The asteroid deflection formula for Earth quasi co-orbiting asteroids

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***Abstract*— A good preliminary estimate of the amount of deflection one may expect to be able to impose over a given asteroid is given by the so-called asteroid deflection formula for the general case. While the expression reduces to an algebraic expression for the impulsive deflection case, it still contains an integral for long duration thrust strategies. We find that in the case of quasi Earth co-orbiting asteroid it is still possible to have a good estimate of the deflection amount in a very simple algebraic form, also in the long duration thrust case. The result is allowed by the underlying hypothesis of small asteroid eccentricity and asteroid orbital period close to one year.**

I. INTRODUCTION

ASTEROID deflection is a subject that is attracting the attention of an increasingly large section of the aerospace engineering community. Thought of as a science fiction subject only a few years ago, the increasingly high number of potentially hazardous asteroid discovered every year and the number of scientific studies reporting feasible concepts and strategies for the impact risk mitigation, is contributing to make people seriously consider asteroid deflection as a feasible and important technology to be developed. The research on this subject ranges from the development of space mission concepts able to deflect asteroids to the development of tools to efficiently assess the risks associated to some observed asteroid or to quickly estimate the deflection amount one could possibly hope to obtain. In this last field, a valid tool recently developed, is the so called asteroid deflection formula [1],[2] which relates, in a generic case, the asteroid deflection startegy to the amount of deflection achieved with a simple expression. The expression generalizes some results valid in specific cases [3], [4] and offers a good approximation to the actual deflection capabilities in the real case.

II. THE ASTEROID DEFLECTION FORMULA

For the convenience of the reader we report here, and briefly explain, the asteroid deflection formula. A complete analysis may be found in [1]. We consider an asteroid on an orbit with semi-major axix a that is due to encounter the Earth at some epoch t_e. Defining the deflection startegy $\mathbf{A}(t)$ as the perturbing acceleration we are able to apply to the asteroid at t, the asteroid image on the encounter b-plane will move approximately by an amount $\Delta\zeta$ given by:

$$\Delta\zeta = \frac{3a}{\mu} v_E \sin\theta \int_0^{t_p} (t_s - \tau)\mathbf{v}_{ast}(\tau) \cdot \mathbf{A}(\tau) d\tau \qquad (1)$$

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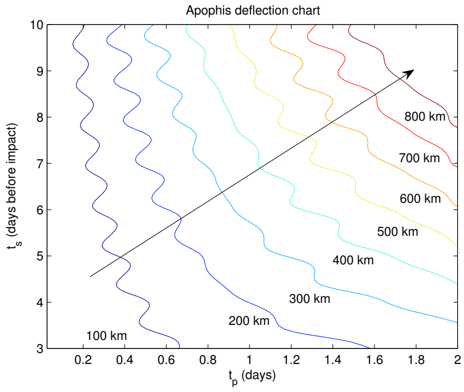

Fig. 1. Deflection chart for Apophis (pre-keyhole)

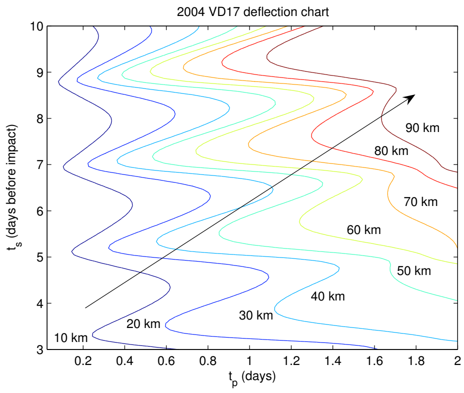

Fig. 2. Deflection chart for 2004 VD17 (pre-keyhole)

where v_E is the Earth velocity at encounter, t_p is the duration of the deflection startegy, t_s is the time between the start of the deflection strategy and the impact and $\mathbf{v}_{ast}$ is the heliocentric asteroid velocity. The angle θ is between the Earth heliocentric velocity and the asteroid relative velocity. The above expression is quite useful for performing preliminary mission design as it can be used as the objective function of the spacecarft trajectory optimiser. It is also useful to quickly build the t_p, t_s chart that gives useful informations on the efforts needed to deflect a given asteroid with long duration thrust strategies.

As an example we reported such charts for two famous asteroids. In Figure 1 the deflection chart for 99942 Apophis is given for a continuous 1N deflection along the velocity vector. The orbital parameters used were $a = .922$ AU, $e = 0.191$, $i = 3.331$ deg., $\Omega = 204.46$ deg., $\omega = 126.39$ deg., $M = 84.78$ deg. at epoch 54000 MJD. The 14th of

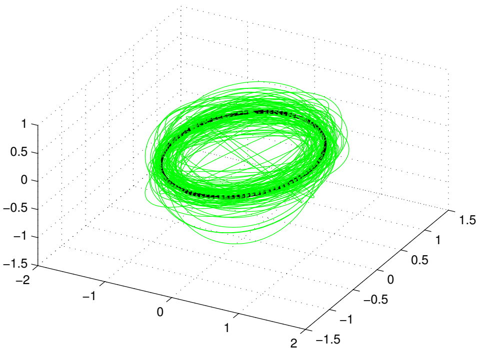

Fig. 3. Visualization of quasi co-orbiting asteroids orbits as of 06/08/2006

April 2029 encounter is considered. The asteroid mass has been set to 4.6e+10 kg. In Figure 2 the deflection chart for 2004 VD17 is given for a continuous 1N deflection along the velocity vector. The orbital parameters used were $a = 1.508$ AU, $e = 0.589$, $i = 4.223$ deg., $\Omega = 224.24$ deg., $\omega = 90.686$ deg., $M = 33.427$ deg. at epoch 54000 MJD. The 4th of May 2102 encounter is considered (SHOULD BE CHANGED). The asteroid mass has been set to 2.6e+11 kg. Clearly the asteroid 2004 VD17 represent a much more challenging deflection case. Its eccentricity warps the contour lines of its deflection chart significantly introducing more strict phasing consideration in the mission design, its mass makes the possible deflection stratgey $\mathbf{A}$ one order of magnitude smaller with respect to the 99942 Apophis case and the relief brought by its larger semi-major axis is completely cancelled by the unfavorable encounter geometry ($\sin\theta$).

Observing Eq.(eq:ADF) we note that the b-plane deflection depends linearly on both the mass and the magnitude of $\mathbf{A}$ (thrust to mass ratio), thus these type chart can be used in many different cases by a simple rescaling of the b-plane deflection amount.

III. The Earth quasi co-orbiting asteroids

As of 06/08/2006 there are 4100 near earth asteroids whose orbits have been observed and determined (source: JPL Near Earth Object program). Not all of these are considered dangerous as many do not intersect the Earth orbit. The potentially hazardous object, objects with a minimal orbit interception distance smaller than 0.05AU, as of the same epoch are 800. These numbers are in continuous growth as more and more objects gets observed and discovered every day. If we consider asteroids with eccentricities smaller than $e_{max} = 0.2$, and with semimajor axis in the range $a \in [.9, 1.1]a_E$ we get that 97 have already been cataloged amongst which 99942 Apophis is probably the most famous. For these asteroids (visualized in Figure 3 and defined here as Earth quasi co-orbiting asteroids) it is possible to reduce Eq.(1) to an algebraic expression by Taylor expansion around the $e = 0$ and $a = a_E$ point. We get:

$$\Delta\zeta = 3Av_E \sin\theta/\mu\left(R_E + \Delta a\right)\int_0^{t_p}(t_s-\tau)\left(V_E + o(\Delta a, \Delta e)\right)d\tau$$

in the case of a startegy constantly aligned with the velocity vector. Integrating and retaining only the zeroth order terms,

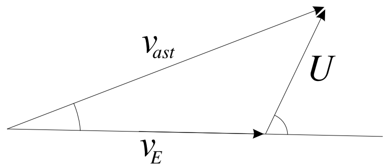

Fig. 4. Encounter geomerty

we get:

$$\Delta\zeta = 3Av_E/\mu R_E \sin\theta \int_0^{t_p}(t_s-\tau)V_E d\tau$$

and, performing the integration:

$$\Delta\zeta = 3/2A\left(2t_s - t_p\right)t_p \sin\theta$$

note that we did not expand $\sin\theta$, as this term does not admit a Taylor series representation as its first derivative is not continuous in $a = R_E$, $e = 0$. With reference to figure 4 we use the sine rule to show that, for small Γ:

$$\tan\theta = \Gamma\frac{v_{ast}}{v_{ast} - v_E}$$

The hypothesis of small Γ is verified by asteroids having small inclinations and small eccentricities.
Considering the vis-viva equation expanded around $a = R_E$ it is easy to show that:

$$v_{ast} = v_E\left(1 + \frac{\Delta a}{R_E}\right) + o(\Delta a)$$

Also, by considering the spherical geometry identity $\cos i \cos e = \cos\Gamma$, we may approximate:

$$\Gamma = \sqrt{(i^2 + e^2)} + o(i, e)$$

hence we may use:

$$\tan\theta = \sqrt{(i^2 + e^2)}\left(\frac{R_E}{\Delta a}\right) \tag{2}$$

and write the asteroid deflection formula as:

$$\Delta\zeta = \frac{3}{2}A\left(2t_s - t_p\right)t_p\frac{\tan\theta}{\sqrt{1 + \tan^2\theta}} \tag{3}$$

this expression is valid for asteroid with small eccentricities, small inclinations and small Δa, that is to what we have previously defined as quasi Earth co-orbiting asteroids. It is interesting to note that according to Eq.(1) if the encounter geometry is such that $\sin\theta = 0$, the deflection amount is zero. This corresponds to asteroids that cannot be deflected efficiently by exploitation of a phase shift strategy. These asetroid are therefore more difficult as their deflection must rely on changing the orbit geometry, a much more difficult task. According to Eq. (2) and Eq.(3) in the class of the Earth quasi co-orbiting asteroids the effectivness of the deflection stratgey based on phase shift is determined uniquely by the ratio $\frac{R_E\sqrt{i^2+e^2}}{\Delta a}$, the larger the better. As an example, an asteroid with zero inclination, $e = .001$ and $a = .9AU$ would have a factor $\sin\theta = 0.001$, and its deflection would have to rely on a substantial orbital geometry change.

IV. Conclusion

We have introduced two new useful tools to analyse asteroid deflection missions. The first one is what we have defined asteroid deflection charts. These are unique for each asteroid and allow to easily assess any long thrust deflection strategy based on phase shifting the asteroid. The second tool introduced is an algebraic expression of the asteroid deflection formula that avoids the numerical integration and explicitly relates the encounter geometry to the asteroid orbital parameters. This last tool is valid under the hypothesis that the asteroid is an Earth quasi co-rotating asteroid.

References

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