

# INVARIANT RELATIVE SATELLITE MOTION

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## ABSTRACT

Formation flying is a key technology enabling a number of missions which a single satellite cannot accomplish: from remote sensing to astronomy and fundamental physics. The design of relative navigation and control systems of the spacecraft is certainly one of the most challenging topic. In order to ease the tasks of these subsystems, a proper reference trajectory must be conceived, and a relative motion, which shows no drift even in presence of a large disturbance as the J2 effect, could be a very attractive solution. This paper describes the research activities for finding invariant relative orbits under J2 effect. Numerical tools as genetic algorithms enabled the discovery of two special inclinations which represent the necessary conditions for periodicity of the motion. These results generated interest, and analytical explanations for the numerical evidence have been proposed: on-going studies face this problem from different points of view, and basic results are reported.

## 1 INTRODUCTION

Formation flying is a key technology enabling a number of missions which a single satellite cannot accomplish: from remote sensing to astronomy and basic physics. In order to keep the satellites of the formation in the designed configuration, and therefore to achieve the mission's goals, control actions are needed. The cost of this orbital control in terms of  $\Delta V$  limits both the mission duration and the expected performances. Advantageous dynamics could reduce the cost of these operations, in particular the possibility to obtain periodic or quasi-

periodic natural relative motion would be a significant saving factor.

Many different approaches to find a periodical relative motion are considered in the recent literature. Inalhan, Tillerson and How [1] found the analytical expression for the initial conditions resulting in periodic motion based on the classical Tschauner-Hempel equations [2]. Kasdin and Koluman [3] used the epicyclic orbital elements theory to derive bounded, periodic orbits in presence of various perturbations. Vaddi, Vadali, and Alfriend [4] studied the Hill-Clohessy-Wiltshire [5] (HCW) modified system to include second order terms. Finally, Schaub and Alfriend [6] formulated the

conditions for invariant  $J_2$  relative orbits introducing relations between the mean orbital elements of the two satellites. The analytical approaches taken in these works lead to initial conditions that ensure exact periodicity in approximated dynamical models or initial conditions resulting in bounded (but not periodic) relative motion in more detailed dynamical models.

In the frame of the 2004 ESA ARIADNA project, the possibility to obtain natural periodic relative motion of formation flying LEO (Low Earth Orbits) satellites has been investigated numerically. The algorithm is based on a genetic strategy (GA), refined by means of nonlinear programming, that rewards periodic relative trajectories. Only the  $J_2$  perturbed case is considered, as presence of a dissipative disturbance like drag does not enable a periodic motion. Using the proposed numerical approach, it has been possible to find two couples of inclinations ( $63.4^\circ$  and  $116.6^\circ$ , the critical inclinations, and  $49^\circ$  and  $131^\circ$ , two new “special” inclinations) that seemed to be favoured by the dynamical system for obtaining periodic relative motion at small eccentricities.

This interesting numerical result still missed a mathematical or physical explanation. Vadali, Sengupta, Yan, and Alfriend in [7] find the values for inclination which enable equal in-plane and out-of-plane fundamental frequencies, resulting in non-precessing and distortion-free relative orbits over the short-run. In this way, however, the results obtained by means of the numerical optimization are only partially validated. In fact, according to [7], all the inclinations which fall in the interval between the special inclinations,  $i \in [49.1^\circ, 63.4^\circ]$ , turn out to be valid for periodic motion, while only the range limits were identified with GA and nonlinear optimization.

Following these results, a new kind of study has been started, on mathematical basis, which aims to show that:

1) a truly periodic relative motion in a  $J_2$  perturbed environment is generally not possible;

2) two special inclinations exist for which at least the projection on a coordinate plane of the relative motion is periodic

## **2 THE NUMERICAL APPROACH: DEFINITION OF THE PROBLEM**

Consider a generic non-autonomous dynamical system  $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t)$ , e.g. the relative dynamics of satellites flying in formation. Define  $\delta\mathbf{x} = \mathbf{x}_0 - \mathbf{x}(T)$ , where  $\mathbf{x}_0$  is the system state at the initial time and  $T$  is a time variable here called ‘candidate period’ for reasons that will soon be clear. Then, the following optimisation problem is defined:

$$\begin{cases} \text{find : } \mathbf{\kappa} = [\mathbf{x}_0 \ T]^T \\ \text{to maximise : } J = J(|\delta\mathbf{x}|) \\ \text{subject to : } \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t) \end{cases} \quad (1)$$

where the objective function  $J$  is constructed in such a way as to have its global maximum at  $\delta\mathbf{x} = \mathbf{0}$ . The optimisation problem above is equivalent to the task of finding as-periodic-as-possible solutions to the system  $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t)$ . These solutions correspond, in our case, to minimal relative orbit drift. As we study a number of systems  $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t)$ , we face different levels of complexity for the optimisation and for objective function properties. Think about the relative motion between satellites moving on keplerian orbits, the problem defined by Eq.(1) has an infinite number of solutions, corresponding to orbits with equal semi-major axis. A similar structure is also expected when the keplerian dynamic is perturbed. As a consequence, a genetic approach, avoiding issues related to domain knowledge and able to cope with multiple local and global minima, has been selected to perform a search in the

solution space. The PIKAIA freely available software [7] was used in this work as genetic optimiser. Table 1 shows the fundamental parameters of the genetic algorithm used in all the simulations.

**Table 1** Parameters used for the Genetic Optimizer

|                              |                            |
|------------------------------|----------------------------|
| Number of individuals        | 20                         |
| Number of generations        | 500                        |
| Number of significant digits | 9                          |
| Crossover Probability        | 0.85                       |
| Initial Mutation Rate        | 0.005                      |
| Minimum Mutation Rate        | 0.0005                     |
| Maximum Mutation Rate        | 1                          |
| Reproduction Plan            | Steady-State-Replace-Worst |

The best solution returned by the genetic algorithm is then refined locally by means of a nonlinear programming solver.

In our simulations the decision vector  $\mathbf{\kappa}$  contains the initial relative position, the initial relative velocity, and the candidate period  $T$ . We consider the relative motion between two satellites: a chief and a deputy to use a popular terminology connected to formation flying research. The absolute dynamics of both the chief and the deputy are simulated propagating the inertial coordinates of the spacecraft in

time,  $\frac{d^2 \mathbf{r}}{dt^2} = -\frac{\mu}{r^3} \mathbf{r} + \mathbf{P}$  where  $\mathbf{P}$  are the perturbing action considered,  $\mu$  is the planetary constant and  $\mathbf{r}$  is the orbital radius vector. The relative state is then evaluated by means of Eq. (2):

$$\begin{cases} [x & y & z]^T = \mathbf{R}([X_d & Y_d & Z_d]^T - [X_c & Y_c & Z_c]^T) \\ [\dot{x} & \dot{y} & \dot{z}]^T = \mathbf{R}([\dot{X}_d & \dot{Y}_d & \dot{Z}_d]^T - [\dot{X}_c & \dot{Y}_c & \dot{Z}_c]^T) - \boldsymbol{\omega} \times [x & y & z]^T \end{cases} \quad (2)$$

where  $\mathbf{R}$  is the rotation matrix from the inertial coordinate system to the Local Vertical Local Horizontal (LVLH) frame in which the relative state is defined. The subscripts  $c$  and  $d$  stand for chief and deputy satellites. The orbit of the chief is

considered known and the initial conditions to propagate the deputy motion are obtained transforming the relative  $\delta \mathbf{x}_0$  position into absolute coordinates inverting Eq.(2).

We then defined  $T = T_{kep} \pm \bar{\kappa}_7 k$ , where  $k$  is a properly chosen constant (some tens of seconds) and  $T_{kep}$  is the orbital

period  $2\pi\sqrt{\frac{a^3}{\mu}}$  of the chief orbit.  $T$  is

clearly a crucial parameter. At  $T$ , the final relative coordinates are compared to the initial relative coordinates, thus determining the quality of the individual. A good individual has a small  $\delta \mathbf{x}$  and its position in the individual ranking is high, therefore it has a larger chance to mate and to generate “good” sons. Its genes will survive in the next generation, and if they will be ranked first in the last generation, they will be further refined by a local optimiser and represent the set of initial conditions that generate the minimum drift relative orbit. The fitness function we used to rank the individuals is:

$$J(\kappa) = \frac{1}{10^{-3} + \sqrt{\left(\frac{x_f - x_0}{x_0}\right)^2 + \left(\frac{y_f - y_0}{y_0}\right)^2 + \left(\frac{z_f - z_0}{z_0}\right)^2 + \left(\frac{\dot{x}_f - \dot{x}_0}{\dot{x}_0}\right)^2 + \left(\frac{\dot{y}_f - \dot{y}_0}{\dot{y}_0}\right)^2 + \left(\frac{\dot{z}_f - \dot{z}_0}{\dot{z}_0}\right)^2}} \quad (3)$$

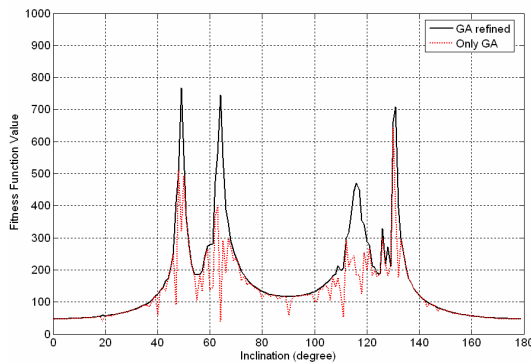
A perfect individual (periodic motion) has a fitness value of 1000, while a percentage difference of 0.1% between the initial and final state, brings down the fitness value to 500, a difference of 1% corresponds to a fitness value of 90, and so on.

### 3 THE NUMERICAL APPROACH: RESULTS

Let us study the solutions of Eq. (1) in the case  $\mathbf{f}$  describes the relative motion between two satellites orbiting around an oblate Earth. As already mentioned, this corresponds to minimising the relative orbital drift. Some previous work has been done to determine the possibility of

invariant relative satellite motion when  $J_2$  is considered as a perturbing term. In particular, the paper by Schaub and Alfriend [6] introduces the so called  $J_2$  invariant relative orbits. In their work mean orbital elements are used and the secular drifts of the longitude of the ascending node and of the sum of the argument of perigee and mean anomaly are set to be equal between two neighbouring orbits. Even though called  $J_2$  invariant orbits, these two conditions are only valid in a first order approximation.

We use our numerical approach based on the solution of the global optimisation problem stated in Eq. (1) to check to what extent the residual drift obtained with this analytical approach is an artifact of the use of mean elements. Repeating the calculation for the entire range of inclinations, the results vary sensibly, disclosing a previously unknown feature of invariant relative motion. In Fig. 1 we report the best fitness function reached for different inclinations ranging from 0 to 180 degrees. The other orbital parameters of the Chief satellite used for this simulation were  $a = 6678$  km,  $e = 0.00118$ ,  $\varpi = 90^\circ$ ,  $\Omega = 270^\circ$ ,  $\theta = 0^\circ$ , with the usual meaning of the symbols. In the Fig. 1 we report both the output from the genetic algorithm and the final solution obtained refining the solution with a local optimisation.

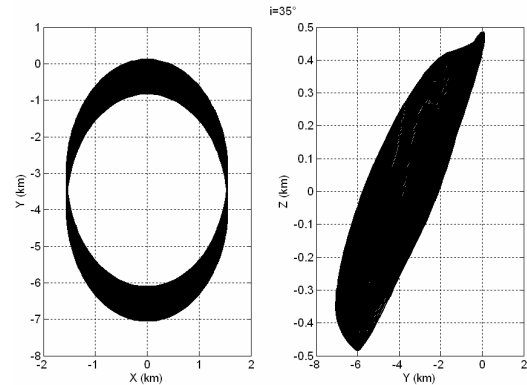


**Fig. 1** Best Individual Fitness Value vs. Inclination of Reference Orbit

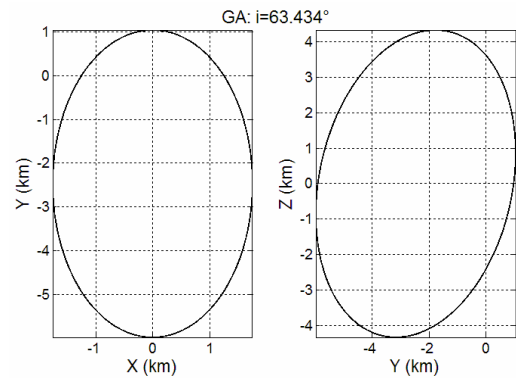
For all inclination the minimal drift is not zero, with two remarkable exceptions:  $49^\circ$  and  $63.4^\circ$ , and their symmetric counterparts (with respect to  $90^\circ$ ), i.e.

$131^\circ$  and  $116.6^\circ$ . In the following we will refer to these as “special inclinations”. The heuristic of the genetic algorithms is definitely not responsible for these peaks, as it turns out by actually propagating the resulting best individuals. At a generic inclination, say  $35^\circ$ , the best individual returned by the optimisation results in a relative motion that is not periodic, as visualised in Fig. 2. The small residual drift is comparable to the one that results using Schaub  $J_2$ -invariant orbit condition.

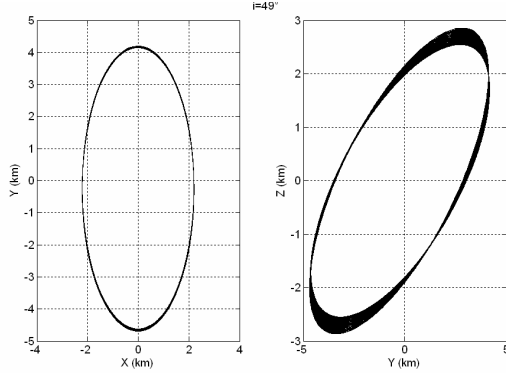
At the special inclinations  $63.4^\circ$  and  $116.6^\circ$  the relative motion turns out to be perfectly periodical (see Fig. 3). The possibility of obtaining a perfectly periodical relative motion at these inclinations is probably related to the cancellation of the secular drift of the perigee argument, which causes the variation of all parameters to happen with same main frequency. At the other two special inclinations ( $49^\circ$ ,  $131^\circ$ ) the relative satellite motion resulting from the best individual has only a very small drift, as shown in Fig. 4.



**Fig. 2** 100 Relative Orbits for a Perturbed case at Inclination (Best Individual)



**Fig. 3** 100 Relative Orbits for a Perturbed case at Inclination (Best Individual)

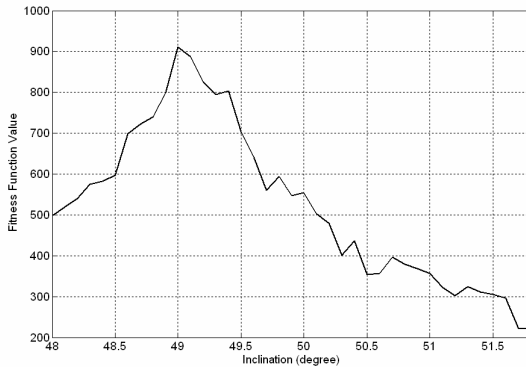


**Fig. 4** Relative Orbits for a Perturbed case at 49° Inclination (Best Individual)

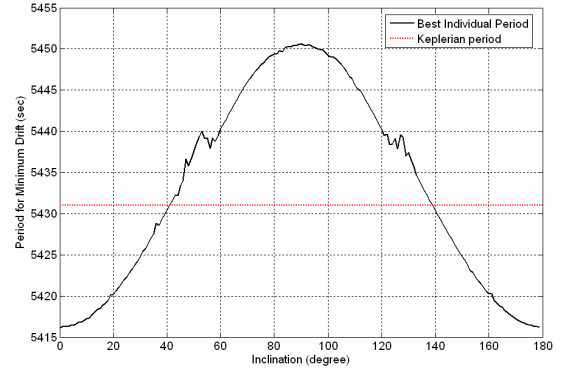
The residual drift does not allow us to conclude that the motion is perfectly periodical at these inclinations. We were in fact unable to find a fitness value of 1000 (meaning perfect periodicity, according to Eq. (3)) at any inclination.

A more detailed plot of the objective function achieved around the special inclination is shown in Fig. 5. The clutter that can be observed in the graph is a consequence of the numerical optimisation process, amplified by the definition of the objective function given by Eq.(3). At higher values of the fitness very small differences in the residual drift cause significant differences in the objective function value.

For completeness we also report in Fig. 6 a plot of the period of the found minimum drift orbits. This is clearly quite different from the keplerian period confirming the importance of having let the optimiser to choose it.



**Fig. 5** Details around a special inclination



**Fig. 6** Best Individual Period (sec)

The existence of the two special inclinations represented one of the main findings of the 2004 ARIADNA project on the search of invariant relative motion of satellites, but it clearly called for some physical or analytical explanation.

#### 4 AN ANALYTICAL EXPLANATION

Vadali et al. in [7] look for a physical explanation of the special inclinations. The first step consists in a geometric description of the relative motion using the unit-sphere approach. The expressions for the in-plane and cross-track motion variables are linearized to extract the fundamental frequencies of motion. A set of classical differential orbital element initial condition formulae are derived, valid even for circular orbits.

In this way the difference between the in-plane and cross-track frequencies ( $n_{xy}$  and  $n_z$ , respectively), over a short time interval (compared to the period of the differential nodal precession rate), is evaluated as:

$$n_z - n_{xy} = \dot{\omega}_1 - \frac{\sin i_0 \Delta \dot{\Omega} \Delta i}{\Delta i^2 + (\sin i_0 \Delta \Omega(0))^2} \quad (4)$$

where  $i_0$  is the reference orbit inclination,  $\Delta \dot{\Omega}$ ,  $\Delta \Omega$ ,  $\Delta i$  the differences in RAAN rate, RAAN and inclination between the formation members. The drift rate for the argument of perigee can be written as

$$\dot{\omega}_0 = -k \left( 2 - \frac{5}{2} \sin^2 i_0 \right) n_0 \quad (5)$$

where:

$$k = -1.5 J_2 \left( \frac{R_E}{a_0} \right)^2 n_0 \quad (6)$$

The differential nodal precession rate for near-circular orbits is

$$\Delta \dot{\Omega} = -k \sin i_0 \Delta i \quad (7)$$

Neglecting the effect of  $\Delta \omega$ , the frequency mismatch is estimated as follows:

$$n_z - n_{xy} = k \sin^2 i_0 \left( \frac{5}{2} + \frac{\Delta i^2}{\Delta i^2 + (\sin i_0 \Delta \Omega(0))^2} \right) \quad (8)$$

For the special case of the projected circular orbit (ref. [9]), it can be shown that:

$$\Delta \Omega(0) = -\frac{\rho(0)}{a_0 \sin i_0} \sin \alpha(0) \quad (9)$$

$$\Delta i = \frac{\rho(0) \cos \alpha(0)}{a_0} \quad (10)$$

where  $\alpha(0)$ , is the desired initial phase angle and  $\rho(0)$  is the initial radius of the relative orbit in the y-z plane

By substituting Eqs. (9) and (10) into Eq. (8), the frequency matching condition is satisfied by two possible values of the inclination:

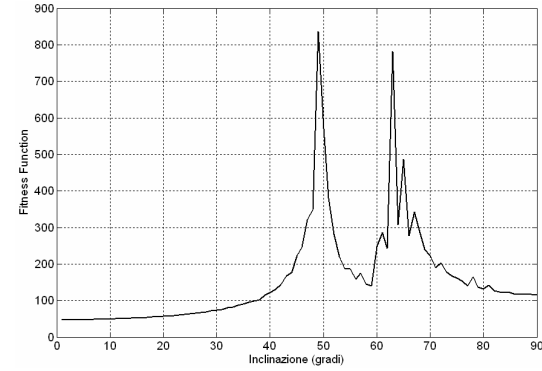
$$i^* = \sin^{-1} \left( \sqrt{\frac{2}{2.5 + \cos^2 \alpha(0)}} \right) \quad (11)$$

The inclinations for  $\alpha(0) = 0$  are  $i^* = 49.11^\circ$  and  $130.89^\circ$  and for  $\alpha(0) = 90^\circ$ , the results are the critical inclination values:  $i^* = 63.43^\circ$  and  $116.57^\circ$ . Anyway,  $\alpha(0)$  is a parameter varying between  $0^\circ$  and  $360^\circ$ , this means that  $i^* \in [49.11^\circ, 63.4^\circ]$ , while the numerical evidence is for  $i^*$  equal to the boundary values of the interval. This

partially contradictory result led to the research of a different approach.

## 5 AN ALTERNATIVE MATHEMATICAL APPROACH

In order to try to solve the contradictions between the numerical and the analytical results, a completely different approach is adopted. In [10] and [11], a linear model is obtained, describing the relative dynamics of a formation in a  $J_2$  perturbed environment (circular reference orbit case). If the GA exploited for the nonlinear formation dynamics shown in section 2 is used now for the linear model, similar results are obtained, as Fig. 7 confirms.



**Fig. 7** Results of the application of the numerical method applied to the linear  $J_2$  model

It is therefore possible to focus the attention on the linearized dynamics, since the causes of the existence of special inclinations are in the first order gradients of gravity and  $J_2$  disturbance. The adoption of a linear system as object of the study allows the use of relevant theorems valid for linear systems, as the one demonstrated by Bose in [12]:

“Let  $\dot{Y} = A(t)Y$  be a  $n$ -dimensional homogeneous linear system and  $\Phi(t)$  its fundamental matrix; let  $k$  be a non-negative integer,  $0 \leq k \leq n$ . There exists a  $k$ -dimensional sub-space  $S_k$  of the solution space  $S_n$  of the linear homogeneous system such that each member of  $S_k$  is periodic of period  $T$  and no member of  $S_n - S_k$  is periodic of period  $T$  if and only if  $\lambda = 1$  is an eigen value of the scalar matrix  $\Phi(0)^{-1} \Phi(t)$  of multiplicity  $k$ ”.

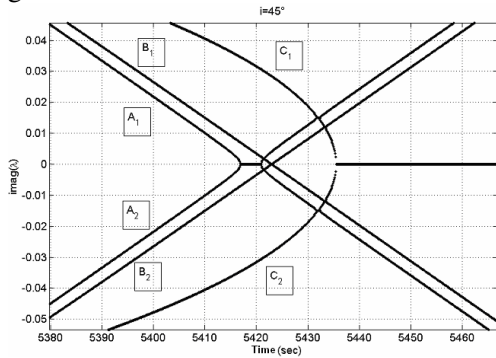
Our case perfectly matches the hypothesis of this theorem, since the linear model adopted is homogeneous; however, the  $T$  period is not known a priori (but clearly in the close proximity of the keplerian period, see again Fig. 6). Therefore, the time behaviour of the six eigen values  $\lambda$  of  $\Phi(t)$  has been analysed.  $\Phi(t)$  is evaluated by means of:

$$\frac{d\Phi(t)}{dt} = A(t)\Phi(t), \quad (12)$$

with  $\Phi(0) = I$  (the identity matrix).

The  $A(t)$  state matrix is a 6-by-6 matrix: according to the Bose's theorem a relative periodic motion is possible only if a time  $t^*$  exists for which the eigen values of  $\Phi(t^*)$  are equal to 1 (i.e.  $\text{imag}(\lambda)=0$  and  $\text{re}(\lambda)=1$ ) with multiplicity 6.

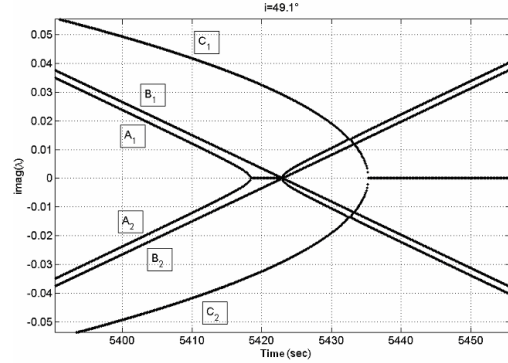
For reference orbits inclined from  $0^\circ$  to  $49.1^\circ$ , there is not a time instant when this is verified; as an example, see Fig. 8, referred to the  $i=45^\circ$  case. The imaginary part of the eigen values is never 0 for more than a pair of eigen values at the same time, therefore  $\lambda=1$  with multiplicity greater than two is never achieved.



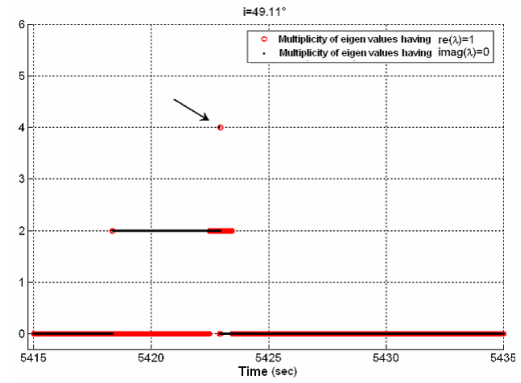
**Fig. 8** Time behaviour of the imaginary part of the 6 eigen values of  $\Phi(t)$  :  $45^\circ$  case

However the pairs  $A_1, A_2$  and  $B_1, B_2$  have an imaginary part which is zero for time instants which get closer as the inclination increases, finally coinciding for  $i$  equal to the special inclination  $49.1^\circ$  (Fig. 9). Fig. 10 confirms that not only 4 eigen values have a zero imaginary part in a certain instant, but that in that instant also the real part of the same eigen values is equal to 1. Following Bose's theorem, this means that there exists a coordinated plane, where the

projection of the relative motion is periodic (a 4-dimensional subspace necessarily is comprehensive of two coordinates and their derivatives, i.e., a coordinated plane).



**Fig. 9** Time behaviour of the imaginary part of the 6 eigen values of  $\Phi(t)$  :  $49.1^\circ$  case

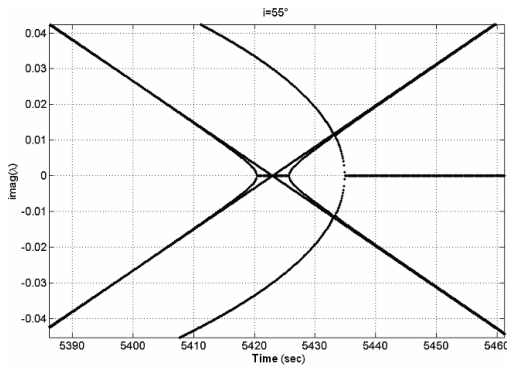


**Fig. 10** Multiplicity of the eigen values having  $\text{imag}(\lambda)=0$ , and multiplicity of eigen values having  $\text{re}(\lambda)=1$  :  $49.1^\circ$  case

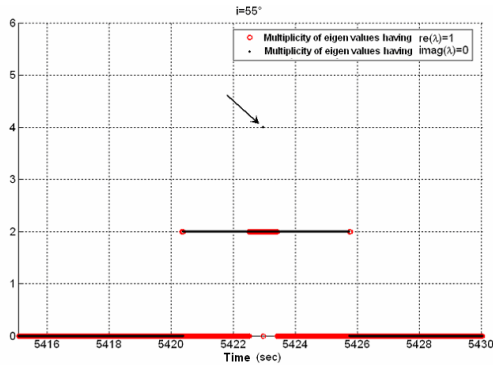
Similar considerations can be done for the other special inclination,  $63.1^\circ$ : at these two special inclinations the motion of the formation is by far more stable than at any other inclination, though it is not mathematically periodic in its six components, yet just in four.

In fact, for the inclinations in the range  $[49.1^\circ, 63.1^\circ]$  while there are still 4 eigen values with imaginary part equal to zero in a certain instant, they do not have real part equal to one (Fig. 11 and Fig. 12), and so the Bose's theorem is not verified, not even for a 4-dimensional subspace. For inclinations greater than  $63.1^\circ$ , even if it is possible that 4 eigen values have imaginary part equal to zero, again they do not have real parts equal to one, and a

periodic motion is not possible, not even a periodic projection of the motion.



**Fig. 11** Time behaviour of the imaginary part of the 6 eigen values of  $\Phi(t)$  : 55° case



**Fig. 12** Multiplicity of the eigen values having  $\text{imag}(\lambda)=0$ , and multiplicity of eigen values having  $\text{re}(\lambda)=1$  : 55° case.

## 6 FUTURE DEVELOPMENT

The proposed mathematical demonstration still have a number of drawbacks: first, it deals with a linearized dynamical model and not with the complete dynamics; moreover, it is not explained *why* the special inclinations do exist. Therefore a physical investigation should be the natural following of these researches. In fact, it is not important just to notice that the phenomenon exists, but it is significant to understand the forces acting on the two satellites which produce it. Only in this way, the hidden potential in the perturbations of the LEO environment could be fully exploited.

A periodic relative orbit represents a very low-cost station keeping solution. Even

though it is true that  $J_2$  is not the only environmental disturbance, it is certainly the largest for a wide range of missions profiles. In addition, the special inclination 49.1° is quite promising since it is not very different from the inclination of GPS, of the future GALILEO, of the International Space Station. Having an almost free-of-control formation could suggest a number of solution, different from the usual formation tasks: it can be useful for occupying the same orbital slot without conflicting, serving as a spare satellite, or for continuous monitoring of the health of the main satellite. Furthermore, once invariant  $J_2$  orbits have been assessed, the effects of other perturbations can be isolated and contrasted in a most efficient way, if needed, or alternatively, they can be fully exploited on purpose.

## 7 CONCLUSIONS

The possibility of a periodic relative motion of satellites in a  $J_2$  perturbed environment has been numerically confirmed in a ESA ARIADNA project in 2004. An invariant motion is found to be not feasible, with two remarkable exception, when the reference orbit is inclined at 49.1° and 63.4°. Those results generated interest in the scientific community, and new studies has been focused on this topic, searching for a physical or analytical explanation of the numerical evidence. Since inevitable approximations are necessary to simplify the dynamic equations, the results are not matching completely the numerical evidence, and a new approach is here outlined: it shows the uniqueness of the special inclinations 49.1° and 63.4° which enable periodic relative trajectories.

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