,f}$$

- Then: $\forall \boldsymbol{x} \in [\boldsymbol{a}, \boldsymbol{b}], \quad f(\boldsymbol{x}) \in P_{n,\boldsymbol{x}_0,f}(\boldsymbol{x}-\boldsymbol{x}_0) + I_{\alpha,f}$

- The pair $(P_{n,\boldsymbol{x}_0,f}, I_{\alpha,f})$ is said a **Taylor Model** (TM) of f :

$$T_{\alpha,f} = (P_{n,\boldsymbol{x}_0,f}, I_{\alpha,f})$$

and it represents a validated enclosure of f on $[\boldsymbol{a}, \boldsymbol{b}]$

VAlidated Global Optimization of MGA

- Non-guaranteed global optimizers are classically used in MGA transfers design
  - They are able to find **good solutions**
  - The solution is **not guaranteed** to be the global optimum

We have only best known solutions

- Validated global optimizers can be used to rigorously identify the **global minimum**
- COSY-GO implements a validated global optimizer based on Taylor Models

Main Problems on MGA Transfers

- The validated solution of parametric implicit equations must be computed
  - Ephemeris evaluation → Kepler´s eq.
  - Lambert´s problem → Lagrange´s eq.
  - Power GA → Bending angle eq.
- A dedicated algorithm is developed
  - Find a reference point solution
  - Expand the solution using DA
  - Validate the solution using TM
- The previous validated solver enables the TM evaluation of the objective function for MGA

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  - Power GA → Bending angle eq.
- A dedicated algorithm is developed
  - Find a reference point solution
  - Expand the solution using DA

  } DA-based GASP
  - Validate the solution using TM
- The previous validated solver enables the TM evaluation of the objective function for MGA

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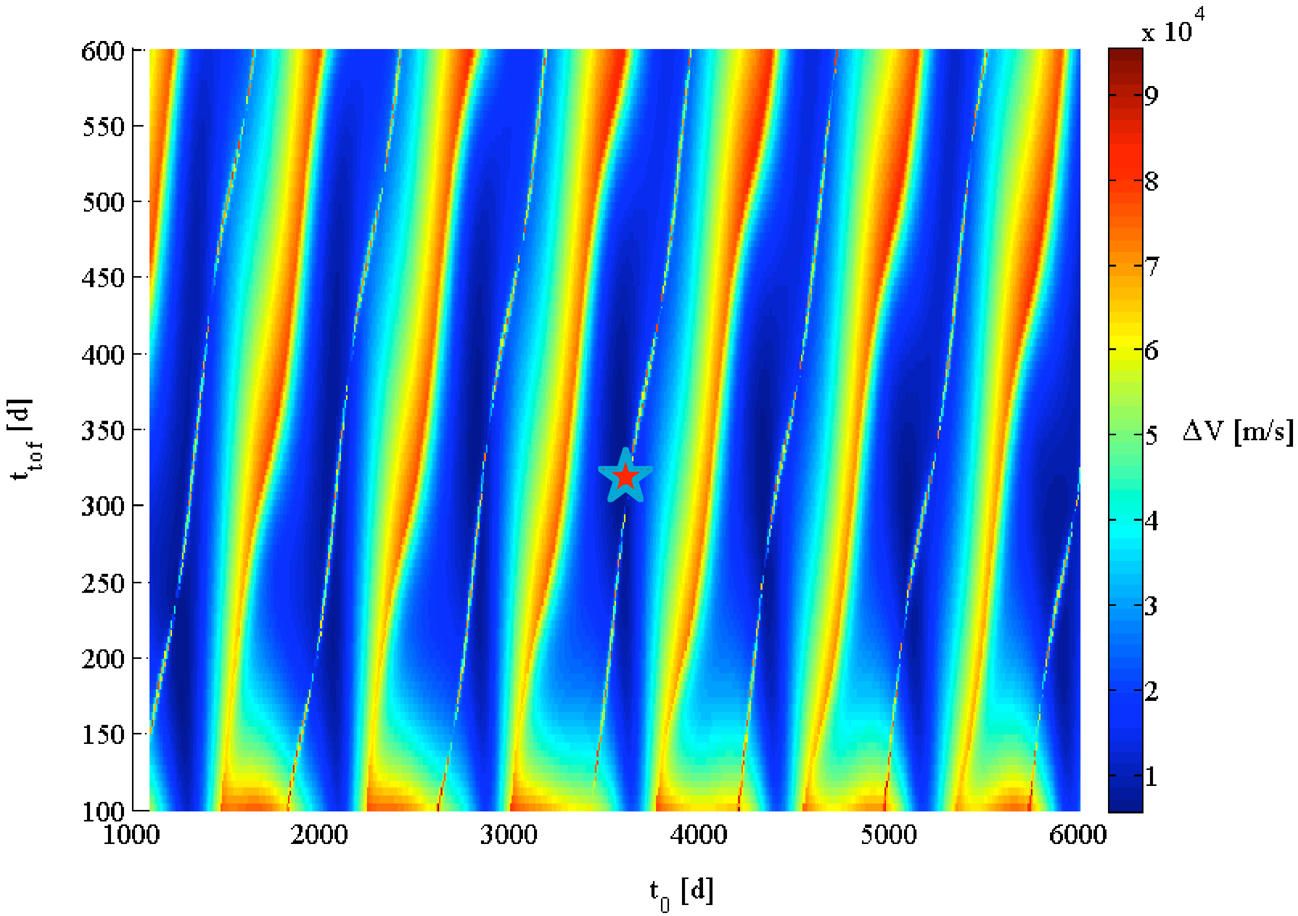
Main Problems on MGA Transfers

- The validated solution of parametric implicit equations must be computed
  - Ephemeris evaluation → Kepler´s eq.
  - Lambert´s problem → Lagrange´s eq.
  - Power GA → Bending angle eq.
- A dedicated algorithm is developed
  - Find a reference point solution } DA-based GASP
  - Expand the solution using DA } DA-based GASP
  - Validate the solution using TM
- The previous validated solver enables the TM evaluation of the objective function for MGA

Earth-Mars 2 impulse Transfer

- COSY-GO was applied to find the verified global optimum of the impulsive Earth-Mars transfer



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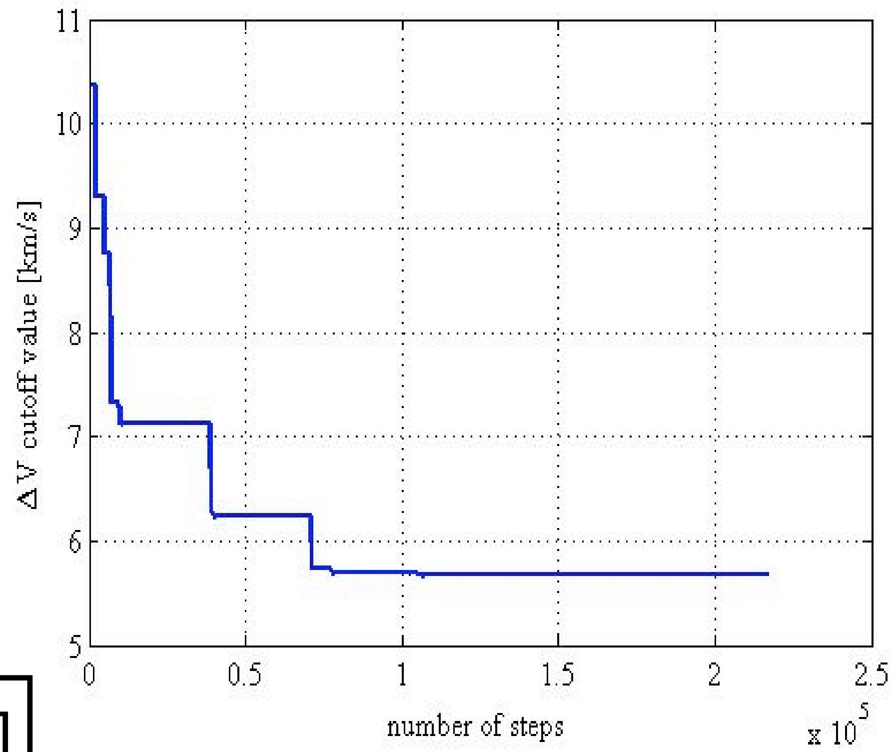
Earth-Mars 2 impulse Transfer

- COSY-GO was applied to find the verified global optimum of the impulsive Earth-Mars transfer
- Platform: Pentium IV 3.06 GHz laptop
- Computational time: 4954.39 s
- Enclosure of the minimum [km/s]: [5.6673264, 5.6673272]
  (GASP-DA: 5.6673270)
- Enclosure of the solution [days]:

t_0	[3573.176, 3573.212]
t_{EM}	[324.034, 324.088]



Earth-Venus-Mars Transfer

- The validated implicit equation solver is used for the **powered gravity assist maneuver**
- The **MGA** problem is reformulated **in absolute times**
- The verified global optimization of the Earth-Venus-Mars transfer is addressed
- Search Space [days] : [5000,6000]x[140,240]x[200,400] (t_0, t_{EV}, t_{VM})
- Results:
  - **Computational time**: about 3 weeks!
  - **Enclosure of the minimum**: [8.5220251,8.5231393] km/s (GASP-DA: 8.5226)
  - **Enclosure of the solution:**

t_0	[5611.475, 5611.512]
t_{EV}	[157.592, 157.623]
t_{VM}	[255.564, 255.620]

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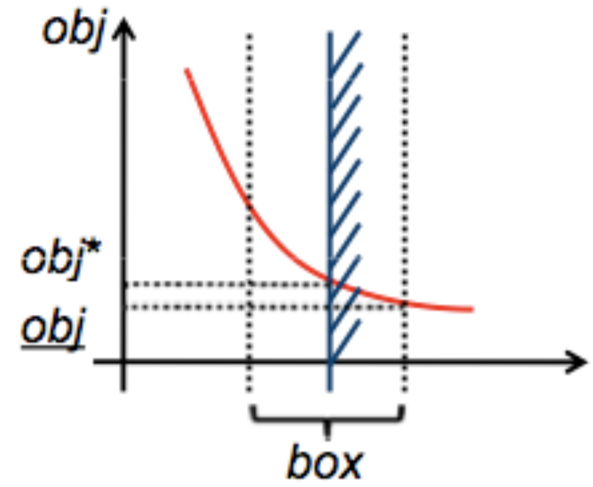
Main Drawbacks and possible solutions

- Management of the constraints
  - Boxes lying on the constraint are kept as feasible

    The computed obj can **underestimate** the actual obj*
  - Possible solution: **validated enclosure of the constraint manifold**



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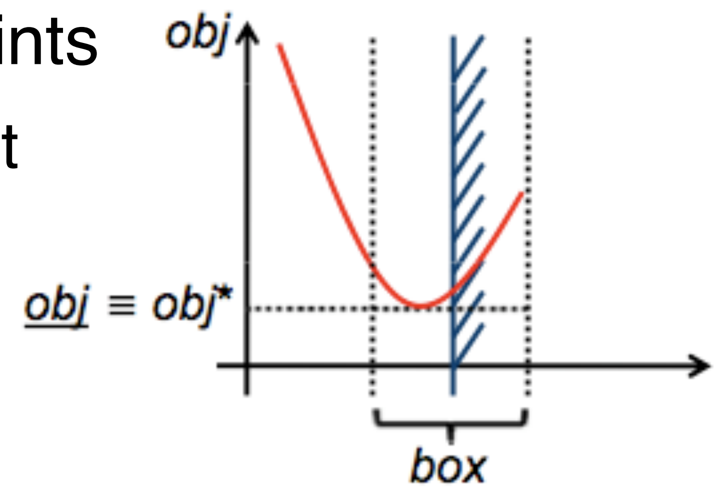

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NOTE: The global optimum of the Earth-Venus-Mars transfer **does not lie** on the constraint manifold

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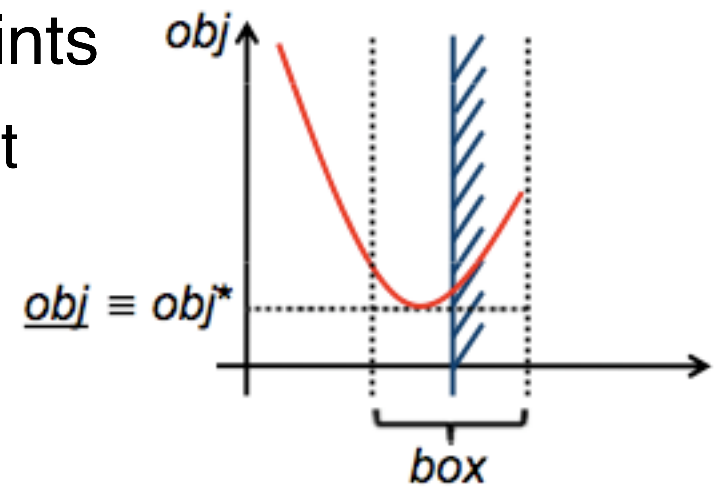

The computed obj can **underestimate** the actual obj*

  - Possible solution: **validated enclosure of the constraint manifold**

NOTE: The global optimum of the Earth-Venus-Mars transfer **does not lie** on the constraint manifold

- High computational time
  - COSY-GO algorithm is fully parallelizable

Possible solution: **Parallel run**

Conference announcement

New Trends in Astrodynamics and Applications V

Politecnico di Milano, Milano, Italy

Summer 2008

More details coming soon ...