Self-assembly of large structures in space using intersatellite Coulomb forces

Dario Izzo[†], Lorenzo Pettazzi[§]

[†]ESA, Advanced Concepts Team, ESTEC Keplerlaan 1, Postbus 299,2200 AG, Noordwijk
[§] ZARM - University of Bremen, Bremen, Germany

contact: dario.izzo@esa.int

Abstract

We discuss on the possibility of aiding the self assembling of large structures in space using the Coulomb forces resulting from satellite charging. In particular we propose an original scheme where the self-assembly procedure is carried out autonomously by large numbers of identical spacecraft and the individual electrostatic charges are also autonomously decided as to aid the process. We prove that it is possible to accomplish such a cooperative task saving propellant and we lay down the basis to study the stability of the resulting control feedback taking into account reasonable saturation levels for the thrust and for the satellite charge. We do not face here the design issues arising from considering controllable currents flowing in and out each agent, but we prove that the non linear control problem that arises is not a show stopper.

Introduction

A number of attractive space concepts are based on the prediction that large structures can be built in space. Solar power satellites, large orbiting reflectors, antennas or telescopes are certainly extremely challenging concepts given the current available technology limits, and thus are dubbed by many as "unfeasible". Nevertheless these concepts do offer the possibility of incredible achievements for the human kind not to be considered seriously. Technologies that today are closer to the realm of science fiction than to the one of engineering, such as weather control, terraforming or power from space, are also the candidates to be revolutions that will radically change the way we perceive space missions. Besides, many less extreme, and therefore nearer-term, concepts still require the ability to construct large structures in space. The ability to perform such an ambitious operation in a fully automated way, is certainly one of the enabling factors for all these concepts. The autonomous self-assembly of structures is a complicated problem that involves automated scheduling, path planning and navigation. Different solutions may be envisaged for each of these parts. In the case of structures formed by regular lattices dispositions of identical parts, the so-called equilibrium shaping technique (see Izzo & Pettazzi [1, 2]) has been recently shown to be able to cope with all these problems at the same time and thus to be compatible to be conveniently used to drive a self-assembly procedure. The equilibrium shaping drives its inspiration from important developments in collective robotics and, in particular, in swarm aggregation (see Gazi [3]). When using these techniques for a space application, the constraints deriving from the design of each single component have to be accounted for carefully. In particular, each component has to be able to sense its orientation and its position and needs some ΔV capability to be able to perform its task. This could be provided in large amounts by means of "propellant-less" propulsion exploiting inter spacecraft Coulomb forces. The concept of using electrostatic forces to control the relative position of a satellite formation in geostationary orbit has first been introduced by Schaub et al. [4] for formation keeping and reconfiguration applications. Being internal forces, the set of manoeuvres allowed by Coulomb forces is limited, thus making the concept of having satellites equipped purely with electrostatic actuation to particular applications. In principle one could also consider an hybrid actuation system using conventional propulsion aided by the Coulomb actuation whenever possible. Such a concept has been introduced by Pettazzi et al. [5] and allows to significantly reduce the fuel consumption in those manoeuvres where internal forces can actually be used.

In this paper we introduce the technical background that can be used to automatize completely the tasks in ambitious missions such as that of self-assembling large structures in space using inter-satellite Coulomb forces. We propose and study a feedback law that allow the satellites to command their charge level as to lower the propellant consumption necessary to complete a given assembly task.

Modeling and notation

Let us consider the motion of N points having mass m_i and electrical charge q_i immersed in a space plasma modeled by a Debye length λ_D and under the influence of a central gravity field with strength μ. We assume that the point masses represent spacecraft able to thrust in the three directions and that the net currents I_i flowing from the spacecraft are controllable. If we neglect the mass variation of each spacecraft, the equations of motion for such a system can be written as:

$$\begin{aligned}&\ddot{\mathbf{r}}_i + \frac{\mu}{r_i^3}\mathbf{r}_i = -\frac{k_c q_i}{m_i}\sum_{j=1,j\neq i}^{N}\frac{q_j}{r_{ij}^2}\exp\left(-\frac{r_{ij}}{\lambda_D}\right)\mathbf{r}_{ij} + \mathbf{u}_{t_i}\\ &\dot{q}_i = u_{q_i}\end{aligned} \tag{1}$$

where $\mathbf{r}_i$ is the position vector of the i-th spacecraft of the swarm, μ is the gravitational parameter of the body whose sphere of influence is containing the swarm, $k_c = 8.9876 \times 10^9$ N m^2 C^{-2} is the Coulomb constant and q_i is the electrostatic charge of the i-th spacecraft. We also take into account the shielding due to the plasma environment by introducing the Debye length λ_D. We assume that the agents are equipped with a generic thruster able to give the acceleration $\mathbf{u}_{t_i}$ to the agent. We also assume that the currents flowing in and out the spacecraft are controllable and equal to u_{q_i}. The system has a dimension of $4N$, the first $3N$ equations are describing the dynamic of the swarm geometry, the last N the charge dynamic of the various satellites. In the form of Eq. 1 the spacecraft dynamic equations are decoupled from the charge dynamic equations. In this form the system is fully controllable but requires each agent to be equipped both with a thruster and with a charge control system. This kind of agent, that we call satellitron because of its charge and swarm dynamic, and the possibility to use the thrusting system together with the charge control system, has been studied by ESA's Advanced Concepts Team in cooperation with Surrey University [6] and Bremen University [7] in arecent joint research effort. Other researchers have taken the approach to study agents equipped with only charging control, called Coulomb satellites from the original work by Schaub et al. [4]. In this last case $\mathbf{u}_{t_i} = \mathbf{0}$ and Eq. 1 becomes uncontrollable. While this approach can be used for some interesting applications, such as formation keeping or Coulomb tethers [8], it seems hopeless for applications such as self-assembly where a fully controllable system is certainly a major asset.

Let us first introduce some notation that will be useful both for the purpose of analytical and numerical analysis of the system. We introduce $\mathbf{q} = [q_1, q_2, ..., q_N]$ as a vector containing the charges of the satellite in the swarm. We write the acceleration due to the electrostatic interactions in the simplified form:

$$\mathbf{a}_i^{el} = -\frac{k_c q_i}{m_i}\sum_{j=1,j\neq i}^{N}\frac{q_j}{r_{ij}^2}\exp\left(-\frac{r_{ij}}{\lambda_D}\right)\mathbf{r}_{ij} = q_i\mathbf{R}_i\mathbf{q}$$

where we have introduced the matrices $\mathbf{R}_i := \mathbf{G}_i\mathbf{\Lambda}_i$ defined as the product of a geometry matrix $\mathbf{G}_i$ defined as:

$$\mathbf{G}_i = -\frac{k_c}{m_i}\left[\begin{array}{cccccc} | & | & & | & & | \\ \mathbf{r}_{i1}/r_{i1}^2 & \mathbf{r}_{i2}/r_{i2}^2 & ... & 0 & ... & \mathbf{r}_{iN}/r_{iN}^2 \\ | & | & & | & & | \end{array}\right]$$

and a shielding matrix defined as:

$$\mathbf{\Lambda}_i = \left[\begin{array}{cccc} e^{-\frac{r_{i1}}{\lambda_D}} & 0 & ... & 0 \\ 0 & e^{-\frac{r_{i1}}{\lambda_D}} & ... & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & ... & e^{-\frac{r_{i1}}{\lambda_D}} \end{array}\right]$$

Note that when the Debye length is much larger than the typical inter spacecraft distance $\mathbf{\Lambda}_i \to \mathbf{I}, \forall i$. We write again the equations of motion:

$$\begin{aligned}&\ddot{\mathbf{r}}_i + \frac{\mu}{r_i^3}\mathbf{r}_i = q_i\mathbf{R}_i\mathbf{q} + \mathbf{u}_{t_j}\\ &\dot{q}_j = u_{q_j}\end{aligned} \tag{2}$$

Following a classical approach we may linearize these equations with respect to a point moving along a perfectly Keplerian and circular orbit and project them onto the relevant LHLV frame. We obtain the Hill-Clohessey-Wiltshire equations perturbed by the electrostatic force [4]:

$$\begin{aligned}&\ddot{\mathbf{r}}_i + \mathbf{D}\dot{\mathbf{r}}_i + \mathbf{K}\mathbf{r}_i = q_i\mathbf{R}_i\mathbf{q} + \mathbf{u}_{t_j}\\ &\dot{q}_j = u_{q_j}\end{aligned} \tag{3}$$

where:

$$\mathbf{D} = \begin{bmatrix} 0 & -2\omega & 0\\ 2\omega & 0 & 0\\ 0 & 0 & 0\end{bmatrix}, \mathbf{K} = \begin{bmatrix} -3\omega^2 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & \omega^2\end{bmatrix}$$

where ω is the reference orbit angular velocity. Here we will focus our attention on this last system, considering the assembly procedure in geostationary orbit where the Debye length is large enough to justify the use of Coulomb satellites. Our results, though, would be equally valid in a interplanetary space environment where gravitational forces can be considered as perturbations and may not therefore appear in the dynamic system considered (formally this would translate into $\mathbf{D} = \mathbf{K} = \mathbf{O}$), where $\mathbf{O}$ is the null matrix.

Feedback Synthesis approach

To drive a self assembly whose dynamic is described by Eq.(3) we want to control the swarm position vectors $\mathbf{r}_i$ using, as much as possible, the controls u_{q_j} which do not require propellant, rather than the controls $\mathbf{u}_{t_j}$ which require propellant and therefore are a scarce resource. In mathematical terms we want to find $\mathbf{u}_{t_j}(\mathbf{r}, \dot{\mathbf{r}}, q_j)$ and $u_{q_j}(\mathbf{r}, \dot{\mathbf{r}}, \mathbf{q})$ in the class of piecewise continuous functions, so that the swarm acquires a final configuration defined by some terminal position vectors $\boldsymbol{\xi}_j$ (permutations of the agents in the final configurations may be allowed to increase the robustness of the assembly procedure) minimising some objective $J(\mathbf{r}(t), \dot{\mathbf{r}}(t), \mathbf{q}(t))$ that can be the total fuel consumption of the agents, the fuel consumption of one particular agent the standard deviation from the mean of the fuel consumption between the agents or some other quantity desired to be small. The problem of optimal feedback synthesis is known to be very difficult for non linear problems (the interested reader is addressed to the famous book by Pontryagin et al. [9]) A numerical reconstruction of an optimal state feedback is certainly possible but it requires a significant computational effort not implementable in a real time navigation scheme. A behavioral approach can, on the other hand, synthesize a state feedback that, though sub-optimal, allows real time swarm navigation [2]. We here take that approach, renouncing to find optimal solutions and trying to *just solve the problem*. We propose to follow the following scheme:

- We first consider the feedback synthesis for a system in the form $\ddot{\mathbf{r}}_i + \mathbf{D}\dot{\mathbf{r}}_i + \mathbf{K}\mathbf{r}_i = \mathbf{u}_{d_j}$. We are completely ignoring the charge dynamics here and focussing on the swarm path planning. We build a feedback $\mathbf{u}_{d_j}(\mathbf{r}, \dot{\mathbf{r}})$ able to drive the swarm to a final configuration. We refer to this part of the control system as to the path planning layer. Note that the symbols $\mathbf{r}, \dot{\mathbf{r}}$ are used to indicate the collection of all the satellite $\mathbf{r}_j$, $\dot{\mathbf{r}}_j$
- We then set $\mathbf{u}_{t_j}(\mathbf{r}, \dot{\mathbf{r}}, \mathbf{q}) = \mathbf{u}_{d_j}(\mathbf{r}, \dot{\mathbf{r}}) - q_i\mathbf{R}_i\mathbf{q}$
- We finally consider the charge dynamic and we synthesize a charge state feedback $u_{q_j}(\mathbf{r}, \dot{\mathbf{r}}, \mathbf{q})$ that tries to fulfill, at least on average across the swarm, the criteria $u_{t_j} < u_{d_j}$

We note here that the first feedback drives the path planning of the swarm, whereas the second feedback drives the charge dynamic and therefore determines, alone, the fuel savings introduced by the use of inter satellite Coulomb forces.

The path planning

As the focus of this paper is not in the feedback synthesis of $\mathbf{u}_{d_j}(\mathbf{r}, \dot{\mathbf{r}})$, we will start from the assumption that such a state feedback is available. The interested reader may consult for example the work by Izzo and

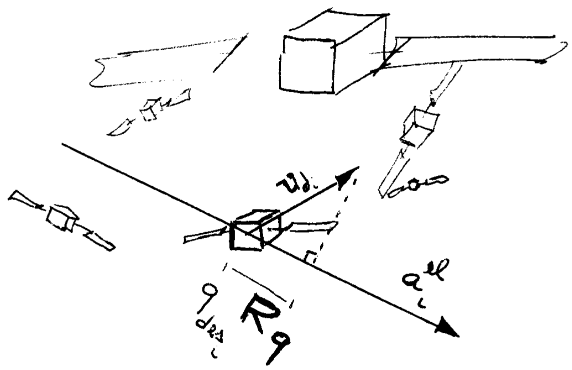

Figure 1: Visualization of the charge strategy.

Pettazzi [2] where behavioral approaches are mixed with sliding mode control to obtain a robust state feedback.

The charge feedback

We assume that at a certain time instant $\mathbf{u}_{d_i}$ is given. At the same instant the electrostatic inter- satellite forces are also well determined, and so is $\mathbf{u}_{t_i}$. Consider now the i-th agent. The electroststic force is $q_i \mathbf{R}_i \mathbf{q}$, and its direction does not depend on the agent charge as the i-th column of $\mathbf{R}_i$ is zero. As a consequence there is a well defined value of the agent electrical charge, that we call desired value q_{des_i} and that results in an electrostatic force of a given magnitude. We stress how the electrostatic force direction is not controllable by the single agent via its charge, only its magnitude. We introduce the following charge feedback on the current emitted by each spacecraft in a group of N.

$$\mathbf{I} = \dot{\mathbf{q}} = k(\mathbf{q}_{des} - \mathbf{q}) \tag{4}$$

where $\mathbf{q} = [q_1, q_2, ..., q_N]$ are the N values of the charges, $\mathbf{q}_{des} = [q_{des_1}, q_{des_2}, ..., q_{des_N}]$ and k is a scalar constant. We assume, in the general case, that q_{des_i} is a function of $\mathbf{q}, \mathbf{r}, \mathbf{u}_{d_i}$.

There are different options to define the desired charge of the i-th satellite (in other words to define the desired magnitude of the electrostatic force acting on the ii-th agent along the direction defined by $\mathbf{Rq}$). We here require that the electric force magnitude equals the $\mathbf{u}_i$ component along the electrostatic force direction (see Figure 1). In this way each agent desires to have the charge that minimises his personal propellant consumption or, in other words, to provide the largest possible portion of the $\mathbf{u}_{d_i}$ by means of the electrostatic actuation. We have:

$$q_{des_i} = \mathbf{u}_{d_i} \cdot \mathbf{R}_i \mathbf{q} / |\mathbf{R}_i \mathbf{q}|^2 \tag{5}$$

In the rest of this section we will try to prove the stability of such a feedback and to assess whether it actually brings advantages in term of fuel consumption. We will consider the $\mathbf{R}_i$ matrices as constant (i.e. the swarm is not moving) and the $\mathbf{u}_{d_i}$ as constant (i.e. the acceleration desired by the different still agents does not vary) and we study the differential system described by Eq.(4) and Eq.(5). An equilibrium point $\hat{\mathbf{q}}$ for the system will be defined by:

$$\mathbf{q}_{des}(\hat{\mathbf{q}}) - \hat{\mathbf{q}} = \mathbf{0}$$

if such a point exists and is reached, each one of the spacecraft desires to have exactly the charge that it has and will not emit any current (the interactions with the plasma environment are not taken into account at this level).

Theorem 1 *The differential system defined by Eq. (4) and Eq.(5) admits the equilibrium position $\hat{\mathbf{q}}$ if and only if $\mathbf{u}_{d_i} = \hat{q}_i \mathbf{R}_i \hat{\mathbf{q}} + \mathbf{u}_\perp$ where $\mathbf{u}_\perp$ represents any vector perpendicular to $\mathbf{R}_i \hat{\mathbf{q}}$*

This theorem is simply stating that according to what the required controls $\mathbf{u}_{d_i}$ are the satellites could fail to agree on what charge distribution to have and would therefore always emit currents. The prove the theorem is quite straightforward and is therefore omitted.

Theorem 2 *If $\hat{\mathbf{q}}$ is an equilibrium condition for the differential system defined by Eq. (4) and Eq.(5) then also $-\hat{\mathbf{q}}$ will be equilibrium position.*

This theorem is a rather trivial statement as changing the sign of all the agents charges leaves the charge products unchanged.

We now study the linear stability of the generic equilibrium position $\hat{\mathbf{q}}$. Without losing generality we assume that $\mathbf{u}_{d_i} = \hat{q}_i \mathbf{R}_i \hat{\mathbf{q}}$ and we study the system:

$$\dot{\mathbf{q}} = k(\mathbf{q}_{des} - \mathbf{q}) \tag{6}$$

$$q_{des_i} = \hat{q}_i \frac{\mathbf{R}_i \hat{\mathbf{q}} \cdot \mathbf{R}_i \mathbf{q}}{\mathbf{R}_i \mathbf{q} \cdot \mathbf{R}_i \mathbf{q}}$$

performing a standard linearization of the dynamic equations around the equilibrium condition $\hat{\mathbf{q}}$ and introducing the variable $\mathbf{x} = \mathbf{q} - \hat{\mathbf{q}}$ we get the linearized system

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$$

with

$$\mathbf{A} = k \left(\left. \frac{\partial \mathbf{q}_{des}}{\partial \mathbf{q}} \right|_{\mathbf{q}=\hat{\mathbf{q}}} - \mathbf{I} \right)$$

Where,

$$\left. \frac{\partial q_{des_i}}{\partial \mathbf{q}} \right|_{\mathbf{q}=\hat{\mathbf{q}}} = -\hat{q}_i \frac{\mathbf{R}_i^T \mathbf{R}_i \hat{\mathbf{q}}}{\mathbf{R}_i \hat{\mathbf{q}} \cdot \mathbf{R}_i \hat{\mathbf{q}}}$$

Having a compact analytical expression for the dynamic matrix $\mathbf{A}$, the linear stability of the feedback is soon evaluated by studying its eigenvalues. Clearly these depend on the vectors $\mathbf{R}_i \hat{\mathbf{q}}$ which essentially depend on the relative positions of the satellites and their requested final controls. The relation between $\mathbf{R}_i \hat{\mathbf{q}}$ and the eigenvalues of $\mathbf{A}$ is non trivial and its rigorous description is well beyond the purposes of this paper. For the purposes of this paper it is enough to describe how to assess the feedback linear stability case by case.

We show as an example a typical simulation of the feedback described by Eq.(4,5) for a selected case. In particular, we consider a group of ten satellites in a regular polygon formation such as that shown in Figure 2 and we select a random $\hat{\mathbf{q}}$, and as a consequence a random $\mathbf{u}_{d_i} = \hat{q}_i \mathbf{R}_i \hat{\mathbf{q}}$, for which the eigenvalues of the systems have all negative real part. During the simulation the satellites were not allowed to move and $\mathbf{u}_{d_i}$ was kept constant. In Figure 3 the charge history is shown starting from a number of randomly selected initial conditions. The time axis is logarithmic. The feedback constant k is set to one. In the particular case selected the only two equilibrium points are $\hat{\mathbf{q}}$ and $-\hat{\mathbf{q}}$ and they are both linearly stable. We note

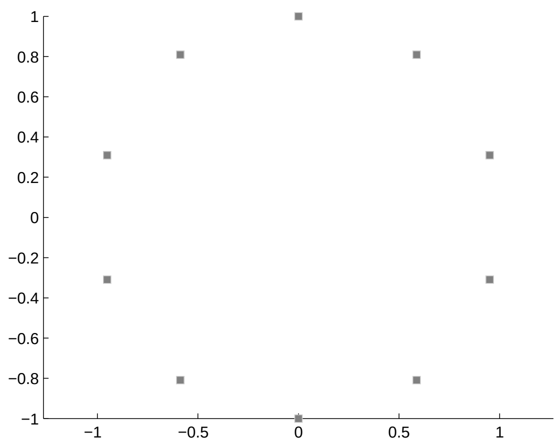

Figure 2: Agents positions in the regular polygon (10 elements)

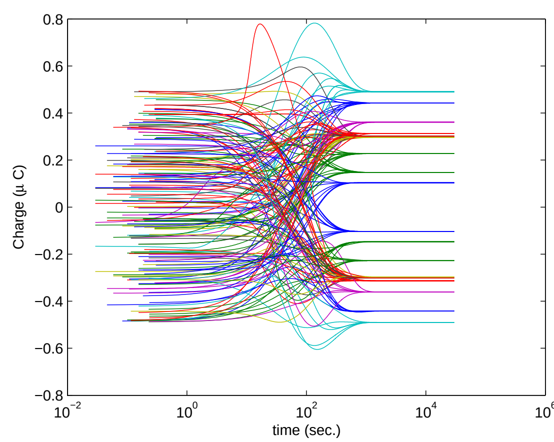

Figure 3: Monte Carlo simulation of the feedback for one test case

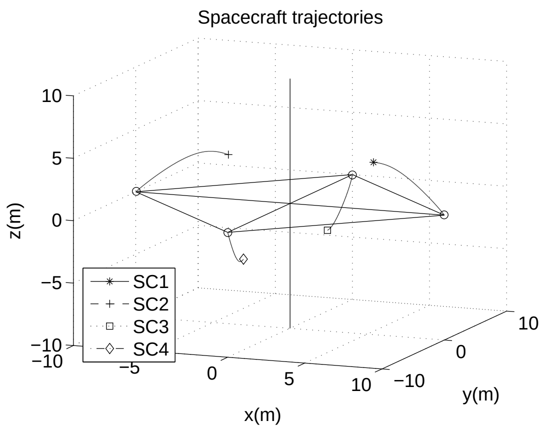

Figure 4: Trajectories visualization during the assembly sequence

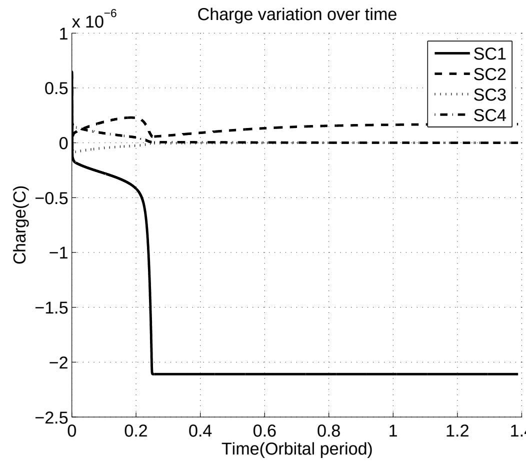

Figure 5: Charge history for the selected case.

how at least one of the stable equilibrium points is always reached at the end of the simulation.

Results for on-orbit assembly

In the previous section we have established some preliminary analytical results on the proposed control system. We have not proved the feedback global stability but we have established a number of tools that may help in anlaysing its behaviour case by case. We have shown under what conditions we attain at least the linear asymptotic stability, but as multiple equilibrium points exist and as the system in non linear, its global behaviour is extremely challenging to be studied. On the other hand a Monte Carlo campaign has revealed that when a linearly stable equilibrium point exist, the proposed feedback is able to drive the system towards it. This seems to suggest that the proposed scheme is suitable to drive the self-assembly procedures of identical agents equipped both with some “conventional ” thruster and with the capability to control its own charge level. Clearly one could argue that in a real simulations things are pretty different as the path planning level of the control system continuously updates the $\mathbf{u}_{d_i}$ and the various agents

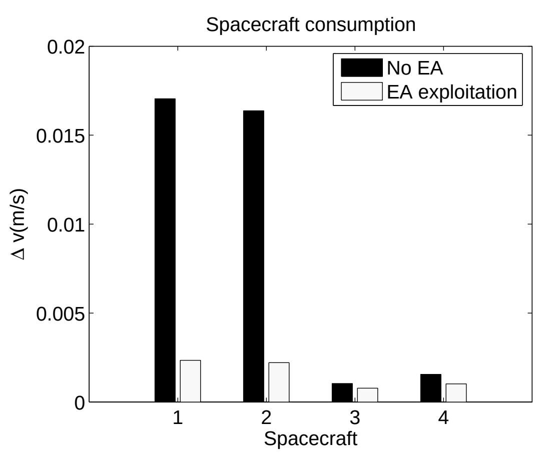

Figure 6: ΔV for the satellites

are actually moving, as a consequence $\mathbf{R}_i\hat{\mathbf{q}}$ is actually not constant. Fortunately the high bandwidth achievable in the charge control allow us to select the control parameter k so that the charge equilibrium is reached in timescales much smaller than those involved in the path planning. As an example we here report a full simulation taken from [7] where the whole control architecture is considered. We consider four satellites that have to acquire a planar quadrangular formation starting from randomly chosen positions. In Figure 4 the resulting final trajectories are shown for a swarm of four satellites. In the simulation the satellite mass is 50 kg, the maximum thrust level allowed 0.005 N, the maximum charges allowed 2 μC. The parameter k of the charge controller was set to 1/50. In the initial state the satellites are randomly placed within a sphere of 10 m centered in the origin, while the final required formation is, in a LHLV frame referred to a geostationary orbit, a square with two satellites placed along the x axis (Earth pointing) and two on the y axis. The path planning used was the one descried in [2]. The charge history is shown in Figure 5, while the comparison between the total DV necessary to perform the manoeuvre and the portion provided by the thrusters is given in Figure 6 (EA stands for Electrostatic Actuation). We do not want here to go into the details of this particular simulation, we only observe that in this case an infinite number of equilibria exist in the final configuration where the control at the path planning level requires to perfectly cancel the forces due to the tidal gravity. The consistent fuel savings confirm that the self-assembly procedure is indeed aided by the addition of the charge feedback proposed. Other simulations for different configurations and with a higher number of satellites confirm this result.

Conclusions

The self assembling of structures in orbit is a process that can be autonomously done by identical agents using sophisticated path planning techniques that allow each satellite to pursue a simple task while remaining unaware that a more complicated undertaking is being accomplished. If the agents are capable of controlling their electrostatic charge, the resulting inter-satellites Coulomb forces can be used to reduce significantly the fuel consumption thus allowing for more frequent reconfigurations to take place. The resulting highly non linear control problem can be approached by implementing a layered scheme that separates the path planning problem from the charge control problem and that results in a suboptimal procedure that exploits a limited knowledge on the global swarm status and that is yet able to conveniently use the Coulomb interactions to alleviate the propellant consumption required for reconfigurations, formation keeping and ultimately for completing an assembly sequence. The proposed algorithm revealed to be able to work in real time and exhibits interesting stability features worth of further investigations.

References

[1] Izzo, D. and Pettazzi, L., "Mission Concept for Autonomous on Orbit Assembly of a Large Reflector in Space," *56th International Astronautical Congress of the International Astronautical Federation, the International Academy of Astronautics, and the International Institute of Space Law, Fukuoka, Japan, Oct. 17-21, 2005, proceedings.*

[2] Izzo, D. and Pettazzi, L., "Autonomous and Distributed motion planning for satellite swarm," *Journal of Guidance Control and Dynamics*, Accepted for publication in September 2006.

[3] Gazi, V., "Swarm Aggregations Using Artificial Potentials and Sliding Mode Control," *42nd IEEE Conference on Decision and Control, 2003, proceedings.*

[4] Schaub, H., Parker, G. G., and King, L. B., "Challenges and Prospect of Coulomb Formations," *Journal of the Astronautical Sciences*, Vol. 52, No. 1–2, Jan.–June 2004, pp. 169–193.

[5] Pettazzi, L., Izzo, D., and Theil, S., "Swarm navigation and reconfiguration using electrostatic forces," *7th International Conference On*

Dynamics and Control of Systems and Structures in Space. The Old Royal Naval College, Greenwich, London, England, 16- 20 July, 2006, proceedings.

[6] Saaj, C. M., Lappas, V., Richie, D., Peck, M., Streetman, B., and Schaub3, H., "Electrostatic forces for satellite swarm navigation and reconfiguration." Tech. Rep. ARIADNA 05/4107b, September 2006. Avialable on-line [www.esa.int/gsp/ACT].

[7] Pettazzi, L., Krüger, H., and Theil, S., "Electrostatic forces for satellite swarm navigation and reconfiguration." Tech. Rep. ARIADNA 05/4107a, September 2006. Avialable on-line [www.esa.int/gsp/ACT].

[8] Natarajan, A. and Schaub, H., "Linear Dynamics and Stability Analysis of a Coulomb Tether Formation," *Journal of Guidance, Control, and Dynamics*, Vol. 29, No. 4, July–Aug. 2006, pp. 831–839.

[9] Pontryagin, L. S., Boltyanskii, V. G., Gamkrelidze, R. V., and Mishchenko, E. F., *The Mathematical Theory of Optimal Processes.*, John Wiley & Sons, Inc.. New York., 1962.

[10] Storn, R. and Price, K., "Differential Evolution - A Simple and Efficient Heuristic for Global Optimisation over Continuous Space," *Journal of Global Optimisation*, Vol. 11, No. 4, 1997, pp. 341–359.