

# A critical analysis of the hydrino model

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**Abstract.** Recently, spectroscopic and calorimetric observations of hydrogen plasmas and chemical reactions with them have been interpreted as evidence for the existence of electronic states of the hydrogen atom with a binding energy of more than 13.6 eV. The theoretical basis for such states, that have been dubbed hydrinos, is investigated. We discuss both, the novel deterministic model of the hydrogen atom, in which the existence of hydrinos was predicted, and standard quantum mechanics. Severe inconsistencies in the deterministic model are pointed out and the incompatibility of hydrino states with quantum mechanics is reviewed.

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## 1. Introduction

Recently experimental results have been published in respectable physics journals that have been interpreted in support of a new model of the hydrogen atom [1, 2, 3, 4]. This model predicts the existence of new orbital states for the electron of the hydrogen atom with enhanced binding energy compared to the known hydrogen ground state. These new states have been named hydrinos. Applications of these alleged states have already been considered. In particular the NASA Institute for Advanced Concepts has funded a study to investigate new propulsion concepts based on the transition of conventional hydrogen states to hydrino states [5].

Although the hydrino model has received considerable public attention, the discussion of the underlying theory has mainly been restricted to the sweeping statement that the hydrino model is in contradiction to quantum mechanics and hence dubious (cf. e.g. [6]). This lack of theoretical consideration is particularly unfortunate in view of the wealth of experimental evidence that has been published in peer reviewed journals in favour of the hydrino model [1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22]. In this paper we attempt to fill this gap by giving a brief review of the theory underlying the hydrino model. We investigate its internal consistency and comment on the possibility that hydrino-like states exist in standard quantum mechanics.

Hydrinos are alleged lower-energetic electronic states of the hydrogen atom. These states are predicted within a new deterministic theory of quantum mechanics called the “grand unified theory of classical quantum mechanics” (CQM) [23]. In this theory the sheath electrons of an atom are orbiting the core at a fixed distance on a so-called orbit sphere. For the well known electronic states of the hydrogen atom the radius of the orbit sphere equals the radius of the corresponding state in Bohr’s model. For the new hydrino states, the radius is  $r = q a_H$ , where  $a_H$  is the Bohr-radius and  $q$  is a pure fraction. The corresponding binding energies are given by  $W_q = W_1/q^2$  where  $W_1 = 13.6 \text{ eV}$  is the energy of the standard hydrogen ground state. The standard ground state of the hydrogen atom is assumed to be metastable and the new hydrino states are only attainable by “non-radiative” transitions [23, 25]. These states are assumed to be reachable in the collision of hydrogen atoms with a catalyst, which can make an electronic transition of the same energy. In the collision the energy is transferred from the hydrogen to the catalyst, which absorbs it by an electronic transition to a more energetic state. Eventually the catalyst will release the acquired energy by the emission of a photon and return to its ground state. The lowest-energy hydrino state, the real ground state of the hydrogen atom, is then determined by the requirement that the orbital velocity of the sheath electron must not exceed the speed of light. The use of the alleged hydrino states for power systems relies on inducing the decay of hydrogen to a hydrino state and using the energy released in this process.

The layout of this paper is the following. In section 2 we assess the underlying theory and review its internal consistency. In section 3 we briefly consider the possibility that states of the hydrogen atom with enhanced binding energy could exist in standard quantum mechanics. We summarise our conclusions in section 4.

## 2. Aspects of the so-called “Grand unified theory of classical quantum mechanics”

This section is devoted to a review of R. Mills’ “grand unified theory of classical quantum mechanics” (CQM), which he claims to be a consistent, deterministic, and Lorentz invariant replacement of standard quantum mechanics [23]. The theory allegedly predicts the existence of new lower-energy states of the hydrogen atom — hydrinos. Brief expositions of the theory can be found in [24, 25]. In this theory, the equation of motion of a charged elementary particle, in particular that of the electron, is given by a scalar wave equation with a peculiar dispersion relation. To obtain the bound states of the electron in the hydrogen atom this equation of motion is augmented by a quantisation condition, which is similar to that of Bohr. In the following we discuss the solutions for the wave equation presented in [24, 25], which are subject to this quantisation condition. We pay particular attention to the solutions associated with alleged new electronic states of the hydrogen atom.

CQM assumes that the dynamics of the electron are described by a classical wave equation for its charge-density function,  $\rho(t, \mathbf{x})$ ,

$$\left( \nabla^2 - \frac{1}{v^2} \frac{\partial^2}{\partial t^2} \right) \rho(t, \mathbf{x}) = 0, \quad (1)$$

where  $v$  is the phase velocity of the wave.

Already this starting point is troublesome in view of the fact that this wave equation is not Lorentz invariant for any other phase velocity than the speed of light. Hence we find, in contrast to the claims of [24], that the theory can at best be the non-relativistic limit of a broader theory, but more probably is inconsistent already from equation (1) of [24].

For the following we will put aside these concerns and focus on the part of the theory which is essential to the existence of hydrinos. This is the solution of the wave equation (1) for the hydrogen atom. CQM postulates a particle–wave duality, in which the wavelength of the solution of the wave equation has to correspond to the classical circumference of the electron orbit as derived from the classical motion of the electron in a Coulomb potential, i. e.

$$2\pi r_n = \lambda_n, \quad (2)$$

where  $r_n$  and  $\lambda_n$  denote the radius of the electron orbit and the wavelength of the electron, respectively, and  $n$  labels the allowed orbits. Furthermore the de Broglie

relation between the wavelength,  $\lambda$ , and momentum,  $p$ , of a particle is assumed to be valid,

$$\lambda = h/p, \quad (3)$$

where  $h$  denotes Planck's constant. Combining the two relations, one obtains for the phase velocity of the electron in the  $n$ th orbit,

$$v_n = \frac{\hbar}{m_e r_n}, \quad (4)$$

where  $m_e$  is the mass of the electron and  $\hbar \equiv h/2\pi$ .

If you combine the relations in equations (2)–(4) with the classical circular motion of an electron in the Coulomb field of a proton, the ground state of Bohr's model is the only solution. No solutions exist for excited states of the hydrogen atom. In order to obtain the whole set of radii of Bohr's model one would have to change equation (2) to  $2\pi r_n = n\lambda_n$ , where  $n$  is a positive integer. Disregarding this fact, R. Mills claims that the solutions to the wave equation for the electron of the hydrogen atom are given by

$$\rho(r, \theta, \phi, t) = \frac{e}{4\pi r^2} \delta(r - r_n) Y_0^0(\theta, \phi) \quad (5)$$

for zero orbital angular momentum, and

$$\rho(r, \theta, \phi, t) = \frac{e}{4\pi r^2} \delta(r - r_n) [Y_0^0(\theta, \phi) + \Re(Y_l^m(\theta, \phi) [1 + \exp(i\omega_n t)])] \quad (6)$$

for non-zero orbital angular momentum,  $l > 0$ , where  $r_n$  is the radius of the  $n$ th orbit in Bohr's model. Here  $\delta(x)$  denotes Dirac's delta function,  $\Re$  denotes taking the real part of the following expression,  $Y_m^l$  denote the spherical harmonics,  $r, \theta, \phi$  are the spherical coordinates in obvious notation and  $e$  is the electron charge. Assuming that the radii  $r_n$  can be obtained by some other procedure (which Mills does not specify), we check if Eqs. (5) and (6) are at least solutions of the wave equation with  $v = v_n$ . In [24] it is stated that the "solutions", Eqs. (5) and (6), can be obtained by a separation ansatz,

$$\rho(r, \theta, \phi, t) = f(r) A(\theta, \phi, t) = f(r) Y(\theta, \phi) k(t). \quad (7)$$

Using this ansatz we can transform the wave equation into

$$\frac{1}{r^2 f(r)} \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} \right) f(r) = -\frac{1}{A(\theta, \phi, t)} \left( \frac{1}{r^2} \Delta_{\theta, \phi} + \frac{1}{v_n^2} \frac{\partial^2}{\partial t^2} \right) A(\theta, \phi, t), \quad (8)$$

where  $\Delta_{\theta, \phi}$  denotes the angular part of the Laplace operator. Furthermore [24] states that the separation yields

$$\left( \frac{1}{r^2} \Delta_{\theta, \phi} + \frac{1}{v_n^2} \frac{\partial^2}{\partial t^2} \right) A(\theta, \phi, t) = 0. \quad (9)$$

We decide to accept this claim for the moment. Continuing the separation into an angular part,  $Y(\theta, \phi)$ , and a time part,  $k(t)$  we find the equation for the function  $k(t)$

$$\frac{1}{\omega_n^2} \frac{d^2}{dt^2} k(t) = \text{const} \times k(t), \quad (10)$$

where we have used  $v_n = \omega_n r_n$ , with  $\omega_n \equiv 2\pi v_n/\lambda_n$ . Equation (10) has the solutions

$$k(t) = \text{const}, \quad \text{and} \quad k(t) = e^{\pm i a \omega t}. \quad (11)$$

where  $a$  is a constant. With these solutions the differential equation for the angular part becomes

$$\Delta_{\theta,\phi} Y(\theta, \phi) = 0 \quad (12)$$

for the time independent solution,  $k = \text{const}$ , and

$$(\Delta_{\theta,\phi} - a^2) Y(\theta, \phi) = 0 \quad (13)$$

for the solution which is harmonic in time. Hence for the time-independent situation the angular function has the solution  $Y(\theta, \phi) = Y_0^0(\theta, \phi) = 1$ . The time-dependent solution  $k(t)$  only allows solutions for  $Y(\theta, \phi)$  in terms of spherical harmonics if  $a$  is imaginary and fulfils the equation  $a = \sqrt{-l(l+1)}$  with  $l$  being a positive integer. Hence, we can recover the angular part of equation (5) but not that of equation (6). In conclusion, equation (6) is not a solution of the wave equation (1). In practical terms this inconsistency of the theory means that the model cannot describe the electron motion in a hydrogen atom with non-minimal angular momentum. Note that the electron states with non-zero angular momentum are well-described in standard quantum mechanics. Hence CQM lacks important features of quantum mechanics and does not describe known physics properly.

More important for our considerations is the radial part of the wave equation because it contends to admit solutions with an orbital radius smaller than that of the ground state of Bohr's model. From equation (8) and equation (9) we know that the radial part of the wave equation is given by

$$\frac{d}{dr} \left( r^2 \frac{d}{dr} \right) f(r) = 0. \quad (14)$$

This is the well known Euler differential equation. The general solution to this is (cf. e.g. [26])

$$f(r) = c_1 + \frac{c_2}{r} \quad (15)$$

However [24] and [25] give the solution

$$f(r) = \frac{1}{r} \delta(r - r_n). \quad (16)$$

Using the standard expression for the  $n$ th derivative of the Dirac function,  $\delta(x)$ , (see e.g. [27]),

$$\frac{d^n}{dx^n} \delta(x) = (-1)^n n! x^{-n} \delta(x), \quad (17)$$

it is straightforward to check that equation (16) is not a solution of the radial part of equation (14). However equation (16) is also claimed to be the radial solution for hydrinos [25]. Since equation (16) is not a solution of the radial part

of the wave equation (14) for any  $r_n$  there is no way of deriving the existence of hydrinos from the wave equation (1).

Our analysis of the theory of [24, 25] has demonstrated that the theory is mathematically inconsistent in several points: the quantisation condition of CQM allows only a solution for the ground state of the hydrogen atom; the radial solutions for the charge density function of the electron, as well as the angular solutions with non-zero angular momentum, differ from those given in the literature on CQM [24, 25]. To uncover the latter problem we did not resort to any physics argument but instead directly checked the alleged solution of the underlying equations of motion. Hence there is no way to cure the flaws of the theory by adding physical assumptions. CQM is obviously inconsistent, and in particular does not contain solutions that predict the existence of hydrinos. Hence we can omit a further discussion of CQM and, in particular, will not discuss the description of “non-radiative” electronic transitions.

### 3. Hydrinos in standard quantum mechanics

Having found that CQM does not predict the existence of hydrinos (and is furthermore inconsistent) it is worth considering if standard quantum mechanics would allow for the existence of new electronic states of the hydrogen atom with enhanced binding energy.

We start with a discussion at the level of the Schrödinger equation. In [25] it was mentioned that the Schrödinger equation has solutions with main quantum number  $n < 1$ . If such states were allowed by standard quantum mechanics then also the existence of hydrinos would also be possible in the standard theory. However, while solutions of the Schrödinger equation with  $n < 1$  indeed exist, they are not square integrable. This does not only violate one of the axioms of quantum mechanics, but in practical terms prohibits that these solutions can in any way describe the probability density of a particle. Thus solutions with  $n < 1$  are meaningless in standard quantum theory and the existence of hydrinos as solution of the Schrödinger equation for a classical Coulomb potential is excluded.

The stability of the hydrogen atom in general is a long discussed topic. For the isolated non-relativistic hydrogen atom, stability has been proven, with a maximal binding energy of approximately 20 eV [28]. This bound prohibits the existence of states with the high binding energies attributed to hydrino states. The stability of the hydrogen atom under the influence of external fields is not easily proven in quantum field theory and the upper bound on the binding energy is difficult to determine (see [29] for a review on this problem). Hence a state of the hydrogen atom that is lower-energetic than the ground state cannot be ruled out completely under some exotic conditions at our current level of understanding. Such conditions are however not likely to be fulfilled in the relatively low-energy, low electromagnetic field environment of the plasmas studied by Mills et al. (cf. [1, 2, 3, 4, 7, 14, 15, 16]). Of course, there is no theoretical indication that

the binding energy of a putative new state of the hydrogen atom should be a multiple of that of the ground state of the free hydrogen atom. Note also that a transition to a new state induced by strong external fields cannot be a non-radiative one, which is in contrast to the interpretation of the experimental data by Mills et al. [1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22]. Hence, whereas the stability of the ground state of the hydrogen atom is not yet proven for all environmental conditions, the hydrinos have alleged properties that make it impossible that their existence can be encompassed by standard quantum mechanics.

#### 4. Conclusion

In this paper we have considered the theoretical foundations of the hydrino hypothesis, both within the theoretical framework of CQM, in which hydrinos were originally suggested, and within standard quantum mechanics. We found that CQM is inconsistent and has several serious deficiencies. Amongst these are the failure to reproduce the energy levels of the excited states of the hydrogen atom, and the absence of Lorentz invariance. Most importantly, we found that CQM does not predict the existence of hydrino states! Also, standard quantum mechanics cannot encompass hydrino states, with the properties currently attributed to them. Hence there remains no theoretical support of the hydrino hypothesis. This strongly suggests that the experimental evidence put forward in favour of the existence of hydrinos should be reconsidered for interpretation in terms of conventional physics. This reconsideration of the experimental data is beyond the scope of the current paper. Also, to understand properly the experimental results presented by Mills et al., it would be helpful if these were independently reproduced by some other experimental groups.

#### References

- [1] R. L. Mills, M. Nansteel and P. C. Ray, "Bright hydrogen-light source due to a resonant energy transfer with strontium and argon ions," *New J. Phys.* **4** (October 2002) 70
- [2] R. L. Mills and P. Ray, "Substantial changes in the characteristics of a microwave plasma due to combining argon and hydrogen," *New J. Phys.* **4** (April 2002) 22
- [3] R. L. Mills, P. C. Ray, B. Dhandapani, R. M. Mayo, and J. He, "Comparison of excessive Balmer alpha line broadening of glow discharge and microwave hydrogen plasmas with certain catalysts," *J. Appl. Phys.* **92** (2002) 7008
- [4] J. Phillips, R. L. Mills, and Xuemin Chen, "Water bath calorimetric study of excess heat generation in "resonant transfer" plasmas," *J. Appl. Phys.* **96** (2004) 3095
- [5] A. J. Marchese, P. Jansson, J. L. Schmalzel, "The blacklight rocket engine, a phase I study funded by the NIAC CP 01-02 advanced aeronautical/space concept," [http://www.niac.usra.edu/files/studies/final\\_report/752Marchese.pdf](http://www.niac.usra.edu/files/studies/final_report/752Marchese.pdf) (cited on 1 May 2005)
- [6] T. Reichhardt, "New form of hydrogen power provokes scepticism," *Nature* **404** 218 (2000)
- [7] R. Mills and P. Ray, "Extreme ultraviolet spectroscopy of helium-hydrogen Plasma," *J. Phys. D* **36** (2003) 1535

- [8] R. L. Mills and P. Ray, "Spectral emission of fractional quantum energy levels of atomic hydrogen from a helium-hydrogen plasma and the implications for dark matter," *Int. J. Hydrogen Energy* **27** (2002) 301
- [9] R. L. Mills and P. Ray, "Vibrational spectral emission of fractional-principal-quantum-energy-level hydrogen molecular ion," *Int. J. Hydrogen Energy* **27** (2002) 533
- [10] R. L. Mills, "Spectroscopic identification of a novel catalytic reaction of atomic hydrogen and the hydride ion product," *Int. J. Hydrogen Energy* **27**, (2002), 927
- [11] R. L. Mills and P. Ray, "Spectroscopic identification of a novel catalytic reaction of rubidium ion with atomic hydrogen and the hydride ion product," *Int. J. Hydrogen Energy* **27**, (2002), 927
- [12] R. L. Mills and P. Ray, "A comprehensive study of spectra of the boundfree hyperfine levels of novel hydride ion  $H^-(1/2)$ , hydrogen, nitrogen, and air," *Int. J. of Hydrogen Energy* **28** (2003) 825
- [13] R. L. Mills and P. Ray, "Spectroscopic identification of a novel catalytic reaction of potassium and atomic hydrogen and the hydride ion product," *Int. J. Hydrogen Energy* **27** (2002) 183-193
- [14] R. L. Mills, N. Greening, S. Hicks, "Optically measured power balances of glow discharges of mixtures of argon, hydrogen, and potassium, rubidium, cesium, or strontium vapor," *Int. J. Hydrogen Energy* **27** (2002) 651
- [15] R. L. Mills, "Measurement of energy balances of noble gas-hydrogen discharge plasmas using Calvet calorimetry," *Int. J. Hydrogen Energy* **27**, (2002), 967
- [16] R. L. Mills, Jinquan Dong and Ying Lu, "Observation of extreme ultraviolet hydrogen emission from incandescently heated hydrogen gas with certain catalysts" *Int. J. Hydrogen Energy* **25** (2000) 919
- [17] R. L. Mills, M. Nansteel and Ying Lu, "Observation of extreme ultraviolet hydrogen emission from incandescently heated hydrogen gas with strontium that produced an anomalous optically measured power balance," *Int. J. Hydrogen Energy* **26** (2001), 309
- [18] R. L. Mills, B. Dhandapani, N. Greenig and Jiliang He, "Synthesis and characterization of potassium iodo hydride," *Int. J. Hydrogen Energy*, **25** (2000) 1185
- [19] R. L. Mills, B. Dhandapani, M. Nansteel, Jiliang He, T. Shannon and A. Echezuria, "Synthesis and characterization of novel hydride compounds," *Int. J. Hydrogen Energy* **26** (2001) 339
- [20] R. L. Mills, B. Dhandapani, M. Nansteel, Jiliang He and A. Voigt, "Identification of compounds containing novel hydride ions by nuclear magnetic resonance spectroscopy," *Int. J. Hydrogen Energy* **26** (2001) 965
- [21] R. Mills, X. He, P. Ray, B. Dhandapani and X. Chen, "Synthesis and characterization of a highly stable amorphous silicon hydride as the product of a catalytic heliumhydrogen plasma reaction," *Int. J. of Hydrogen Energy*, **28** (2003), 1401
- [22] R. L. Mills, Jinquan Dong, W. Good and A. Voigt, "Minimum heat of formation of potassium iodo hydride," *Int. J. Hydrogen Energy*, **26** (2001) 1199
- [23] R. L. Mills, *The Grand Unified Theory of Classical Quantum Mechanics* Blacklight Power Inc, January 1999 edition (Cranbury, USA, 1999)
- [24] R. L. Mills, "The grand unified theory of classical quantum mechanics," *Int. J. of Hydrogen Energy* **27** (2002) 565
- [25] R. L. Mills, "The nature of free electrons in superfluid helium a test of quantum mechanics and a basis to review its foundations and make a comparison to classical theory," *Int. J. of Hydrogen Energy*, **26** (2001) 1059
- [26] Eric W. Weisstein. "Euler Differential Equation," from MathWorld – A Wolfram Web Resource, <http://mathworld.wolfram.com/EulerDifferentialEquation.html> (cited on 1 May 2005)
- [27] I. M. Gel'fand and G. E. Shilov, *Generalized functions*, vol. 1 (Academic Press, New York, USA, 1964) p. 232f.



- [28] E. H. Lieb and W. E. Thirring, “Bound For The Kinetic Energy Of Fermions Which Proves The Stability Of Matter,” Phys. Rev. Lett. **35**, 687 (1975) [Erratum-ibid. **35**, 1116 (1975)]
- [29] E. H. Lieb, “Quantum mechanics, the stability of matter and quantum electrodynamics,” arXiv:math-ph/0401004