ASTEROID DEFLECTION BY MEANS OF ELECTROMAGNETIC FORCES DURING AN EARTH FLYBY

Sanjurjo-Rivo, M., Peláez, J.[1], Izzo, D., Purcell, O., Yam, C. H. [2], and Scheeres, D.J.[3]

[1] *Universidad Politécnica de Madrid, Pza. Cardenal Cisneros 3, 28040 Madrid, Spain.*
E-mail address: m.sanjurjo@upm.es
[2] *ESA, Advanced Concepts Team, ESTEC, Keplerlaan 1, Postbus 299, 2200 AG, Noordwijk*
[3] *The University of Colorado at Boulder, 429 UCB, Boulder, CO 80309-0429*

Abstract

In recent years, substantial efforts have been carried on in the sake of avoiding a possible impact of a threatening object on the Earth. New technologies have been suggested in this field, investigating the utilization of innovative applications for this purpose. In this context, this paper is devoted to take the most of the fact that, during an Earth fly-by, there will be several distinct forces acting on an asteroid due to the interaction with the Earth magnetic field. Among these interactions, this paper is focused on the Lorentz force, assessing under what assumptions and conditions it can provide an effective deflection of the asteroid's orbit.

INTRODUCTION

Near Earth Objects (NEO) pose a significant hazard to Earth, mitigate this hazard requires the use of a variety asteroid deflection techniques. The aim of any deflection mission is to alter the asteroid orbit in order to avoid possible dangerous encounters. The scenarios usually considered for those missions in literature refer to actions carried out during the whole heliocentric asteroid path.

Nevertheless, it is practicable to explore the possibilities of asteroid deflection in a narrower time and space scale i.e. during an Earth flyby. Furthermore, the specific conditions of this trajectory leave some open issues. In particular, we will be focused on answering the question whether it is feasible to take advantage of the interaction between the Earth's magnetic field and the asteroid. In previous works the feasibility of utilizing electrostatic forces has been consider in [1], where the estimation of the needed charge to obtain noticeable force is provided, or in the design of the Electrostatic Tractor [2]. In addition estimations of electromagnetic perturbations of asteroid's orbits can be found in [3].

In this paper, we present a comprehensive method to study the effect of disturbing accelerations during the asteroid flyby. This method will allow us to estimate the force needed to achieve a desired deflection. Furthermore, four different scenarios are devised in this study. Firstly, a mission contemplating a conventional kinetic impactor. We will see that the obtained results suggest that this strategy can lead to a efficient deflection. Secondly, an electrostatically charged asteroid evolving under the effect of the Lorentz force will be considered, supposing a fixed charge during the flyby. The effect of this perturbation force on the orbit changes is analyzed in terms of the charge and the initial characteristics of the orbit. Thirdly, the utilization of electrodynamic tethers is considered as a thruster option in close encounter events. The operation of these devices is highly conditioned by the capacity of collect electrons from a extremely rarified plasma environment. Finally, the significance of long electrodynamic tethers in the attitude motion of the asteroid during the flyby is analyzed, studying the possible enhancement of the change of the angular momentum the asteroid undergoes. The feasibility of these strategies is discussed in the light of numerical results.

In summary, this paper is devoted to assess the feasibility of deflection strategies during a previous encounter of a hazardous body.

ANALYSIS OF ASTEROID DEFLECTION DURING A FLYBY

The purpose of this paper is to deal with the analysis of the asteroid deflection during an Earth flyby. Thus, first of all, we will be focused on establishing the needed tools to study the effect of the perturbation forces acting during the swingby on the asteroid long term orbit. In order to do so, the patched conic approximation will be used. Besides, we will analyze, on the one hand, the Gauss' planetary equations to determine the effect of the perturbing forces on the hyperbolic fly by trajectory and, on the other hand, the modification which comes up on the b-plane.

Within the validity of the patched conic approximation, the initial orbit will be a hyperbola. To analyze the problem three reference frames will be used: 1) local osculating polar reference frame $(\vec{i}_r, \vec{i}_\theta, \vec{i}_h)$. The unit vector $\vec{i}_r$ pointing in the direction of the position vector and $\vec{i}_h$ pointing in the direction perpendicular to both position and velocity vectors; 2) osculating plane reference frame $(\vec{i}_e, \vec{i}_p, \vec{i}_h)$. The unit vector $\vec{i}_e$ lying in the direction of the eccentricity vector and $\vec{i}_h$ defined in the same way as before; 3) pseudo inertial reference frame $(\vec{i}_x, \vec{i}_y, \vec{i}_z)$. The origin of this reference frame would be the center of mass of the Earth, the $\vec{i}_x$ and $\vec{i}_y$ axis define the equatorial plane and $\vec{i}_z$ is coincident with the Earth's rotation axis. Besides,

the projection of $\vec{i}_x$ on the ecliptic has the same direction as the position vector of the Earth respect to the Sun (We are assuming the Earth follows a circular orbit around the Sun).The initial hyperbola can be characterized either by the initial osculating orbital elements $(\Omega_0, i_0, \omega_0, a_0, e_0, \tau_0$, where $a < 0$ and $e > 1)$ or by the arrival velocity vector $\vec{v}_\infty$ and the intended position vector of the periapsis $\vec{r}_p$. The osculating elements used to describe the change of the hyperbola will be $(\Omega, i, \omega, a, e, \lambda)$ where $\lambda = n\,\tau$, and $n = \sqrt{\mu/(-a)^3}$. If the perturbation vector expressed in the local osculating polar reference frame has the form: $\vec{a} = \{a_{d,r}, a_{d,\theta}, a_{d\,h}\}^T$, the formulation of the Gauss' planetary equations for hyperbolas is:

$$\frac{d\,\Omega}{d\,t} = \frac{r\,\sin\theta}{h\,\sin i}\,a_{d,h} \qquad \frac{d\,i}{d\,t} = \frac{r\,\cos\theta}{h}\,a_{d,h} \tag{1}$$

$$\frac{d\,\omega}{d\,t} = \frac{1}{h\,e}\left[-p\,\cos f\,a_{d,r} + (p+r)\,\sin f\,a_{d,\theta}\right] - \frac{r\,\cos i\,\sin\theta}{h\,\sin i}\,a_{d,h} \tag{2}$$

$$\frac{d\,a}{d\,t} = -\frac{2\,a^2}{h}\left[e\,\sin f\,a_{d,r} + (1 + e\,\cos f)\,a_{d,\theta}\right] \tag{3}$$

$$\frac{d\,e}{d\,t} = -\frac{1}{h}\left[p\,\sin f\,a_{d,r} + \left(p\,\cos f + p\,\frac{(a+r)}{a\,e}\right)a_{d,\theta}\right] \tag{4}$$

$$\frac{d\,\lambda}{d\,t} = a_{d,r}\left[\frac{b}{h\,a\,e}(2\,r\,e - p\,\cos f) - \frac{3\,t\,e\,\sin f}{b}\right] - a_{d,\theta}\left[\frac{3\,t\,(1 + e\,\cos f)}{b} - \frac{b\,\sin f}{h\,a\,e}(p+r)\right] \tag{5}$$

Together with the Kepler's equation: $(n\,t + \lambda = e\,\sinh H - H)$, the planetary equations allow to compute the evolution of the osculating orbital elements given the initial conditions and the perturbing force law $\{a_{d,r}(t), a_{d,\theta}(t), a_{d,h}(t)\}$.

Concerning the b-plane, we can establish the relation between the b-vector and the elements of an osculating hyperbola. In fact, calling $\vec{s}$ the unit vector in the direction of the incoming asymptote and $\vec{n}$ the unit vector in the direction perpendicular to the orbital plane $(\vec{n} = (\vec{r} \times \vec{v})/|\vec{r} \times \vec{v}|)$, the impact parameter is computed as:

$$\vec{B} = (-b)\,(\vec{s} \times \vec{n}) \tag{6}$$

The expression of the vector on the inertial reference frame will be : $\vec{s} = \mathcal{R}\,\{\cos\alpha, \sin\alpha, 0\}^T$ and $\vec{n} = \mathcal{R}\,\{0, 0, 1\}^T$ where $\mathcal{R}(\Omega, i, \omega)$ is the transformation matrix from the osculating plane coordinates to the inertial reference coordinates and α is the half bend angle defined as: $\alpha = \arccos(1/e)$

Asymptotic Analysis

In this paragraph we take advantage from the fact that the variations of the orbital elements will be small as it will be the disturbing acceleration. We start establishing the non dimensional formulation for the planetary equations. The characteristic length will be the initial semi major axis in absolute value, $|a_i|$. In turn, the characteristic time, τ_c, will be $\tau_c = \sqrt{|a_i|^3/\mu_\oplus}$. The non dimensional variables will be marked with a hat, e.i., $a = |a_i|\,\hat{a}$.

Furthermore, assuming the non dimensional disturbing acceleration is small, it is possible to derive a simpler formulation to compute the variation of the osculating orbital elements. Let $\varepsilon \ll 1$ be a small parameter, satisfying the relation for the non dimensional disturbing acceleration $\hat{a}_d = \varepsilon\,\tilde{a}_d$. Thus, we consider the asymptotic expansion of the osculating orbital elements in powers of ε: $\hat{\vec{\alpha}} = \hat{\vec{\alpha}}_0 + \varepsilon\,\hat{\vec{\alpha}}_1 + \varepsilon^2\,\hat{\vec{\alpha}}_2 + \mathcal{O}(\varepsilon^2)$, where $\hat{\vec{\alpha}}_0$ corresponds to the solution of the unperturbed problem. Besides, we consider the quantity H related to the true anomaly can be expanded in terms of the same parameter ε: $H(\tau) = H_0(\tau) + \varepsilon\,H_1(\tau) + \mathcal{O}(\varepsilon)$. The system of differential equations which governs the evolution of $\hat{\vec{\alpha}}_1$ can be computed from the non dimensional planetary equations. The initial conditions to this set of differential equations is $\hat{\vec{\alpha}}_1 = \vec{0}$, since initially the osculating values of the orbit elements correspond to the unperturbed problem.

Therefore, assuming either constant thrust $\tilde{a}_d = \tilde{A}_d$ or impulsive thrust $\tilde{a}_d = \delta(\tau_M)\,\tilde{A}_d$, we can express the final osculating orbital parameters of the disturbed hyperbola as functions of the unperturbed ones:

$$\Omega_1^F = \frac{\tilde{A}_{d,h}}{\sqrt{e_0^2 - 1}\,\sin i_0}\left[\cos\omega_0\,\mathcal{I}_2 + \sin\omega_0\,\mathcal{I}_1\right] \qquad i_1^F = \frac{\tilde{A}_{d,h}}{\sqrt{e_0^2 - 1}}\left[\cos\omega_0\,\mathcal{I}_1 - \sin\omega_0\,\mathcal{I}_2\right] \tag{7}$$

$$\omega_1^F = \frac{\sqrt{e_0^2 - 1}}{e_0}\,\tilde{A}_{d,r}\mathcal{I}_3 - \frac{\tilde{A}_{d,\theta}}{e_0\,\sqrt{e_0^2 - 1}}\left[(e_0^2 - 1)\,\mathcal{I}_4 - \mathcal{I}_2\right] - \frac{\cos i_0\,\tilde{A}_{d,h}}{\sqrt{e_0^2 - 1}\,\sin i_0}\left[\cos\omega_0\,\mathcal{I}_1 + \sin\omega_0\,\mathcal{I}_2\right] \tag{8}$$

$$\hat{a}_1^F = \frac{2(e_0\,\mathcal{I}_4\,\tilde{A}_{d,r} + (\mathcal{I}_5 + e_0\,\mathcal{I}_3)\,\tilde{A}_{d,\theta})}{\sqrt{e_0^2 - 1}} \qquad e_1^F = \sqrt{e_0^2 - 1}\left[\mathcal{I}_4\,\tilde{A}_{d,r} + \left(\mathcal{I}_3 - \frac{\mathcal{I}_5 + \mathcal{I}_6}{e_0}\right)\tilde{A}_{d,\theta}\right] \tag{9}$$

where $\mathcal{I}_i$ stands for the value of the integrals appearing in the previous equations. It is possible to obtain

the quadratures in a simple manner when we consider a impulsive manoeuvre. In such a case:

$$\mathcal{I}_1 = -\hat{r}_M \cos f_M \qquad\qquad \mathcal{I}_2 = -\hat{r}_M \sin f_M \qquad\qquad \mathcal{I}_3 = \cos f_M \qquad\qquad (10)$$

$$\mathcal{I}_4 = \sin f_M \qquad\qquad \mathcal{I}_5 = \tau_M \qquad\qquad \mathcal{I}_6 = -\hat{r}_M \qquad\qquad (11)$$

where the subindex M refers to the instant when the impulse is provided and $\hat{r}_M = (e_0 \cosh(H_0(\tau_M)) - 1)$. For continuous thrust, the expressions are more complex but still it is possible to find an analytical formula for the integrals. We haven't include the closed expression of the parameter λ since the secular terms yield to more complex quadratures.

The non dimensional asymptotic approach can be also used to calculate the variation on the b-vector due to the perturbing acceleration. The non dimensional values will be: $\hat{\vec{B}} = \frac{\vec{B}}{|a_i|}$ and $\hat{b} = \frac{b}{|a_i|}$. On the other hand, the asymptotic expansion has the same form as before, i.e., $\hat{\vec{B}} = \hat{\vec{B}}_0 + \varepsilon \hat{\vec{B}}_1 + \mathcal{O}(\varepsilon)$, where $\hat{\vec{B}}_0$ is the non dimensional b-vector of the unperturbed problem. Thus, the variation of the b-vector with respect to time has the following form:

$$\frac{d\hat{\vec{B}}_1}{d\tau} = -\frac{d\hat{b}_1}{d\tau}(\vec{s}_0 \times \vec{n}_0) - b_0 \left(\frac{d\vec{s}_1}{d\tau} \times \vec{n}_0\right) - b_0 \left(\vec{s}_0 \times \frac{d\vec{n}_1}{d\tau}\right) \qquad (12)$$

The previous set of equations can be integrated as function of the final values of the osculating orbital elements. And substituting the expressions (7-9) in those results, we can finally reach a direct relation between the variation of the b-vector and the non dimensional accelerations applied in two cases: continuous thrust and impulsive manoeuvre.

These results allow us to compute the vectorial difference between the initial and final b-vectors: $\vec{B}^F - \vec{B}_0 = \varepsilon \, |a_i| \, \hat{\vec{B}}_1$,

$$\hat{\vec{B}}_1 = -\hat{b}_1^F (\vec{s}_0 \times \vec{n}_0) - \hat{b}_0 \left(\vec{s}_1^F \times \vec{n}_0\right) - \hat{b}_0 \left(\vec{s}_0 \times \vec{n}_1^F\right) \qquad (13)$$

$$\vec{n}_1^F = \left(\frac{\partial \mathcal{R}_0}{\partial \Omega_0}\Omega_1^F + \frac{\partial \mathcal{R}_0}{\partial i_0}i_1^F + \frac{\partial \mathcal{R}_0}{\partial \omega_0}\omega_1^F\right) \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad (14)$$

$$\vec{s}_1^F = \left(\frac{\partial \mathcal{R}_0}{\partial \Omega_0}\Omega_1^F + \frac{\partial \mathcal{R}_0}{\partial i_0}i_1^F + \frac{\partial \mathcal{R}_0}{\partial \omega_0}\omega_1^F\right) \begin{bmatrix} \cos\alpha_0 \\ \sin\alpha_0 \\ 0 \end{bmatrix} + \mathcal{R}_0 \begin{bmatrix} -\sin\alpha_0 \\ \cos\alpha_0 \\ 0 \end{bmatrix} \frac{\partial \alpha_0}{\partial e_0}e_1^F \qquad (15)$$

$$\hat{b}_1^F = 3\,e_0\,\mathcal{I}_4\,\tilde{A}_{d,r} + (3\,e_0\,\mathcal{I}_3 + \mathcal{I}_5 - \mathcal{I}_6)\,\tilde{A}_{d,\theta} \qquad (16)$$

Eventually, the variation of the b-vector can be expressed in terms of the applied forces. Therefore, given the initial trajectory, the mass of the asteroid (and the mass of the impactor) and the force applied (or the velocity relative to the asteroid), we can compute directly the variation of the b-vector. For the continuous thrust we obtain that the inclination of the orbit has a relative importance when tangential or normal accelerations are applied. However the behavior differs between both cases: while the greatest variations on the modulus of the b-vector are achieved for near polar orbits when the thrust is applied in the direction normal to the trajectory, for tangential thrust the best results are obtained for inclinations of about 45°. The largest variation of the modulus of the b-vector for all the different orientation strategies are attained for normal disturbing acceleration, and they are of the order of 60 cm when the continuous thrust acting on the asteroid is 1 N. We can find the reason why the variations are so small in the small span of time the thrust is active. On the other hand, the efficiency of the normal thrust can be explained by the fact that the effect of that acceleration is a change on the angular momentum of the orbit, and the arm of the torque provided is large. This outcome is more interesting for the impulsive manoeuvre also for tangential impulses, since it allows to change the orientation of the orbit inside the orbital plane. In fact, the impact strategy presents one more degree of freedom: the position of the encounter. There are two extreme privileged locations along the hyperbolic orbit, the pericenter and the initial position at distance of the radius of the sphere of influence. At the perigee the relative velocity is greater while at the sphere of influence the arm to change the angular momentum of the orbit is maximum. To clarify which of those is the dominant effect, we have carried out simulations assuming the impulsive manoeuvre is provided by a spacecraft traveling along the asteroid's orbit in opposite direction. The results show the maximum deflection achieved impacting at the distance of the sphere of influence is between one and two orders of magnitude greater than those obtained at pericenter. Clearly, for the same impact velocity, the remote location of the impact is more efficient.

Next section deals with the detailed analysis of the possible effect of a kinetic impactor during an Earth fly by, extending the results considered so far.

KINETIC IMPACTOR DURING EARTH FLY-BY

In this section we use the equations stated earlier in the paper in order to calculate the effect of a kinetic impactor deflecting Aphosis during its Earth fly-by in 2029. We will study the possible deflection distances
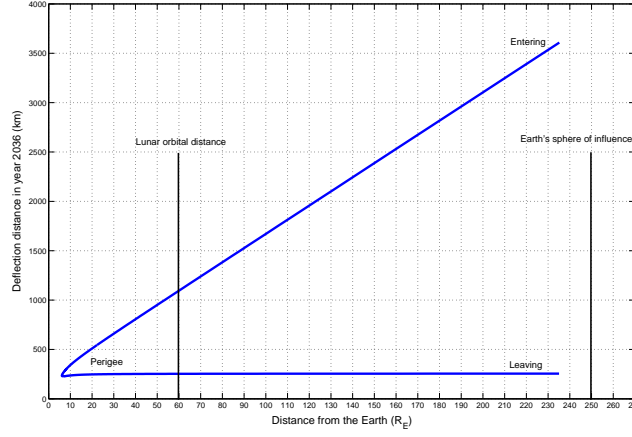
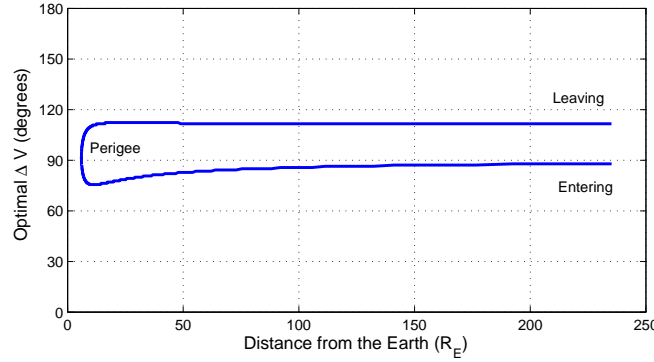Figure 1: Possible deflection distances achievable

Figure 2: Optimal impact angles for the spacecraft

achievable and also the optimal impact angles for the spacecraft. This deflection strategy has not been studied in previous literature and is worthy of further research. The kinetic impactor could even be a satellite in a geosynchronous orbit being sent off its predetermined orbit in order to impact the approaching Aphosis in 2029. An older operation satellite, with some remaining fuel, for instance would be ideal for such a scenario. The nominal data of the asteroid Aphosis was obtained from the JPL website and a velocity change was applied along the nominal hyperbola. The velocity change of the asteroid is obtained from:

$$\Delta V = k \left(\frac{Msc}{Msc + Mast} \right) U \tag{17}$$

The following assumptions have been made for the variables: mass of the satellite = 2000kg (M_{sc}), mass of Aphosis = $4.6 \cdot 10^{10}$kg (M_{ast}), impact coefficient = 1 (k) and a relative velocity to the asteroid = 10km/s (U) (relative velocity is entirely mission dependant). Propagating the hyperbola after the ΔV, we obtain the velocity change of the asteroid in the heliocentric frame caused by the impact of the spacecraft during Aphosis Earth fly-by in 2029. This is then transformed into the deflection achievable on the b-plane of the 2036 encounter. Using the asteroid deflection equation [2], various angles of velocity change along the hyperbola have been analysed and the optimum angles have been chosen which give the maximum deflection for the year 2036. Fig 3. illustrates that the highest deflection for Aphosis is obtained when the impact takes place at a distance furthest away from Earth, as the asteroid is entering the sphere of influence. Fig.4 shows that the optimal angle for impact of the spacecraft lies within the range of 75-112 degrees. These results show that the deflection of an asteroid during a previous Earth fly-by could be a concrete possibility as the amount of deflection achievable is significant and already orbiting satellites could be used.

MODEL OF THE ELECTROSTATIC FORCE

The possibility of charging the surface of the asteroid to provide a deflection of the orbit has already been consider in previous works [3, 1]. Nevertheless, it has been always studied in the context of the heliocentric orbit. Here, we will assume the behavior of the asteroid is as a charged particle in a hyperbolic trajectory. Moreover, we suppose the asteroid charge is constant and equal to Q during the whole flyby. The electrostatic force acting upon the asteroid can then be computed as:

$$\vec{F} = Q \left(\vec{v} \times \vec{B}_{Mg} \right) \tag{18}$$

The magnetic field $\vec{B}_{Mg}$ corresponds to the geomagnetic field, in this case. In this work we will consider a simple dipole model:

$$\vec{B}_{Mg} = \frac{\mu_m \, R_{\oplus}^3}{r^3} \left(3 \left(\hat{m} \cdot \hat{r} \right) \hat{r} - \hat{m} \right) \tag{19}$$

where $\hat{m}$ is the unit vector in the direction of the dipole. Assuming the dipole is not titled respect to the Earth's rotation axis, $\hat{m} = \vec{i}_z$.

Finally, the expression for the disturbing acceleration due to electrostatic forces can be calculated as:

$$\vec{a}_d = \frac{Q}{M} \left(\vec{v} \times \vec{B}_{Mg} \right) \tag{20}$$

where M is the mass of the asteroid.

We have tested a reference case in which the asteroid charge is 10^9 C. The numerical results obtained for different initial hyperbolas and different inclinations show that the maximum deviation on the modulus of the b-vector corresponds to lower inclinations. Besides, despite the huge charge, the effects on the b-vector are of the order of millimeters. A simple computation provides an estimation of the needed electric potential to get that charge on the asteroid: $Q = C \, V$ where C is its capacitance and V the electric potential. The capacitance of a sphere is $C = 4 \pi \varepsilon_0 \, r$ and therefore: $V = \frac{Q}{4 \pi \varepsilon_0 \, r}$. To obtain $Q = 10^9$ C, the electric potential should be of the order of $V \sim 10^{17}$ kV. The power required to maintain that voltage at the asteroid's surface is unfeasible, regardless of the method used to estimate the current of repelled species or emitted particles [2].

MODEL OF THE ELECTRODYNAMIC FORCE

The model of the electrodynamic force is needed to estimate the effect of an electrodynamic tether acting upon the asteroid. The attachment of the electrodynamic device to the asteroid is not studied in detail. The objective of this section is to provide a first approximation of the accelerations which can be achieved if all the electrodynamic force generated on the cable was applied directly on the center of mass of the asteroid.

The electrodynamic force acting on a tether element, F_{ed}^{ds}, can be written as:

$$\vec{F}_{ed}^{ds} = I(s, \vec{r}, \vec{v}, \vec{q}, \dot{\vec{q}}) \, \vec{u}(\vec{q}) \times \vec{B}_{Mg}(s, \vec{r}) \tag{21}$$

where $I(s, \vec{r}, \vec{v}, \vec{q}, \dot{\vec{q}})$ is the current along the tether, $\vec{B}_{Mg}(s, \vec{r})$ is the magnetic field at the tether element, s is the variable along the tether and $\vec{r}, \vec{v}, \vec{q}, \dot{\vec{q}}$ is the state of the system. In order to simplify that expression, some hypothesis are made: 1) both the magnetic field and the velocity do not vary along the tether: $\vec{B}_{Mg}(s, \vec{r}) \approx \vec{B}_{Mg}(\vec{r})$, $\vec{v}(s, \vec{r}) \approx \vec{v}(\vec{r})$; 2) the unit vector in the direction from the cathode to the anode, $\vec{u}$, is always pointing the center of the pseudo inertial reference frame. Hence, $\vec{u} = \hat{\vec{r}}$; 3) the intensity along the tether is computed as the average in a given instant: $L_t \, I_{av}(\vec{r}, \vec{v}, \vec{u}) = \int_{s_0}^{s_f} I(\xi, \vec{r}, \vec{v}, \vec{u}) \, d\xi$.

The magnetic field is defined as in the previous section. Hence, the Lorentz force is written as:

$$\vec{F}_{ed}\Big|_{OPC} = L_t \, I_{av} \left(\{1, 0, 0\}^T \times \vec{B}_{Mg}\Big|_{OPC} \right) \tag{22}$$

The parameter $L_t \, I_{av}$ can be used to estimate the effect of the electrodynamic forces on the asteroid deflection. Nevertheless, it is also possible to establish a model for the averaged intensity along the tether as a function of the state of the system and environmental parameters. In this approach we used the model of the so-called orbital-motion-limited (OML) regime as it is defined in [5]:

$$I_{av,OML}(\vec{r}, \vec{v}, \phi) = \frac{2}{5} \, e \, N_\infty \, \frac{L_t \, p}{\pi} \sqrt{\frac{2 \, e \, E_m \, L_t}{m_e}}$$

where

$$E_m = \vec{u} \cdot [(\vec{v} - \vec{v}_{pl}) \times \vec{B}_{Mg}] = \{1, 0, 0\}^T \cdot \left[\left(\vec{v}\Big|_{OPC} - \mathcal{R}^T \, \mathcal{R}_f^T \, \{1, 0, 0\}^T \right) \times \vec{B}_{Mg}\Big|_{OPC} \right]$$

is the electric motional field, e is the electron charge, N_∞ is the plasma density, m_e is the mass of the electron and p is the perimeter of the tether. It should be noted that the utilization of the OML in this scenario is not completely accurate since the ambient plasma does not fulfil completely the conditions in which the OML current has been derived. In particular, the velocity distribution of the solar wind electrons is not Maxwellian [6].

The reference case considered to evaluate the performance of the electrodynamic forces consist on a 100 km tether when a current of 1 A is flowing along it. In such situation, the variations of the modulus of the b-vector are even smaller than the previous case of the electrostatic forces. Moreover, the assessment about if it is feasible to have an average intensity of 1 A during the flyby shows that it is not feasible for reasonable tether length. In fact, using the following values: plasma velocity : $v_{pl} = 500$ km/s [3](1 AU); electron plasma density at 1 AU: $N_\infty = 3 \cdot 10^6$ m^{-3} [7] (1 AU); Earth magnetic field at pericenter: $B_{Mg} = 2.5 \ 10^{-7}$ T; and the expression of the averaged intensity for the OML regime, we reach a relation which links the tether length and perimeter:

$$L_t^{3/2} \, p \approx 8 \cdot 10^7 \quad \text{m}^{5/2}$$

Assuming the hypothesis that p is of the order of centimeters, the needed tether length to achieve a current of 1 A during the fly by turns out to be $L_t \sim 4 \cdot 10^3$ km.

CHANGE ON ANGULAR MOMENTUM

In previous works the relevance of the alteration of the spin state of asteroids during flybys has been pointed out [8, 9]. This phenomenon is controlled by the gravitational coefficient of the asteroid C_{22}, related to its equatorial ellipticity [10]. This natural effect can lead to a disruption or a reshaping of the asteroid under the action of gravitational torques. Furthermore, the variation of the spin state of the asteroid will modify the Yarkovsky acceleration it will undergo in the subsequent heliocentric motion.

The feasibility of enhancing this process by means of electrodynamic tethers is explored in what follows. The contribution of these devices to the change on the angular momentum of the asteroid is twofold. On the one hand, they change the shape of the body, and consequently its C_{22} coefficient. On the other hand, they can provide a torque due to electrodynamic forces.

To estimate the significance of the first contribution, it is necessary to compute the influence of the tether on the C_{22} coefficient. It will be assumed the tether and the asteroid constitute a rigid body when they are jointed. Within this approximation, the expression of the coefficient gathered in [11] is valid: $C_{22} = (A - B)/(4 M_a)$, where A and B are the moments of inertial of the body about the axis Ox and Oy in the principal body fixed reference frame. If the relative configuration of tether and asteroid is such that the tether lies in the y axis of the body fixed reference frame, the new moment of inertia can be computed as: $A' = A + A_t$, being A_t the moment of inertia of the tether respect to the Ox axis. Calling d the distance between the center of mass of the asteroid (it is implicitly supposed that the position of the center of mass does not vary due to the presence of the tether) and the point of attachment of the tether in the asteroid's surface, supposing the mass of the tether is lumped at its both ends and those masses are equal, the value of A_t will be:

$$A_t = \frac{1}{2} \, m_t \, L_t^2 \left[1 + 2 \frac{d}{L_t} + 2 \left(\frac{d}{L_t} \right)^2 \right] \tag{23}$$

Thus the variation of the gravitational coefficient will be:

$$\Delta C_{22} = \frac{m_t \, L_t^2}{8 \, M_a} \left[1 + 2 \frac{d}{L_t} + 2 \left(\frac{d}{L_t} \right)^2 \right] \tag{24}$$

In turn, the effect of the electrodynamic forces will be measured by the change on the rotational angular momentum of the asteroid, $\vec{H}$. In this preliminary study, the nature of the attachment won't be discussed in detail. As before, it is supposed the tether is acting as a part of a rigid solid composed by the tether itself and the asteroid. Therefore, the contribution on the change of $\vec{H}$ due to electrodynamic forces will be: $\frac{d\vec{H}}{dt} = \vec{M}_{ed}$. In turn, the electrodynamic torque due to the Lorentz force acting over the tether can be computed as:

$$\vec{M}_{ed} = L_t^2 \, I_{av} \left(\frac{1}{2} + \frac{d}{L_t} + \frac{I_{m1}}{I_{av}} \right) \vec{u} \times \left(\vec{u} \times \vec{B} \right) \tag{25}$$

if the tether is deployed in a way that its direction $\vec{u}$ is coincident with the direction of the radius vector connecting the centers of mass of tether and asteroid. Moreover, the tether is assumed to be homogeneous with equal end masses attached at the asteroid's surface, being d the distance from the center of mass of the asteroid to the point of attachment.

For a uniform intensity along a tether with its center of mass located at the center of the tether, the value of I_{m1} would be zero. For less restrictive hypothesis $I_{m1} \neq 0$ but still $I_{m1} \ll I_{av}$. Furthermore, given the dependence of the torque on the tether length, the larger the tether the grater the moment about the

center of mass of the system. Hence, we will consider tethers longer than the distance d, $L_t \gg d$. In that way, the expression of the torque reads:

$$\vec{M}_{ed} \approx \frac{1}{2} L_t^2 I_{av} \vec{u} \times \left(\vec{u} \times \vec{B} \right) \quad (L_t \gg d) \tag{26}$$

Finally, it is necessary to establish a model to calculate the cross product $\vec{u} \times \left(\vec{u} \times \vec{B} \right)$. For the magnetic field, we will assume the non-tilted dipole description. In turn, for the direction of the tether, the considered hypothesis will be: 1) the asteroid is rotating uniformly around its maximum moment of inertia; 2) the maximum moment of inertial is perpendicular to the orbital plane; 3) the tether is contained in a plane perpendicular to the angular velocity. With these restrictive assumptions it is possible to compute a first approximation of the change on the angular momentum due to the electrodynamic torque as a function of the parameters L_t and I_{av}. Nevertheless, it seems more interesting to compare the estimation of the change of angular momentum due to this electrodynamic torque and the correspondent to the modification of the mutual gravitational interaction. Based on [8], it is possible to estimate such change in angular momentum as a function of the change on the gravitational coefficient ΔC_{22} of the asteroid. In fact, we have $\Delta H_g \sim \Delta C_{22} \mathcal{F}_g(r_p, v_\infty, i, \Omega, \omega)$ for the change due to gravitational interaction and $\Delta H_{ed} \sim (L_t^2 I_{av})/2 \, \mathcal{F}_{eg}(r_p, v_\infty, i, \Omega, \omega)$ for the electrodynamic case. The computations which have been carried on show that the the order of magnitude of the ratio

$$\frac{\Delta H_{ed}}{\Delta H_g} \frac{2 \Delta C_{22}}{L_t^2 I_{av}} = \frac{\mathcal{F}_g(r_p, v_\infty, i, \Omega, \omega)}{\mathcal{F}_{eg}(r_p, v_\infty, i, \Omega, \omega)}$$

is $\sim 10^{-7}$ 1/A, in the most favorable case for the electrodynamic forces. The calculation has been doing considering different flybys characterized by radius of periapsis and excess hyperbolic speed.

From those results, we can derive the order of magnitude of the tether parameters in order to obtain a change in the angular momentum due to electrodynamic forces of the same order as the gravitational one, imposing $\Delta H_{ed} \sim \Delta H_g$. The condition reads

$$\frac{2 \Delta C_{22}}{L_t^2 I_{av}} \sim 10^{-7} \quad \frac{1}{A}$$

for the better ratio of the electrodynamic torque. Besides, taking into account the expression (24), a relation can be found which links tether intensity and mass needed to provide a change on the angular momentum of the same order for both contributions. Furthermore, from the estimation of the intensity of the OML regime, we can obtain the needed length of the tether for which the contributions to the change in angular momentum of the gravitational and electrodynamic effects are of the same order of magnitude:

$$L_t \sim \sqrt{\frac{2 \rho_l}{M_a}} \, 10^8 \, \text{m} \tag{27}$$

where a tether perimeter of centimeters has been taken into account. For a tether made of aluminium ($\rho_l \sim 2.7 \cdot 10^{-1}$ kg/m) attached to Apophis ($M_a \sim 4.6 \cdot 10^{10}$ kg), the length which would provide the same change on angular momentum due to electrodynamic torque and as a consequence of the change on the shape of the body will be: $L_t \sim 350$ m. Therfore, it can be stated that the presence of long electrodynamic tethers will be more significative because of the electrodynamic torque, although the model of the collected intensity drives an uncertainty in the previous estimations.

Finally, we explore the change on the angular velocity provided by the modification of the gravitational interaction due to the presence of the tether. The maximum change on angular velocity in a equatorial flyby can be estimated [8] as:

$$\Delta \Omega_z \sim \Omega_H \frac{e^2 - 1}{e^2} \tag{28}$$

where Ω_H is defined as follows:

$$\Omega_H = 2 \frac{\nu}{1 + \nu} \frac{C_{22}}{(I_z/M_a)} \frac{v_\infty}{r_p \, (1 + r_p \, v_\infty/\mu)}$$

being $\nu = M_{\text{☽}}/M_a$. When the tether is part of the body, the parameters of the asteroid will vary. Assuming $\Delta C_{22}/C_{22} \ll 1$, $\Delta I_z/I_z \ll 1$ and $m_t/M_a \ll 1$, we can consider only the first order variations on the value of Ω_H. Modeling the asteroid as a triaxial ellipsoid [12], whose width, length and height will be a, b, c, the ratio between the change on angular velocity due to the gravitational interaction with and without tether turns out to be:

$$\frac{\Delta \Omega^{'}}{\Delta \Omega} \sim 1 + \frac{3 \rho_l}{4 \pi \rho_v \, b \, c} \left(\frac{L_t}{a} \right) \left(1 + \frac{5}{2} \left(\frac{L_t}{a} \right)^2 \left(\frac{1}{(b/a)^4 - 1} \right) \right) \tag{29}$$

where ρ_v is the volumetric density of the asteroid. Assuming the change of angular velocity is small respect to the ininitial angular velocity $\Delta \Omega/\Omega \ll 1$ and the contribution of the tether to that change is small

$\Delta\Omega^{'}/\Delta\Omega = 1 + \mathcal{O}(1)$, the ratio of the rotational periods has the same form as the correspondent ratio of angular velocities.

The relative change on the rotational period after is therefore a function of the ratio L_t/a. From (29), we observe that the effect of the presence of the tether in the gravitational interaction is noticeable for values of L_t/a greater than $3\cdot10^2$. In the case of Apophis this means tether lengths of the order of hundreds of kilometers. Moreover, from the previous analysis is expected that the significance of the electrodynamic torque in that situation will be greater than the gravitational interaction. As a conclusion, it is possible to affirm that electrodynamic tethers with lengths of the order of thousands of kilometers could alter the rotational state of the asteroid in an appreciable manner.

CONCLUSIONS

In this paper, a method to analyze the effect of deflection techniques during Earth flybys has been developed. The above mentioned method allow to estimate the variation of an asteroid's orbit as a function of the strategy used to perform the deflection. It has been confirmed that small forces, (e.g. electromagnetic and electrostatic) acting during the flyby do not provide great changes in the asteroid orbit, since the time span is small. On the other hand, the specific characteristics of the hyperbolic orbit around the main body can be exploited for deflection purposes. Particularly, the possibility of changing the orientation of the orbital plane by means of a impulse manoeuvre even with not so large impact velocities seems feasible, Also, the deflection achieved in the Apophis case by impacting the asteroid with a 10km/s relative velocity during its 2029 Earth fly-by is compatible with the possible deflection requirements.

The potential interactions of the asteroid with the Earth magnetic field is also studied. For both electrostatic and electrodynamic forces are small, the results are similar to those obtained for small continuous thrust. In fact, to obtain even small forces it is necessary to charge the asteroid or provide a current that is not affordable. This results settle the low importance of the magnetic effect on the asteroid's trajectory.

Finally, the effect of an electrodynamic tether on the rotational dynamics of an asteroid is analyzed. In previous works, the relevance of the rotational state of an asteroid on the trajectory is stated. Unlike on the dynamics of the CM, small forces acting at a great distance of the CM can provide an important torque and play a role on the change of the angular momentum of the asteroid. Moreover, as the tether modifies the body shape, it can enhance the gravitational interaction between the primary and the asteroid. Both effects are simultaneous.

References

[1] D. Fargion. Asteroid Deflection: How, Where and When? *Chinese Journal of Astronomy and Astrophysics Supplement*, 8:399–411, October 2008.

[2] N. Murdoch, D. Izzo, C. Bombardelli, A. Hilgers, D. Rodgers, and I. Carnelli. Electrostatic tractor for near Earth object deflection. In *Proceedings of the 59th International Astronautics Conference, Glasgow, UK*, 2008.

[3] D. Steel. Electromagnetic perturbations of the orbits of asteroids. In B. Warmbein, editor, *Asteroids, Comets, and Meteors: ACM 2002*, volume 500 of *ESA Special Publication*, pages 433–436, November 2002.

[4] D. Izzo, A. Bourdoux, R. Walker, and F. Ongaro. Optimal trajectories for the impulsive deflection of near earth objects. *Acta Astronautica*, 59:294–300, July 2006.

[5] Sanmartín J.R., Martínez-Sánchez M., and Ahedo E. Bare Wire Anodes for Electrodynamic Tethers. *Journal of Propulsion and Power*, 9(3):353–360, MayJune 1993.

[6] W. G. Pilipp, K.-H. Muehlhaeuser, H. Miggenrieder, M. D. Montgomery, and H. Rosenbauer. Characteristics of electron velocity distribution functions in the solar wind derived from the HELIOS plasma experiment. *Journal of Geophysical Research*, 92:1075–1092, February 1987.

[7] Meyer-Vernet, N. *Basics of the Solar Wind*. Cambridge University Press, 2007.

[8] D. J. Scheeres, S. J. Ostro, R. A. Werner, E. Asphaug, and R. S. Hudson. Effects of Gravitational Interactions on Asteroid Spin States. *Icarus*, 147:106–118, September 2000.

[9] D. J. Scheeres. Changes in Rotational Angular Momentum due to Gravitational Interactions between Two Finite Bodies*. *Celestial Mechanics and Dynamical Astronomy*, 81:39–44, 2001.

[10] D. J. Scheeres. The Effect of C22 on Orbit Energy and Angular Momentum. *Celestial Mechanics and Dynamical Astronomy*, 73:339–348, January 1999.

[11] A.E. Roy. *Orbital Motion*. Institute of Physics, 2005.

[12] D. J. Scheeres, L. A. M. Benner, S. J. Ostro, A. Rossi, F. Marzari, and P. Washabaugh. Abrupt alteration of Asteroid 2004 MN4's spin state during its 2029 Earth flyby. *Icarus*, 178:281–283, November 2005.