

EQUILIBRIUM SHAPING: DISTRIBUTED MOTION PLANNING FOR SATELLITE SWARM

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ABSTRACT

A satellite path planning algorithm inspired by collective robotics is presented. The algorithm exploits a behaviour-based approach to achieve a highly distributed control over the relative geometry of a satellite swarm. The desired velocity is defined for each satellite as a sum of different contributions coming from high level behaviours. Several control feedbacks able to track the desired velocities are then introduced and discussed. The resulting architecture is able to solve autonomously the target selection problem.

Key words: Satellite Swarm, Collision Avoidance, Distributed Navigation, Autonomous Navigation, Equilibrium Shaping, Artificial Potential.

1. INTRODUCTION

“Does a coherent group behaviour require an explicit mechanism of cooperation?” “Can useful tasks be accomplished by a homogeneous team of mobile robots without direct communication and using decentralized control?” These sort of questions [1] have been addressed by an increasingly large community of computer scientists, engineers and scientists in general working in a field of research that we may call swarm intelligence or collective robotics. The relevance of the possible answers to the aerospace community is significant. Space engineers are currently developing autonomous systems and are envisaging space missions that would certainly benefit from a deeper understanding of the collective behaviour of similar, but also dissimilar agents. Multi-robot planetary exploration, on-orbit self-assembly and satellites swarm for coordinated observations are just examples of what could be achieved if our technology level was proved to be sufficient to provide spacecraft swarm with autonomous decision capabilities. As recently proposed by Ayre et al. [2] it is possible to build large solar panels or large antennas exploiting collective emerging behaviours. The use of collective robotics is also very relevant for advanced missions architectures such as those recently studied by ESA (APIES mission, [3]) and NASA (ANTS mission, [4]) making use of satellite swarm to explore the asteroid belt.

Because of these synergies between collective robotics and space mission design, it is useful to design a decen-

tralized control for a satellite swarm relying upon the lesson learned from collective robotics. Drawing inspiration from this research field we will often use the name agent to indicate a certain spacecraft belonging to some group. When a complex system of many agents has to act in a coordinated manner, the action selected by each component may or may not take into account the decisions taken by the others. The smallest the number of communications required between the agents, the smallest is the degree of coordination of the system. On the other hand a small amount of communications leads to a simple and robust system. The information exchanged with the other swarm components are useful but not necessary in defining the geometric and kinematical representation of the time varying environment that will then influence the agent action selection. Many works dealing with terrestrial robots navigation [5] with spacecraft proximity and rendezvous operation [6] and self-assembly structures in space [7] have taken the approach of defining an artificial potential field to model the environment. With this method the action selection is then made by following the local gradient of the artificial potential field. Although this method allows a precise modelling of the external environment it also introduces local undesired equilibrium configurations that the system may reach. A laplacian-based potential field [8] or the use of harmonic functions [9] along with the introduction of random walks [10] have been proposed in order to alleviate the problem. Another approach to the action selection problem was introduced by Schoner, [11] based on the dynamic systems theory. In this approach the state space contains behavioural variables such as heading directions or velocities. All the contributions given by each behaviour are combined by means of weighting parameters into a final dynamical system that defines the course of behaviours that each agent will follow. The weighting parameters can be evaluated by solving a competitive dynamics operating at a faster time scale. Recently, also other approaches have been proposed for space applications in the attempt to obtain some degree of decentralized coordination in a group of satellites. Lawton and Beard [12, 13] introduced what they call a Virtual Structure method to design a decentralized formation control scheme while Campbell [14] applied some results of the optimal control theory in order to design a coordinated formation re-configuration manoeuvre. These methods aim at reaching a unique final configuration in which each satellite has its position preassigned. When a swarm of homogeneous agents is considered and the task is given to acquire a certain final geometry, the final positions occupied by each agent in the target configurations should be chosen

in an autonomous way and should be part of the global behaviour emerging from the individual tasks assigned.

In this paper we investigate the possibility of using the limited and local sensing capabilities of each single spacecraft to coordinate the individual responses and achieve a common task without making use of any direct inter satellite communication. The common task we try to achieve is the acquisition of a given relative geometry. We develop a novel behaviour based control [15] able to achieve this. Based on selective sensory information, each selected behaviour contributes to the final decision taken by the spacecraft control system. We show how this approach may drive a self-assembly process of homogeneous agents in space and it scales well with the number of satellites belonging to the swarm. With respect to previously developed techniques the new approach presented has two advantages: it does not make use of any inter-satellite communication and it autonomously assigns the final satellites positions. Due to its behavior based nature the procedure is sub-optimal.

2. METHODOLOGY

Consider the relative motion of N spacecraft randomly distributed in the space neighboring N targets and subject to the gravitational attraction of a near planet. In some relevant Local Horizontal Local Vertical (LHLV) reference frame we define the target positions ξ_i , $i = 1..N$ and the initial states \mathbf{x}_i^0 , \mathbf{v}_i^0 , $i = 1..N$ of each spacecraft. Our goal is to build a real time navigation scheme allowing each agent to autonomously decide what final target to acquire relying just upon its limited sensor information, and to safely navigate to it without conflicting with the other spacecraft. We will follow a two-step:

- First, a method is developed that defines for each target disposition and each agent neighborhood configuration the desired velocity vector of the agent as a sum of different weighted contributions named behaviours.
- Then, several control techniques are considered that allow each spacecraft to track the desired kinematical field.

In this way the control design is completely independent from the design of the desired velocity field and may be faced separately.

3. DESIGN OF THE UNDERLYING KINEMATICAL FIELD: THE EQUILIBRIUM SHAPING

The approach we propose and that we call Equilibrium Shaping, draws inspiration from past published works on robot path planning and artificial intelligence. In the work

by Gazi [16, 17] some theoretical results have been introduced on the dynamics of aggregating swarm of robots. Each agent of the swarm is there asked to follow a certain velocity field defined as the sum of two different contributions, both solely dependent from the inter-agent distance $\mathbf{x}_{ij} = \mathbf{x}_j - \mathbf{x}_i$. The first contribution defines a linear global gather behaviour whereas the second one introduces an avoidance behaviour. The mathematical definition used by Gazi for the desired velocity of the i -th agent is:

$$\mathbf{v}_{d_i} = - \sum_j \mathbf{x}_{ij} \left[c_i - b_i \exp \left(- \frac{\mathbf{x}_{ij} \cdot \mathbf{x}_{ij}}{k_1} \right) \right]$$

where c_i, b_i are coefficients whose value are uniquely determined by the formation geometry. This method produces a swarm in which each agent is preassigned to a particular place in the final formation. It is possible to achieve a global swarm behaviour that also solves the target assignment problem autonomously. This may be achieved defining the desired kinematical field according to the Equilibrium Shaping approach proposed in this paper. This technique consists in building a dynamical system that has as equilibrium points all the possible agents permutations in the final target formation and using it to define the desired velocities. Let us consider the simple example of a swarm of two satellites that aims at reaching a final configuration made up of the two geometric positions given by $\xi_1 = [1, 0, 0]$ and $\xi_2 = [-1, 0, 0]$. We have to build a dynamical system that admits two equilibrium configurations, one in which the agent 1 is in ξ_1 and agent 2 in ξ_2 and one in which the final positions are inverted.

We design our dynamic as a sum of three different behaviours that we name: gather, avoid and dock. The mathematical expression of each kind of behaviour along with some brief comments are listed below:

- **Gather Behaviour** This behaviour introduces N different global attractors towards the N targets. The analytical expression of this behaviour contribution to the i -th agent desired velocity may be written in the following form:

$$\mathbf{v}_i^{Gather} = \sum_j c_j \psi_G(\|\xi_j - \mathbf{x}_i\|)(\xi_j - \mathbf{x}_i)$$

where ψ_G is a mapping from positive real to positive reals that introduces some non linear dependency from the target distance. This behaviour may also be designed to account for the gravitational field as we shall see in the next section. There are some important choices that we implicitly make when we choose this form of the gather behaviour. By allowing the c_j coefficient to depend solely on the targets position and not on the agents' we make sure that each component of the swarm is identical to the others so that agent permutations do not change the swarm behaviour. We also write the function ψ_G as dependant only on the distance so that an isotropy

of the desired velocity field around each hole is imposed. This may not be desired in some particular problems in which case some angular dependency could be introduced.

- **Dock Behaviour** This behaviour introduces N different local attractors towards the N targets. The component of the desired velocity field due to each dock behaviour has a non-negligible value only if the agent is in the neighborhood of the sink. The k_D parameter determines the radius of the sphere of influence of the dock behaviour. The expression used for this behaviour is:

$$\mathbf{v}_i^{Dock} = \sum_j d_j \psi_D(\|\boldsymbol{\xi}_j - \mathbf{x}_i\|, k_D)(\boldsymbol{\xi}_j - \mathbf{x}_i)$$

where ψ_G is a mapping from positive reals to positive reals that vanishes outside a given radius from the target. The same comments made for the gather behaviour still apply and the dock behaviour is similar to the gather one except that it is a local attractor and it therefore governs the final docking procedure.

- **Avoid Behaviour** This behaviour establishes a relationship between two different agents that are in proximity one with each other. In such a case a repulsive contribution will contribute to the desired velocity field. The expression that describes the desired velocity for this kind of behaviour is given below:

$$\mathbf{v}_i^{Avoid} = \sum_j b \psi_A(\|\mathbf{x}_i - \mathbf{x}_j\|, k_A)(\mathbf{x}_i - \mathbf{x}_j)$$

where ψ_A is a mapping from positive real to positive reals that vanishes whenever the mutual distance is considered to be not dangerous according to the value k_A . In order to maintain the symmetry between all the agents the b parameter does not depend on the particular agent.

According to the definitions given before, the desired velocity field for a swarm of N agents and for a final formation made of N target is defined as follows:

$$\mathbf{v}_{d_i} = \mathbf{v}_i^{Avoid} + \mathbf{v}_i^{Dock} + \mathbf{v}_i^{Gather} \quad (1)$$

This builds a dynamical system defined by the weighted sum of different and often conflicting behaviours and can be written in the simple form:

$$\dot{\mathbf{x}} = \mathbf{v}_{d_i} = \mathbf{f}(\mathbf{x}, \boldsymbol{\lambda}).$$

where we introduced $\mathbf{v}_d = [\mathbf{v}_{d_1}, \dots, \mathbf{v}_{d_N}]$, $\mathbf{x} = [\mathbf{x}_1, \dots, \mathbf{x}_N]$ and $\boldsymbol{\lambda} = [c_j, d_j, b]$. This last vector contains the parameter that have to be decided so that all the final desired configurations are equilibrium points. As we took care of retaining the symmetry of the dynamical system with respect of agent permutations the only relation that has to be fulfilled in order to impose the existence of such equilibria can be written in the compact form:

$$\mathbf{f}(\mathbf{x}_e; \boldsymbol{\lambda}) = 0 \quad (2)$$

where $\mathbf{x}_e = [\xi_1, \dots, \xi_N]$. All the other configurations, obtained by permutation of the ξ_i , are granted to be also equilibrium points. This equation will be referred to as the Equilibrium Shaping formula, as it effectively allow to find the value of $\boldsymbol{\lambda}$ able to shape the equilibria of the dynamical system represented by Eq.(2). The study of what possible equilibria may be shaped with the previous equation reveals to be intriguing and well described by the theory of symmetry groups. Let us take a closer look at the Equilibrium Shaping formula. It is a set of N vectorial equation each one related to a particular target position ξ_i :

$$\sum_{\substack{j=1 \\ i \neq j}}^N \{ [c_j \psi_G(\|\boldsymbol{\xi}_j - \boldsymbol{\xi}_i\|) + d_j \psi_D(\|\boldsymbol{\xi}_j - \boldsymbol{\xi}_i\|, k_D) - b \psi_A(\|\boldsymbol{\xi}_j - \boldsymbol{\xi}_i\|, k_A)] (\boldsymbol{\xi}_j - \boldsymbol{\xi}_i) \} = 0 \quad (3)$$

It is convenient to think to b as a parameter, and to write the Equilibrium Shaping formula in the form:

$$\mathbf{A} [c_1, \dots, c_N, d_1, \dots, d_N]^T = \mathbf{g}$$

The matrix \mathbf{A} and the vector \mathbf{g} depend on the functions ψ chosen to represent the various behaviours and on the target positions. This set of equations represent, for each target disposition and for each choice of the parameters k_D and k_A , a linear set of equations in the $2N$ unknowns c_j, d_j . Depending on the spatial distribution of the target points we might be able to find solutions. Let us further investigate the general case: a set of $3N$ equations in $2N$ unknowns and no possible solution. We must rely upon the linear dependency of some of the equations. We therefore introduce the punctual symmetry group G of the target positions. Whenever two target points ξ_i, ξ_j are equivalent with respect to G (i.e. it exists a punctual symmetry belonging to G that maps ξ_i into ξ_j) then the two corresponding equations are linearly dependent if we set $c_i = c_j, d_i = d_j$. This statement can be easily proven pre-multiplying Eq.(3) by a matrix $R \in G$ and using the identity between the coefficients and the isometric property of R . This simple trick allow us to count the number of independent vectorial equations, each of them counting as three, two or just one scalar equation according to a simple rule. Each independent vectorial equation is equivalent to a single scalar equation whenever a symmetry axes passes through the considered point, as two scalar equations if a symmetry plan passes through it and it is an identity if more than one symmetry axis passes through it.

Let us consider the Hexagonal Bravais Lattice shown in Figure 1. The target positions belonging to this formation can be divided in two different groups (the prism vertices and the bases centers) belonging to two different symmetry classes. As a symmetry plane passes through the vertices and a symmetry axis through the hexagons centers the Equilibrium Shaping formula may be reduced to three scalar equations if we set $c_i = c_j, d_i = d_j$ among the positions belonging two the same group.

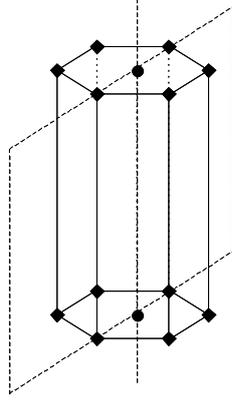


Figure 1. Formation with an Hexagonal-P shape.

Formation shape	Number of equations	Number of unknowns
Regular solids	1	2
Regular polygons	1	2
Pyramids with a regular basis	3	4
Cubic-P	1	2
Cubic-I	1	4
Cubic-F	2	4
Tetragonal-P	2	2
Tetragonal-I	2	4
Orthorhombic-P	3	2
Orthorhombic-I	3	4
Orthorhombic-C	4	4
Orthorhombic-F	6	4
Hexagonal-P	3	4

Table 1. Count of the equations and the unknowns for different formations.

In Table 1 some formations are visualized together with the number of independent equations and unknowns that characterize them. Some of the formations presented in Table 1 draw the inspiration from the well known Bravais lattice spatial configurations. The visualization of them is provided in Figure 2. Whenever the number of equations is less than the number of unknowns the choice of the various parameters may be done in many different ways. Exploiting this fact the choice may be made differently by each agent and may be seen as the “subjective” view that the i -th spacecraft has of the equilibrium condition. There is no need to agree on the particular solution chosen and the swarm does not need to communicate.

Let us consider the case in which the target positions ξ_i form an icosahedron. For this particular target geometry (see Table 1 under regular solids) the Equilibrium Shape

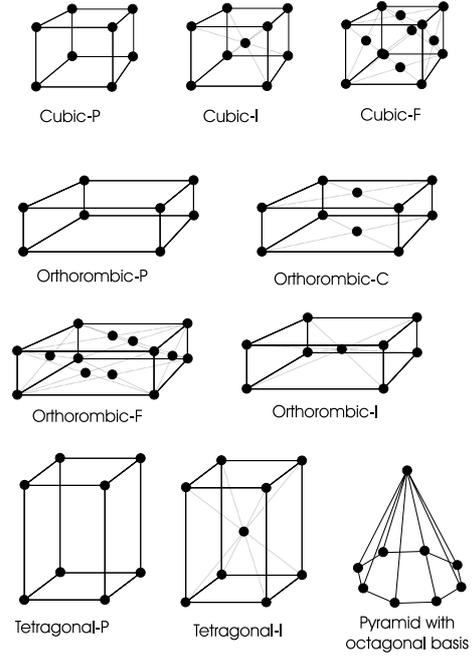


Figure 2. Visualization of some formations.

ing formula reduces to a single scalar relation:

$$c = [(R - h)e^{-\frac{\ell^2}{k_1}} + (R + h)e^{-\frac{\ell'^2}{k_1}} + 2Re^{-\frac{2R^2}{k_1}}]b - [(R - h)e^{-\frac{\ell^2}{k_2}} + (R + h)e^{-\frac{\ell'^2}{k_2}} + 2Re^{-\frac{2R^2}{k_2}}]d \quad (4)$$

where the quantities ℓ and R are respectively the edges of the icosahedron and the radius of the sphere in which the icosahedron can be inscribed, whereas ℓ' and h can be written as follows

$$h = \sqrt{R - \left(\frac{\ell}{2 \sin(\frac{\pi}{10})}\right)^2}$$

$$\ell' = \sqrt{(R + h)^2 + \left(\frac{\ell}{2 \sin(\frac{\pi}{10})}\right)^2}.$$

For a fixed parameter b , Eq.(4) defines a relation between the two remaining parameters c and d . Any choice of these two parameters leads to a dynamical system describing the desired velocity having the icosahedron as equilibrium point. Such a dynamical system has the following form:

$$\dot{\mathbf{x}}_i = \sum_{j=1}^N [-b \exp(-\frac{\mathbf{x}_{ij} \cdot \mathbf{x}_{ij}}{k_1})] \mathbf{x}_{ij} + \sum_{j=1}^N [-c - d \exp(-\frac{\xi_{ij} \cdot \xi_{ij}}{k_2})] \xi_{ij} \quad (5)$$

where we have defined $\xi_{ij} = \xi_i - \mathbf{x}_j$. The expression used for the function ψ is the one used by Gazi in [16, 17]. It is possible to define ψ in different ways in order to decrease the computational load as done for example in [18] where a simple sine function is used. In Figure 3 the outcome of a numerical integration of this dynamical system is shown for a particular choice of the parameters c and d . The initial positions of the various agents have been randomly generated. The lines shown in Figure 3 are the

trajectories that each agent having chosen d and c foresees and uses to evaluate its desired velocity. We note that the agents do not perform any numerical integration, only a simple algebraic calculation, in order to obtain its desired velocity. During various simulations the rise of emerging behaviors due to the interaction between different conflicting behaviors (see [15]) may be observed.

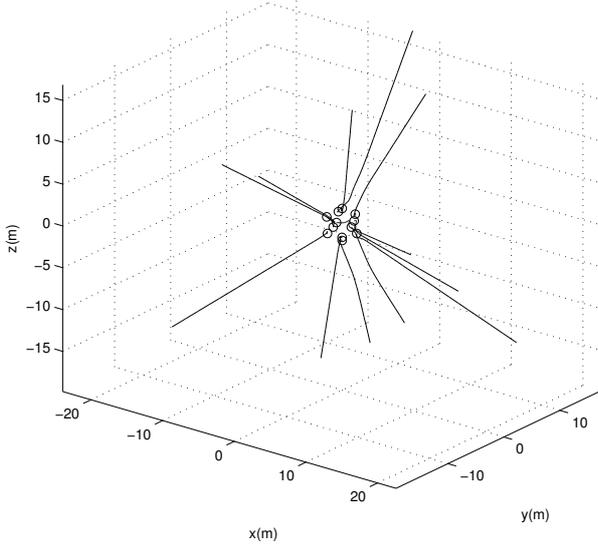


Figure 3. Desired trajectories of a swarm reaching an icosahedron-shaped target formation.

4. EXPLOITING THE GRAVITATIONAL ENVIRONMENT

The desired kinematical field designed in the previous section allows to reach the final formation following forced trajectories that are not geodesics. It is not difficult to imagine that the control system will struggle to follow these trajectories using unnecessarily large amount of propellant whenever the gravitational forces become significant. A modification may be introduced that takes into account and exploits the geodesics to reduce the overall mass consumption. We start from the well-known system of Hill equations:

$$\begin{cases} \ddot{x} - 2\omega\dot{y} - 3\omega^2x = 0 \\ \ddot{y} + 2\omega\dot{x} = 0 \\ \ddot{z} + \omega^2z = 0 \end{cases}$$

which admit an exact analytical solution in the form:

$$\begin{bmatrix} \boldsymbol{\rho} \\ \dot{\boldsymbol{\rho}} \end{bmatrix} = \begin{bmatrix} \mathbf{A}(\tau) & \mathbf{B}(\tau) \\ \mathbf{C}(\tau) & \mathbf{D}(\tau) \end{bmatrix} \begin{bmatrix} \boldsymbol{\rho}_0 \\ \dot{\boldsymbol{\rho}}_0 \end{bmatrix}$$

where $\boldsymbol{\rho} = [\rho, \dot{\rho}]$ is the non dimensional state space vector, τ the non dimensional time and the matrices \mathbf{A} , \mathbf{B} , \mathbf{C} and \mathbf{D} have a well-known expression (see [7]). The above solution can be used in order to define a new gather behavior that exploits the gravitational force to reach the

final desired configuration. Requiring that a given satellite has to reach a certain point $\boldsymbol{\rho}_d$ after a fixed time τ_d the following relation has to be fulfilled:

$$\boldsymbol{\rho}_d = \boldsymbol{\rho}(\tau_d) = \mathbf{A}(\tau_d)\boldsymbol{\rho}_0 + \mathbf{B}(\tau_d)\dot{\boldsymbol{\rho}}_0.$$

Taking this into account we may assign for each position in the space \mathbf{x}_i and each target belonging to the final formation $\boldsymbol{\xi}_j$ a new gather velocity vector given by:

$$\mathbf{v}_i^{Gather} = \frac{1}{N} \sum_j \mathbf{B}^{-1}\boldsymbol{\xi}_j - \mathbf{B}^{-1}\mathbf{A}(\hat{\tau}_d - t)\mathbf{x}_i \quad (6)$$

where $\hat{\tau}_d$ is the time in which, at the beginning of the simulation, the agent is required to reach the center of the desired formation. Even though the resulting desired velocity vector depends explicitly on the time in a practical application the agent clocks need not to be synchronized. This contribution is added to the dock behaviour and the avoid behaviour in order to build the final desired kinematical field. Unfortunately Eq.(6) is singular when t approaches $\hat{\tau}_d$, i.e. in the final part of the target acquisition. Besides, near the targets the desired velocity due to this new gather behaviour is higher than needed (the spacecraft has to get out of a ballistic trajectory to acquire the targets). For these reasons the desired kinematical field will be divided in two different parts, one, far from the desired final configuration, in which the gather behaviour takes into account the gravitational force and one, close to the desired final formation, in which the space can be considered flat. The geometrical shape of the edge of these two different zones of the space can be easily set as a sphere of radius R_s that can be considered, together with the desired gather time $\hat{\tau}_d$ as parameters to be decided from the system designer.

5. FEEDBACK SYNTHESIS

In general the agent will not possess the desired velocity and a control system has to be designed that is able to reduce the error between the actual velocity and the desired one. In this section different feedbacks achieving this will be derived and discussed.

5.1. Q-guidance

The first feedback we develop is inspired by the Q-guidance steering law introduced formally by Battin [19] for rockets guidance. It is based on the definition of the velocity to be gained vector \mathbf{v}_{g_i} that represents, in our case, the instantaneous difference between each agent's actual \mathbf{v}_i and desired velocity \mathbf{v}_{d_i} . The objective of the control system is to drive the velocity to be gained vector to zero. From now on each quantity will be related to each agent but, in order to simplify the notation the subscript will be omitted. We define, for each agent, the following function:

$$V = \frac{1}{2} \mathbf{v}_g \cdot \mathbf{v}_g$$

the velocity to be gained vector decreases along the trajectories followed by each agent if and only if:

$$\dot{V} = \mathbf{v}_g \cdot \dot{\mathbf{v}}_g < 0. \quad (7)$$

The time derivative of \mathbf{v}_g during the motion has the expression $\dot{\mathbf{v}}_g = \dot{\mathbf{v}}_d - \dot{\mathbf{v}}$. We substitute into this relation the momentum balance of each spacecraft written in the LHLV frame introduced:

$$\dot{\mathbf{v}} = \mathbf{f}_{in} + \mathbf{u}$$

where \mathbf{f}_{in} are the external inertial forces and \mathbf{u} is our control vector. The following expression is obtained:

$$\dot{\mathbf{v}}_g = \dot{\mathbf{v}}_d - \mathbf{f}_{in} - \mathbf{u}.$$

We now express the desired velocity derivative using the chain rule:

$$\dot{\mathbf{v}}_d = \frac{\partial \mathbf{v}_d}{\partial t} + \frac{\partial \mathbf{v}_d}{\partial \mathbf{x}} \mathbf{v} = \frac{\partial \mathbf{v}_d}{\partial t} + \frac{\partial \mathbf{v}_d}{\partial \mathbf{x}} (\mathbf{v}_d - \mathbf{v}_g)$$

The various terms can be found by deriving Eq.(1) and taking into account the expressions chosen for the different behaviors. As an example we consider the contribution given by the gather behavior defined by Eq.(6). In this particular case the time derivative of the desired velocity has the expression:

$$\frac{d\mathbf{v}_d}{dt} = \mathbf{f}_{in} - \frac{\partial \mathbf{v}_d}{\partial \mathbf{x}} \mathbf{v}_g$$

and the resulting total time derivative of the velocity to be gained (under the hypothesis that the avoid behavior contribution is negligible) may be written in the following compact form:

$$\dot{\mathbf{v}}_g = -\mathbf{u} - \frac{\partial \mathbf{v}_d}{\partial \mathbf{x}} \mathbf{v}_g = -\mathbf{u} - \mathbf{B}^{-1} \mathbf{A} \mathbf{v}_g. \quad (8)$$

To make $V < 0$ we introduce the following feedback:

$$\mathbf{u} = \kappa \mathbf{v}_g - \dot{\mathbf{v}}_d - \mathbf{f}_{in} \quad (9)$$

in which $\kappa > 0$ is a positive real parameter whose choice will be discussed below. We then have:

$$\dot{V} = -\kappa \mathbf{v}_g \cdot \mathbf{v}_g$$

that assure us the global stability of this controller. In real cases the agents will have an upper limit to the thrust magnitude, we may therefore have to saturate the feedback defined above. As soon as we introduce a saturation level we lose the mathematical result on the feedback global stability, but the controller is still able to drive the velocity to be gained to zero as confirmed by numerical simulations. In particular we here note how the geometrical interpretation of the saturated feedback shown in Figure 4 allow us to rederive, for particular choices of the positive parameter κ the analogous to known steering laws based on the Q-guidance (see Battin [19]).

If $\kappa \rightarrow \infty$ the control strategy is to thrust in the direction of the velocity to be gained vector regardless of the contribution to the $\dot{\mathbf{v}}_g$ due to the uncontrollable terms $\dot{\mathbf{v}}_d$ and

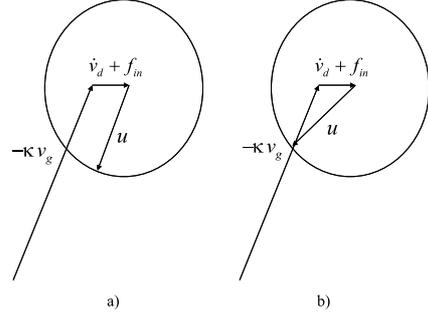


Figure 4. Vectorial diagrams representing different strategies.

\mathbf{f}_{in} . A different strategy can be achieved if the thrust direction is chosen in order to try aligning the time derivative of the velocity to be gained vector to the \mathbf{v}_g vector itself, as expressed by the following relation:

$$\dot{\mathbf{v}}_g \times \mathbf{v}_g = 0. \quad (10)$$

We have reduced to the known cross product steering that, in our notation, may be implemented by finding that value of κ for which (see Figure 4):

$$(\dot{\mathbf{v}}_d + \kappa \mathbf{v}_g) \cdot (\dot{\mathbf{v}}_d + \kappa \mathbf{v}_g) = u_{sat}$$

where u_{sat} is the saturation considered for the thrust vector modulus.

5.2. Sliding mode control

A feedback can be also obtained using the results of the sliding-mode control theory (see for example [20]). The aim of this approach is to design a control law able to drive the system trajectory on a predetermined manifold and keep it there once reached. The control design procedure can be broken down in two different steps: first we design a sliding manifold (or switching manifold) such that the motion of the system restricted to it leads the swarm of satellites towards the desired equilibrium configuration. Then a control law has to be derived to force the trajectories of the system to collapse on the sliding manifold and to belong to it during the whole simulation.

The dynamical system to be controlled for each agent is

$$\begin{cases} \dot{\mathbf{v}} = \mathbf{f}_{in} + \mathbf{u} \\ \dot{\mathbf{x}} = \mathbf{v} \end{cases} \quad (11)$$

where \mathbf{f}_{in} and \mathbf{u} are respectively the inertial acceleration acting upon the agent and the control vector. The sliding manifold may be written in the following form:

$$\sigma(\mathbf{x}, \mathbf{v}, t) = 0$$

where

$$\sigma(\mathbf{x}, \mathbf{v}, t) = [\sigma_1(\mathbf{x}, \mathbf{v}, t), \dots, \sigma_3(\mathbf{x}, \mathbf{v}, t)] = 0.$$

The relations $\sigma_i(\mathbf{x}, \mathbf{v}, t)$ have to be chosen such as the trajectory that the system follows “sliding” on the manifold reaches the final desired formation. The Equilibrium Shaping technique we introduced in this paper is in fact a method to build such a sliding manifold. For the system in Eq.(11) the expression we use for $\sigma(\mathbf{x}, \mathbf{v}, t)$ is:

$$\sigma(\mathbf{x}, \mathbf{v}, t) = \mathbf{v}_d - \mathbf{v} = 0 \quad (12)$$

where \mathbf{v}_d is defined in Eq.(1). Whenever the system is on the sliding manifold it will stay on it if and only if the following relation is satisfied at each instant:

$$\dot{\sigma}(\mathbf{x}, \mathbf{v}, t) = 0$$

that is, according to Eq.(11) and to Eq.(12):

$$\dot{\mathbf{v}}_d - \dot{\mathbf{v}} = \dot{\mathbf{v}}_d - \mathbf{f}_{in} - \mathbf{u} = 0.$$

It is then possible to define the equivalent control \mathbf{u}_{eq} as a feedback that keeps the state of the system on the manifold for all the time instants:

$$\mathbf{u}_{eq} = -\mathbf{f}_{in} + \dot{\mathbf{v}}_d. \quad (13)$$

A particular case is when the gravitational gather behavior is the only contribution to the desired velocity. Then the control force reduces to zero since on the sliding manifold we have:

$$\dot{\mathbf{v}}_d = \frac{\partial \mathbf{v}_d}{\partial t} + \frac{\partial \mathbf{v}_d}{\partial \mathbf{x}} \mathbf{v}_d = \mathbf{f}_{in}.$$

The dynamical system in Eq.(11) subject to the equivalent control will never leave the sliding manifold after having intersected it once. It is now necessary to add to the equivalent control another term that acts when $\sigma \neq 0$ and able to drive the system trajectory to intersect the sliding manifold. The total control vector can then be expressed as the sum of two contributions:

$$\mathbf{u} = \mathbf{u}_{eq} + \mathbf{u}_N \quad (14)$$

where \mathbf{u}_{eq} has been defined in Eq.(13). The vector \mathbf{u} applied to the system Eq.(11) couples the dynamics of each single spacecraft to the other components of the swarm that the spacecraft can sense. A switched control law in the form:

$$u_{N_i} = \begin{cases} u_{N_i}^+(\mathbf{x}, \mathbf{v}, t) \longleftrightarrow \sigma_i(\mathbf{x}, \mathbf{v}, t) > 0 \\ u_{N_i}^-(\mathbf{x}, \mathbf{v}, t) \longleftrightarrow \sigma_i(\mathbf{x}, \mathbf{v}, t) < 0 \end{cases}$$

will enforce the system to fall onto the sliding manifold if the values $u_{N_i}^+, u_{N_i}^-$ are chosen so that the velocities of the system will point always towards it. The value chosen for the feedback gains is determined according to the sign of the components of $\sigma(\mathbf{x}, \mathbf{v}, t)$, also called switching surface for this reason. The control u_{N_i} is set to zero on the switching surface. A Lyapunov method can be used to find the values of the switching gains. Let's define the following Lyapunov function

$$V = \frac{1}{2} \sigma \cdot \sigma$$

then a control feedback must be derived thus to impose the time derivative of V to be negative definite along the trajectories of the system. The condition on the total time derivative of the Lyapunov function can be imposed by

$$\dot{V} = (\mathbf{v}_d - \mathbf{v}) \cdot (\dot{\mathbf{v}}_d - \dot{\mathbf{v}}) < 0$$

that, recalling Eq.(13) and Eq.(14), becomes

$$\dot{V} < 0 \iff (\mathbf{v}_d - \mathbf{v}) \cdot \mathbf{u}_N > 0.$$

The latter equation can be written in term of the velocity to be gained vector already defined

$$\mathbf{v}_g \cdot \mathbf{u}_N > 0. \quad (15)$$

Each additional feedback law \mathbf{u}_N that meets this condition can be used in order to drive the motion of the system towards the sliding manifold. Consistently with the work presented by Gazi [16] and to keep the derivation close to the classical sliding mode approach, the so called “relays with constant gain” (see [20]) thrusting strategy for \mathbf{u}_N is introduced:

$$\mathbf{u}_N = u_0 \text{sign}(\sigma) = u_0 \text{sign}(\mathbf{v}_g) \quad (16)$$

where the *sign* function is defined componentwise. This definition clearly satisfies Eq.(15) and leads the system to reach the sliding manifold and then the desired equilibrium configuration. As a final remark we note that the thrusting strategy $\mathbf{u} = \kappa \mathbf{v}_g + \dot{\mathbf{v}}_d - \mathbf{f}_{in}$ inspired by the Q-guidance method can be written as $\mathbf{u} = \mathbf{u}_{eq} + \mathbf{u}_N$ with $\kappa \mathbf{v}_g$ satisfying Eq.(15). In this sense the sliding mode theory and the velocity to be gained approach reveal to be equivalent.

5.3. Artificial Potential Approach

A different thrusting strategy can be obtained starting from the definition of an artificial potential function ([6]) for the whole swarm $V(\mathbf{x}_1, \dots, \mathbf{x}_n, \mathbf{v}_1, \dots, \mathbf{v}_n)$ that has minimum points in all the possible agents permutation in the final desired formation. Such a function of the state of the system can be written as:

$$V = \frac{1}{2} \sum_i \mathbf{v}_i \cdot \mathbf{v}_i + \sum_i \sum_{j \neq i} \phi_A^{ij}(\mathbf{x}_{ij}) + \sum_i \sum_j \phi_G^{ij}(\xi_{ij}) + \sum_i \sum_j \phi_D^{ij}(\xi_{ij}) \quad (17)$$

where ϕ_A^{ij} , ϕ_D^{ij} and ϕ_G^{ij} are defined according to the Equilibrium Shaping technique such as:

$$\begin{aligned} \frac{\partial \phi_A^{ij}}{\partial \mathbf{x}_i} &= -\mathbf{v}_i^{\text{Avoid}} \\ \frac{\partial \phi_G^{ij}}{\partial \mathbf{x}_i} &= -\mathbf{v}_i^{\text{Gather}} \\ \frac{\partial \phi_D^{ij}}{\partial \mathbf{x}_i} &= -\mathbf{v}_i^{\text{Dock}} \end{aligned}$$

and each quantity labeled with the i pedex is related to the i -th agent. The swarm will reach the final formation avoiding the inter-vehicles collisions if the function

$V(\mathbf{x}_1, \dots, \mathbf{x}_n, \mathbf{v}_1, \dots, \mathbf{v}_n)$ decreases during the motion. We get:

$$\dot{V} = \sum_i \left(\frac{\partial V}{\partial \mathbf{x}_i} \dot{\mathbf{x}}_i + \frac{\partial V}{\partial \mathbf{v}_i} \dot{\mathbf{v}}_i \right) = \sum_i (\dot{\mathbf{v}}_i - \mathbf{v}_{d_i}) \cdot \mathbf{v}_i < 0.$$

Taking into account the i -th agent equation of motion it is possible to use the following feedback $\mathbf{u}_i = \mathbf{v}_{d_i} - \kappa_i \mathbf{v}_i - \mathbf{f}_{in_i}$ that written in term of the velocity to be gained vector becomes:

$$\mathbf{u}_i = \kappa_i \mathbf{v}_{g_i} + (1 - \kappa_i) \mathbf{v}_{d_i} - \mathbf{f}_{in_i}. \quad (18)$$

With this feedback the time derivative of the potential function is:

$$\dot{V}(\mathbf{x}_1, \dots, \mathbf{x}_n, \mathbf{v}_1, \dots, \mathbf{v}_n) = - \sum_i \kappa_i \mathbf{v}_i \cdot \mathbf{v}_i$$

which is definite negative as long as the κ_i parameters are chosen positive. With respect to the previous presented feedback design methods the one showed in this subsection relies upon a slightly different approach. First a global artificial potential function is defined for the entire swarm of satellites. This function is required to have minimal positions in all the possible swarm target formations and this may be obtained using the Equilibrium Shaping approach. Then a control law is imposed such as the potential function decreases along the trajectories followed by the system. The feedback derived in this section cannot be obtained from the Q-guidance or the sliding mode and therefore represents an alternative to be considered.

6. SIMULATION RESULTS

In this section we present some numerical simulations we performed to study the performances of the discussed behaviour based control. We randomly placed N satellites within a certain range and we activated the controller to study the swarm behaviour. We performed our simulations for different relative geometries. We also considered different feedbacks given by:

- Eq.(9) with κ such that Eq.(10) is satisfied. We called this feedback CPSL (Cross Product Steering Law)
- Eq.(14) with u_0 tuned in such a way as to make the acquisition time of the final targets equal to the CPSL case. We called this feedback SMC (Sliding Mode Control).
- Eq.(9) with κ tuned as for the SMC. We called this feedback VTBG (Velocity To Be Gained)
- Eq.(18) with κ tuned as for the SMC. We called this feedback APF (Artificial Potential Feedback).

Typically, as a consequence of the control actuation, each agent path consisted of different phases:

- A powered part in which the initial velocity to be gained vector is driven to zero and in which a ballistic trajectory is reached.
- A coasting phase in which the desired velocity and the actual velocity are identical and the control system does not use the actuators.
- A last phase, activated within the sphere of radius R_s , in which the agent is near to the final targets and navigates towards one of them and the final geometry is acquired. In this phase the gather behaviour does not take into account the gravitational effect.

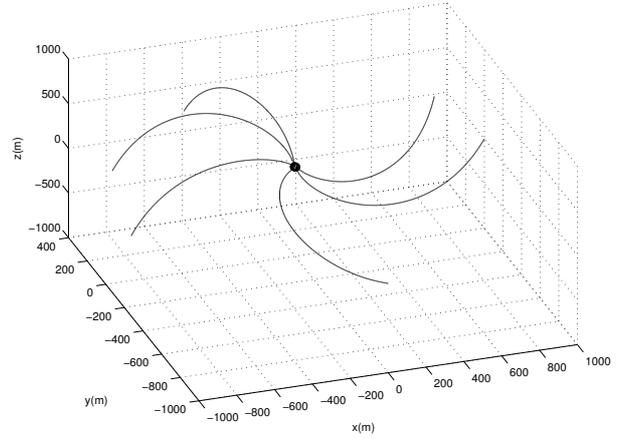


Figure 5. Simulation of a swarm of satellites reaching an hexagonal regular formation. First approaching phase

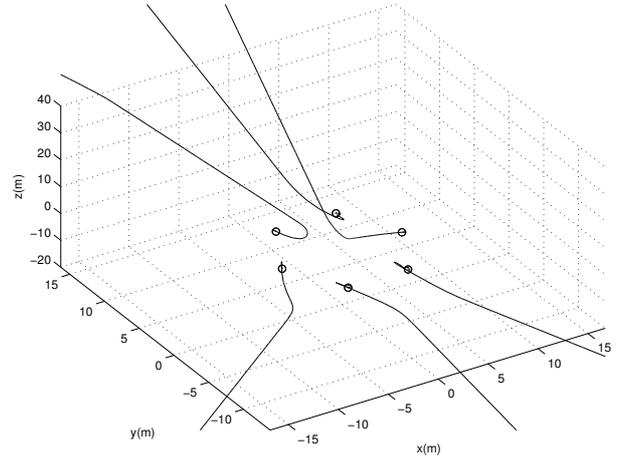


Figure 6. Simulation of a swarm of satellites reaching an hexagonal regular formation. Second approaching phase

The final relative geometry is achieved only when every agent occupies a target position so that the Equilibrium Shaping formula is satisfied and the desired velocities are all zero. As an example we show the trajectories followed by six satellites achieving an hexagonal formation with a $6m$ radius. Each spacecraft belonging to the swarm starts from an average distance of $1000m$ with respect to the center of the final configuration. The saturation

value used for the thrust acceleration modulus is $0.005 \frac{m}{s^2}$ and the feedback law used is the VTBG. The final formation was achieved after roughly $20000sec$ corresponding to roughly a quarter of the reference orbital period. In Figure 5 and 6 the trajectories followed by the spacecraft belonging to the swarm are displayed. Figure 5 shows the motion of the swarm in the the outer part of the kinematical field where gravity is accounted for, while Figure 6 shows the motion of the swarm in the very last phase. In this particular simulation the center of the desired formation was on a geostationary orbit. The thrust profiles of each spacecraft are shown in Figure 7. The different phases we described at the beginning of this section are visible in this chart. The expensive phases of the whole procedure in terms of propellant consumption are at the very first seconds when the engines are constantly saturated in order to reach a ballistic trajectory and at the last part of the formation acquisition when the gravitational force is no more considered in the definition of the desired velocities. It is in this phase that each agent chooses its final position and navigates towards it. An average Δv of about $0.8 \frac{m}{s}$ was required in this particular manoeuvre by each agent.

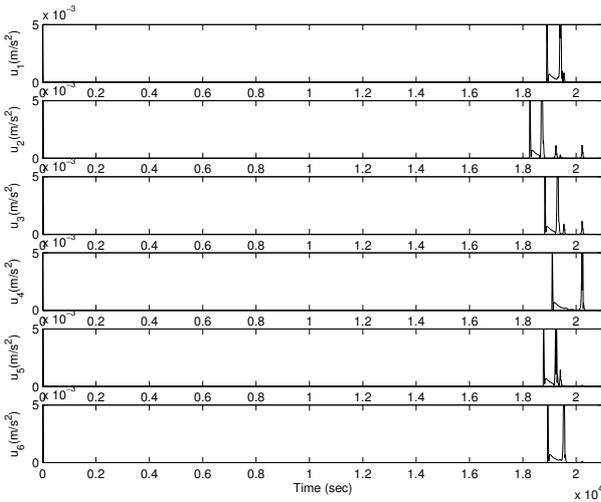


Figure 7. Control profiles for the satellites performing the maneuver.

In Figure 8 the different feedbacks introduced are considered and compared in terms of propellant consumption for this particular simulation. The numerical campaign performed showed that the SMC and the VTBG feedback are always outperforming the CPSL and the APF feedbacks.

7. LOCAL MINIMA ESCAPE PROCEDURE.

In some simulations the agents could not reach the final desired relative geometry and got stuck in an undesired equilibrium position. From the Equilibrium Shaping we

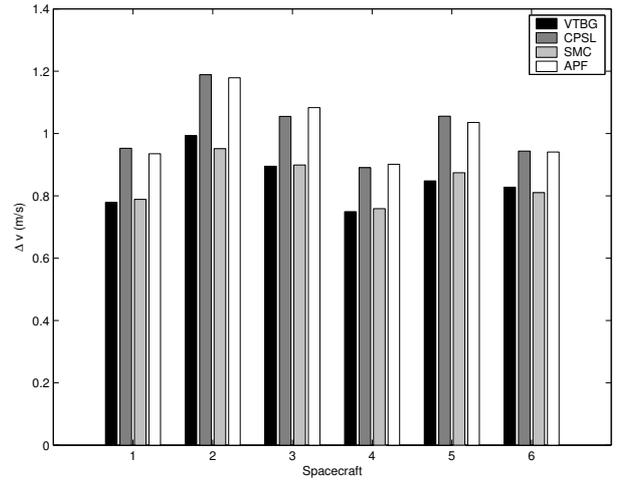


Figure 8. Comparison between the different thrusting strategies proposed.

know that the final relative geometry is an equilibrium point for the system, but we do not have any guarantee that it is globally stable. Most probably, depending on the choice of the functions ψ , other equilibrium points may exist and be stable attracting the swarm in an undesired configuration. There are several ways one can deal with this problem commonly referred to as local minima problem. In all those cases in which Eq.(3) has more than one solution (see Table 1) there is a very effective procedure to deal with this problem. The procedure (sketched in a compact form in Figure 9) is autonomously activated when the agent evaluates its desired velocity to be zero. Then the agent checks whether its current position is one of the target positions. If not it starts changing the parameters λ in the space of the solutions of Eq.(3) that keeps the final target positions stable. The process is iterated in order to find that set of parameters λ that make the current occupied position unstable keeping the final desired one stable. We note that the agent does not communicate the new parameters to the rest of the swarm because the Equilibrium Shaping formula can be considered subjective, i.e. to represent the subjective kinematical field “desired” by each agent. Inevitably the occurrence of these local minima increase the fuel consumption of the agent that gets stuck in one of them.

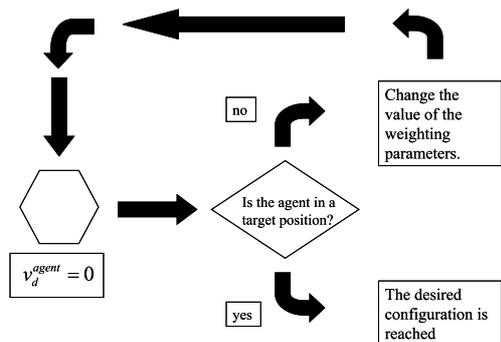


Figure 9. Local minima avoid procedure.

8. CONCLUSIONS

We show that the task of acquiring certain formations may be achieved by a satellite swarm using only local sensory information and without communications. At the same time also the target selection problem may be solved. Using a behaviour based approach it is found that the introduction of three simple behaviours that have been named gather, avoid and dock, allow this result for a number of interesting formation geometries. These are found to be described properly by the symmetry group theory applied to the solutions of the Equilibrium Shaping formula here introduced. Some simulations of various feedbacks show that the requirements in terms of propellant for real applications are well within our technological capabilities. The lesson learned from collective robotics has been successfully transferred to the design of a completely autonomous and distributed control system for satellite swarm navigation.

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